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# Random Coefficients and Unbalanced Panels: An Application on Data from Norwegian Chemical Plants

**Abstract:**
A framework for analyzing substitution and scale properties, and technical change from plant-level panel data is presented. Focus is on comparing the constant and random coefficient specification of
the substitution and scale parameters and investigating the potential variation of the parameters across firms. Characteristics of the model framework are (i) an equation system consisting of a three-factor translog cost function and the corresponding cost-share equations, (ii) random firm specific heterogeneity in coefficients, and (iii) a Maximum Likelihood procedure allowing for unbalanced panel data. The empirical results, based on data from Norwegian chemical plants, indicate substantial firm specific heterogeneity in substitution and scale properties.

**Keywords:** Panel Data. Random Coefficients. Unbalanced Data. Heterogeneity. Production technology

**JEL classification:** C 33, D21, D24, L65

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# 1 Introduction

A common challenge in microeconometric analyses of economic relationships is how to treat heterogeneity concerning the form of the relationships across the micro units. Such heterogeneity may be modelled and analyzed when panel data are available, unlike the situation when only cross section data are at hand. However, even in a panel data context, most researchers have tended to assume a common coefficient structure, possibly allowing for unit specific (or time specific) differences in intercept terms of the equations ('fixed' or 'random' effects) only. If the heterogeneity has a more complex form, this modelling approach may lead to inefficient (and even inconsistent) estimation of the slope coefficients, and hence invalid inference.

A more general approach is to allow for heterogeneity also in slope coefficients of the equation. The challenge then becomes how to obtain a model which is sufficiently flexible while avoiding overparametrization. The fixed effects slope coefficient approach, in which each unit has its distinct coefficient vector, with no *a priori* assumptions made about its variation between units, is very flexible, but may easily suffer from this degrees of freedom problem. The *random coefficients* approach, in which specific assumptions are made about the distribution from which the unit specific coefficients are drawn, is far more parsimonious in this respect. At the same time, the expectation vector in this distribution represents, in a precise way, the coefficients of an average unit, while its second order moments matrix gives easily interpretable measures of the degree of heterogeneity. The random coefficients approach may also be considered a particular way of representing disturbance heteroskedasticity in panel data analysis, since the random effects enter multiplicatively to the covariates of the equation.

While there is a growing body of methodological articles in the econometric literature dealing explicitly with this random coefficient problem for balanced panel data situations, far less has been done with *unbalanced panel data*. This is somewhat surprising, since in practice the latter is rather the rule than the exception. Because working with complete panels is mathematically more convenient, a common procedure for empirical researchers is to leave out the units for which the time series are incomplete and use the balanced sub-sample of the original, less tidy data-set. This may, however, involve a of loss of efficiency, see Mátyás and Lovrics (1991) and Baltagi and Chang (1994).

In this paper, we consider a general framework for analyzing the production process, *i.e.*, factor substitution, scale properties and technical change, from unbalanced plant-level panel data. The paper discusses the importance of choosing a random versus the more common constant coefficients specification of plant heterogeneity in econometric analyses of factor demand systems. The translog cost function approach suggested by

Christensen *et al.* (1971, 1973) is acommodated to an application on plant-level panel data for Norwegian chemical industries for the years 1972 – 1993.

The model framework of the present analysis has the following basic characteristics: (i) A translog cost-function with three variable inputs and technical change and derived cost-share equations. Capital input is treated as a fourth, 'quasi-fixed' input, reflecting the assumed long lead time needed to build new capacity in the sector under consideration, and this mechanism cannot be adequately represented by static optimization. (ii) Plant specific heterogeneity in some coefficients of the cost function is allowed for and treated as random. (iii) The model is designed for an unbalanced plant-level panel data set. While (i) is rather standard, the combination of (ii) and (iii) is not, at least in applied econometrics. Mixed regression models with unbalanced design, however – both uni- and multivariate – have, to some extent, been discussed in the statistical literature denoted as $X$ and $K$ respectively, and both, like the three input prices, are treated as exogenous variables, *i.e.*, capital is treated as a 'quasi-fixed' factor. A deterministic trend, $\tau$, representing, *inter alia*, technical change is included.

The production technology, represented by its dual cost function, is assumed to be a *translog* function in $(X, K, \tau, Q_L, Q_E, Q_M)$ [see, *e.g.*, Christensen, Jorgenson, and Lau (1971, 1973) and Jorgenson (1986)]:

$$\begin{aligned}
\ln C \;=\;& \beta_0 + \beta_X \ln X + \beta_K \ln K + \beta_\tau \tau + \textstyle\sum_g \gamma_g \ln Q_g \\
&+ \textstyle\sum_g \beta_{Xg} (\ln X)(\ln Q_g) + \sum_g \beta_{Kg} (\ln K)(\ln Q_g) \\
&+ \textstyle\sum_g \beta_{\tau g}\, \tau \ln Q_g + \frac{1}{2} \sum_g \sum_h \gamma_{gh} (\ln Q_g)(\ln Q_h) \\
&+ \tfrac{1}{2}\beta_{XX}(\ln X)^2 + \tfrac{1}{2}\beta_{KK}(\ln K)^2 + \tfrac{1}{2}\beta_{\tau\tau}\tau^2 \\
&+ \beta_{XK}(\ln X)(\ln K) + \beta_{\tau X}\tau \ln X + \beta_{\tau K}\tau \ln K, \qquad g, h = L, E, M.
\end{aligned}$$

Cost minimization with respect to $V_g$, conditional on $Q_g$ $(g = L, E, M)$, $X$ and $K$ gives, according to Shephard's lemma $(\partial C / \partial Q_g = V_g)$, the following expression for the optimal cost share of factor $g$:

$$\frac{Q_g V_g}{C} = \frac{\partial \ln C}{\partial \ln Q_g} = \gamma_g + \beta_{Xg} \ln X + \beta_{Kg} \ln K + \textstyle\sum_h \gamma_{gh} \ln Q_h + \beta_{\tau g}\tau.$$

The homogeneity and symmetry conditions on the cost function and the adding-up of cost shares imply

$$\textstyle\sum_g \gamma_g = 1, \quad \sum_g \beta_{Xg} = \sum_g \beta_{Kg} = \sum_g \beta_{\tau g} = \sum_g \gamma_{gh} = 0, \quad \gamma_{gh} = \gamma_{hg}.$$

We represent these restrictions in the model by letting

$$\begin{aligned}
&\gamma_M = 1 - \gamma_L - \gamma_E, \\
&\beta_{XM} = -\beta_{XL} - \beta_{XE}, \quad \beta_{KM} = -\beta_{KL} - \beta_{KE}, \quad \beta_{\tau M} = -\beta_{\tau L} - \beta_{\tau E}, \\
&\gamma_{LM} = -\gamma_{LL} - \gamma_{LE}, \quad \gamma_{EM} = -\gamma_{EE} - \gamma_{LE}, \quad \gamma_{MM} = \gamma_{LL} + 2\gamma_{LE} + \gamma_{EE},
\end{aligned}$$

and can write the cost function and the cost-share equations of labour and energy as

$$\begin{aligned}
(1) \qquad \ln(C/Q_M) \;=\;& \beta_0 + \beta_X \ln X + \beta_K \ln K + \beta_\tau \tau + \textstyle\sum_g \gamma_g \ln(Q_g/Q_M) \\
&+ \textstyle\sum_g \beta_{Xg} (\ln X)(\ln(Q_g/Q_M)) + \sum_g \beta_{Kg} (\ln K)(\ln(Q_g/Q_M)) \\
&+ \textstyle\sum_g \beta_{\tau g} \tau \ln(Q_g/Q_M) + \frac{1}{2} \sum_g \sum_h \gamma_{gh} (\ln(Q_g/Q_M))(\ln(Q_h/Q_M)) \\
&+ \tfrac{1}{2}\beta_{XX}(\ln X)^2 + \tfrac{1}{2}\beta_{KK}(\ln K)^2 + \tfrac{1}{2}\beta_{\tau\tau}\tau^2 \\
&+ \beta_{XK}(\ln X)(\ln K) + \beta_{\tau X}\tau \ln X + \beta_{\tau K}\tau \ln K, \qquad g, h = L, E,
\end{aligned}$$

$$(2) \qquad \frac{Q_g V_g}{C} = \gamma_g + \beta_{Xg} \ln X + \beta_{Kg} \ln K + \beta_{\tau g}\tau + \textstyle\sum_h \gamma_{gh} \ln(Q_h/Q_M), \qquad g, h = L, E,$$

The cost elasticity of output, capital, and the rate of increase of cost with time are, respectively,

