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Mortality, fertility and old age care in a two-sex growth model

Abstract:
The paper discusses the importance of decreasing mortality in explaining demographic change over the last century. A two-sex overlapping generations model is used where care both for children and the elderly is modeled. Assuming that the main costs of care are tied to time use (and thereby fairly invariant to income changes), the paper illustrates how exogenous changes in mortality, the cost of children and the bargaining power of women can explain fluctuations in both the level and timing of births. The interaction between declining mortality and the expansion of care for the elderly is of special importance. As a consequence, mortality affects fertility differently according to how much the government sector has expanded and how much human capital has been accumulated. At an early development stage, when public care is little developed, the effect of decreasing mortality on fertility is found to be positive, while at a later stage, with higher levels of public care, the effect is found to be negative.

Keywords: mortality, fertility, old age care, olg

JEL classification: D1 D9 J1

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1 Introduction

The aim of the paper is to illustrate how important decreasing mortality has been for demographic change over the last century. The model used is theoretically very simple, but utilizes a fairly detailed representation of each generations mortality and other demographic events (based on each generation potentially living for five equally long periods). The fairly large number of periods makes it possible to see how complex the effects of mortality can be. For example, living longer means you wish more care from your children, but this need may in part be met by their living longer too. Within the following unified framework, the working of mortality is such that at the same time it is possible to have income growth, fluctuating fertility (decreasing mortality can increase or decrease fertility according to developed public old age care is), increasing education, convergence in education between women and men and an increasing average age at birth. Despite the many periods, it should be noted that the model is entirely recursive and fully solvable analytically.

It is notable that mortality not only has changed a great deal over the last century, but also that there have been and still are large differences between women and men. In addition, giving birth is one of the few activities which by necessity is segregated. It is therefore natural in a model of mortality and fertility to attempt to include the two sexes. Doing so requires some type of coordination between women and men concerning fertility. In the following, a simple Nash bargaining situation is used to reconcile their wishes. The bargaining framework introduces relative negotiation as an exogenous faktor in the model. A result due to the negotiation framework is that an increase in the cost of children may have opposite effects on the educational levels of women and men, increasing womens' levels and decreasing mens'.

An important aspect of the model is that most of the demographic decisions are not connected directly to income but to use of time. Since time costs generally increase at about the same rate as income (wages), income growth does not lead to a large increase in the desire for children. This leaves room for other factors such as mortality and womens' emancipation to play a large role in determining fertility.

The model allows the analysis of the timing of births. Both decreasing mortality and increasing influence of women generally increase the average age when giving birth. An important factor in how these factors influence the timing of births is the discounted wage profile over each generations life cycle. In the model this is exogeously determined through age specific experience parameters. Changes in interest rates can also influence the wage profile and thereby the level and timing of fertility.

The model is consistent with observed developments over the last century including fluctuations and decline in fertility, increases in the average age of giving birth, increasing levels of education with lessening differences in the education levels of women and men, increasing incomes, and increased public care for the elderly. During the last century fertility has fallen, risen, and fallen again. At present it is at a level close to, but below, the reproduction level of 2.1 children per women. The number of live births per women in Norway fell from 4.5 for the cohort born 1850 to 1.96 for the cohort born 1905. After this it rose again to 2.58 for the cohort born in 1934. Since then the number of live births have fallen to 2.09 for the cohort born in 1950 and is expected to further fall to the present total fertility rate of around 1.86.

At the same time as fertility was fluctuating, there was a large increase in the stock of human capital. From 1962 to 1992 the proportion of 16-year-olds under education has increased from 53.8 percent to 93,5 percent and the proportion of 20-year-olds under education has increased from 16.6 percent to 43.8 percent. Women are now in the majority at universities as well as at colleges.

During the last 15 years a large literature has evolved discussing different aspects of fertility choice within a dynamic framework. Excellent surveys of the literature can be found in for example Holz, Klerman and Willis (1997) or Arroyo and Zhang (1997). The paper is closely related to a recent paper by Blackburn and Cipriani (2002), where economic and demographic outcomes are jointly determined in a dynamic general equilibrium model of longevity, fertility and growth. The present paper does not follow Blackburn and Cipriani (2002) in assuming longevity to be exogenous, but extends their analysis by looking at the interaction between mortality, fertility and growth in publicly provided old age care. The present paper shares with Blackburn and Cipriani (2002) the assumption that parents are non-altruistic, deriving utility only from the production of children and not the children's welfare. In addition the present paper also includes utility derived from the care received from children. Mortality is modelled as in Blackburn and Cipriani (2002), but including mortality at working ages in addition to mortality at retirement age.

Another related paper discussing morality and fertility is Yakita (2001), which finds that an increase in life expectancy lowers the fertility rate and raises life-cycle savings, and that pay-as-you-go social security does not reverse the effect on fertility. This is the result of workers wanting more consumption as old, at the cost of reducing other consumption, including consumption of children. In the following, decreased mortality among workers will decrease fertility, but the effect of decreased mortality among the elderly is uncertain, depending on the state of the economy.

2 The general economic environment

As mentioned in the introduction, the paper presents a two-sex overlapping-generations model that examines how exogenous changes in mortality, the cost of children and the bargaining power of women influence fertility, public and private care for the elderly, and the length of education taken by women and men.

The framework employed in the following is an overlapping-generations model of a small open economy where individuals live for a maximum of five periods with uncertain lifetimes. All individuals face a probability of dying at different ages. Time is discrete and indexed by t, while generation i is defined as those born at time $t = i$. The age of an individual of generation i is thereby given by $t - i$ with ages numbered from 0 to 4.

Age 0 consists of childhood, education is taken at age 1, children are born at ages 1 and 2, work extends over the ages from 1 to 3, and retirement is at age 4. Each child is cared for over two periods, and parents are cared for when they reach age 4.

It should be noted that even though individuals are economically active at age 1 (taking an education and having children) they are still being cared for by their parents since care of children spans two periods. This is a result of the long time interval each age spans.

Production is the product of a constant returns function in real capital and skills subject to exogenous technological progress. Skills are assumed to be the product of an age specific experience parameter and the level of human capital. The accumulation of human capital is assumed to be a function of earlier generations' level of human capital and the amount of education undertaken. Since a small open economy is assumed, the real interest rate is given exogenously by international capital markets.

Following Zhang, Zhang and Lee (2001), it is assumed that there are actuarially fair annuity markets that distributes the savings of those who die to those who survive from one age to another. This avoids either introducing unintended bequests to children or assuming that the savings of those who die before reaching old age are wasted.

2.1 Negotiation and signaling

Each generation is modeled by a representative female and a representative male. Assuming equal numbers of female and male individuals at the time of household formation, all individuals become members of a representative household.

Differing mortality and costs of bearing children lead women and men to have different wishes about the number of children they will have. Disagreements about such life determining decisions are made through negotiation, with negotiation positions being determined by what the individuals could achieve by themselves (though assuming child care is shared in the same manner as within the union). The time cost of young children differs between the sexes, both due to women's extra time costs connected with carrying a child and breast feeding it, and to the relative negotiation strength of the sexes.

Differing mortality leads also to disagreements about the level of care given to their parents, because individuals are assumed to believe that the level of care they give their parents will in turn determine the care they themselves will receive. Government provided old age care is determined as the average of the wishes of women and men (can be thought of as a type of voting mechanism).

Education is chosen to maximize incomes conditional on the chosen levels of fertility and private old age care. Since educational choice maximizes income, it will be both individually and collectively optimal (given fertility levels). Finally, total consumption is determined by maximizing the couples average utility given common household budget constraints, ensuring that even though a negotiation framework is chosen, budget constraints are observed.

In the paper, the choices a society makes about number of children and old age care is viewed as involving negotiation and signaling based on what is individually optimal.

2.2 Keeping track of the population

Since the timing of births is one of the issues addressed in this paper, each generation is assumed to be able to have children during two ages, age 1 and age 2. The number of children generation i gives birth to at age 1 is denoted n_{1i} and the number at age 2 is denoted n_{2i}. The probability of surviving from birth (age 0) to age k for a female child born at time t is denoted π_{ktf} and for a male child denoted π_{ktm}. The number of persons at age k at time t is denoted N_{kt}. The number of persons born into generation i will be

$$N_{0i} = \frac{N_{1,i-1}}{2} \cdot n_{1,i-1} + \frac{N_{2,i-2}}{2} \cdot n_{2,i-2}, \tag{1}$$

where $N_{1,i-1}/\,2$ and $N_{2,i-2}/\,2$ are the number of households having reached respectively age 1 and age 2 in year $t = i$ (there at two persons in each household), while $n_{1,i-1}$is the number born early to the generation having reached age 1 and $n_{2,i-1}$is the number born late to the generation having reached age 2.

Even though differences in adult female and male mortality will be prominent in the following, for simplicity it is assumed that the number of females born equals the number of males and that child mortality, the probability of living until age 1, is equal between the sexes ($\pi_{1tf} = \pi_{1tm}$). At age 1 there will thereby be an equal number of females and males, who we further assume all match into households, modeled by a representative household (individuals are also represented by a representative female and a representative male). For further simplicity, it is assumed that mortality does not affect the couples ability to have the planned number children when they reach age 2.

Note that the index i in the child variables n_{2i} and n_{2i} refer to the parents generation (time of birth of the parents), while the index t in mortality parameters π_{ktf} and π_{ktm} refer to the time the child was born (one or two periods after the parents where born) and the aggregate population variable N_{kt} refers to the present time. For example as generation i (those born at time i) progresses through the ages we go from having N_{0i} persons being born into generation i to

$$N_{j,i+j} = \pi_{jif} \cdot \frac{N_{0i}}{2} + \pi_{jim} \cdot \frac{N_{0i}}{2} = \pi^*_{ji} \cdot N_{0i}$$

being the number of generation i surviving to age j, where the the total mortality parameter π^*_{kt} is defined as

$$\pi^*_{kt} = \frac{\pi_{ktf} + \pi_{ktm}}{2}.$$

In the following it will be assumed that the institutional care for those reaching old age will be provided through taxation by those who are at age 2 at that time. The dependency ratio at time $t = i$,

$$\frac{N_{4i}}{N_{2i}} = \frac{\pi^*_{4,i-4} N_{0,i-4}}{\pi^*_{2,i-2} N_{0,i-2}},$$

therefore gives an expression of the burden of pensioners on those of working age at this time. As one can see, it depends both on the size and the mortality of the two generations.

For later use, the proportion of parents of generation i (consisting of both young and old parents) surviving to old age is defined as

$$\eta_{1i} = \pi^*_{4,i-1} \cdot \frac{n_{1,i-1}}{n_{1,i-1} + n_{2,i-2}} \quad \text{and} \quad \eta_{2i} = \pi^*_{4,i-2} \frac{n_{1,i-2}}{n_{1,i-1} + n_{2,i-2}},$$

where η_{2i} is the proportion of parents reaching old age when generation i is at age 2 and η_{1i} is the proportion reaching old age when generation i is at age 3.

3 Old age care

Desired levels of care provided to parents by individuals in generation i is denoted E_i if privately supplied and G_i if supplied indirectly by the government through taxes. Individuals derive utility from the care they expect to receive from their children in old age. The level of care is determined by a social contract (in the spirit of Kant) stipulating that individuals must provide to their own parents the level of care they themselves wish to receive.

