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Targeting Public Services through Unequal Treatment of Unequals

Abstract:
When private goods are publicly provided, government authorities have to determine the distribution of services on recipients. In this paper, the public service provider is assumed to maximize utility defined over service supply to different target groups, given a budget constraint. The production technology is target group specific and depends on the ability of each target group to produce service outcomes. Three benchmark allocation principles are identified: equality of treatment (ET), equality of outcome (EO) and equality of marginal cost (EMC). These principles can be considered to be consistent with special cases of a public preference model, which allows for compromises between different allocation principles. The condition of technological dominance implies that there is a clear-cut equity-productivity trade-off, whereas violations of this condition may reduce the significance of the trade-off.

Keywords: treatment targeting, technological dominance, equity-productivity trade-off, publicly provided private goods, in-kind transfers

JEL classification: H42

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1. Introduction

The economic literature on targeting has largely been dealing with poverty alleviation, see e.g. Besley and Kanbur (1993). In this context the purpose of targeting is to identify who is poor and target benefits towards that group. Among others, Akerlof (1978) and Nichols and Zeckhauser (1982) have argued that targeting can improve the efficiency of income redistribution. However, this approach requires that public authorities are able to observe some characteristic that is correlated with ability and income.

Not only cash transfers, but also in-kind transfers can be targeted on selected groups. One of the most striking aspects of in-kind programs is how widespread and important they are, see Currie and Gahvari (2008). A property of public service provision is that information constraints might be overcome in the production process. This is due to the extensive information that public authorities collect in order to adjust services to recipients and to monitor the outcome of service production.

Although in-kind benefits can indeed be targeted in order to alleviate poverty, the distribution of public services is frequently motivated by reference to "needs", where "needs" can be interpreted as a disadvantage in the capability to function. According to Sen (1992) capabilities represent the various combinations of functionings (beings and doings) that the person can achieve. Examples of such functionings are to move about, being adequately nourished, being in good health, having self-respect and to participate in social life. In so far as functionings are constitutive of well-being, capability represents a person's freedom to achieve well-being. Thus one can argue that important public services like education and health care provide citizens with basic capabilities. People that lack education or suffer from serious health problems are disadvantaged in terms of capabilities, even if they do rather well in terms of commodities, income and utility. As noted by Sen (1992) and Roemer (1998), the basic diversity of human beings implies that equal consideration for all may demand very unequal treatment in favor of the disadvantaged.

Targeting of public services can be defined as any unequal distribution of public services in a given population. Van de Walle (1998) discusses the distinction between broad targeting and narrow targeting. Broad targeting denotes the allocation of budgets among categories of public spending, and narrow targeting entails targeting categories of people. For the purpose of this paper it is also relevant to introduce a conceptual distinction between take-up targeting and treatment targeting. *Take-up targeting* means that transfers are provided to a selected group of people either based on observable

characteristics or based on self-targeting.[1] By contrast, *treatment targeting* entails that different recipients are given unequal treatment. A common assumption in theoretical analysis of publicly provided goods is that recipients receive equal spending and are given equal treatment. This assumption is, however, at odds with the observation that important public services like health care, child care, education and care for the elderly and disabled are normally provided unequally to recipients with different diagnoses and/or abilities. In order to accommodate this salient feature of public service provision, this paper aims to discuss different allocation principles that may justify the widespread practice of unequal treatment. The analysis focuses on treatment targeting, while take up is assumed to be universal or based on exogenous observable characteristics.

Treatment targeting has earlier been analyzed by Arrow (1971). In his approach a basic assumption is that utility of each individual depends on public expenditure on the individual and some personal characteristics, termed ability. A utilitarian social welfare function is applied to the study of optimal public expenditure policy. The present paper employs a similar framework, where public expenditures can be targeted on individuals with different ability to produce service outcomes. Public expenditure is treated as an input in the production of service outcomes or final output, where final output is a good or a functioning that is contributing to well-being. Individuals with different ability are assumed to be sorted into case-mix groups or target groups that constitute different ability types. This assumption facilitates the comparison of the production functions of different target groups. However, unlike Arrow's approach the present paper is not based on utilitarianism.