(3) $(\partial \ln C)/(\partial \ln X) = \varepsilon_X = \beta_X + \beta_{XX} \ln X + \sum_g \beta_{Xg} \ln(Q_g/Q_M) + \beta_{XK} \ln K + \beta_{\tau X}\tau,$

(4) $(\partial \ln C)/(\partial \ln K) = \varepsilon_K = \beta_K + \beta_{KK} \ln K + \sum_g \beta_{Kg} \ln(Q_g/Q_M) + \beta_{XK} \ln X + \beta_{\tau K}\tau,$

(5) $(\partial \ln C)/(\partial \tau) = \varepsilon_\tau = \beta_\tau + \beta_{\tau\tau}\tau + \sum_g \beta_{\tau g} \ln(Q_g/Q_M) + \beta_{\tau K} \ln K + \beta_{\tau X} \ln X.$

Cross-price and own-price elasticities of substitution in the demand for factor $g$ with respect to the price of factor $h$, defined as the Slutsky analogues (output constrained price elasticities of input quantities), are

$$\varepsilon_{gh} = \begin{cases} \dfrac{\gamma_{gh}}{s_g} + s_h, & g \neq h, \\ \dfrac{\gamma_{gg}}{s_g} + s_g - 1, & g = h, \end{cases} \tag{6}$$

which satisfy $\sum_h \varepsilon_{gh} = 0$. The corresponding (symmetric) Allen-Uzawa elasticities of substitution are

$$\eta_{gh} = \eta_{hg} = \frac{\varepsilon_{gh}}{s_h} = \begin{cases} \dfrac{\gamma_{gh}}{s_g s_h} + 1, & g \neq h, \\ \dfrac{\gamma_{gg}}{s_g^2} + 1 - \dfrac{1}{s_g}, & g = h. \end{cases} \tag{7}$$

Denoting the cost shares as $s_g = (Q_g V_g)/C \quad (g = L, E)$, and using lower case letters to symbolize logarithms, *i.e.*, $c = \ln C, x = \ln X, q_g = \ln Q_g$ etc., the cost-share equations of labour and energy, (2), can be written as[1]

$$s_L = \gamma_L + \beta_{XL}x + \beta_{KL}k + \beta_{\tau L}\tau + \gamma_{LL}(q_L - q_M) + \gamma_{LE}(q_E - q_M) + u_L, \tag{8}$$

$$s_E = \gamma_E + \beta_{XE}x + \beta_{KE}k + \beta_{\tau E}\tau + \gamma_{LE}(q_L - q_M) + \gamma_{EE}(q_E - q_M) + u_E, \tag{9}$$

and the cost function, (1), as

$$\begin{aligned} (c - q_M) = {} & \beta_0 + \beta_X x + \beta_K k + \beta_\tau \tau + \gamma_L(q_L - q_M) + \gamma_E(q_E - q_M) \\ & + \beta_{XL}x(q_L - q_M) + \beta_{XE}x(q_E - q_M) \\ & + \beta_{KL}k(q_L - q_M) + \beta_{KE}k(q_E - q_M) \\ & + \beta_{\tau L}\tau(q_L - q_M) + \beta_{\tau E}\tau(q_E - q_M) \\ & + \tfrac{1}{2}\gamma_{LL}(q_L - q_M)^2 + \gamma_{LE}(q_L - q_M)(q_E - q_M) + \tfrac{1}{2}\gamma_{EE}(q_E - q_M)^2 \\ & + \tfrac{1}{2}\beta_{XX}x^2 + \tfrac{1}{2}\beta_{KK}k^2 + \tfrac{1}{2}\beta_{\tau\tau}\tau^2 + \beta_{XK}xk + \beta_{\tau X}\tau x + \beta_{\tau K}\tau k + u_C, \end{aligned} \tag{10}$$

[1]To these equations may be added, for symmetry reasons, the corresponding equation for materials, but to avoid singularity of the disturbance covariance matrix, it is omitted from the econometric model. Using, as here, a Maximum Likelihood estimation procedure, this involves no loss of efficiency.

where we have added the disturbances $u_L$, $u_E$, and $u_C$.

The data are from an unbalanced panel, in which the plants are observed in at least 1 and at most $P$ years. We assume that the plants are arranged in groups according to the number of years the plants are observed. This will be convenient when presenting the estimation procedure (cf. Section 3). Let $N_p$ be the number of plants which are observed in exactly $p$ years (not necessarily the same and not necessarily consecutive), let $(ip)$ index the $i$'th plant among those observed in $p$ years ($i = 1, \dots, N_p$; $p = 1, \dots, P$), and let $t$ index the observation number ($t = 1, \dots, p$). The total number of plants in the panel is $N = \sum_{p=1}^{P} N_p$ and the total number of observations is $n = \sum_{p=1}^{P} N_p p$. To capture heterogeneity of the technology, some coefficients are allowed to be *plant dependent* and treated as *random*.

Two *model classes* will be considered:

**Model A:** The two cost-share equations, (8) and (9).

**Model B:** The two cost-share equations and the cost function, (10).

Identification of the scale properties of the technology and the trend effects is possible within the full Model B only. The substitution properties can be identified from both models. Model A contains 11 and Model B contains 21 (fixed or random) coefficients. Since the latter incorporates more prior information, it leads to more efficient parameter estimation within a full Maximum Likelihood procedure, provided that all restrictions are valid.

The two model versions can be written compactly as

$$\boldsymbol{y}_{(ip)t} = \boldsymbol{X}_{(ip)t}\boldsymbol{\beta}_{(ip)} + \boldsymbol{u}_{(ip)t}, \qquad p = 1, \dots, P;\ i = 1, \dots, N_p;\ t = 1, \dots, p, \tag{11}$$

where $\boldsymbol{\beta}_{(ip)}$ is the coefficient vector of plant $(ip)$, in which at least some elements are be random and the other are fixed constants common to all plants. *Model A* is characterized by

$$\boldsymbol{X}' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ x & 0 \\ 0 & x \\ k & 0 \\ 0 & k \\ (q_L - q_M) & 0 \\ (q_E - q_M) & (q_L - q_M) \\ 0 & (q_E - q_M) \\ \tau & 0 \\ 0 & \tau \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \gamma_L \\ \gamma_E \\ \beta_{XL} \\ \beta_{XE} \\ \beta_{KL} \\ \beta_{KE} \\ \gamma_{LL} \\ \gamma_{LE} \\ \gamma_{EE} \\ \beta_{\tau L} \\ \beta_{\tau E} \end{bmatrix},$$

$$\boldsymbol{y}' = [s_L \quad s_E], \qquad \boldsymbol{u}' = [u_L \quad u_E],$$

and *Model B* by

$$
\boldsymbol{X}' = \begin{bmatrix}
0 & 0 & 1 \\
0 & 0 & x \\
0 & 0 & k \\
0 & 0 & \tau \\
1 & 0 & (q_L - q_M) \\
0 & 1 & (q_E - q_M) \\
x & 0 & x(q_L - q_M) \\
0 & x & x(q_E - q_M) \\
k & 0 & k(q_L - q_M) \\
0 & k & k(q_E - q_M) \\
(q_L - q_M) & 0 & \frac{1}{2}(q_L - q_M)^2 \\
(q_E - q_M) & (q_L - q_M) & (q_L - q_M)(q_E - q_M) \\
0 & (q_E - q_M) & \frac{1}{2}(q_E - q_M)^2 \\
\tau & 0 & \tau(q_L - q_M) \\
0 & \tau & \tau(q_E - q_M) \\
0 & 0 & \frac{1}{2}x^2 \\
0 & 0 & \frac{1}{2}k^2 \\
0 & 0 & \frac{1}{2}\tau^2 \\
0 & 0 & xk \\
0 & 0 & \tau x \\
0 & 0 & \tau k
\end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix}
\beta_0 \\ \beta_X \\ \beta_K \\ \beta_\tau \\ \gamma_L \\ \gamma_E \\ \beta_{XL} \\ \beta_{XE} \\ \beta_{KL} \\ \beta_{KE} \\ \gamma_{LL} \\ \gamma_{LE} \\ \gamma_{EE} \\ \beta_{\tau L} \\ \beta_{\tau E} \\ \beta_{XX} \\ \beta_{KK} \\ \beta_{\tau\tau} \\ \beta_{XK} \\ \beta_{\tau X} \\ \beta_{\tau K}
\end{bmatrix},
$$

$$
\boldsymbol{y}' = [s_L \quad s_E \quad c - q_M], \qquad \boldsymbol{u}' = [u_L \quad u_E \quad u_C],
$$

In the applications, the following *model versions* will be considered:

**Model A1:** All 11 coefficients are fixed.

**Model A2:** The coefficients $\gamma_L, \gamma_E$ are random. The other 9 coefficients are fixed.

**Model B1:** All 21 coefficients are fixed.

**Model B2:** The coefficients $\beta_0, \gamma_L, \gamma_E$ are random. The other 18 coefficients are fixed.

**Model B3:** The coefficients $\beta_0, \beta_X, \beta_K, \beta_\tau, \gamma_L, \gamma_E$ are random. The other 15 coefficients, representing second-order terms in the cost function, are fixed.