Assumption 1 (Self-referring expectations concerning children's behavior). The social contract between generations is such that individuals shall behave as if the level of care provided to parents equals the level of care they will receive from their own children.

This social contract ensures that each generation receives old age care in accordance with the economic situation of their children. The behavior of parents can be seen as a signal of what they expect from their children. Assumption 1 implies that the variables E_i and G_i denote both care given and care received in the individuals decision problem. In addition it is assumed that individuals take the implicit price of the government providing old age care as given, even though it will be assumed to depend on wages.

The level of public care is decided by the median voting generation (those at age 2) and for simplicity is assumed that they also pay for this care through taxes. Public care is received in any case, while private care is only received if ones children survive long enough to take care of their parents. Each generation is only taxed once, at age 2. Since there are perfect financial markets and we use lump-sum taxation, it doesn't really matter whether we collect all taxes at one age or spread them out over different ages.

Since the modeling of the provision of old age care is perhaps the most important part of the model, it is discussed fairly detailed using a slightly different notation from that used later on. Assume that the care an individual receives in old age has two components, a physical production component, E^{physical}, and an emotional component, $E^{\text{emotional}}$. The emotional component is produced jointly with the physical component according to the simple linear form

$$E^{\text{emotional}} = \varsigma_0 \cdot E^{\text{physical}},$$

where ς is a parameter. An example would be dinner made by your children (providing E^{physical}), also giving you the enjoyment of their company (receiving an additional $E^{\text{emotional}}$). Care provided by the government, denoted G^{physical}, is only of a physical nature. If the government provides dinner instead of the children, the elderly do not receive any emotional support.

For a person reaching age, the utility of care is given by

$$u^*_{\text{care}} = \left(E^{\text{emotional}} \cdot \sqrt{\Pi_1 n_1 \cdot \Pi_2 n_2}\right)^{\zeta_1} \cdot \left[\left(E^{\text{physical}} \cdot \sqrt{\Pi_1 n_1 \cdot \Pi_2 n_2}\right)^{\zeta_E} \left(G^{\text{physical}}\right)^{\zeta_G}\right]^{\zeta_2},$$

where ζ_1, ζ_2, ζ_E and ζ_G are parameters. The number of early born surviving children at this age is denoted $\Pi_1 n_1$ and the number of surviving late born is denoted $\Pi_2 n_2$ (the parameters Π_1 and Π_2 are the survival probabilities). The geometric weighing of the number of surviving children is used to derive the amount of private care received from each child. This is in the spirit of the general assumption that the utility function favors spacing children evenly instead of having all children at one age. The consequence is that receiving care from children of differing ages gives greater utility than receiving it from children all of the same age.

The subutility of care, u_E, is given by:

$$u_E = \pi_4 \cdot \ln u^*_{\text{care}}.$$

Inserting from the equation for u^*_{care}, this can be written

$$u_E = \frac{\zeta_1 + \zeta_2 \cdot \zeta_E}{2} \pi_{4ig} \ln (\Pi_1 \cdot \Pi_2) + \pi_{4ig} \ln u_{\text{care}}$$

where

$$u_{\text{care}} = \left(E^{\text{emotional}} \cdot \sqrt{n_1 \cdot n_2}\right)^{\zeta_1} \cdot \left[\left(E^{\text{physical}} \cdot \sqrt{n_1 \cdot n_2}\right)^{\zeta_E} \left(G^{\text{physical}}\right)^{\zeta_G}\right]^{\zeta_2}.$$

If the parameters are constant, this implies that there is full separability between $E^{\text{emotional}}$, E^{physical} and G^{physical} (so that for example $\partial^2 (\log u_{\text{care}}) / \partial E^{\text{physical}} \, \partial G^{\text{physical}}$ equals zero). In the following, non-separability between different types of physical care is introduced at low levels of either private or public care by assuming that the parameters ζ_E and ζ_G are not constant, but determined by

$$\zeta_E = \begin{cases} \zeta_{E0} + \zeta_{E1} \cdot \frac{1}{1+G^{\text{physical}}} & \text{if} \quad 0 \leq G^{\text{physical}} < G^* \\ \zeta_{E0} & \text{if} \quad G^{\text{physical}} \geq G^* \end{cases}$$

and

$$\zeta_G = \begin{cases} \zeta_{G0} + \zeta_{G1} \cdot \frac{1}{1+E^{\text{physical}}} & \text{if} \quad 0 \leq E^{\text{physical}} < E^* \\ \zeta_{G0} & \text{if} \quad E^{\text{physical}} \geq E^* \end{cases},$$

where ζ_{E0}, ζ_{E1}, ζ_{G0} and ζ_{G1} are parameters. The cut off points G^* and E^* are exogenous levels of public (private) care above which public (private) care no longer affects the utility of private (public) care. At low levels of publicly provided care, $G_{i-2} < G^*$, the utility of private care is high. As publicly provided care increases the variable determining the utility of private care, x_i, decreases until reaching the level ξ_1. Note that G_{i-2} is the amount of care being given to the generation born in $i-4$ by the generation born in $i-2$. Generation i will provide G_i public care when reaching the age of two (in period $i+2$). Having x_i depend on the level of care observed when born means that this part of utility is set at birth and is not changed by the decisions of the individual. The marginal utility of private care thereby becomes higher when there is little government care than when government care is well established. An increase in government provided care from a low level will decrease the marginal utility of privately provided care ($\partial^2 (\log u_{\text{care}}) / \partial E^{\text{physical}} \, \partial G^{\text{physical}} < 0$). This property will prove to be important in the later discussion of how changes in mortality affect fertility. Low levels of government care lead to a high marginal utility of private care and indirectly of children (who provide the care). As government care is increased, the marginal utility of children in this respect decreases.

Inserting for emotional care $E^{\text{emotional}}$ and rearranging, u_{care} can be written as

$$u_{\text{care}} = (\zeta_0)^{\zeta_1} \cdot \left(\sqrt{n_1 \cdot n_2} \cdot E^{\text{physical}}\right)^{\zeta_1 + \zeta_2 \zeta_E} \left(G^{\text{physical}}\right)^{\zeta_2 \zeta_G}$$

Assuming that E^{physical} is always larger than the threshold level X, reparameterizing and introducing subscripts for generation, i, and for gender, g, the subutility of care, u_E, can be written

$$u_{E\,ig} = \xi_{3ig} + \pi_{4ig}\left[\frac{x_i}{2}\left(\ln n_1 + \ln n_2\right) + x_i \ln E_{ig} + \alpha_G \ln G_i\right] \tag{2}$$

and

$$x_i\left(G_{i-3}\right) = \begin{cases} \xi_1 + \xi_2 \frac{1}{1+G_{i-2}} & \text{if} \quad 0 \leq G_{i-2} < X \\ \xi 1 & \text{if} \quad G_{i-2} \geq X \end{cases}, \tag{3}$$

with E_{ig} being private and G_i government provided physical care of the elderly.[1] It is assumed that the level of government care at the age a person is born (G_{i-2} for a person born in period i) determines x_i. The variable x_i is thereby a predetermined variable in the agents' utility maximization problem. This assumption introduces some simple dynamics to the model with government care G evolving over time.

Financing of public care for the elderly is done through lump-sum taxation of individuals at age 2. Denote p^*_{Gi} as the price of delivering one unit of government care to one individual at time i (consisting mainly of wages). The total cost of delivering G_i at time $i+2$ to all the elderly in the population who need care is

$$p^*_{G,i+2} G_i \cdot N_{4i-1}.$$

Since generation i numbers N_{2i} at age 2, the tax needed to finance government care will be given by

$$\tau_{i+2} = p^*_{G,i+2} \cdot G_i \cdot \frac{N_{4i-1}}{N_{2i}} = p_{G,\,i+2} \cdot G_i.$$

where the price of delivering one unit of G_i (adjusted for the dependency ratio $N_{4i-1}/\,N_{2i}$) is given by

$$p_{G,\,i+2} = p^*_{G,i+2} \cdot \frac{N_{4i-1}}{N_{2i}}.$$

Figure 1 shows the interaction between generation i's timeline and the timelines of the two parent generations $i-1$ and $i-2$. In the figure e_i denotes education and $p_{G\,i+2}$ is the perceived price of providing public care G_i to each elderly person in period $i-2$ through a lump-sum tax. The figure shows the number of children each of these two parent generations had, how these combine into generation i, and gives an illustration of when care of children, care of parents, education, and bequests occur during generation i's lifetime.

Individual utility consists of the expected utility of consumption, the direct utility of having children and the utility of receiving care from one's children in old age. The subutility of old age care for a person in generation i, $u_{E\,i}$, is given by

$$u_{E\,i} = \left(\frac{\pi^*_{3,i+1} n_{1i}}{2} \cdot E_i\right)^{x_i/2} \cdot \left(\frac{\pi^*_{2,i+2} n_{2i}}{2} \cdot E_i\right)^{x_i/2} \cdot \left(1 + G_i\right)^{\alpha_G} \tag{2}$$

[1] The reparameterization is as follows: $E^{\text{physical}} = E_{ig}$, $G^{\text{physical}} = G_i$, $\Pi_1 = n^*_{3,i+1}$, $\Pi_2 = n^*_{2,i+2}$, $\xi_1 = \zeta_1 + \zeta_2\zeta_{E0}$, $\xi_2 = \zeta_2\zeta_{E1}$, $\alpha_G = \zeta_2\zeta_G$, and $\xi_{3ig} = \frac{\zeta_1+\zeta_2\cdot\zeta_E}{2} \cdot \pi_{4ig} \cdot \ln\left(\left(n^*_{3,i+1} \cdot n^*_{2,i+2}\right)\right) + \zeta_1 \ln \zeta_0$.

where $\pi^*_{3,i+1} \cdot n_{1i}$ is the number of children born at an early age who survive to age 3 when the parent reaches age 4, $\pi^*_{2,i+2} \cdot n_{2i}$ is the number of children born at an late age who survive to age 2 when the parent reaches age 4, and α_G is a parameter.

Figure 1. Generation i's timeline in relationship to parents' timelines*

t	$i-2$	$i-1$	i	$i+1$	$i+2$	$i+3$	$i+4$
$i-2$	0	1	2	3	4		
births		$n_{i-2,1}$	$n_{i-2,,2}$				
$i-1$		0	1	2	3	4	
births			$n_{i-1,1}$	$n_{i-1,2}$			
i			0	1	2	3	4
education				e_i			
births				n_{i1}	n_{i2}		
care of children				$\varphi_1 n_{i1}$	$\varphi_2 n_{i1}$		
					$\varphi_1 n_{i2}$	$\varphi_2 n_{i2}$	
care of parents					$\eta_{2i} E_i$	$\eta_{1i} E_i$	
publ. old age care					$p_{G\,i+2} G_i$		

* The subscript g has been dropped in the table

The variable x_i has the same role towards determining the utility of private care as the parameter α_G has determining the utility of public care, but in the present context it is assumed to depend on the level of publicly provided care observed at the time an individual is born,

$$x_i\left(G_{i-2}\right) = \begin{cases} \xi_1 + \xi_2 \cdot \left(1 + G_{i-2}\right)^{-1} & \text{if} \quad 0 \leq G_{i-2} < G^* \\ \xi_1 & \text{if} \quad G_{i-2} \geq G^* \end{cases}, \tag{3}$$

where G^* is an exogenous level of public care above which public care no longer affects the utility of private care. At low levels of publicly provided care, $G_{i-2} < G^*$, the utility of private care is high. As publicly provided care increases the variable determining the utility of private care, x_i, decreases until reaching the level ξ_1. Note that G_{i-2} is the amount of care being given to the generation born in $i-4$ by the generation born in $i-2$. Generation i will provide G_i public care when reaching the age of two (in period $i+2$). Having x_i depend on the level of care observed when born means that this part of utility is set at birth and is not changed by the decisions of the individual.