Based on a review of the limited empirical evidence, Currie and Gahvari (2008) argue that paternalism is a leading overall explanation for the existence of in-kind transfers. Paternalism may signify that government decision-makers care about the distribution of some goods on individuals. Thus in-kind transfers become instruments to depart from consumer sovereignty in the allocation of merit goods. A related idea is the notion of specific egalitarianism. Tobin (1970) argues that while many people have no problem with income inequality per se, they would like to see that all individuals receive adequate food, medical services, or housing. Similarly, Kelman (1986) postulates that individuals have rights to certain specific things, not to the cash equivalent of these things.

If public authorities are motivated by paternalism, specific egalitarianism or provision of capabilities, it is relevant to develop a decision-model where public authorities care about service outcomes rather

[1] In the case of self-targeting, the government relies on individuals to identify themselves as needy. By imposing costs on the recipients or by the nature of the good itself, there are created disincentives for the non-targeted group to participate.

than individual utilities derived from these outcomes. Such a model differs from the utilitarian approach of Arrow (1971) by being non-welfarist.[2] An advantage of the non-welfarist approach is that it does not require interpersonal comparability of utility, unlike social evaluation based on the utilitarian decision model.

In the non-welfarist model it is assumed that public authorities are concerned about the distribution of service outcomes. However, to achieve equality of outcome is not necessarily the only objective in the allocation of public services. Another important concern might be efficiency, which is achieved by maximization of aggregate production that is provided for a given total expenditure. When disadvantaged groups have comparably low productivity in the production of service outcomes, there is a conflict between equity in service outcomes and high aggregate production. This is due to the fact that the marginal cost for increased production in low-ability groups is higher than marginal cost in high-ability groups, provided that equality of outcome is satisfied. Consequently a redistribution of spending that leads to a small decrease in service outcomes for low-ability types may facilitate a comparably large increase in service outcomes for high-ability types.

This type of trade-off has previously been discussed by Shoup (1964) and Behrman and Craig (1987), who provide analyses of the distribution of police resources across neighborhoods. Behrman and Craig (BC) define the trade-off between equity and productivity as a property of the local governmental utility function. The equity goal requires that the safety level is equalized across neighborhoods, whereas the concern for productivity requires focus on the aggregate city-wide safety. This paper demonstrates that the discussion of equity-productivity trade-off can be generalized to other types of public services than police services. This paper, however, deviates from the approach of BC by assuming that target groups are homogeneous when the relevant characteristic is ability. In BC heterogeneous neighborhoods play the role of target groups.

The purpose of the present paper is to combine the idea of Arrow (1971) that public services can be targeted on ability types with the notion of equity-productivity trade-off developed by Shoup (1964) and Behrman and Craig (1987). The implications of a government that is motivated by high aggregate production of the publicly provided good and/or equity in the distribution of public service outcomes are explored. Furthermore, the analysis exposes the conditions that determine the significance of the equity-productivity trade-off. The decisive condition is called technological dominance, which entails

[2] Welfarism is the view that the goodness of a state of affairs can be judged entirely by the goodness of utilities in that state, see Sen (2006).

that there is a positive relationship between ability to produce service outcomes (output level) and marginal productivity of the production functions of the respective ability types. Suppose, on the other hand, that target groups with a high initial output have a low marginal productivity, and that target groups with a low initial output have a high marginal productivity in the production of service outcomes. Then the technological dominance condition is violated, and the significance of the equity-productivity trade-off is reduced or in some cases is even entirely removed.

Finally, governmental preference for public services is applied to the linear expenditure system derived from a Stone-Geary utility function. It is demonstrated that this specification provides attractive interpretations of the preference parameters. Public authorities may determine the model parameters in order to satisfy different allocation principles. The community preference framework is also shown to allow compromises between different allocation principles. It will be justified that this modeling framework will form an attractive basis for empirical applications.

The paper is organized as follows. Section 2 discusses the technology for producing service outcomes or final output. Section 3 presents the allocation problem faced by public authorities when they have to prioritize among target groups, and introduces several principles of allocation. Section 4 demonstrates that the benchmark allocation principles are consistent with special versions of a community preference model. It is assumed that public authorities have preferences that are defined over the distribution of expenditures on target groups, and that utility is maximized subject to a budget constraint. A brief conclusion is given in Section 5.