In addition, a 'constant returns to scale' version of Model B2, denoted as B2R is also considered. Model B3 implies that in the two cost-share equations, only the intercepts are random, whereas the cost equation from which they are derived, have five random slope coefficients, and a random intercept. Hence, both the cost elasticity function with respect to output and the substitution elasticity function, conditional on the exogenous variables, will contain random coefficients.

The model is formally a system of $G$ regression equations with random coefficients and with a total of $K$ (fixed or random) coefficients. In Model A, $G = 2, K = 11$, in

Model B, $G = 3, K = 21$. The $(G \times 1)$ vector of observations of the regressands in the $G$ equations from plant $(ip)$, observation $t$ is $\boldsymbol{y}_{(ip)t}$, and the corresponding $(G \times K)$ regressor matrix is $\boldsymbol{X}_{(ip)t}$. The $(K \times 1)$ coefficient vector of plant $(ip)$ is

$$\boldsymbol{\beta}_{(ip)} = \boldsymbol{\beta} + \boldsymbol{\delta}_{(ip)}, \tag{12}$$

where $\boldsymbol{\beta}$ is the common expectation vector of $\boldsymbol{\beta}_{(ip)}$ for all plants, and $\boldsymbol{\delta}_{(ip)}$ a zero mean vector specific to plant $(ip)$. By inserting (11) in (12), the $G$ equations for plant $(ip)$, observation $t$, can thus be written as

$$\boldsymbol{y}_{(ip)t} = \boldsymbol{X}_{(ip)t}\boldsymbol{\beta} + \boldsymbol{\eta}_{(ip)t}, \qquad \boldsymbol{\eta}_{(ip)t} = \boldsymbol{X}_{(ip)t}\boldsymbol{\delta}_{(ip)t} + \boldsymbol{u}_{(ip)t}. \tag{13}$$

We further assume that

$$\boldsymbol{X}_{(ip)t},\ \boldsymbol{u}_{(ip)t},\ \boldsymbol{\delta}_{(ip)} \ \text{ are all stochastically independent}, \tag{14}$$

and that

$$\boldsymbol{u}_{(ip)t} \sim \mathsf{IIN}(\mathbf{0}_{G1}, \boldsymbol{\Sigma}^u), \qquad \boldsymbol{\delta}_{(ip)} \sim \mathsf{IIN}(\mathbf{0}_{K1}, \boldsymbol{\Sigma}^\delta), \tag{15}$$

where $\mathsf{IIN}$ signifies independently, identically, normally distributed, $\mathbf{0}_{m,n}$ is a $(m \times n)$ zero matrix and

$$\boldsymbol{\Sigma}^u = \begin{bmatrix} \sigma^u_{11} & \cdots & \sigma^u_{1G} \\ \vdots & & \vdots \\ \sigma^u_{G1} & \cdots & \sigma^u_{GG} \end{bmatrix}, \qquad \boldsymbol{\Sigma}^\delta = \begin{bmatrix} \boldsymbol{\sigma}^\delta_{11} & \cdots & \boldsymbol{\sigma}^\delta_{1K} \\ \vdots & & \vdots \\ \boldsymbol{\sigma}^\delta_{K1} & \cdots & \boldsymbol{\sigma}^\delta_{KK} \end{bmatrix}.$$

The latter two matrices may be singular, reflecting for instance that some of the coefficients may be fixed. In all model versions we consider below, $\boldsymbol{\Sigma}^u$ is a full positive definite $(G \times G)$ matrix, while the $(K \times K)$ matrix $\boldsymbol{\Sigma}^\delta$ has reduced rank and is partioned as

$$\boldsymbol{\Sigma}^\delta = \begin{bmatrix} \overline{\boldsymbol{\Sigma}}^\delta & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

where in, *e.g., Model A2*,

$$\boldsymbol{\Sigma}^u = \begin{bmatrix} \sigma^u_{LL} & \sigma^u_{LE} \\ \sigma^u_{EL} & \sigma^u_{EE} \end{bmatrix}, \qquad \overline{\boldsymbol{\Sigma}}^\delta = \begin{bmatrix} \sigma^\delta_{LL} & \sigma^\delta_{LE} \\ \sigma^\delta_{EL} & \sigma^\delta_{EE} \end{bmatrix},$$

and in, *e.g., Model B3*,

$$\boldsymbol{\Sigma}^u = \begin{bmatrix} \sigma^u_{LL} & \sigma^u_{LE} & \sigma^u_{LC} \\ \sigma^u_{EL} & \sigma^u_{EE} & \sigma^u_{EC} \\ \sigma^u_{CL} & \sigma^u_{CE} & \sigma^u_{CC} \end{bmatrix}, \qquad \overline{\boldsymbol{\Sigma}}^\delta = \begin{bmatrix} \sigma^\delta_{00} & \sigma^\delta_{0X} & \sigma^\delta_{0K} & \sigma^\delta_{0\tau} & \sigma^\delta_{0L} & \sigma^\delta_{0E} \\ \sigma^\delta_{X0} & \sigma^\delta_{XX} & \sigma^\delta_{XK} & \sigma^\delta_{X\tau} & \sigma^\delta_{XL} & \sigma^\delta_{XE} \\ \sigma^\delta_{K0} & \sigma^\delta_{KX} & \sigma^\delta_{KK} & \sigma^\delta_{K\tau} & \sigma^\delta_{KL} & \sigma^\delta_{KE} \\ \sigma^\delta_{\tau 0} & \sigma^\delta_{\tau X} & \sigma^\delta_{\tau K} & \sigma^\delta_{\tau\tau} & \sigma^\delta_{\tau L} & \sigma^\delta_{\tau E} \\ \sigma^\delta_{L0} & \sigma^\delta_{LX} & \sigma^\delta_{LK} & \sigma^\delta_{L\tau} & \sigma^\delta_{LL} & \sigma^\delta_{LE} \\ \sigma^\delta_{E0} & \sigma^\delta_{EX} & \sigma^\delta_{EK} & \sigma^\delta_{E\tau} & \sigma^\delta_{EL} & \sigma^\delta_{EE} \end{bmatrix}.$$

We stack the $p$ realizations from plant $(ip)$ in

$$\boldsymbol{y}_{(ip)} = \begin{bmatrix} \boldsymbol{y}_{(ip)1} \\ \vdots \\ \boldsymbol{y}_{(ip)p} \end{bmatrix}, \quad \boldsymbol{X}_{(ip)} = \begin{bmatrix} \boldsymbol{X}_{(ip)1} \\ \vdots \\ \boldsymbol{X}_{(ip)p} \end{bmatrix}, \quad \boldsymbol{u}_{(ip)} = \begin{bmatrix} \boldsymbol{u}_{(ip)1} \\ \vdots \\ \boldsymbol{u}_{(ip)p} \end{bmatrix}, \quad \boldsymbol{\eta}_{(ip)} = \begin{bmatrix} \boldsymbol{\eta}_{(ip)1} \\ \vdots \\ \boldsymbol{\eta}_{(ip)p} \end{bmatrix},$$

which, in general, have dimensions $(Gp\times 1)$, $(Gp\times K)$, $(Gp\times 1)$, and $(Gp\times 1)$, respectively. Then we can write (13) as

$$\text{(16)} \qquad \boldsymbol{y}_{(ip)} = \boldsymbol{X}_{(ip)}\boldsymbol{\beta} + \boldsymbol{\eta}_{(ip)}, \quad \boldsymbol{\eta}_{(ip)} = \boldsymbol{X}_{(ip)}\boldsymbol{\delta}_{(ip)} + \boldsymbol{u}_{(ip)}.$$

It follows from (14), (15), and (16) that

(17) All $\boldsymbol{\eta}_{(ip)}|\boldsymbol{X}_{(ip)}$ are stochastically independent, and $\boldsymbol{\eta}_{(ip)}|\boldsymbol{X}_{(ip)} \sim \mathsf{N}(\mathbf{0}_{Gp,1}, \boldsymbol{\Omega}_{(ip)})$,

where $\boldsymbol{\Omega}_{(ip)}$ is the $(Gp \times Gp)$ matrix

$$\text{(18)} \qquad \boldsymbol{\Omega}_{(ip)} = \boldsymbol{X}_{(ip)}\boldsymbol{\Sigma}^{\delta}\boldsymbol{X}'_{(ip)} + \boldsymbol{I}_p \otimes \boldsymbol{\Sigma}^{u}.$$

We see from (18) that the 'gross disturbance' vector $\boldsymbol{\eta}_{(ip)}$ exhibits a particular kind of heteroskedasticity.