The subutility of having children, u_{ni}, is assumed to have the form

$$u_{ni} = (n_{1i})^{\alpha_n/2} \cdot (n_{2i})^{\alpha_n/2} \tag{4}$$

where α_n is a parameter. The total individual expected lifetime utility of the individuals of gender g belonging to generation i, U_{ig}, depends on consumption and the two subutility components connected with having children and old age care. It is assumed to have the logarithmic form

$$U_{ig} = \ln c_{1i} + \pi_{2ig}\beta \ln c_{2i} + \pi_{3ig}\beta^2 \ln c_{3i} + \pi_{4ig}\beta^3 \ln c_{4i} + \ln u_{ni} + \pi_{4ig} \ln u_{E\,i} \tag{5}$$

where β and α_B are parameters and π_{4g} is the probability of surviving to the fifth period (age 4). The utility function depends on the individual's own longevity, π_{2ig}, π_{3ig} and π_{4ig}, as well as on the longevity of the individual's female (π_{2i+1f}, π_{3i+1f}, π_{2i+2f} and π_{3i+2f}) and male children (π_{2i+1m}, π_{3i+1m}, π_{2i+2m} and π_{3i+2m}). Since the utility function has a Cobb-Douglas structure, it simplifies the notation later to define the four share parameters $\Lambda_{C\,ig}$, Λ_{Nig}, $2\Lambda_{E\,ig}$ and $\Lambda_{G\,ig}$ as

$$\begin{aligned} \Lambda_{C\,ig} &= \tfrac{1}{\Lambda_{ig}} \left(1 + \pi_{2ig}\beta + \pi_{3ig}\beta^2 + \pi_{4ig}\beta^3\right), & \Lambda_{E\,ig} &= \tfrac{1}{\Lambda_{ig}} \left(\pi_{4ig} \cdot x_i\right), \\ \Lambda_{Nig} &= \tfrac{1}{\Lambda_{ig}} \alpha_n, & \Lambda_{Gig} &= \tfrac{1}{\Lambda_{ig}} \left(\pi_{4ig} \cdot \alpha_G\right), \end{aligned}$$

where

$$\Lambda_{ig} = 1 + \pi_{2ig}\beta + \pi_{3ig}\beta^2 + \pi_{4ig}\beta^3 + \alpha_n + 2\pi_{4ig}x_i + \pi_{4ig}\alpha_G$$

so that $\Lambda_{C\,ig} + \Lambda_{Nig} + 2\Lambda_{E\,ig} + \Lambda_{G\,ig} = 1$.

4 Care of children

At each working age (ages 1 to 3) individuals have at their disposal T hours which can be used for work, education, taking care of children and providing private care for the elderly. Education is taken at age 1, with the education taken by individual of type k of generation i being denoted e_{ik} (where the subscript k denotes male or female) .

Children are cared for over two periods with the time used per child varying between the sexes for care of young children (of age 0), but not for care of older children (children of age 1). The time used caring for each younger child by generation i is denoted φ_{1if} and φ_{1im} for females and male respectively, while the time used per older child is denoted is φ_{2i}. Time usage is constant within a generation, but can change between generations due to changes in housekeeping technologies (the introduction of the washing machine for example) or in the relative negotiation strength of women and men. Time used on young children by females and males is given by

$$\begin{aligned}\varphi_{1if} &= \varphi_f + (1-\Phi_i)\cdot\varphi_{1i} \\ \varphi_{1im} &= \Phi_i\cdot\varphi_{1i},\end{aligned}$$

where φ_f is time only the female can provide (used for example for breast feeding) and φ_{1i} is time which can be shared between mothers and fathers. The parameter $0 < \Phi_i < 1$ reflects the relative negotiating strength within the household union. An increase in Φ_i constitutes an increase in the relative negotiating power of women.

The time parameters can be interpreted as net time costs, being the difference between the time used on children and the time children themselves are productively employed in the household. In the following, it will be supposed that the cost of older children, φ_{2i}, has been increasing both because urbanization has lead them to contribute less to the household and because of increases in the cost of their upkeep. On the other hand it is supposed that the cost of young children faced by women, φ_{1if}, has been falling and converging towards that of men due to technological advances.

5 Education and human capital

The human capital acquired through education by an individual of gender g belonging to generation i is denoted h_{ig}. It accrues to the individual in the period the education is taken and is determined by the aggregate level of human capital in the previous period, $\bar{h}_{i-1}$ and the amount of education, e_{ig}, the individual has taken. The individuals' realized level of human capital will be

$$\bar{h}_{ig} = \left(\bar{h}_{i-1}\right)^{\theta_1} \left(\bar{e}_{ig}\right)^{\theta_2}, \tag{6}$$

where θ_1 and θ_2 are parameters.

The level of aggregate human capital for generation i is denoted $\bar{h}_i$ and is determined by the previous level, $\bar{h}_{i-1}$, and the actual amount of education taken by women, $\bar{e}_{if}$, and men, $\bar{e}_{im}$,

$$\bar{h}_i = \left(\bar{h}_{i-1}\right)^{\theta_1} \left(\frac{\bar{e}_{if} + \bar{e}_{im}}{2}\right)^{\theta_2}. \tag{7}$$

In the following, we will distinguish between human capital, which in our use of the term only takes education into account, and skills, which also take experience into account. The skill of an individual of gender g of generation i at age j, S_{jig}, is assumed to be the product of an age specific experience parameter λ_j and the level of human capital. An individual of generation $i = t$ is assumed to have the following skill levels at different ages (at time $t = i + j$),

$$S_{1ig} = \lambda_1 h_{ig}, \qquad S_{2ig} = \lambda_2 h_{ig}, \qquad S_{3ig} = \lambda_3 h_{ig} \tag{8}$$

If the discounted value of these experience parameters increases with age for a generation born at time i, we have:

$$\lambda_1 < (1+r)^{-1}\lambda_2 < (1+r)^{-2}\lambda_3. \tag{9}$$

The number of persons of age j at time t will be N_{t-j}. Letting l_{jig} be the amount of labor supplied by individuals of gender g belonging to cohort i at age j, the total quantity of efficiency-labor employed in production at time t, H_t, is assumed to be given by:

$$H_t = \sum_{j=1}^{3} N_{t-j} \sum_{g=f,m} l_{j,t-j,g} S_{j,t-j,g} - \sum_{g=f,m} L^G_{2,t-2,g} S_{2,t-2,g} \tag{10}$$

where $\sum_{g=f,m} L^G_{2,t-j,g} S_{2,t-2,g}$ is the amount of effective labor used by the government in providing old care (the loss to the private sector is in efficiency units, while later it will be assumed that only labor in hours worked, $L^G_{2,t-j,g}$, enters the production function for public old age care). In the above skills are assumed to be perfect substitutes in production. Males and females work in general different hours due to time used on education and caring for children, but at age 3 they work equal amount of hours.

6 Production and factor prices

Production Y_t occurs within a period according to a standard one-sector production function F that exhibits constant returns to scale. Letting K_t be capital, output at time t is assumed to be given by

$$Y_t = F(K_t, H_t) = AK_t^{\mu} (H_t)^{1-\mu} = H_t f(k_t), \tag{11}$$

where $k_t = K_t/H_t$, $f(k_t) = Ak_t^{\mu}$, and A and μ are parameters.

A small open economy is assumed, so that the real interest rate, r, is given exogenously by international capital markets. The solution to the firms' optimization problem sets factor costs equal to their marginal productivity. For capital this gives

$$r_t = \partial Y_t / \partial K_t = f'(k_t), \tag{12}$$

which implies that the capital ratio k_t is determined exogenously in the capital markets, $k_t = \left(\frac{r_t}{\mu A}\right)^{1/(\mu-1)}$.

Profit maximization implies that the marginal productivity of efficiency-labor H_t is given by $\partial Y_t/\partial H_t = f(k_t) - k_t f^{'}(k_t)$. At the disaggregated level, marginal productivity determines the following sequence of per capita wages for generation i (born at time $t = i$)

$$\begin{aligned} w_{1ig} &= \overline{w}(r)\lambda_1 \cdot \left(\bar{h}_{i-1}\right)^{\theta_1} \left(e_{ig}\right)^{\theta_2}, \\ w_{2ig} &= \overline{w}(r)\lambda_2 \cdot \left(\bar{h}_{i-1}\right)^{\theta_1} \left(e_{ig}\right)^{\theta_2}, \\ w_{3ig} &= \overline{w}(r)\lambda_3 \cdot \left(\bar{h}_{i-1}\right)^{\theta_1} \left(e_{ig}\right)^{\theta_2}, \end{aligned} \tag{13}$$

where $w_{jig} = \frac{1}{N_{t-j}} \cdot (\partial Y_t/\partial H_t)(\partial H_t/\partial l_{j,t-j,g})$ for $t = i + j$ is the per capita wage for individuals of generation i at age j and $\overline{w}(r_t)$ is defined by the function

$$\overline{w(r)} = f(k_t(r)) - k_t(r)f'(k_t(r)) = (1-\mu)A\,(k_t(r))^{\mu} = (1-\mu)A\left(\frac{r}{\mu A}\right)^{\mu/(\mu-1)}.$$

To see how this looks at a point in time, consider for example time $t = i+2$ in which firms employ the three cohorts $i-1$, i and $i+1$. They pay different wages to the three cohorts based on the cohort's different levels of experience and human capital in the following manner:

$$\begin{aligned} w_{1,i+1,g} &= \overline{w}(r)\lambda_1 \cdot \left(\bar{h}_{i}\right)^{\theta_1} \left(e_{i+1,g}\right)^{\theta_2} \\ w_{2ig} &= \overline{w}(r)\lambda_2 \cdot \left(\bar{h}_{i-1}\right)^{\theta_1} \left(e_{ig}\right)^{\theta_2} \\ w_{3,i-1,g} &= \overline{w}(r)\lambda_3 \cdot \left(\bar{h}_{i-2}\right)^{\theta_1} \left(e_{i-1,g}\right)^{\theta_2}. \end{aligned}$$

In the following, the interest rate will be considered constant (but will be allowed to vary in the proofs in the appendixes). Introducing the notation $\lambda^*_{ki} = (1+r)^{-(k-1)} \cdot \overline{w}(r)\lambda_k$ discounted wages of an individual of gender g in generation i can be written

$$(1+r)^{-(k-1)} \cdot w_{kig} = \lambda^*_{ki} \cdot \left(\bar{h}_{i-1}\right)^{\theta_1} \left(e_{ig}\right)^{\theta_2}.$$

Assuming a constant interest rate, if the discounted value of the experience parameters increases with age as in equation (9), then $\lambda^*_{1i} < \lambda^*_{2i} < \lambda^*_{3i}$.