2. Production of a publicly provided private good

Assume that a private good is produced by a subordinate public organization, like a public school, hospital, nursing home, kindergarten or local government. This lower level of government has the discretion to determine the service allocation on recipients, whereas the budget constraint (total expenditure) of the service producing entity is determined by a higher level of government. Thus the higher level of government decides the general priorities across different types of services and/or across geographic areas. However, the decision of how much to spend on different recipients requires detailed knowledge of each recipient. Adjustment of production to recipients is therefore decentralized to the lower level of government. The public service is assumed to be provided free of charge. Furthermore, the service is provided exclusively by the public sector, so there is little scope for topping up or opting out with private provision.

The public service producer may divide potential recipients into target groups according to socio-demographic characteristics. Such characteristics may include age, gender, family background, neighborhood, country background, diagnoses, abilities and skills. This kind of information is assumed to be collected as part of the production process. Depending on the degree of homogeneity, members of a given target group are given similar treatment by the service producer. This means that members of a sufficiently homogenous target group receive an equal amount of spending and output. Such a distribution is based on the principle of *horizontal equity*, which requires equal treatment of equals.[3]

Total income y equals total expenditure, and is measured per person in the population for which the public service producer is responsible. Spending is allocated on r target groups subject to the budget constraint

$$\sum_{j=1}^{r} z_j y_j = y, \tag{1}$$

where y_j is spending per person in target group j, and z_j is the population share that belongs to target group j.

What remains is to determine the distribution of expenditure on target groups. Following Arrow (1971), it is assumed that recipients use public expenditure as an input in the production of service outcomes (or final output). This transformation process is described by the production functions

$$\begin{aligned} &x_j = f_j\left(y_j\right), \quad f_j' > 0, \quad f_j'' < 0, \quad \left(j = 1,2,...,r\right), \\ &\varepsilon_j\left(y_j\right) = \frac{y_j f_j'\left(y_j\right)}{f_j\left(y_j\right)}, \quad \left(j = 1,2,...,r\right), \end{aligned} \tag{2}$$

where x_j is the production of final output per person in target group j and ε_j is the elasticity of the production function. The production function is increasing and concave in expenditure. Moreover, the production function is specific to the target group, which means that final output and marginal productivity may differ across target groups for a given expenditure per person.[4]

[3] As a special case each target group may include only one individual. Thus the assumption of horizontal equity is not a necessary condition in the model. However, this paper focuses on inequities across rather than within target groups.

[4] The target group specific production functions correspond to the assumption in Arrow (1971) that production depends on public expenditure on a given individual and ability of the individual. However, the specification in this paper does not preclude intersecting production functions, although non-intersecting production functions are imposed as an additional restriction defined by ability dominance.

In order to allocate total expenditure on target groups, it is required to compare the production functions of different target groups. A striking feature of public service allocation is that target groups have different ability in producing final output. For instance, the multi-handicapped or severely disabled may need much more aid than other target groups to produce a given level of knowledge as measured by test scores in public schools. Moreover, the same groups also need intensive care to achieve normal functionings of daily living.

In order to compare target groups, it is useful to introduce the following concepts.

Definition 1: *Conditional ability ranking*. Assume that $f_j(g)$ and $f_k(g)$ are production functions of two different target groups *j* and *k*, and *g* is expenditure per person. Then target group *j* is said to have conditionally higher ability than target group *k* for a given level of *g* if $f_j(g) > f_k(g)$.

Note that definition 1 presupposes that the target groups are treated equally (receive equal spending per person). Hence the definition does not preclude intersecting production functions, which implies that conditional ability rankings are reversed at some expenditure level.

Definition 2: *Ability dominance*. Assume that $f_j(g)$ and $f_k(g)$ are production functions of two different target groups *j* and *k*, and g is expenditure per person. Then target group *j* is said to be ability dominant and to have generally higher ability than target group *k* if $f_j(g) > f_k(g) \forall g > 0$.

Definition 3. *Productivity dominance*. Assume that $f_j'(g)$ and $f_k'(g)$ are marginal productivities of the production functions of two different target groups *j* and *k*, and *g* is expenditure per person. Then target group *j* is said to be productivity dominant and to have higher productivity than target group *k* if $f_j'(g) > f_k'(g) \forall g > 0$.