# 3 Estimation procedure and data

The joint log-density function of plant $(ip)$, *i.e.* of $\boldsymbol{y}_{(ip)}$ conditional on $\boldsymbol{X}_{(ip)}$, is

$$L_{(ip)} = -\frac{Gp}{2}\ln(2\pi) - \frac{1}{2}\ln|\boldsymbol{\Omega}_{(ip)}| - \frac{1}{2}[\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\boldsymbol{\beta}]'\boldsymbol{\Omega}^{-1}_{(ip)}[\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\boldsymbol{\beta}],$$

so that by utilizing the ordering of the observations in the $P$ groups, we can write the log-likelihood function of all observations on $\boldsymbol{y}$ conditional on all observations on $\boldsymbol{X}$ as

$$\text{(19)} \qquad \begin{aligned} L = \sum_{p=1}^{P}\sum_{i=1}^{N_p} L_{(ip)} = & -\frac{Gn}{2}\ln(2\pi) - \frac{1}{2}\sum_{p=1}^{P}\sum_{i=1}^{N_p}\ln|\boldsymbol{\Omega}_{(ip)}| \\ & - \frac{1}{2}\sum_{p=1}^{P}\sum_{i=1}^{N_p}[\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\boldsymbol{\beta}]'\boldsymbol{\Omega}^{-1}_{(ip)}[\boldsymbol{y}_{(ip)} - \boldsymbol{X}_{(ip)}\boldsymbol{\beta}]. \end{aligned}$$

The *Maximum Likelihood (ML)* estimators of $(\boldsymbol{\beta}, \boldsymbol{\Sigma}^{u}, \boldsymbol{\Sigma}^{\delta})$ are obtained by maximizing $L$ with respect to (the unknown elements of) these parameter matrices, as given in Section 2.[2] The structure of this problem is more complicated than the ML problem for systems

[2] The soluton conditions may be simplified by concentrating $L$ over $\boldsymbol{\beta}$ and maximizing the resulting function with respect to the unknown elements of the $\boldsymbol{\Omega}$ matrices.

of regression equations in more standard situations with balanced panel data sets and fixed slope coefficients for two (related) reasons. First, the various $\boldsymbol{y}$, $\boldsymbol{X}$, and $\boldsymbol{\Omega}$ matrices have different number of rows, reflecting the different number of observations of the plants in the panel. Although the dimensions of $\boldsymbol{\Sigma}^{u}$, and $\boldsymbol{\Sigma}^{\delta}$ in (18) are the same for all plants, the dimensions of $\boldsymbol{X}_{(ip)}$, and hence of $\boldsymbol{\Omega}_{(ip)}$, differ. Second, different plants have different disturbance covariance matrices, since $\boldsymbol{\Omega}_{(ip)}$ depends on $\boldsymbol{X}_{(ip)}$ when $\boldsymbol{\Sigma}^{\delta}$ is non-zero, *i.e.*, when at least one of the coefficients (in addition to the intercepts) are random.

Primarily we use *data* from the Manufacturing Statistics database of Statistics Norway, supplemented, to a minor extent by data from the Norwegian *National Accounts*. All industries classified under *SIC-code 351 Manufacture of Industrial Chemicals* are included. The data set is unbalanced and covers $T = 22$ years, with a total number of plants $N = \sum_p N_p = 88$, and a total number of observations $n = \sum_p N_p p = 1101$, so that on average, the plants are observed in $11 - 12$ years. Of these plants, $N_{22} = 30$ are observed in all the 22 years – representing 660 observations or about 60 percent of the data set – and $N_1 = 16$ are observed in one year only. All times series used for the estimation and testing reported below are *contiguous*, *i.e.*, plants for which there are gaps in the time series in the original database, 220 observations in all, are excluded. The output measure is tonnes output. This implies that systematic, gradual quality changes in output over time will be represented by the trend variable in the model. The capital input data are constructed from information on fire insurance values, in combination with information on gross investment flows from the Manufacturing Statistics and in the National Accounts. Details on the data and data construction are given in the Appendix.

## 4 Empirical results

Maximum Likelihood estimates of the six models described in Section 2 have been obtained by using the PROC MIXED-procedure in SAS/STAT software [SAS (1992)].

In addition to the estimated elements of the $\boldsymbol{\beta}$ vector, cf. equation (12), Tables 1 and 3 include the coefficients calculated by using the adding-up conditions described in Section 2. Apart from the results for (the restrictive) Model B2R, the tables show that several coefficient estimates are relatively stable across models. This supports our stochastic assumptions, which implies that neglecting coefficient heterogeneity does not cause inconsistent estimates of $\boldsymbol{\beta}$ asymptotically. However, there is a loss of efficiency. A clear majority of the coefficient estimates are significant according to the asymptotic standard-error estimates.

Tables 2 and 4 give some measures of overall model fit. Both the Akaike and the

Schwartz Bayesian information criteria support the most flexible models, *i.e.*, A2 among the A-models and B3 among the B-models. Furthermore, it is evident that substantial explanatory power is lost when the output-elasticity *a priori* is set to one, which is the case for Model B2R. The Log-Likelihood values of the different models are also given in Tables 2 and 4, and the Log-Likelihood Ratios of interest can be calculated from the tables. These test-statistics are, however, not asymptotically $\chi^2$-distributed under the null, because the coefficients then are on the border of the admissible parameter space, see Shin (1995, p. 321). Thus, for making formal inference, other test procedures are needed. For a discussion of statistical tests for mixed linear models see Khuri *et al.* (1998).

To facilitate the economic interpretation of the models, we have calculated various elasticities. In Table 5, own-price and cross-price elasticities of (variable) input demand are reported. These are the analogues of the Slutsky-elasticities from consumer demand, and the cross-price elasticities are not symmetric. In addition, we also report the Allen-Uzawa elasticities, which are symmetric, see Table 6. By construction, the two sets of elasticities have the same sign. In accordance with the chosen translog functional form, the elasticities will be functions not only of the estimated coefficients, but also of the exogenous variables. Typically, the elasticities will therefore vary across plants and over time. In Tables 5 and 6, the different elasticities are reported at the overall sample means of the exogenous variables. The elasticities in Models A2, B2, B3 and B2R are functions of both the exogenous variables and the random coefficients, and these models will in general predict variation in elasticities even across plants with the same exogenous input. When calculating the elasticities reported in the tables, we use the estimated expectations of the random coefficients. Because the predicted (variable) cost shares depend on unknown coefficients, all the estimated elasticities in Tables 5 and 6 are non-linear functions of the coefficients. To calculate standard deviations, a first order Taylor-expansion of the non-linear functions is utilized (cf. Kmenta (1986)).

According to the results in Table 5, none of the variable inputs are price-elastic. This is in line with the findings in Lindquist (1995), where a dynamic translog factor-demand system assuming fixed effects is estimated on Norwegian manufacturing plant-level panel data. Except for energy in Model B1, the own-price elasticities in the six models are all less than 1 in absolute value. From Table 6 it is seen that all the cross-price Allen-Uzawa elasticities are positive (at the global mean of the exogenous variables), which means that all the three variable inputs in average are substitutes. The Allen-Uzawa own-price elasticity for energy is fairly high in absolute value in all models and higher than the estimates usually reported in studies of input demand. The price elasticities are relatively equal in the A- and B-models. Even the price elasticties from Model B2R are rather similiar to those obtained for the other models. Hence, in our case, little gain is obtained in adding the cost function to the cost-share equations if the sole interest is in factor-price elasticities. However, in this paper we are also interested in the scale properties of the technology, which can only be obtained within a framework where the cost function is an integral part of the model, as in our B-models.