If individuals have T hours available in each period, then the discounted value of total potential wage income of an individual, R_{ig}, will be

$$\begin{aligned} R_i &= \sum_{j=1}^{3} (1+r)^{-(j-1)} w_{ji} - w_{1i} \cdot e_i \\ &= \left[T^*_i - \lambda^*_{1i} \cdot e_i\right] \cdot \left(\bar{h}_{i-1}\right)^{\theta_1} \left(e_i\right)^{\theta_2} \end{aligned} \tag{14}$$

where $T^*_i = \lambda^*_{1i} \cdot T + \lambda^*_{2i} \cdot T + \lambda^*_{3i} \cdot T$. The variable R_i gives the value of an individual's time during working ages minus the time spent on education.

The variable G_i is the amount of publicly provided care provided by generation i to the elderly. By assumption this care is provided at time $i+2$ when generation i is at age 2. The generation receiving care consists of those who at this time have reached age 4, namely generation $i-2$. The publicly provided care to generation $i-2$ at time $i+2$ is provided through a linear production function

$$G_i = \min\left[\frac{L^G_{2,i+2,f}}{(\mu_G/\,2)}, \frac{L^G_{2,i+2,m}}{(\mu_G/\,2)}\right]$$

so that employment is given by

$$L^G_{2,i+2,k} = \frac{\mu_G}{2}\cdot G_i, \quad k = f, m.$$

To be able to entice individuals to work in the sector providing old age care, they must receive the same wage as in the normal production sector. In addition it is assumed that there is a fixed cost component p_{0G} which does not change over time. We thereby have that the price of a unit government care for the elderly is

$$p^*_{G,i+2} = p_{0G}\cdot\frac{\bar{h}_{if}+\bar{h}_{im}}{2\bar{h}_i} + \left(\frac{w_{2,i+2,f}+w_{2,i+2,m}}{2}\right)\mu_G.$$

The fixed cost component can play a large effect at low income levels, becoming less and less important as incomes increase. It can thereby explain low initial levels of government care. The fixed cost component p_{0G} is multiplied by the adjustment term $(h_{if}+h_{im})/\,2\bar{h}_i$ so that all elements in the equation relate to the same human capital notions. The adjustment term can be seen as an index of the inequality in education between women and men,

$$\frac{\bar{h}_{if}+\bar{h}_{im}}{2\bar{h}_i} = \frac{\frac{(\bar{e}_{if})^{\theta_2}+(\bar{e}_{im})^{\theta_2}}{2}}{\left(\frac{\bar{e}_{if}+\bar{e}_{im}}{2}\right)^{\theta_2}}.$$

If women and men take equal amounts of education it will equal 1 and the less equal education is, the smaller it becomes (leading to a lower price of education). While the adjustment term is introduced to simplify the expressions derived below, it can be argued that inequality makes it easier to pressure part of the work force to work harder, leading to a lowering of the fixed cost component.

At time $i+2$ there will be N_{4i-2} elderly receiving care G_i which is paid by individuals belonging to generation i (according to the simple tax assumption made here). The total paid by each member of generation i will be the lump-sum tax

$$\tau_{i+4} = \frac{p^*_{G,i+2}\cdot G_i\cdot N_{4i-2}}{N_{2i}} = p_{G,\,i+2}\cdot G_i.$$

where $p_{G,\,i+2}$ is defined as the implicit price of G_i as perceived by this generation,

$$p_{G,\,i+2} = \frac{p^*_{G,i+4}\cdot N_{4i-2}}{N_{2i}} = \left[p_{0G}\cdot\frac{\bar{h}_{if}+\bar{h}_{im}}{2\bar{h}_i} + \left(\frac{w_{2,i+2,f}+w_{2,i+2,m}}{2}\right)\mu_G\right]\cdot\frac{N_{4i-2}}{N_{2i}}. \tag{15}$$

7 Individual budget constraints

Letting r be the interest rate and denoting c_{jik} as the consumption, w_{jik} as the wages, and s_{jik} as the savings of individuals of gender g of generation i at age j, the individual constraints faced by males and females of generation i can be written

$$\begin{aligned} c_{1i} &= w_{1i}\left(T - e_i - \varphi_{1ig} n_{1i}\right) - s_{1i} \\ c_{2i} &= w_{2i}\left(T - \varphi_{1ig} n_{2i} - \varphi_{2i} n_{1i} - \eta_{2i} E_i\right) - p_{G\,i+2} G_i + (1+r)\, s_{1i} - s_{2i} \\ c_{3i} &= w_{3i}\left(T - \varphi_{2i} n_{2i} - \eta_{1i} E_i\right) + (1+r)\, s_{2i} - s_{3i} \\ c_{4i} &= (1+r)\, s_{3i} \end{aligned} \quad (16)$$

where $\tau_i = p_{G\,i+2} G_i$ is the lump-sum tax paid for public care of the elderly. The cost of providing public old age care to a generation is thereby met by the working population at age 2 and not just the generation's children.

It should be noted that in the above formulation the cost of children increases proportionally with income (wages). It is implicitly assumed that the number of children and level of education do not violate the time constraints.

Maximization of the individual utility functions given these individual constraints lead to males' and females' demand for children and care for the elderly. Negotiation then determines the realized effective demands.

For later use we define φ^*_{1ig} and φ^*_{2ig} as the discounted cost of having children at respectively age 1 and age 2 divided by the level of human capital,

$$\begin{aligned} \varphi^*_{1ig} &= \left[w_{1ig}\varphi_{1ig} + (1+r)^{-1} w_{2ig}\varphi_2\right]/h_{ig} \\ \varphi^*_{2ig} &= \left[(1+r)^{-1} w_{2ig}\varphi_{1ig} + (1+r)^{-2} w_{3ig}\varphi_2\right]/h_{ig}\ . \end{aligned} \quad (17)$$

The total discounted cost of having children at any age will be equal to the age specific time costs, φ_j, multiplied by the wage at the point in time the time cost is incurred.

In the same manner let η^*_i be the discounted cost of taking care of ones parents

$$\eta^*_i = \left[(1+r)^{-1} w_{2ig}\eta_{2i} + (1+r)^{-2} w_{3ig}\eta_{1i}\right]/h_{ig}\ . \quad (18)$$

Using the notation introduced earlier, these expressions can also be written

$$\varphi^*_{1ig} = \lambda^*_{1i}\varphi_{1ig} + \lambda^*_{2i}\varphi_{2i}, \qquad \varphi^*_{2ig} = \lambda^*_{2i}\varphi_{1ig} + \lambda^*_{3i}\varphi_{2i},$$

and

$$\eta^*_i = \lambda^*_{2i}\cdot\eta_{2i} + \lambda^*_{3i}\cdot\eta_{1i}.$$

8 Utility maximization

In the following individual demands which establish the initial negotiating position will be termed notional demands, while the realized demands which are determined after negotiation will be called effective demands. In particular the notional demands for children are denoted $\tilde{n}_{1ig}$ and $\tilde{n}_{2ig}$. Individual utility maximization leads to the conditional demand for education given in Lemma 1.

Lemma 1 (Conditional demand for education) *Maximizing (5) with respect to s_{1i}, s_{2i}, s_{3i}, n_{1i}, n_{2i}, E_i, G_i and e_i subject to the subutility equations (2), and (4), the budget constraints (16), and the wage equations (13) leads to the demand for education*

$$e_{ig} = \frac{1}{\lambda_{1i}^*} \cdot \frac{\theta_2 \left(1 - \Lambda_{Nig} - 2\Lambda_{E\,ig}\right)}{1 + \theta_2 \left(1 - \Lambda_{Nig} - 2\Lambda_{E\,ig}\right)} \cdot T_i^*, \qquad g = f, m \tag{19}$$

The demand for education conditioned on the number of children chosen is given by

$$e_{ig}\left(n_{1i}, n_{2i}\right) = \frac{\theta_2}{\lambda_{1i}^* \left(1 + \theta_2\right)} \left[T_i^* - \frac{\Lambda_{Nig} + 2\Lambda_{E\,ig}}{\Lambda_{Nig} + \Lambda_{E\,ig}} \cdot \left(\varphi_{1ig}^* n_{1i} + \varphi_{2ig}^* n_{2i}\right)\right], \qquad g = f, m. \tag{20}$$

Proof. See appendix A. ■

In the negotiation between females and males the number of children is determined as the Nash bargaining solution and then equation (20) determines the education each individual takes. Note that the above conditional demand equation for education maximizes income, and therefore is optimal both before and after negotiation about children has taken place.

The numerator in the expression for the demand for education (19) contains the elasticity of education on human capital, θ_2, multiplied by $(1 - \Lambda_{Nig} - 2\Lambda_{E\,ig})$, which takes into account that increasing wages also increase the costs of caring for children and the elderly.

The denominator $1 + \theta_2 \left(1 - \Lambda_{Nig} - 2\Lambda_{E\,ig}\right)$ determines how much (the proportion) the value of available working time $\sum_{j=1}^{3} d_{i+1,i+j} \cdot w_{jig} \cdot T_i^*$ is reduced by using time on taking an education. This can be seen by inserting equation (19) into the equation for the value of potential wage income, R_{ig}, given in equation (14),

$$R_{ig} = \sum_{j=1}^{3} d_{i+1,i+j} \cdot w_{jig} \cdot T_i^* - w_{1ig} \cdot e_{ig} = \frac{1}{1 + \theta_2 \left(1 - \Lambda_{Nig} - 2\Lambda_{E\,ig}\right)} \sum_{j=1}^{3} d_{i+1,i+j} \cdot w_{jig} \cdot T_i^*.$$

The notional demands for children are given in Lemma 2.

Lemma 2 (Notional demands for children) *Maximizing (5) with respect to s_{1ig}, s_{2ig}, s_{3ig}, n_{1ig}, n_{2ig}, E_{ig}, G_{ig} and e_{ig} subject to the subutility equations (2), and (4), the budget constraints (16), and the wage equations (13) leads to the notional demand equations for children,*

$$\tilde{n}_{1ig} = \frac{1}{\varphi_{1ig}^*} \cdot \frac{\left(\Lambda_{Nig} + \Lambda_{E\,ig}\right)/2}{1 + \theta_2 \left(1 - \Lambda_{Nig} - 2\Lambda_{E\,ig}\right)} \cdot T_i^*, \qquad g = f, m \tag{21}$$

and

$$\tilde{n}_{2ig} = \frac{1}{\varphi^*_{2ig}} \cdot \frac{(\Lambda_{Nig} + \Lambda_{E\,ig})/\,2}{1 + \theta_2\,(1 - \Lambda_{Nig} - 2\Lambda_{E\,ig})} \cdot T^*_i, \qquad g = f, m. \tag{22}$$

Proof. See appendix A. ■

The notional demands for children say, as usual with Cobb-Douglas demand systems, that the expenditure on children equal a constant budget share of income. This can be seen by rearranging equations (21) and (22), then inserting from (17) to get

$$\left(w_{1ik}\varphi_{1k} + (1+r)^{-1} w_{2ik}\varphi_2\right) \cdot \tilde{n}_{1ik} = \frac{\Lambda_{Nik} + \Lambda_{E\,ik}}{2} \cdot R_{ig}$$

and

$$\left((1+r)^{-1} w_{2ik}\varphi_{1k} + (1+r)^{-2} w_{3ik}\varphi_2\right) \cdot \tilde{n}_{2ik} = \frac{\Lambda_{Nik} + \Lambda_{E\,ik}}{2} \cdot R_{ig}.$$

The left hand side of these expressions is the discounted cost of having children, while the right hand side gives the budget share.