Definition 4. *Elasticity dominance*. Assume that $\varepsilon_j(g)$ and $\varepsilon_k(g)$ are elasticities of the production functions of two different target groups *j* and *k*, and *g* is expenditure per person. Then target group *j* is said to be elasticity dominant and to have higher output elasticity than target group *k* if $\varepsilon_j(g) > \varepsilon_k(g) \forall g > 0$.

Definition 5. *Weak technological dominance*. Target group *j* is said to exert weak technological dominance over target group *k*, if (i) target group *j* is ability dominant to target group *k*, and (ii) target group *j* is productivity dominant to target group *k*. (See definitions 2 and 3.)

Definition 6. *Strong technological dominance*. Target group *j* is said to exert strong technological dominance over target group *k*, if (i) target group *j* is ability dominant to target group *k*, and (ii) target group *j* is elasticity dominant to target group *k*. (See definitions 2 and 4.)

The definition of weak technological dominance means that the vertical distance between two production functions is always increasing as the level of expenditure is increasing. If there is strong technological dominance, the percentage increase in final output as expenditure per person increases with 1 percent, is always larger for a high-ability group than for a low-ability group. Weak technological dominance is implied by strong technological dominance, but the reverse implication is not true.

3. Allocation of expenditure and production on target groups

Public authorities may want to compensate low-ability groups for their disadvantage in producing service outcomes. Thus, the principle of *vertical equity* is defined by unequal treatment of unequals in order to achieve equal outcomes. The distribution of expenditure across target groups is not necessarily based on equal treatment. As alternatives, consider the following principles of allocation among target groups.

Definition 7. *Equality of treatment* (ET): Members of different target groups receive equal expenditure per person, $y_j = y_k \forall (j,k)$.

Definition 8. *Equality of outcome* (EO): Members of different target groups receive equal final output per person, $x_j = x_k \forall (j,k)$.

Definition 9. *Equality of marginal cost* (EMC): The cost of a marginal increase in final output is equal for all target groups, $1/f_j'(y_j) = 1/f_k'(y_k) \forall (j,k)$.

It follows from definition 9 that under EMC, the marginal productivity is equalized for different target groups. Thus EMC focuses exclusively on productivity and is not concerned with vertical equity. At

the other extreme, EO is concerned with vertical equity and disregards differences in the marginal productivity.

The meaning of EMC can be further explored by introducing aggregate production per person in the population

$$x = \sum_{j=1}^{r} z_j x_j \,, \tag{3}$$

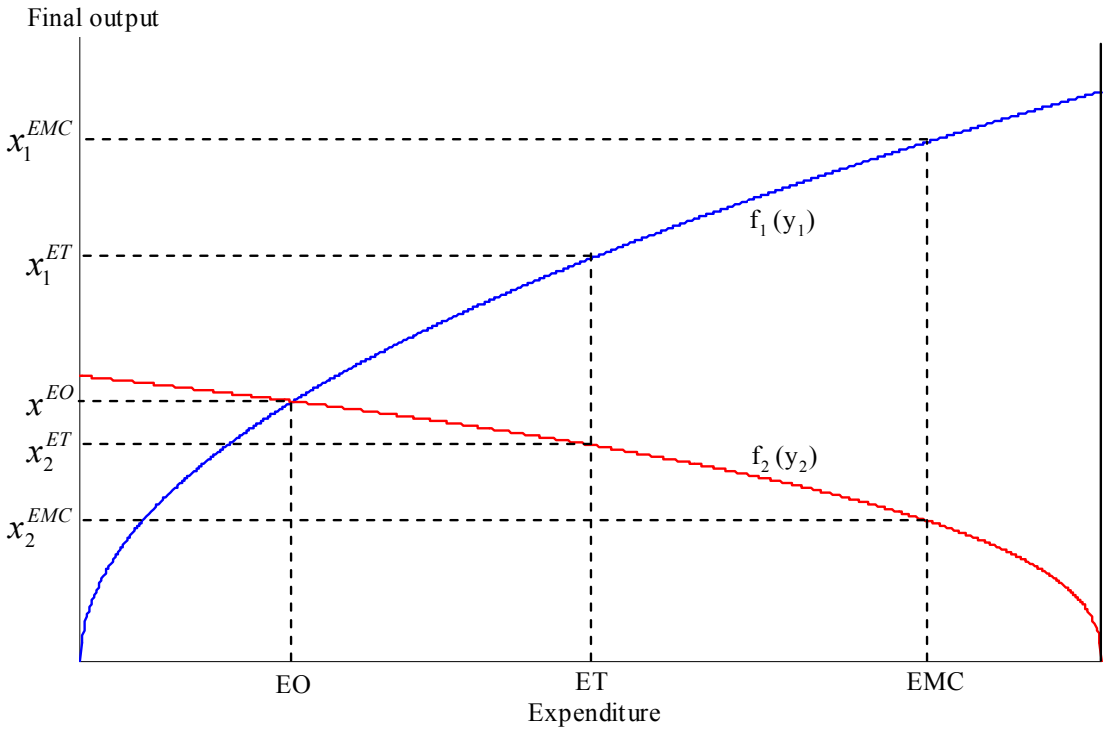
where x is the average of production per person in different target groups weighted by their respective population shares. Assume that the public service producer is maximizing average production for a given total cost y, or alternatively, that the public service producer is minimizing the cost to produce a given average production. It can be shown that either optimizing problem leads to first order conditions that require EMC to be fulfilled. This solution means that the marginal productivity is equal for all target groups. Moreover, high-ability groups may receive more spending per person than low-ability groups. High priority of high-ability groups follows from the assumptions of weak technological dominance and concave production functions. When the cost of producing an extra unit of final output is comparably low for high-ability groups at a given level of spending per person, the cost minimizing or average production maximizing service provider is distributing a high share of resources to high-ability groups, while low-ability groups are given a low priority.