The first row of Table 7 contains the output elasticity in the B-models. This elasticity, like the price elasticities, depends on the value of the exogenous variables (except in Model B2R where it is restricted to 1 *a priori*), and again we use the overall mean of the exogenous variables and the estimated expected values of the random coefficients as a reference point when calculating the elasticities. The output elasticity is between 0.25 and 0.3 in the random coefficient models B2 and B3. In Model B1, in which all coefficients are assumed to be plant-invariant, the estimated output elasticity is about 0.15. The inverse of the output elasticity can be interpreted as 'a variable-input scale elasticity' of the underlying (three factor) production function. These estimated 'economies of scale effects' are rather high compared with the unitary elasticity assumed in Model B2R. The loss of explanatory power for Model B2R is closely linked to higher output elasticity for this model than for the other B-models.

In the second row of Table 7, we report the elasticity of variable costs with respect to an increase in the capital stock calculated at the global empirical mean. To be well behaved, the cost function should be non-increasing and convex in the levels of any fixed factor, cf. Brown and Christensen (1981, pp. 217 – 218). The intuition is that as capital stock increases, one needs less variable inputs to maintain the given level of output. This will accordingly reduce variable costs. However, for Models B1, B2 and B3 this elasticity has wrong sign, implying that an increase in the capital stock increases variable costs at a given level of output. The capital elasticity has the correct sign in Model B2R, however, although it is not significantly different from zero according to the asymptotic t-value. In the last row of Table 7, we report the derivative of the logarithm of variable costs with respect to the trend variable. The trend variable is included in order to pick up the effect of technological progress, which may be both neutral and non-neutral. This derivative should be of negative sign, since technological progress means that a given output level can be obtained with less variable inputs. As for the response to an increase in the capital stock, we get a positive sample mean estimate of this derivative for Models B1, B2 and B3, whereas the correct negative sign is obtained for Model B2R, and the effect is significant.

In order to study the robustness of the results obtained for Model B3 with respect to the production technology, several alternative calculations have been performed. First, insignificant coefficients have been constrained to zero and the model reestimated following a general to specific modelling approach. Second, an alternative capital measure was constructed, based on fire insurance values, while the first measure was very much based on data on investments. However, the results using this alternative capital measure came out very similiar to the results in Table 7, and they are therefore not reported.

Our interpretation of the 'theory-inconsistent' results is that we are unable to properly identify simultaneously the effect of changes in production, capital stock, and technological progress on variable costs. This may again be due to our proxies for these two variables. In our context there are no generally accepted way to calculate the capital stock. A lot of different capital stock measures can be established, and empirically, there are no operational way to distinguish between them. Our results are, however, robust to which of the alternative measures we include in our information set (see Appendix). Technological change is proxied by a deterministic time trend which is, admittedly, a very rough way of picking up technological progress, and the need for additional information about technological progress is highly desirable. The problem of identifying the different properties of the production process is not particular to our study. Morrison (1988, p. 278) for instance, reports within a Generalized Leontief framework that "empirical researchers have found it difficult to identify independently the impacts of technology, quasi-fixed inputs and returns to scale".

In Table 8, we report the estimated covariance matrix of the random coefficient-vector for Models A2, B2, B3, and B2R, respectively. These matrices are of different dimensions, because the number of random coefficients varies across the models. The covariance matrix of the genuine disturbances of the six estimated models are given in Table 9. The covariance matrices of the genuine error-terms are of dimension $(2 \times 2)$ and $(3 \times 3)$ for the A- and B-models, respectively. All the estimated covariance matrices satisfy the positive-definiteness requirement, since all the calculated eigenvalues of the different matrices are positive. Both the A- and B-models which do not allow for coefficient heterogeneity have higher estimated variances of the genuine disturbances than the more flexible models. This is as expected when coefficient heterogeneity is important, because Models A1 and B1 are then misspecified and the estimated variances of the disturbances are influenced by the coefficient heterogeneity. By comparing the estimated covariance matrices for Model B2R with those of Model B2, we see that constraining the output elasticity to 1 has a significant impact on both the covariance matrix for the random coefficients and the covariance matrix of the genuine error-terms. However, the submatrix consisting of only the second order moments of the genuine errors in the two cost-share equations looks very similiar to that in Model B2.

Table 1. Coefficient estimates and standard deviations in A-models

| | Model A1[1] | | Model A2[2] | |
|---|---|---|---|---|
| Coefficient | Value | Std. dev. | Value | Std. dev. |
| $\gamma_L$ | 0.3351 | 0.0465 | 0.2261 | 0.0557 |
| $\gamma_E$ | 0.0825 | 0.0290 | -0.0027 | 0.0354 |
| $\gamma_M$ | 0.5824 | 0.0622 | 0.7766 | 0.0711 |
| $\beta_{XL}$ | -0.0257 | 0.0024 | -0.0171 | 0.0022 |
| $\beta_{XE}$ | 0.0115 | 0.0019 | 0.0060 | 0.0016 |
| $\beta_{XM}$ | 0.0142 | 0.0034 | 0.0111 | 0.0027 |
| $\beta_{KL}$ | 0.0021 | 0.0029 | 0.0030 | 0.0049 |
| $\beta_{KE}$ | 0.0018 | 0.0023 | 0.0069 | 0.0033 |
| $\beta_{KM}$ | -0.0040 | 0.0042 | -0.0099 | 0.0064 |
| $\gamma_{LL}$ | 0.0445 | 0.0088 | 0.0466 | 0.0057 |
| $\gamma_{LE}$ | -0.0192 | 0.0045 | -0.0089 | 0.0026 |
| $\gamma_{LM}$ | -0.0253 | 0.0103 | -0.0377 | 0.0062 |
| $\gamma_{EE}$ | -0.0112 | 0.0044 | 0.0053 | 0.0024 |
| $\gamma_{EM}$ | 0.0303 | 0.0068 | 0.0036 | 0.0035 |
| $\gamma_{MM}$ | -0.0050 | 0.0146 | 0.0341 | 0.0080 |
| $\beta_{\tau L}$ | 0.0003 | 0.0007 | 0.0008 | 0.0004 |
| $\beta_{\tau E}$ | 0.0017 | 0.0006 | 0.0015 | 0.0003 |
| $\beta_{\tau M}$ | -0.0020 | 0.0011 | -0.0023 | 0.0005 |

[1] Model A1: None of the coefficients are assumed to be random.

[2] Model A2: $\gamma_L$ , $\gamma_E$ and $\gamma_M$ are the expectations of $\gamma_{L(i,p)}$ , $\gamma_{E(i,p)}$ and $\gamma_{M(i,p)}$ respectively.

Table 2. Overall measures of fit in A-models

| | Model A1 | Model A2 |
|---|---|---|
| Number of estimated parameters | 14 | 17 |
| Log-likelihood value | 1546.398 | 2877.179 |
| Akaike's information criterion | 1543.398 | 2871.179 |
| Schwartz's Bayesian criterion | 1534.853 | 2854.088 |