The demand equations for children, (21) and (22), imply that increases in the costs of having children, φ^*_{1ig} and φ^*_{2ig}, will decrease notional demand. Differences in the discounted costs of having children early, φ^*_{1ig}, and having them late, φ^*_{2ig}, determine the distribution between early born and late born children. The fact that women experience higher costs than men ($\varphi^*_{jif} > \varphi^*_{jim}$), will in isolation lead to women wanting fewer children than men. On the other hand, as will be seen later, the parameters of the utility function (especially if the relative size of $x_{\,i}$ is large and $\pi_{i+4,f} > \pi_{i+4,m}$) can counteract this effect. If $x_{\,i}$ is small enough, φ_{1if} is larger than φ_{1im} and $\pi_{i+4,f}$ is larger than $\pi_{i+4,m}$, then women will always want less children than men, $\tilde{n}_{jif} < \tilde{n}_{jim}$.

The conditional demand for private care of elderly and the demand for public care of the elderly is given in lemma 3..

Lemma 3 (Demand for private and public care of the elderly) *Maximizing (5) with respect to s_{1ig}, s_{2ig}, s_{3ig}, n_{1ig}, n_{2ig}, E_{ig}, G_{ig} and e_{ig} subject to the subutility equations (2), and (4), the budget constraints (16), and the wage equations (13) leads to the conditional demand equation for privately provided old age care*

$$E_{ig}\,(n_{1i}, n_{2i}) = \frac{1}{\eta^*_i} \cdot \frac{\Lambda_{E\,ig}}{\Lambda_{Nig} + \Lambda_{E\,ig}} \cdot \left(\varphi^*_{1ig} n_{1ig} + \varphi^*_{2ig} n_{2ig}\right), \qquad g = f, m, \tag{23}$$

(conditional on the number of children) and the demand equation for publicly provided old age care

$$\tilde{G}_{ig} = \frac{1}{p_{G,\,i+2}} \cdot \frac{\Lambda_{Gig}}{1 + \theta_2\,(1 - \Lambda_{Nig} - 2\Lambda_{E\,ig})} \cdot T^*_i \cdot \left(\bar{h}_{i-1}\right)^{\theta_1} (e_{ig})^{\theta_2}, \qquad g = f, m. \tag{24}$$

Proof. See appendix A. ■

Privately provided care is determined along with number of children and education in the negotiation process between females and males. The demand for publicly provided old age care will be determined by a type of voting mechanism.

9 Bargaining

It is assumed that the effective (actual) demand for children is the outcome of negotiation between the sexes. As mentioned in the introduction, negotiation between the sexes is based on the notional demands for children found in the previous section. The couple negotiate on the basis of these notional demands, agreeing on the number of children they wish to have; their collective effective demand for children.

Individuals wish to minimize the distance between the agreed upon number of children born at time j and their notional demand, $\tilde{n}_{jig}$:

$$\min_{n_{ji}} \left(\tilde{n}_{jig} - n_{ji}\right)^2, \quad j = 1, 2, \quad g = f, m.$$

The Nash product gives the n_{ji} that is the solution to the bargaining problem

$$\min \left(\left(\tilde{n}_{jif} - n_{ji}\right)^2\right)^{\Phi^*_{fi}} \left(\left(\tilde{n}_{jim} - n_{ji}\right)^2\right)^{\Phi^*_{mi}}, \quad j = 1, 2,$$

where Φ^*_{mi} is a parameter reflecting the negotiation strength of males and Φ^*_{fi} the negotiation strength of females.

The effective demand for children born early, $\bar{n}_{1i}$, is then given by

$$\bar{n}_{1i} = \Phi_i \tilde{n}_{1if} + \left(1 - \Phi_i\right) \tilde{n}_{1im}$$

and for children born later in life

$$\bar{n}_{2i} = \Phi_i \tilde{n}_{2if} + \left(1 - \Phi_i\right) \tilde{n}_{2im}$$

where $\Phi_i = \Phi^*_{fi} \Big/ \left(\Phi^*_{fi} + \Phi^*_{mi}\right)$. The realized number of children, $\bar{n}_{ji}$, is the weighted average of the desires of the male and the female weighted with their relative bargaining power.

Lemma 4 (Effective demand for children) *The realized number of births determined by the Nash product is given by*

$$\bar{n}_{1i} = \Omega_{1i} \cdot T^*_i \tag{25}$$

$$\bar{n}_{2i} = \Omega_{2i} \cdot T^*_i \tag{26}$$

with

$$\begin{aligned} \Omega_{1i} &= \frac{\Phi_i}{2\varphi^*_{1if}} \cdot \frac{\Lambda_{Nif} + \Lambda_{E\,if}}{1 + \theta_2 \left(1 - \Lambda_{Nif} - 2\Lambda_{E\,if}\right)} + \frac{1 - \Phi_i}{2\varphi^*_{1im}} \cdot \frac{\Lambda_{Nim} + \Lambda_{E\,im}}{1 + \theta_2 \left(1 - \Lambda_{Nim} - 2\Lambda_{E\,im}\right)} \\ &= \sum_{g=f,m} \frac{\Phi_{ig}}{2\varphi^*_{1ig}} \cdot \left[\frac{\left(\alpha_n + \pi_{4ig} \cdot x\left(G_{i-2}\right)\right)}{\left(1 + \theta_2\right)\left(1 + \sum_{j=2}^4 \pi_{jig}\beta^{j-1} + \pi_{4ig} \cdot \alpha_G\right) + \alpha_n + 2\pi_{4ig} \cdot x\left(G_{i-2}\right)}\right] \end{aligned}$$

and

$$\Omega_{2i} = \frac{\Phi_i}{2\varphi^*_{2if}} \cdot \frac{\Lambda_{Nif} + \Lambda_{E\,if}}{1 + \theta_2 \left(1 - \Lambda_{Nif} - 2\Lambda_{E\,if}\right)} + \frac{1 - \Phi_i}{2\varphi^*_{2im}} \cdot \frac{\Lambda_{Nim} + \Lambda_{E\,im}}{1 + \theta_2 \left(1 - \Lambda_{Nim} + 2\Lambda_{E\,im}\right)}$$

$$= \sum_{g=f,m} \frac{\Phi_{ig}}{2\varphi^*_{2ig}} \cdot \left[\frac{\left(\alpha_n + \pi_{4ig} \cdot x\left(G_{i-2}\right)\right)}{\left(1+\theta_2\right)\left(1 + \sum_{j=2}^{4} \pi_{jig} \beta^{j-1} + \pi_{4ig} \cdot \alpha_G\right) + \alpha_n + 2\pi_{4ig} \cdot x\left(G_{i-2}\right)} \right]$$

where $\Phi_{if} = \Phi_i$ *and* $\Phi_{im} = 1 - \Phi_i$.

Proof. Follows directly by inserting for notional demands from equations (21) and (22) into the Nash bargaining equations for $\bar{n}_{1i}$ and $\bar{n}_{2i}$ above. ■

An increase in the relative negotiation strength of women, Φ_i, will have two effects. One is to shift some of the cost of having children towards men (as mentioned above, φ_{1if} will decrease and φ_{1im} will increase when Φ_i increases), increasing the number of children women want and decreasing the number men want. The other is to give the preferences of the female more weight.

The number of children given by the bargaining process determines education and the demand for old age care of females and males as given in equations (20) and (23):

$$\bar{e}_{ig} = e_{ig}\left(\bar{n}_{1i}, \bar{n}_{2i}\right) = \frac{\theta_2}{\lambda^*_{1i}\left(1+\theta_2\right)} T^*_i \left[1 - \frac{\Lambda_{Nig} + 2\Lambda_{E\,ig}}{\Lambda_{Nig} + \Lambda_{E\,ig}} \cdot \left(\varphi^*_{1ig}\Omega_{1i} + \varphi^*_{2ig}\Omega_{2i}\right)\right], \quad g = f, m. \tag{27}$$

and

$$\bar{E}_{ig} = E_{ig}\left(\bar{n}_{1i}, \bar{n}_{2i}\right) = \frac{1}{\eta^*_i} T^*_i \cdot \frac{\Lambda_{E\,ig}}{\Lambda_{Nig} + \Lambda_{E\,ig}} \cdot \left(\varphi^*_{1ig}\Omega_{1i} + \varphi^*_{2ig}\Omega_{2i}\right), \quad g = f, m,$$

with

$$\frac{\Lambda_{Nig} + 2\Lambda_{E\,ig}}{\Lambda_{Nig} + \Lambda_{E\,ig}} = \frac{\alpha_n + 2 \cdot \pi_{4ig} \cdot x\left(G_{i-2}\right)}{\alpha_n + \pi_{4ig} \cdot x\left(G_{i-2}\right)}$$

$$\frac{\Lambda_{E\,ig}}{\Lambda_{Nig} + \Lambda_{E\,ig}} = \frac{\pi_{4ig} \cdot x\left(G_{i-2}\right)}{\alpha_n + \pi_{4ig} \cdot x\left(G_{i-2}\right)}$$

being dependent on the variable $x_i = x\left(G_{i-2}\right)$.

The realized level of government provided care delivered to generation $i-2$ by generation i, $\bar{G}_i$, is assumed to be determined as the average of male and female notional demands weighted by their relative human capital levels,

$$\bar{G}_i = \frac{1}{2}\left(\tilde{G}_{if} \cdot \frac{\left(\bar{h}_{if} + \bar{h}_{im}\right)/2}{h_{if}} + \tilde{G}_{im} \cdot \frac{\left(\bar{h}_{if} + \bar{h}_{im}\right)/2}{h_{im}}\right) \tag{28}$$

where

$$\tilde{G}_{ig} = \frac{1}{p_{G,\,i+2}} \cdot \frac{\Lambda_{Gig}}{1 + \theta_2\left(1 - \Lambda_{Nig} - 2\Lambda_{E\,ig}\right)} \cdot T^*_i \cdot \bar{h}_{ig}, \quad g = f, m.$$

This can be viewed as a type of mean voter theorem where negotiation strength plays no role. Replacing the individual human capital levels, h_{if} and h_{im}, by the average level of the generation, $\left(\bar{h}_{if}+\bar{h}_{im}\right)/2$, is a normalization done for simplicity.

Lemma 5 (Effective demand for public old age care) *Interpreting equation (28) as a voting mechanism, the effective supply of public old age care is given by*

$$\bar{G}_i=\frac{1}{\frac{p_{0G}}{\bar{h}_i}+\overline{w}(r_{i+2})\lambda_2\cdot\mu_G}\cdot T_i^*\cdot\frac{N_{2i}}{N_{4i-2}}\cdot\sum_{g=f,m}\frac{1}{2}\cdot\left(\frac{\Lambda_{Gig}}{1+\theta_2\left(1-\Lambda_{Nig}-2\Lambda_{E\,ig}\right)}\right)\tag{29}$$

with

$$\frac{\Lambda_{Gig}}{1+\theta_2\left(1-\Lambda_{Nig}-2\Lambda_{E\,ig}\right)}=\frac{\pi_{4ig}\cdot\alpha_G}{(1+\theta_2)\left(1+\pi_{2ig}\beta+\pi_{3ig}\beta^2+\pi_{4ig}\beta^3+\pi_{4ig}\alpha_G\right)+\alpha_n+2\pi_{4ig}x_i\left(G_{i-2}\right)}.$$

Proof. Inserting equations (15) and (24) into (28) gives the above. ■

Finally, it is assumed that given the levels of children, education, and private and public old age care determined through negotiation and voting, the couple maximizes average utility subject to household budget constraints that are the sum of the individual constraints. These levels of consumption are not the main interest of the paper and will not be commented further. This final stage is important only in that it ensures that even though signaling, negotiation and voting have been introduced to the model, the budget constraints of the individuals will be observed.