3.1. Technological dominance

When the conditions for technological dominance are fulfilled, the service provider has to decide whether he should target more spending on high-ability groups or on low-ability groups. In the former case, high aggregate production is achieved at the cost of high outcome inequality, whereas in the latter case welfare of disadvantaged groups is improved at the cost of lower aggregate production. This is the equity-productivity trade-off for public services.

The equity-productivity trade-off is displayed in Figure 1. For a given level of total income, it is assumed that spending is allocated on two different target groups with production functions $f_1(y_1)$ and $f_2(y_2)$. For simplicity the two target groups are assumed to have equal size $(z_1 = z_2)$. Moreover, it is assumed that target group 1 is exerting technological dominance over target group 2. Expenditure received is measured from the left to the right for group 1 and from the right to the left for group 2. Thus expenditure received by the two groups is restricted by the total resource constraint, given by equation (1).

Figure 1. Allocation of expenditure on two target groups based on equal outcome (EO), equal treatment (ET) or equal marginal cost (EMC)



The EO solution is where the two production functions intersect in the diagram. In this case the two groups achieve equal final output. This requires a high priority of the low-ability group, which receives a high share of total expenditure.

The EMC solution is where the slopes of the two production functions are equal (in absolute value), and where the sum of the two production functions reaches its maximum. This requires a high priority of the high-ability group, which receives a high share of total expenditure. Accordingly, the distribution of final output exhibits large outcome inequality.

The ET solution allocates half of total expenditure to group 1, and the other half to group 2. Due to technological dominance group 1 produces more final output than group 2, but inequality in the distribution of final output is smaller than under EMC.