Table 3. Coefficient estimates and standard deviations in B-models

| | Model B1[1] | | Model B2[2] | | Model B3[3] | | Model B2R[2] | |
|---|---|---|---|---|---|---|---|---|
| Coef. | Value | Std. dev. | Value | Std. dev. | Value | Std. dev. | Value | Std. dev. |
| $\beta_0$ | 1.5676 | 0.3001 | 3.8314 | 0.5606 | 3.6053 | 1.0168 | 0.9511 | 0.9224 |
| $\beta_X$ | 0.7355 | 0.0449 | 0.2856 | 0.0557 | 0.4704 | 0.1282 | 1[4] | |
| $\beta_K$ | -0.1458 | 0.0481 | 0.0070 | 0.0863 | 0.1374 | 0.1634 | -0.0425 | 0.2010 |
| $\beta_\tau$ | 0.0544 | 0.0195 | 0.0248 | 0.0120 | -0.0434 | 0.0232 | 0.0672 | 0.0316 |
| $\gamma_L$ | 0.3725 | 0.0431 | 0.2401 | 0.0558 | 0.2104 | 0.0561 | 0.1931 | 0.0610 |
| $\gamma_E$ | 0.1285 | 0.0271 | -0.0206 | 0.0352 | 0.0002 | 0.0350 | 0.0144 | 0.0358 |
| $\gamma_M$ | 0.4990 | 0.0570 | 0.7805 | 0.0706 | 0.7894 | 0.0711 | 0.7925 | 0.0732 |
| $\beta_{XL}$ | -0.0237 | 0.0023 | -0.0156 | 0.0022 | -0.0158 | 0.0022 | 0[4] | |
| $\beta_{XE}$ | 0.0111 | 0.0018 | 0.0060 | 0.0016 | 0.0058 | 0.0016 | 0[4] | |
| $\beta_{XM}$ | 0.0126 | 0.0034 | 0.0096 | 0.0027 | 0.0100 | 0.0027 | 0[4] | |
| $\beta_{KL}$ | 0.0017 | 0.0029 | 0.0031 | 0.0049 | 0.0096 | 0.0049 | -0.0061 | 0.0053 |
| $\beta_{KE}$ | 0.0020 | 0.0022 | 0.0089 | 0.0033 | 0.0087 | 0.0032 | 0.0102 | 0.0032 |
| $\beta_{KM}$ | -0.0037 | 0.0041 | -0.0120 | 0.0063 | -0.0183 | 0.0064 | -0.0040 | 0.0064 |
| $\gamma_{LL}$ | 0.0368 | 0.0081 | 0.0409 | 0.0055 | 0.0349 | 0.0054 | 0.0465 | 0.0058 |
| $\gamma_{LE}$ | -0.0270 | 0.0040 | -0.0086 | 0.0025 | -0.0118 | 0.0024 | -0.0101 | 0.0026 |
| $\gamma_{LM}$ | -0.0098 | 0.0093 | -0.0323 | 0.0059 | -0.0231 | 0.0058 | -0.0364 | 0.0063 |
| $\gamma_{EE}$ | -0.0151 | 0.0042 | 0.0042 | 0.0024 | 0.0027 | 0.0023 | 0.0055 | 0.0024 |
| $\gamma_{EM}$ | 0.0421 | 0.0062 | 0.0044 | 0.0034 | 0.0091 | 0.0033 | 0.0045 | 0.0035 |
| $\gamma_{MM}$ | -0.0323 | 0.0129 | 0.0279 | 0.0075 | 0.0140 | 0.0074 | 0.0318 | 0.0080 |
| $\beta_{\tau L}$ | 0.0005 | 0.0007 | 0.0008 | 0.0004 | 0.0010 | 0.0004 | -0.0002 | 0.0004 |
| $\beta_{\tau E}$ | 0.0020 | 0.0006 | 0.0015 | 0.0003 | 0.0016 | 0.0003 | 0.0019 | 0.0003 |
| $\beta_{\tau M}$ | -0.0025 | 0.0011 | -0.0023 | 0.0005 | -0.0026 | 0.0005 | -0.0017 | 0.0005 |
| $\beta_{XX}$ | 0.0205 | 0.0041 | 0.0573 | 0.0044 | 0.0184 | 0.0082 | 0[4] | |
| $\beta_{XK}$ | -0.0548 | 0.0041 | -0.0355 | 0.0051 | -0.0254 | 0.0122 | 0[4] | |
| $\beta_{KK}$ | 0.1127 | 0.0066 | 0.0569 | 0.0099 | 0.0160 | 0.0189 | 0.0068 | 0.0232 |
| $\beta_{\tau\tau}$ | -0.0015 | 0.0010 | -0.0008 | 0.0005 | -0.0019 | 0.0004 | 0.0006 | 0.0014 |
| $\beta_{\tau X}$ | -0.0082 | 0.0016 | -0.0089 | 0.0012 | -0.0039 | 0.0018 | 0[4] | |
| $\beta_{\tau K}$ | 0.0051 | 0.0022 | 0.0067 | 0.0014 | 0.0096 | 0.0025 | -0.0098 | 0.0027 |

[1] Model B1: None of the coefficients are assumed to be random.

[2] Model B2 and B2R: $\beta_0$, $\gamma_L$ , $\gamma_E$ and $\gamma_M$ are the expectations of $\beta_{0(i,p)}$, $\gamma_{L(i,p)}$, $\gamma_{E(i,p)}$ and $\gamma_{M(i,p)}$ respectively.

[3] Model B3: $\beta_0$, $\beta_X$, $\beta_K$, $\beta_\tau$, $\gamma_L$, $\gamma_E$ and $\gamma_M$ are the expectations of $\beta_{0(i,p)}$, $\beta_{X(i,p)}$, $\beta_{K(i,p)}$, $\beta_{\tau(i,p)}$, $\gamma_{L(i,p)}$, $\gamma_{E(i,p)}$ and $\gamma_{M(i,p)}$ respectively.

[4] A priori restriction.

Table 4. Overall measures of fit in B-models

| | Model B1 | Model B2 | Model B3 | Model B2R[1] |
|---|---|---|---|---|
| Number of estimated parameters | 27 | 33 | 48 | 27 |
| Log-likelihood value | 662.440 | 2521.881 | 2884.850 | 1388.758 |
| Akaike's information criterion | 656.440 | 2509.881 | 2857.850 | 1376.758 |
| Schwartz's Bayesian criterion | 638.133 | 2473.266 | 2775.465 | 1340.143 |

[1] The value of the two information criterias for model B2R is not comparable with those of the three other models since the former has six less fixed coefficients than the others.

Table 5. Own- and cross price-elasticties of input demand.[1] Standard deviations in parentheses

| Elast. | A1 | A2 | B1 | B2 | B3 | B2R |
|---|---|---|---|---|---|---|
| $\varepsilon_{L,L}$ | -0.556 | -0.538 | -0.584 | -0.555 | -0.570 | -0.530 |
| | (0.030) | (0.021) | (0.028) | (0.020) | (0.021) | (0.021) |
| $\varepsilon_{E,E}$ | -0.992 | -0.838 | -1.031 | -0.845 | -0.860 | -0.839 |
| | (0.041) | (0.022) | (0.040) | (0.022) | (0.022) | (0.023) |
| $\varepsilon_{M,M}$ | -0.409 | -0.370 | -0.449 | -0.386 | -0.414 | -0.382 |
| | (0.025) | (0.025) | (0.023) | (0.026) | (0.027) | (0.027) |
| $\varepsilon_{E,L}$ | 0.116 | 0.237 | 0.038 | 0.244 | 0.221 | 0.237 |
| | (0.042) | (0.031) | (0.038) | (0.029) | (0.031) | (0.034) |
| $\varepsilon_{L,E}$ | 0.044 | 0.088 | 0.014 | 0.093 | 0.081 | 0.080 |
| | (0.016) | (0.015) | (0.014) | (0.014) | (0.014) | (0.015) |
| $\varepsilon_{M,L}$ | 0.249 | 0.248 | 0.271 | 0.258 | 0.280 | 0.263 |
| | (0.018) | (0.019) | (0.016) | (0.019) | (0.020) | (0.022) |
| $\varepsilon_{L,M}$ | 0.513 | 0.450 | 0.570 | 0.462 | 0.489 | 0.450 |
| | (0.036) | (0.027) | (0.033) | (0.026) | (0.028) | (0.027) |
| $\varepsilon_{M,E}$ | 0.160 | 0.123 | 0.178 | 0.128 | 0.134 | 0.119 |
| | (0.012) | (0.013) | (0.011) | (0.013) | (0.013) | (0.013) |
| $\varepsilon_{E,M}$ | 0.876 | 0.601 | 0.994 | 0.601 | 0.639 | 0.602 |
| | (0.064) | (0.040) | (0.059) | (0.038) | (0.041) | (0.042) |
| $s_L$ | 0.291 | 0.314 | 0.287 | 0.316 | 0.321 | 0.328 |
| $s_E$ | 0.110 | 0.116 | 0.108 | 0.120 | 0.118 | 0.111 |
| $s_M$ | 0.600 | 0.570 | 0.604 | 0.564 | 0.561 | 0.561 |

[1] All elasticities and cost shares ($s_L$, $s_E$ and $s_M$) are computed at the global mean of the exogenous variables.

Table 6. Allen-Uzawa partial elasticities of substitution.[1] Standard deviations in parentheses

| Elast. | A1 | A2 | B1 | B2 | B3 | B2R |
|---|---|---|---|---|---|---|
| $\eta_{L,L}$ | -1.912 | -1.714 | -2.035 | -1.759 | -1.773 | -1.618 |
| | (0.109) | (0.141) | (0.105) | (0.145) | (0.151) | (0.151) |
| $\eta_{E,E}$ | -9.056 | -7.201 | -9.521 | -7.026 | -7.310 | -7.549 |
| | (0.484) | (0.803) | (0.495) | (0.762) | (0.812) | (0.884) |
| $\eta_{M,M}$ | -0.682 | -0.650 | -0.743 | -0.685 | -0.738 | -0.681 |
| | (0.044) | (0.069) | (0.040) | (0.071) | (0.077) | (0.075) |
| $\eta_{E,L}$ | 0.399 | 0.756 | 0.131 | 0.774 | 0.687 | 0.722 |
| | (0.142) | (0.078) | (0.131) | (0.071) | (0.074) | (0.080) |
| $\eta_{M,L}$ | 0.855 | 0.789 | 0.944 | 0.818 | 0.872 | 0.802 |
| | (0.059) | (0.035) | (0.054) | (0.034) | (0.033) | (0.035) |
| $\eta_{M,E}$ | 1.462 | 1.055 | 1.644 | 1.065 | 1.139 | 1.073 |
| | (0.104) | (0.053) | (0.095) | (0.050) | (0.052) | (0.056) |
| $s_L$ | 0.291 | 0.314 | 0.287 | 0.316 | 0.321 | 0.328 |
| $s_E$ | 0.110 | 0.116 | 0.108 | 0.120 | 0.118 | 0.111 |
| $s_M$ | 0.600 | 0.570 | 0.604 | 0.564 | 0.561 | 0.561 |

[1] All elasticities and cost shares ($s_L$, $s_E$ and $s_M$) are computed at the global mean of the exogenous variables.