10 Dynamics

The dynamics of the model is determined by the following three equations

$$\log\bar{h}_i=\theta_1\log\bar{h}_{i-1}+\theta_2\log\left(\Gamma_i\left(x\left(G_{i-2}\right)\right)\right)\tag{30}$$

$$N_{0i}=\frac{N_{1,i-1}}{2}\cdot\Omega_{1,i-1}\left(G_{i-3}\right)T_{i-1}^*+\frac{N_{2,i-2}}{2}\cdot\Omega_{2,i-2}\left(G_{i-4}\right)T_{i-2}^*,\tag{31}$$

$$\log\bar{G}_i=\log\left(\sum_{g=f,m}T_i^*\cdot\frac{1}{2}\cdot\frac{\Lambda_{Gig}}{1+\theta_2\left(1-\Lambda_{Nig}-2\Lambda_{E\,ig}\right)}\right)-\log\left(\frac{p_{0G}}{\bar{h}_i}+\overline{w}(r_{i+2})\lambda_2\cdot\mu_G\right)+\log N_{2i}-\log N_{4i-2}\tag{32}$$

with

$$
\begin{aligned}
\Gamma_i\left(x\left(G_{i-2}\right)\right) = & \frac{\theta_2}{\lambda_{1i}^*\left(1+\theta_2\right)} \cdot T_i^* \\
& \cdot \left[1 - \left(\frac{1}{2}\sum_{g=f,m} \frac{\alpha_n + 2\cdot\pi_{4ig}\cdot x\left(G_{i-2}\right)}{\alpha_n + \pi_{4ig}\cdot x\left(G_{i-2}\right)} \cdot \varphi_{1ig}^*\right) \cdot \Omega_{1i} \right. \\
& \left. - \left(\frac{1}{2}\sum_{g=f,m} \frac{\alpha_n + 2\cdot\pi_{4ig}\cdot x\left(G_{i-2}\right)}{\alpha_n + \pi_{4ig}\cdot x\left(G_{i-2}\right)} \cdot \varphi_{2ig}^*\right) \cdot \Omega_{2i} \right]
\end{aligned}
$$

The human capital equation (30) is derived by inserting the equation for education, (27), into the equation for aggregate human capital, (7). It is easily seen that $\partial\Omega_{ji}/\partial x > 0$ and $\partial\Gamma_i/\partial x < 0$ as long as own consumption of goods matters more than children in the utility function: $1 + \sum_{j=2}^{4} \pi_{jig}\beta^{j-1} > \alpha_n$ (this is a sufficient, but not a necessary condition). At low levels, increases in G_i will decrease the utility of private care by decreasing x_i and thereby increase Γ_i. Define the minimum level of $\Gamma_i\left(x\left(G_{i-2}\right)\right)$ as $\Gamma^{\min} = \Gamma_i\left(\xi_1 + \xi_2\right)$ and the maximum level as $\Gamma^* = \Gamma_i\left(\xi_1\right)$.

The population equation (31) is found by inserting the child equations (25) and (26) into the aggregate population equation (1).

Equation (32) is the logarithm of the public old age care equation (29). It gives the development in public old age care. Note that big swings in the population will result in changes in public old age care through a price effect. Remember that $\bar{G}_i$ denotes care per person, while total care will be the number of elderly times the amount of care given to each person, $N_{4i-2}\cdot\bar{G}_i$.

Equation (3) discussed earlier is also important, stating how utility of private health care changes as public health care is expanded.

The human capital equation (30) is a first order difference equation with a time-varying constant term ($\theta_2 \log\Gamma_i$). It is well known (see for example Gandolfo (1997) that in general the long run equilibrium state is a moving equilibrium given by

$$\log \bar{h}_i^* = \sum_{t=0}^{\infty} \left(\theta_1\right)^t \cdot \theta_2 \ln \Gamma_{i-t}.$$

Note that the long run equilibrium state can never be smaller than $h^{\min} = \theta_2 \ln\Gamma^{\min} / \left(1-\theta_1\right)$, since this would be the equilibrium level if Γ_i always stayed at its lower bound.

The aim of the model is to highlight how developments during the last century can be explained by transitional dynamics brought on by the introduction of public health care for the elderly. It is therefore natural in the present context to restrict the analysis to the case where the effects of increasing public care of the elderly die out as the level of human capital increases. A sufficient condition for this is assumption 2.

Assumption 2. It is assumed that G^* is set so that

$$G^* \leq \left(\frac{p_{0G}}{h^{\min}} + \overline{w}(r_{i+2})\lambda_2 \cdot \mu_G\right)^{-1} \cdot T_i^* \cdot N^{\min}$$
$$\cdot \sum_{g=f,m} \frac{1}{2} \cdot \left(\frac{\pi_{4ig} \cdot \alpha_G}{(1+\theta_2)\left(1+\pi_{2ig}\beta + \pi_{3ig}\beta^2 + \pi_{4ig}\beta^3 + \pi_{4ig}\alpha_G\right) + \alpha_n + 2\pi_{4ig}\left(\xi_1 + \xi_2\right)}\right)$$

with $\frac{N_{2i}}{N_{4i-2}}$ being bounded below by $N^{\min}$.

Since $\bar{h}_i$ at some point must reach at least its minimum equilibrium level $h^{\min}$, G_i must under assumption 2 at some point become larger than G^*. As soon as G_i becomes equal to G^*, Γ_i becomes a constant and the human capital relationship becomes a first order difference equation with a time invariant constant term with equilibrium state

$$\log \bar{h}^* = \frac{\theta_2 \ln \Gamma^*}{1-\theta_1}.$$

Having assumed that $0 < \theta_1 < 1$, the equilibrium is stable and as long as $\log \bar{h}_0 < \log \bar{h}^*$ there will be a monotonic convergence of human capital towards the the equilibrium level. In this case human capital will be growing over time. The larger θ_1 is, the larger this growth will be. Galor and Tsiddon (1997) assume that θ_1 is a decreasing function of h so that one can have an increasing growth rate at low level of human capital ($\theta_1 > 1$), and a decreasing rate at high levels ($\theta_1 < 1$).

In the situation where $G_{i-2} \geq G^*$ ($x_i = \xi_1$) the variables Γ_i, Ω_{1i}, Ω_{2i} and $\Lambda_{Gig}/\left(1+\theta_2\left(1-\Lambda_{Nig} - 2\Lambda_{E\,ig}\right)\right)$ all become constants. This in turn means that the number of children, $\bar{n}_{1i} + \bar{n}_{2i}$, born to each generation is constant. The population then grows if $\bar{n}_{1i} + \bar{n}_{2i} > 2$, falls if $\bar{n}_{1i} + \bar{n}_{2i} < 2$, and converges to a stable population if $\bar{n}_1 + \bar{n}_2 = 2$. If the size of the population in the two periods preceding the initial periods are different, then the relative size of cohorts will oscillate before approaching a constant value. At this point the population may grow as a whole, but the relative distribution between different age groups remains fixed.

Even though assumption 2 assures us of a long term time invariant equilibrium state, an exogenous shock that leads to a sudden decrease in $\frac{N_{2i}}{N_{4i-2}}$ can lead to complex dynamics in the shorter term (notice that Γ_i is a function of all the parameters in the model). Population size can begin to oscillate and $\bar{G}_i$ can change rapidly (remember it is care *per* person). If this also changes x_i then the development in human capital is also affected.

11 Changes in mortality, bargaining power of women and the cost of children

Decreasing mortality in old age (increasing longevity π_{4ig}) has two effects on the notional demand for children. The wish for more consumption in the last period will lead to fewer children, while the wish for more care will lead to more children.

Proposition 1 (Falling mortality in old age) *Falling mortality in old age (increased longevity π_{4ig}) will increase the number of children born if*

$$1 + \pi_{2ig}\beta + \pi_{3ig}\beta^2 > \left(\frac{(\beta^3 + \alpha_G)}{x_i} + \frac{1}{(1 + \theta_2)} \right) \alpha_n,$$

otherwise falling mortality will decrease the number of children. A sufficiently low utility of private old age care x_i will lead to falling mortality decreasing the number of children born. If the number of children increases when mortality drops, the amount of education taken by women and men decreases, otherwise it will increase.

Proof. The fertility result follows directly from derivation of Ω_{ji} with respect to π_{4ig}, while the education result is easily seen by considering how fertility affects the education function (27). ■

If receiving care when old is not appreciated ($x_i = \alpha_G = 0$), then a decrease in mortality (increase in longevity) leads to an increase in the desire for consumption at age 4 and thereby a decrease in the number of children. Individuals reallocate resources away from children toward consumption in the last period. On the other hand, if care when old is the only aspect of children one cares about ($\alpha_n = 0$), then a decrease in mortality (increase in longevity) makes receiving care when old more important, leading to an increase in the number of children. In this case individuals reallocate resources towards children.

That the effect of mortality on fertility depends on the amount of old age care delivered by the government can give an interesting interpretation of the development in fertility during the 20th century. As noted in the introduction, in Norway births first decreased, then increased and finally started decreasing again. If early in the last century x_i was large (due to a lack of government provided old age care), decreasing mortality would lead to an increase in the number of children. To begin it could be argued that this effect was counteracted by the effect of increasing child costs (especially increases in the cost of older children, φ_{2i}), but over time it would come to dominate and births would start to increase. If then x_i shifted downward as more old age care was provided by government, the effect of decreasing mortality would be reversed, leading to a decrease in births.

Proposition 2 (Falling mortality among those of working age) *Falling mortality among those of working age (increased longevity π_{2ig} or π_{3ig}) will always decrease the number of children born and increase the amount of education taken by women and men.*

Proof. Follows from derivation of Ω_{ji} and considering how a decrease in children affects the education function (27). ■

Falling mortality in working age will increase the expected utility of consumption so that individuals increase consumption and reduce the number of children they have. Higher life expectancy raises the return to human capital, so that it is in the interest of the agents to allocate less time to children and more time on education. There is no income effect due to the assumption that there are actuarially fair annuity markets. Under this assumption an increase in expected wage income due to falling mortality is canceled by a decrease in annuity payments to those surviving from one age to another.