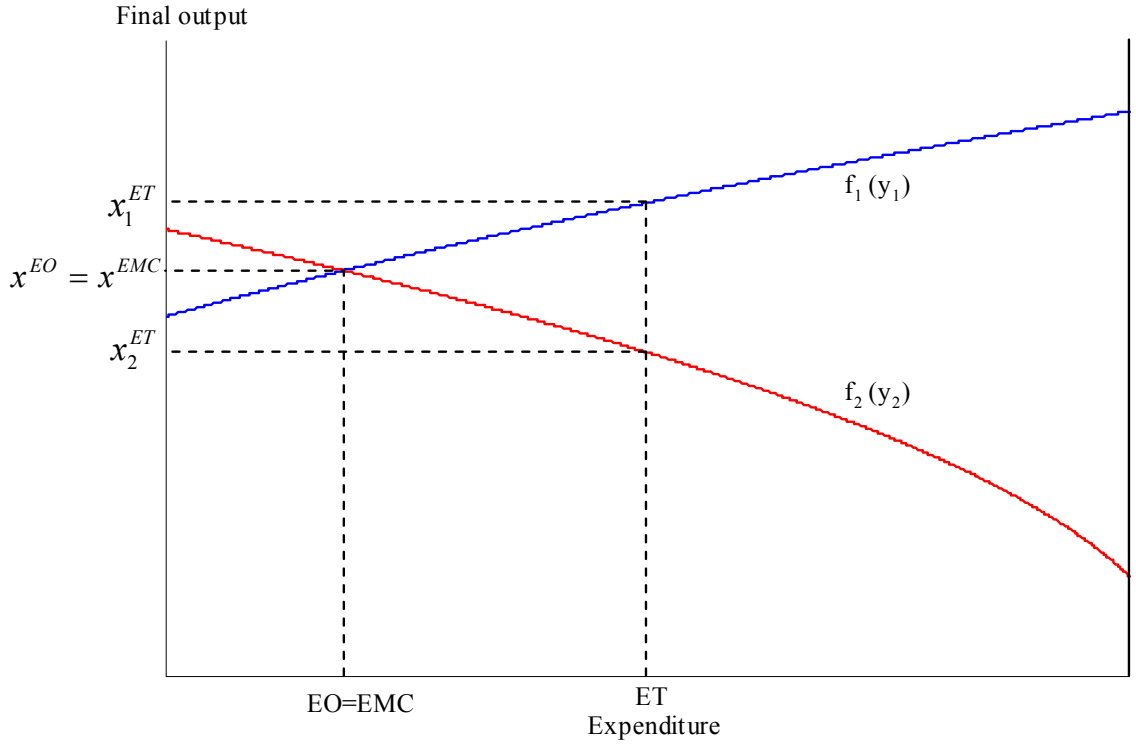
3.2. Violation of technological dominance

The relationship between ability and marginal productivity is not necessarily positive. Define the initial endowment of a target group by the outcome that is produced at zero public expenditure. This

initial endowment may vary across target groups and ability types, either because of different inherited ability or because of different prior investments in health and/or human capital that affect ability. Moreover, in the case of health related services, variation in initial endowments may result from negative health shocks that are more or less accidental.

If a target group with a low initial endowment has a high marginal productivity, whereas another target group with high initial endowment has a low marginal productivity in the production of service outcomes, the equity-productivity trade-off may vanish. Such a case is shown in Figure 2, which is defined similarly as Figure 1, except that the production functions $f_1(y_1)$ and $f_2(y_2)$ have been changed. In Figure 2, it is assumed that target group 1 has a higher initial endowment and is ability dominant to target group 2, whereas target group 2 is productivity dominant to target group 1.

Figure 2. Allocation of expenditure on two target groups based on equal outcome (EO), equal treatment (ET) or equal marginal cost (EMC)



The EO solution is where the two production functions intersect in the diagram. However, the production functions have been constructed such that the EO solution in this case coincides with the EMC solution. Thus the figure illustrates that there is not necessarily a conflict between EO and EMC. Both a low initial endowment and a high marginal productivity (for a given expenditure per person)

require that target group 2 should be given a high priority. Correspondingly, target group 1 should be given a low priority due to a high initial endowment and a low marginal productivity in the production of final output.

The ET solution allocates half of total expenditure to group 1, and the other half to group 2. This solution gives a higher priority to target group 1 and a lower priority to target group 2 as compared to the EO and EMC solution. At the ET solution, it is possible to increase both aggregate production and equity in the distribution of final output by a transfer of spending from target group 1 to target group 2.

4. Preferences for targeting

It is not clear whether any of the allocation principles ET, EO or EMC is predominant in the decisions made by public authorities. Targeting policies may vary across public service sectors, and also across government jurisdictions and service producing agents. Moreover, it is likely that public sector behavior involves compromises between the benchmark allocation principles. As is demonstrated by Figure 1, the EO and EMC principles might produce very different results. Provided that target group rankings satisfy technological dominance, the goals of high aggregate final output and low inequality in final output have to be balanced against each other. Even in cases where EO and EMC are not in conflict, as shown in Figure 2, service producers might still wish to balance ET against EO (and EMC).