Table 7. Output elasticity, capital elasticity and trend effect on costs in B-models[1]. Standard deviations in parentheses

| | B1 | B2 | B3 | B2R |
|---|---|---|---|---|
| $\varepsilon_X$ | 0.142 | 0.276 | 0.257 | 1[2] |
| | (0.013) | (0.014) | (0.035) | |
| $\varepsilon_K$ | 0.650 | 0.411 | 0.248 | -0.082 |
| | (0.016) | (0.037) | (0.046) | (0.094) |
| $\varepsilon_\tau$ | 0.022 | 0.012 | 0.013 | -0.031 |
| | (0.003) | (0.002) | (0.005) | (0.005) |

[1] Computed at the global mean of the exogenous variables.
[2] A priori restriction.

Table 8. The covariance matrix for the random coefficients. Standard deviations in parentheses[1]

Model A2

| | $\gamma_{L(i,p)}$ | $\gamma_{E(i,p)}$ |
|---|---|---|
| $\gamma_{L(i,p)}$ | 2.45 | |
| | (0.41) | |
| $\gamma_{E(i,p)}$ | 0.43 | 1.00 |
| | (0.19) | (0.16) |

Model B2

| | $\beta_{0(i,p)}$ | $\gamma_{L(i,p)}$ | $\gamma_{E(i,p)}$ |
|---|---|---|---|
| $\beta_{0(i,p)}$ | 212.45 | | |
| | (36.24) | | |
| $\gamma_{L(i,p)}$ | -18.12 | 2.50 | |
| | (3.46) | (0.42) | |
| $\gamma_{E(i,p)}$ | -8.31 | 0.40 | 1.01 |
| | (1.96) | (0.19) | (0.16) |

Model B3

| | $\beta_{0(i,p)}$ | $\beta_{X(i,p)}$ | $\beta_{K(i,p)}$ | $\mathbf{b}_{\mathbf{t}}$ (i,p) | $\gamma_{L(i,p)}$ | $\gamma_{E(i,p)}$ |
|---|---|---|---|---|---|---|
| $\beta_{0(i,p)}$ | 1024.45 | | | | | |
| | (341.69) | | | | | |
| $\beta_{X(i,p)}$ | -42.81 | 6.53 | | | | |
| | (16.28) | (1.47) | | | | |
| $\beta_{K(i,p)}$ | -46.48 | 0.03 | 4.69 | | | |
| | (23.25) | (1.39) | (1.99) | | | |
| $\beta_{\tau\ (i,p)}$ | 4.37 | -0.42 | -0.24 | 0.12 | | |
| | (1.88) | (0.18) | (0.17) | (0.04) | | |
| $\gamma_{L(i,p)}$ | -15.22 | -0.82 | -0.23 | 0.00 | 2.65 | |
| | (8.81) | (0.60) | (0.76) | (0.12) | (0.45) | |
| $\gamma_{E(i,p)}$ | -9.09 | 0.24 | 0.00 | -0.04 | 0.45 | 0.98 |
| | (5.89) | (0.33) | (0.47) | (0.05) | (0.20) | (0.16) |

Model B2R

| | $\beta_{0(i,p)}$ | $\gamma_{L(i,p)}$ | $\gamma_{E(i,p)}$ |
|---|---|---|---|
| $\beta_{0(i,p)}$ | 454.30 | | |
| | (71.21) | | |
| $\gamma_{L(i,p)}$ | 6.05 | 3.45 | |
| | (4.41) | (0.54) | |
| $\gamma_{E(i,p)}$ | -11.27 | 0.19 | 1.05 |
| | (2.73) | (0.21) | (0.17) |

[1] All second order moments and their standard deviations are multiplied by 100.

Table 9. The covariance matrix of the genuine error terms. Standard deviations in parentheses[1]

Model A1

| | $u_L$ | $u_E$ |
|---|---|---|
| $u_L$ | 1.94 | |
| | (0.08) | |
| $u_E$ | 0.49 | 1.19 |
| | (0.05) | (0.05) |

Model A2

| | $u_L$ | $u_E$ |
|---|---|---|
| $u_L$ | 0.43 | |
| | (0.02) | |
| $u_E$ | -0.02 | 0.24 |
| | (0.01) | (0.01) |

Model B1

| | $u_L$ | $u_E$ | $u_C$ |
|---|---|---|---|
| $u_L$ | 1.96 | | |
| | (0.08) | | |
| $u_E$ | 0.49 | 1.19 | |
| | (0.05) | (0.05) | |
| $u_C$ | -4.87 | -3.14 | 44.57 |
| | (0.33) | (0.24) | (1.93) |

Model B2

| | $u_L$ | $u_E$ | $u_C$ |
|---|---|---|---|
| $u_L$ | 0.43 | | |
| | (0.02) | | |
| $u_E$ | -0.01 | 0.24 | |
| | (0.01) | (0.01) | |
| $u_C$ | -0.65 | -0.17 | 9.20 |
| | (0.07) | (0.05) | (0.42) |

Model B3

| | $u_L$ | $u_E$ | $u_C$ |
|---|---|---|---|
| $u_L$ | 0.43 | | |
| | (0.02) | | |
| $u_E$ | -0.01 | 0.24 | |
| | (0.01) | (0.01) | |
| $u_C$ | -0.67 | -0.16 | 3.94 |
| | (0.06) | (0.04) | (0.23) |

Model B2R

| | $u_L$ | $u_E$ | $u_C$ |
|---|---|---|---|
| $u_L$ | 0.44 | | |
| | (0.02) | | |
| $u_E$ | -0.02 | 0.24 | |
| | (0.01) | (0.01) | |
| $u_C$ | 0.17 | -0.47 | 61.2 |
| | (0.16) | (0.12) | (2.74) |

[1]All second order moments and their standard deviations are multiplied by 100.

## APPENDIX

### A1. Definition of variables

Some variables are observed directly, others are calculated from the information available. In the latter case, the formulae used are given below. The exception is capital stock, where the calculations are described in Section A2. Further details on all variables are given in Section A2. MS indicates that the data are from the Manufacturing Statistics database of Statistics Norway, and the data are plant specific. NNA indicates that the data are from the Norwegian National Accounts. In this case, the data are identical for all plants classified in the same National Account industry. While the plants in our unbalanced panel are collected from 4 different industries at the 5-digit level according to the Standard Industrial Classification (SIC) system, the plants are classified in 3 different National Accounts industries. Data in value terms are measured in 1000 Norwegian kroner (NOK). Subscript $i$ refers to plant.

$CL_i$: Total labour cost (MS)
$CM_i$: Total material cost (incl. motor gasoline) (MS)
$CE_i$: Total energy cost (MS)
$C_i = CL_i + CM_i + CE_i$: Total factor cost, excluding capital
$VL_i$: Labour input, man-hours (MS)
$QL_i = CL_i/(1000 * VL_i)$: Labour cost, NOK per man-hour
$QM_i$: Price of materials (incl. motor gasoline), 1991=1 (NNA)
$VM_i = CM_i/QM_i$: Input of materials (incl. motor gasoline), 1000 1991-kroner;
$VE_i$: Energy input, 1000 kWh.; electricity plus fuels (excl. motor gasoline) (MS)
$QE_i = 100 * CE_i/VE_i$: Price of energy, øre per kWh
$X_i$: Output, tonnes (MS)
$K_i = KB_i + KM_i$: Total capital stock (buildings plus machinery), 1000 1991-kroner.