Proposition 3 (Increased bargaining power for women) *An increase in the bargaining power of women, Φ_i, will decrease the number of children born if women have a smaller notional demand for children than men, $\tilde{n}_{jif} < \tilde{n}_{jim}$, and if*

$$\frac{\varphi^*_{jif}}{\varphi^*_{jim} + \varphi^*_{jif}} > \Phi_i,$$

where

$$\frac{\varphi^*_{1if}}{\varphi^*_{1im} + \varphi^*_{1if}} = \frac{\lambda^*_{1i}\varphi_f + \lambda^*_{1i}\varphi_{1i} + \lambda^*_{2i}\varphi_{2i} - \Phi_i \cdot \lambda^*_{1i}\varphi_{1i}}{\lambda^*_{1i}\varphi_f + \lambda^*_{1i}\varphi_{1i} + 2\lambda^*_{2i}\varphi_{2i}}$$

and

$$\frac{\varphi^*_{2if}}{\varphi^*_{2im} + \varphi^*_{2if}} = \frac{\lambda^*_{2i}\varphi_f + \lambda^*_{2i}\varphi_{1i} + \lambda^*_{3i}\varphi_{2i} - \Phi_i \cdot \lambda^*_{2i}\varphi_{1i}}{\lambda^*_{2i}\varphi_f + \lambda^*_{2i}\varphi_{1i} + 2\lambda^*_{3i}\varphi_{2i}}.$$

If an increase in Φ_i decreases the demand for children, the amount of education taken by women and men, $\bar{e}_{if}$ and $\bar{e}_{im}$, will increase.

Proof. Derivation of $\bar{n}_{ji}$ with respect to bargaining power leads to

$$\frac{\partial \bar{n}_{ji}}{\partial \Phi_i} = \left(1 + \frac{\Phi_i}{\varphi^*_{jif}} \cdot \lambda^*_{ji}\varphi_{1i}\right) \cdot \tilde{n}_{1if} - \left(1 + \frac{(1-\Phi_i)}{\varphi^*_{jim}} \cdot \lambda^*_{ji}\varphi_{1i}\right) \cdot \tilde{n}_{1im}$$

and the proposition is easily seen to hold. ■

Women will always wish fewer children than men if there is no mortality difference between women and men at working ages, if there is little government provided old age care so that x_i is small enough, if women face higher costs of having children, $\varphi^*_{jf} > \varphi^*_{jm}$, and if women have a higher probability of reaching old age, $\pi_{4if} > \pi_{4im}$. At low levels of female bargaining power, it is safe to assume that the relative weight of female child bearing costs, $\varphi^*_{jif} \Big/ \left(\varphi^*_{jim} + \varphi^*_{jif}\right)$, will be higher than the bargaining parameter Φ_i.

In other words, early increases in the bargaining power of women would seem to lead to fewer children being born, but as government provides increasing levels of care for the elderly and bargaining power reaches higher levels, there is an increased possibility that increased bargaining power can lead to more children.

The cost of having older children, φ_{2i}, can increase both due to falling economic benefits of having older children (they do less household work) or increasing costs (care becoming more expensive) of having older children.

Proposition 4 (Increased costs of children) *An increase in the cost of having older children, φ_{2i}, will lead to a fall in the number of births, $\bar{n}_i$. If women use more time taking care of young children than men, an increase in the cost of older children φ_{2i} will increase the amount of education taken by women, $\bar{e}_{if}$, and decrease the amount taken by men, $\bar{e}_{im}$.*

Proof. It is easily seen that $\frac{\partial\left(1/\varphi^*_{1ig}\right)}{\partial\varphi_{2i}} < 0$ and $\frac{\partial\left(1/\varphi^*_{2ig}\right)}{\partial\varphi_{2i}} < 0$ and it then follows from the equations for children (25) and (26) that the number of children must fall.

Women and men use equal time on young children if the bargaining parameter is as follows:

$$\begin{aligned} \varphi_{1if} &= \varphi_{1im} \\ \varphi_f + (1-\Phi_i)\cdot\varphi_{1i} &= \Phi_i \cdot \varphi_{1i} \\ \Phi_i &= \frac{1}{2}\left(1+\frac{\varphi_f}{\varphi_{1i}}\right) > \frac{1}{2} \end{aligned}$$

From the equation for education (27) it can be seen women will increase their education if

$$\partial\left(\frac{\varphi^*_{1if}}{\varphi^*_{1im}}\right)\Big/\partial\varphi_{2i} < 0 \text{ and } \partial\left(\frac{\varphi^*_{2if}}{\varphi^*_{2im}}\right)\Big/\partial\varphi_{2i} < 0.$$

From the derivatives one finds that these conditions are satisfied only if women take more care of children than men,

$$\Phi_i < \frac{1}{2}\left(1+\frac{\varphi_f}{\varphi_{1i}}\right).$$

For men, the relationships are reversed, so that men increase their education if

$$\partial\left(\frac{\varphi^*_{1im}}{\varphi^*_{1if}}\right)\Big/\partial\varphi_{2i} < 0 \text{ and } \partial\left(\frac{\varphi^*_{2im}}{\varphi^*_{2if}}\right)\Big/\partial\varphi_{2i} < 0,$$

taking less education if women take more care of children than men. ■

This lemma gives as an interesting result that increases in the cost of older children (including reduced benefit of work done by older children) leads to women increasing their education and men reducing theirs. As such, it provides an argument for the convergence of educational levels between women and men. Note that for women and men to devote the same amount of time to young children, women must have more bargaining power than men.

The result is due to child costs only being modeled as time costs and to having fertility determined within a negotiating framework. This leads to relative time costs being important in the effective demands for education, while playing no roll in the nominal demand functions. In the nominal demand for education the number of children one wants is proportional to the cost of taking care of these children. An increase in the cost of older children reduces the demand of men more than women, but the negotiating framework does not allow men to reduce the number of children as much as they want. This implies that men's total time spent on children increases (the number of children they have is reduced by less than the cost of children) and that women's total time spent on children decreases (the number of children is reduced by more than the cost of children).

Finally, our model lets us look closer at how mortality and bargaining power influence the spacing of children. In the following, average age when giving birth will be considered, defined as

$$\frac{1 \cdot \bar{n}_{1i} + 2 \cdot \bar{n}_{2i}}{\bar{n}_{1i} + \bar{n}_{2i}} = 1 + \frac{1}{1 + \frac{\bar{n}_{1i}}{\bar{n}_{2i}}}.$$

Note that the an increase in a parameter z will increase the average age of women giving birth if

$$\frac{\partial \left(\frac{\bar{n}_{1i}}{\bar{n}_{2i}} \right)}{\partial z} < 0 \iff \frac{\frac{\partial \bar{n}_{1i}}{\partial z}}{\bar{n}_{1i}} < \frac{\frac{\partial \bar{n}_{2i}}{\partial z}}{\bar{n}_{2i}}.$$

Proposition 5 (Changes in average age when giving birth) *More children will be born early than late ($\bar{n}_{1i} > \bar{n}_{2i}$) if $\varphi^*_{1ig} < \varphi^*_{2ig}$ which will always be the case if discounted wages increase over time*

$$w_{1ig} < (1+r)^{-1} w_{2ig} < (1+r)^{-2} w_{3ig}.$$

An equal marginal increase in old age longevity π_{4ig} for women and men (falling mortality) will increase average age when giving birth as long as women live longer than men,

$$\pi_{4if} > \pi_{4im}.$$

Increased bargaining power for women increases the average age when giving birth as long as men's notional demand for children is larger than women's and discounted wages rise more strongly at age 3 than at age 2,

$$\frac{(1+r)^{-1} w_{3ig}}{w_{2ig}} > \frac{(1+r)^{-1} w_{2ig}}{w_{1ig}}.$$

Proof. See appendix B. ■

The model thereby has that increases in longevity and increased bargaining power for women leads to an increase in the average age when giving birth, as one has observed in many western countries. An important assumption behind this result is that individuals face increasing wages over the life time, which has been modeled using the age specific experience parameters λ_j. According to the model, changes in the return to experience (or in interest rates) can reverse our findings on the timing of births.

12 Conclusions

The paper has used a very simple combination of an overlapping generations model and Nash bargaining among spouses to analyze the factors that can affect the number and timing of children. The model is able to explain changes in fertility as being the result of the net cost of having children, the utility of being cared for by ones children when one is old, and the bargaining strength of women in marriage.

The stylized facts discussed earlier can now be explained using the above model. Early in the century the benefits of having older children were decreasing fast due to fewer people living on farms in rural environments, leading to a decrease in the number of children. As this effect tapered off, declining mortality and a high utility of receiving care from one's children lead to the number of children rising again. Finally a fall in the utility of receiving care from one's children and increasing bargaining power among women lead to a further fall in fertility.

The increased bargaining power of women combined with less returns to seniority have recently lead to a delay of births. Education and the level of human capital have been increasing throughout the last century. As a result of the increasing time cost of older children and of time costs of young children falling more rapidly for women than for men, the educational level of women has been approaching that of men.

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Appendix A. Proof of Lemmas 1, 2 and 3

Proof. In the following we expand slightly on the model in the main part of the paper by introducing assuming that the production function is subject to exogenous technological progress through a technological coefficient ψ_t, reflecting labor augmenting technological change at time t. The technological coefficient is an externality that is not taken into account by the firms or individuals when they maximize profits and utility. The production function will thereby be

$$Y_t = F(K_t, \psi_t H_t) = AK_t^{\mu}(\psi_t H_t)^{1-\mu} = \psi_t H_t f(k_t).$$

In addition variable interest rates are allowed. To simplify the notation, the interest operator $d_{ij} = (1+r_i)\prod_{k=i}^{j}(1+r_k)^{-1}$ is used. The optimization problem is:

$$\begin{aligned}
\max \quad & U_i = \ln c_{1i} + \pi_{2ig}\beta \ln c_{2i} + \pi_{3ig}\beta^2 \ln c_{3i} + \pi_{4ig}\beta^3 \ln c_{4i} + \pi_{4ig}\cdot x_i \ln E_i \\
& \qquad + \tfrac{\alpha_n + \hat{\pi}_{4ig}\cdot x_i}{2} \ln n_{1i} + \tfrac{\alpha_n + \hat{\pi}_{4ig}\cdot x_i}{2} \ln n_{2i} + \pi_{4ig}\cdot \alpha_G \ln(1+G_i) \\
& \qquad + \tfrac{\pi_{4ig} x_i}{2} \ln\left(\tfrac{\pi^*_{2,i+2}\cdot \pi^*_{3i+1}}{2}\right) \\
\text{w.r.t} \quad & s_{1i}, s_{2i}, s_{3i}, e_i, n_{1i}, n_{2i}, E_i, G_i \\
\\
\text{s.t.} \quad & c_{1i} = w_{1i}\left(T - e_i - \varphi_{1ig} n_{1i}\right) - s_{1i} \\
& c_{2i} = w_{2i}\left(T - \varphi_{1ig} n_{2i} - \varphi_{2i} n_{1i} - \eta_{2i} E_i\right) - p_{G,i+2} G_i + (1+r_{i+2}) s_{1i} - s_{2i} \\
& c_{3i} = w_{3i}\left(T - \varphi_{2i} n_{2i} - \eta_{1i} E_i\right) + (1+r_{i+3}) s_{2i} - s_{3i} \\
& c_{4i} = (1+r_{i+4}) s_{3i} \\
\\
& w_{1i} = \psi_{i+1}\overline{w}(r_{i+1})\lambda_1 \cdot \left(\bar{h}_{i-1}\right)^{\theta_1}(e_i)^{\theta_2} \\
& w_{2i} = \psi_{i+2}\overline{w}(r_{i+2})\lambda_2 \cdot \left(\bar{h}_{i-1}\right)^{\theta_1}(e_i)^{\theta_2} \\
& w_{3i} = \psi_{i+3}\overline{w}(r_{i+3})\lambda_3 \cdot \left(\bar{h}_{i-1}\right)^{\theta_1}(e_i)^{\theta_2}
\end{aligned}$$