The calculations of capital stock data are based on the perpetual inventory method assuming constant retirement rates. We combine plant data on gross investment with fire insurance values for each of the categories KB=buildings and KM=machinery and equipment (MS). The data on investment and fire insurances are deflated using price indices (1991=1) for investment in the two categories from the Norwegian National Account (NNA)

### A2. A presentation of the data

Primarily, we use data from the Manufacturing Statistics data base at Statistics Norway. The Manufacturing Statistics follow the Standard Industrial Classification (SIC) and gives annual data for large plants at the 5-digit code. Until 1992 plants with at least five employees were defined as large, while from 1992 on the limit is 10 employees. In 1993 the activity classification was revised according to EU's NACE Rev. 1 and UNs ISIC Rev. 3, while previously based on UN's ISIC Rev. 2. For this analysis we use data over the period 1972 – 1993. While the revision of the activity classification does not cause inconsistency problems in our data, the change in the definition of large plants cause a break in the time series for plants with 5 – 9 employees in 1992. Our data includes all industries classified under SIC-code 351 Manufacture of industrial chemicals. We use the classification terminology before 1993:

35111 Manufacture of carbide
35119 Manufacture of other industrial chemicals
35120 Manufacture of fertilizers and pesticides
35130 Manufacture of synthetic resins, plastic materials and man-made fibres

For some variables it has been necessary to use data from the Norwegian National Accounts, which use the Manufacturing Statistics as an important source. In this case the data are identical for all plants classified in the same National Accounts industry. While the plants in our panel are collected from 4 different SIC-industries, the plants are classified in 3 different National Accounts industries. For simplicity, the data set used in this analysis includes only plants with contiguous time series. This reduces the panel data set from 1321 observations to 1101 observations. The number of plants per year, which ranges from 43 to 55, shows a negativ trend over time, but the average production per plant increases rather rapidly. The unbalance in our data set is illustrated in Table A1, which gives the number of plants sorted by the number of observations. For example, 30 plants are observed in all 22 years (1972 – 1993), while 16 plants are observed in one year only. The total number of plants in our sample is 88.

**Output:** The plants in the Manufacturing Statistics are in general multi-output plants and report output of a number of products. (Most plants produce less than 10 products.) The classification of products follows The Harmonized Commodity Description and Coding System (HS). In the statistics, two output-measures are available. The plants report (i) total output in value terms (Norwegian kroner (NOK)) as well as (ii) output of each product in both value terms and physical measures. For our analysis, output measure (i) is preferable for two reasons. First, there is a small difference in the definition of output in favour of measure (i) due to the treatment of changes in stocks of finished products, second, the coverage of measure (i) is more complete because some plants do not report their production of all products in both value and volume terms. We have, however, chosen to use measure (ii) to calculate an aggregate price per tonne output for each plant, and then deflate measure (i) by these prices to get the preferable output-measure in tonne.

TABLE A1. NUMBER OF PLANTS CLASSIFIED BY NUMBER OF REPLICATIONS

$p$ = no. of observations per plant, $N_p$ = no. of plants

$\sum_p N_p = 88, \quad \sum_p N_p p = 1101$

| $p$ | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $N_p$ | 30 | 0 | 3 | 0 | 1 | 3 | 3 | 6 | 0 | 2 | 1 |
| $N_p p$ | 660 | 0 | 60 | 0 | 18 | 51 | 48 | 90 | 0 | 26 | 12 |

| $p$ | 11 | 10 | 09 | 08 | 07 | 06 | 05 | 04 | 03 | 02 | 01 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $N_p$ | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 2 | 3 | 5 | 16 |
| $N_p p$ | 11 | 10 | 9 | 16 | 14 | 18 | 15 | 8 | 9 | 10 | 16 |

**Factor input costs and prices:** From the Manufacturing Statistics we get the number of man-hours used, total labour costs in NOK, total electricity consumption in kWh and in NOK, the consumption of a number of fuels in various denominations and NOK, and total material costs in NOK for each plant. We use this to calculate labour costs per man-hour and total energy costs (excl. motor gasoline) in NOK for each plant. The different fuels, such as coal, coke, fuelwood, petroleum oils and gases, and aerated waters, are reduced to the common denominator kWh by using estimated average energy content of each fuel [Statistics Norway (1995)]. This enables us to calculate an energy price per kWh for each plant. For most plants, the energy aggregate is dominated by electricity. The price of material inputs (incl. motor gasoline) is from the Norwegian National Accounts, and the price is identical for all plants classified in the same National Account industry. The input data measures "gross" input, and hence does not refer to the technical production process alone. This means for example that labour costs includes administration, and electricity consumption includes lighting and office heating.

**Capital stock:** We calculate capital stock data for the two categories M=machinery and equipments and B=buildings separately. Plant data from the Manufacturing Statistics on gross investment and fire insurance values measured at the beginning of each period for the two categories enter the calculations. The data on investment and fire insurances are deflated using industry specific prices of investment goods from the Norwegian National Accounts. Fire insurance values in a chosen base year, which vary across plants, are deflated and used to determine benchmark levels of capital stocks. These insurance values are assumed to reflect the replacement values of the existing capital stock. It was necessary to choose plant specific base years for each capital category for two reasons: First, the fact that the panel data set is unbalanced made it impossible to choose one common base year for all plants. Second, we have clear indications that the quality of the fire insurance values varies over time for each plant and capital category. This data quality problem involves partly extreme values, partly missing observations, and partly an increasing trend in the insurance coverage over time for many plants. To minimize the effect of this data quality problem when choosing base years, we sorted the plant observations by increasing fire insurance values and chose as a plant specific base year the observation that is closest to the 75 per cent fractile. From the chosen base year, the level of capital stock for each category is extrapolated backwards and forwards, using the perpetual inventory method with depreciation rates taken from the Norwegian National Accounts.

Let $Fj_{it}$ be the fire insurance value of category $j$ ($j = B, M$) reported by plant $i$ at the beginning of period $t$, and $Pj_{it}$ the National Account price index for investment goods of category $j$ in period $t$, 1991 = 1. These price indices vary over four National Account industries, and we use the same price index for all plants classified in the same National Account industry. Furthermore, let $Ij_{it}$ be the deflated gross investment of category $j$ reported by plant $i$ in period $t$, after deflation by the National Accounts price index $Pj_{it}$. Let $f_i$ and $l_i$ be the first and the last year, respectively, in which plant $i$ is observed and let $\theta = \theta(i, j)$ represent the base year chosen for capital category $j = B, M$ for plant $i$.

The formulae used to calculate capital stock data are:

$$
\begin{aligned}
Kj^*_{i\theta} &= \frac{Fj_{i\theta}}{Pj_{i\theta}}, \\
Kj^*_{it} &= \sum_{s=0}^{t-\theta-1} (1-\delta_j)^s Ij_{i,t-s} + (1-\delta_j)^{t-\theta} Kj^*_{i\theta} \qquad t = \theta+1, \theta+2, \ldots, l_i - 1, l_i, \\
Kj^*_{it} &= (1-\delta_j)^{t-\theta} \left[ Kj^*_{i\theta} - \sum_{s=0}^{\theta-t-1} (1-\delta_j)^s Ij_{i,\theta-s} \right] \quad t = f_i, f_i+1, \ldots, \theta-2, \theta-1,
\end{aligned}
$$

where $\delta_B = 0.040$ and $\delta_M = 0.135$. Since we apply the same method and depreciation rates as in the Norwegian National Accounts, the corresponding industry aggregates should be relatively close. The National Account data are based on long time series on gross investment and need not use fire insurance values as indicators for the level of capital stocks, however. Because of measurement errors in the fire insurance values, as explained above, we adjust the capital stock figures $Kj^*_{it}$ proportionally, at the plant level, to make them consistent with the corresponding National Accounts figures at the industry level. The formula is

$$
Kj_{i_N t} = \frac{Kj^N_t}{\sum_{i_N} Kj^*_{i_N t}} Kj^*_{i_N t},
$$

where $i_N$ represent the plants classified in the National Account industry $N$ and $Kj_t^N$ is the capital stock estimate for industry $N$ in the Norwegian National Accounts.

In Table A2 we give the overall means and standard deviations of variables used in this analysis.

TABLE A2. OVERALL MEANS AND STANDARD DEVIATIONS OF BASIC VARIABLES

| Variable | $\ln(C)$ | $\ln(X)$ | $\ln(K)$ | $\tau$ |
|---|---|---|---|---|
| Mean | 10.532 | 9.519 | 11.077 | 11.121 |
| Std.dev. | 1.885 | 2.626 | 2.132 | 6.266 |

| Variable | $s_L$ | $s_E$ | $s_M$ | $\ln(Q_L)$ | $\ln(Q_E)$ | $\ln(Q_M)$ |
|---|---|---|---|---|---|---|
| Mean | 0.291 | 0.110 | 0.599 | 4.545 | 1.999 | -0.378 |
| Std.dev. | 0.153 | 0.115 | 0.205 | 0.612 | 1.001 | 0.354 |

## References


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