The first order conditions for s_{1i}, s_{2i} and s_{3i} can be written

$$c_{2i} = \tfrac{\pi_{2ig}\beta}{d_{i+1,i+2}} c_{1i}, \qquad c_{3i} = \tfrac{\pi_{3ig}\beta^2}{d_{i+1,i+3}} c_{1i}, \qquad c_{4i} = \tfrac{\pi_{4ig}\beta^3}{d_{i+1,i+4}} c_{1i}.$$

Inserting these first order conditions into the discounted value of total expected consumption $D_i = c_{1i} + d_{i+1,i+2}c_{2i} + d_{i+1,i+3}c_{3i} + d_{i+1,i+4}c_{4i}$ leads to

$$D_i = \left(1 + \pi_{2ig}\beta + \pi_{3ig}\beta^2 + \pi_{4ig}\beta^3\right) c_{1i}.$$

Inserting the budget constraints for c_{1i}, c_{2i}, c_{3i} and c_{4i} into D_i leads to

$$\begin{aligned}
D_i &= \left(1 + \pi_{2ig}\beta + \pi_{3ig}\beta^2 + \pi_{4ig}\beta^3\right) c_{1i} \\
&= \left(\bar{h}_{i-1}\right)^{\theta_1}(e_i)^{\theta_2}\left[T_i^* - \lambda_{1i}^*\cdot e_i - \varphi_{1ig}^* n_{1i} - \varphi_{2ig}^* n_{2i} - \eta_i^* E_i\right] - p_{G,i+2} G_i. \qquad \text{(A.1)}
\end{aligned}$$

Inserting from the first order conditions for s_{1i}, s_{2i} and s_{3i} into the first order conditions for e_i, n_{1i}, n_{2i}, E_i and G_i leads to the following five first order conditions

$$e_i = \frac{\theta_2}{\lambda^*_{1i}(1+\theta_2)}\left[T^*_i - \varphi^*_{1ig}n_{1i} - \varphi^*_{2ig}n_{2i} - \eta^*_i E_i\right] \tag{A.2}$$

$$n_{1i} = \frac{\alpha_n + \pi_{4ig}\cdot x_i}{2}\cdot\frac{1}{\varphi^*_{1ig}}\cdot\left(\bar{h}_{i-1}\right)^{-\theta_1}(e_i)^{-\theta_2}c_{1i} \tag{A.3}$$

$$n_{2i} = \frac{\alpha_n + \pi_{4ig}\cdot x_i}{2}\cdot\frac{1}{\varphi^*_{2ig}}\cdot\left(\bar{h}_{i-1}\right)^{-\theta_1}(e_i)^{-\theta_2}c_{1i} \tag{A.4}$$

$$E_i = \pi_{4ig}x_i\cdot\frac{1}{\eta^*_i}\cdot\left(\bar{h}_{i-1}\right)^{-\theta_1}(e_{ig})^{-\theta_2}c_{1i} \tag{A.5}$$

$$G_i = \pi_{4ig}\alpha_G\cdot\frac{1}{p_{G,\,i+1}}\cdot c_{1i} \tag{A.6}$$

which together with equation (A.1) give six equations in the six variables c_{1i}, e_i, n_{1i}, n_{2i}, E_i and G_i. From equations A.3 and A.4 we get

$$c_{1i} = \frac{\left(\bar{h}_{i-1}\right)^{\theta_1}(e_i)^{\theta_2}}{\alpha_n + \pi_{4ig}\cdot x_i}\cdot\left(\varphi^*_{1ig}n_{1i} + \varphi^*_{2ig}n_{2i}\right)$$

which inserted into the first order condition for E_i gives the conditional demand for old age care

$$E_i(n_{1i}, n_{2i}) = \frac{1}{\eta^*_i}\cdot\frac{\pi_{4ig}x_i}{\alpha_n + \pi_{4ig}\cdot x_i}\cdot\left(\varphi^*_{1ig}n_{1i} + \varphi^*_{2ig}n_{2i}\right). \tag{20}$$

Inserting this into the first order condition for education we get that the conditional demand for education can be written

$$e_i(n_{1i}, n_{2i}) = \frac{\theta_2}{\lambda^*_{1i}(1+\theta_2)}\left[T^*_i - \frac{\alpha_n + 2\pi_{4ig}x_i}{\alpha_n + \pi_{4ig}\cdot x_i}\cdot\left(\varphi^*_{1ig}n_{1i} + \varphi^*_{2ig}n_{2i}\right)\right]. \tag{17}$$

Solving the six equations (A.1) - (A.6) with respect to c_{1i}, e_i, n_{1i}, n_{2i}, E_i and G_i leads to the demand functions

$$c_{1i} = \frac{1}{\Lambda_{ig}}\left(\bar{h}_{i-1}\right)^{\theta_1}(e_i)^{\theta_2}\left(T^*_i - \lambda^*_{1i}\cdot e_i\right)$$

$$e_i = \frac{1}{\lambda^*_{1i}}\cdot\frac{\theta_2\left(1-\Lambda_{Nig}-2\Lambda_{E\,ig}\right)}{1+\theta_2\left(1-\Lambda_{Nig}-2\Lambda_{E\,ig}\right)}\cdot T^*_i \tag{16}$$

$$n_{1i} = \frac{1}{\varphi^*_{1ig}}\cdot\frac{\left(\Lambda_{Nig}+\Lambda_{E\,ig}\right)/\,2}{1+\theta_2\left(1-\Lambda_{Nig}-2\Lambda_{E\,ig}\right)}\cdot T^*_i \tag{18}$$

$$n_{2i} = \frac{1}{\varphi^*_{2ig}}\cdot\frac{\left(\Lambda_{Nig}+\Lambda_{E\,ig}\right)/\,2}{1+\theta_2\left(1-\Lambda_{Nig}-2\Lambda_{E\,ig}\right)}\cdot T^*_i \tag{19}$$

$$E_i = \frac{1}{\eta^*_i}\cdot\frac{\Lambda_{E\,ig}}{1+\theta_2\left(1-\Lambda_{Nig}-2\Lambda_{E\,ig}\right)}\cdot T^*_i$$

$$G_i = \frac{1}{p_{G,\,i+1}}\cdot\frac{\Lambda_{Gig}}{1+\theta_2\left(1-\Lambda_{Nig}-2\Lambda_{E\,ig}\right)}\cdot\left(\bar{h}_{i-1}\right)^{\theta_1}(e_i)^{\theta_2}\cdot T^*_i. \tag{21}$$

■

Appendix B. Proof of Proposition 5

Proof. That the condition $\varphi^*_{1ig} < \varphi^*_{2ig}$ is necessary for more early than late births is seen from the demand functions for children. Inserting for φ^*_{2ig} and φ^*_{1ig} in $\varphi^*_{2ig} > \varphi^*_{1ig}$ and multiplying by the individuals human capital level leads to $\left((1+r)^{-1} w_{2ig} - w_{1ig}\right) \cdot\varphi_{1ig} > \left((1+r)^{-1} w_{2ig} \ - (1+r)^{-2} w_{3ig}\right)$ $\cdot\varphi_{2i}$.

That increasing longevity increases average age at birth can be seen by inserting in the expression

$$\frac{\frac{\partial \bar{n}_{1i}}{\partial \pi_{4i}}}{\bar{n}_{1i}} < \frac{\frac{\partial \bar{n}_{2i}}{\partial \pi_{4i}}}{\bar{n}_{2i}}$$

and rearranging to find the condition

$$\frac{x}{(\alpha_n + \pi_{4im} \cdot x)} + \frac{(1+\theta_2)\left(\beta^3 + \alpha_G\right) + 2x}{(1+\theta_2)\left(1 + \sum_{j=2}^{4} \pi_{jim}\beta^{j-1} + \pi_{4im} \cdot \alpha_G\right) + \alpha_n + 2\pi_{4im} \cdot x}$$
$$> \frac{x}{(\alpha_n + \pi_{4if} \cdot x)} + \frac{(1+\theta_2)\left(\beta^3 + \alpha_G\right) + 2x}{(1+\theta_2)\left(1 + \sum_{j=2}^{4} \pi_{jif}\beta^{j-1} + \pi_{4if} \cdot \alpha_G\right) + \alpha_n + 2\pi_{4if} \cdot x}$$

which is always the case as long as $\pi_{4if} > \pi_{4im}$.

Finally, the effect of increased bargaining power for women (increased Φ_i) is found in the same manner, leading to the expression

$$a_{1f}\left(1 + \frac{\Phi_i \tilde{n}_{2if}}{(1-\Phi_i)\,\tilde{n}_{2im}}\right) - a_{2f}\left(\frac{\varphi^*_{2im}}{\varphi^*_{1im}} \cdot \frac{\varphi^*_{1if}}{\varphi^*_{2if}} + \frac{\Phi_i \tilde{n}_{2if}}{(1-\Phi_i)\,\tilde{n}_{2im}}\right)$$
$$< a_{1m}\left(\frac{\varphi^*_{2im}}{\varphi^*_{1im}} \cdot \frac{\varphi^*_{1if}}{\varphi^*_{2if}} + \frac{(1-\Phi_i)\,\tilde{n}_{1im}}{\Phi_i \tilde{n}_{1if}}\right) - a_{2m}\left(1 + \frac{(1-\Phi_i)\,\tilde{n}_{1im}}{\Phi_i \tilde{n}_{1if}}\right)$$

where

$$a_{1f} = \left(\frac{1}{\Phi_i} + \frac{\lambda^*_{1i}}{\varphi^*_{1if}} \cdot \varphi_{1i}\right) \qquad a_{1m} = \left(\frac{1}{(1-\Phi_i)} + \frac{\lambda^*_{1i}}{\varphi^*_{1im}} \cdot \varphi_{1i}\right)$$
$$a_{2f} = \left(\frac{1}{\Phi_i} + \frac{\lambda^*_{2i}}{\varphi^*_{2if}} \cdot \varphi_{1i}\right) \qquad a_{2m} = \left(\frac{1}{(1-\Phi_i)} + \frac{\lambda^*_{2i}}{\varphi^*_{2im}} \cdot \varphi_{1i}\right).$$

Since it is apparent that $\frac{(1-\Phi_i)\tilde{n}_{1im}}{\Phi_i \tilde{n}_{1if}} > \frac{\Phi_i \tilde{n}_{2if}}{(1-\Phi_i)\tilde{n}_{2im}}$ as long as $\tilde{n}_{kim} > \tilde{n}_{kif}$ (assuming $\Phi < 0,5$) and easily established that $(a_{1f} - a_{2f}) < (a_{1m} - a_{2m})$, a sufficient condition for the above expression to be true is that

$$\frac{\varphi^*_{2im}}{\varphi^*_{1im}} > \frac{\varphi^*_{1if}}{\varphi^*_{2if}}$$

which is the case if

$$\frac{(1+r)^{-1} w_{3ig}}{w_{2ig}} > \frac{(1+r)^{-1} w_{2ig}}{w_{1ig}}.$$

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