In order to analyze targeting policies, a decision model that provides a solution to the allocation problem is called for. This paper assumes that in-kind transfers are motivated by paternalism, specific egalitarianism or a version of non-welfarism, in which public authorities care about the distribution of the publicly provided good on individuals. To account for such preferences the "community preference" model emerges as an appropriate candidate.[5] The model treats public authorities like households that maximize utility for a given budget constraint. Standard applications of this model include studies of resource allocation on private and public consumption, and resource allocation on different public service sectors. Expenditure composition problems in the public sector are treated similarly to consumer choices in the private sector.

Assume that preferences of the public service producer are represented by the Stone-Geary utility function

[5] For a discussion of the community preference model, see Wildasin (1986).

$$W = \sum_{j=1}^{r} \beta_j \log\left(y_j - \alpha_j\right), \qquad \sum_{j=1}^{r} \beta_j = 1, \tag{4}$$

where W is utility and β_j and α_j are preference parameters. The Stone-Geary utility function is a convenient choice, since it keeps the model simple and parsimonious in parameters. As will be demonstrated below the model parameters have attractive interpretations in terms of targeting behavior. Moreover, as demonstrated in Section 4.5 the model is useful for empirical applications even in cases where data on final output are lacking.

The purpose of this part of the analysis is to examine how preferences for different allocation principles can be captured by a linear expenditure system. Note that preferences of the public authority are defined over expenditure per person received by different target groups. Thus preferences for different allocation principles are assumed to be captured indirectly by the preference parameters. This assumption makes it possible to retain standard properties of the expenditure system.

Equation (4) states that utility is increasing with higher group-specific spending. However, since total expenditure is restricted by the budget constraint, the public authority has to make priorities across target groups. Thus, utility in equation (4) is maximized subject to the budget constraint (1). This leads to the linear expenditure system

$$z_j y_j = \alpha_j z_j + \beta_j \left(y - \sum_{j=1}^{r} \alpha_j z_j \right), \quad \left(j = 1, 2, ..., r\right). \tag{5}$$

Each α_j-parameter is interpreted as minimum expenditure per person in target group j, and $\alpha_j z_j$ is group specific minimum expenditure per person in the population. Discretionary income is the income above total minimum expenditure, which is given by $\left(y - \sum_{j=1}^{r} \alpha_j z_j \right)$. Discretionary income is distributed on target groups in line with marginal budget shares β_j. For further analysis it is convenient to define two special cases of the linear expenditure system.

Condition 1: *Exhaustive minimum expenditures* (EME). In this case total incomes are just sufficient to cover total minimum expenditures, which means that discretionary income equals zero, or $\sum_{j=1}^{r} \alpha_j z_j = y$.

Condition 2: *Income elasticities equal to unity*. In this case marginal budget shares equal total budget shares for all target groups, or $\beta_j = z_j y_j / y, \left(j = 1,2,...,r\right)$.

Exhaustive minimum expenditures signify that the sum of minimum expenditures across target groups is set as high as possible given the budget constraint. Income elasticities equal to unity implies that each target group's budget share is kept constant as total income is increasing. Moreover, it can be shown that either of the conditions 1 or 2 leads to the simplified version of the expenditure system

$$s_j = \frac{z_j y_j}{y} = \frac{\alpha_j z_j}{\sum_{j=1}^{r} \alpha_j z_j}, \quad \left(j = 1,2,...,r\right), \tag{6}$$

where s_j is the budget share that is allocated to target group j. Equation (6) states that the budget share allocated to each target group is determined entirely by the distribution of minimum expenditure shares. The intuition behind equation (6) is that either are total minimum expenditures exhaustive, or else discretionary incomes are allocated on target groups in proportion to minimum expenditures. Thus the relative priorities across target groups are not assumed to change significantly in response to increased total incomes. As will be demonstrated below, this restrictive version of the model has sufficient flexibility to accommodate different allocation principles. It follows that the less restrictive version in equation (5) allows for the same flexibility. However, the interpretation of minimum expenditures is modified if the allocation of discretionary incomes is based on distributional preferences that differ significantly from the preferences that determine the distribution of minimum expenditures. Such a possibility is discussed briefly in Section 4.5. Yet the analysis of minimum expenditures under condition 1 and/or condition 2 proves to throw light on how different types of targeting preferences may lead to different expenditure allocations.

4.1. Equality of treatment

From equation (6) it follows that y_j is equalized across target groups, given the following condition

Condition 3. *Joint minimum expenditure*. In this case minimum expenditures per person are equal for all target groups, $y_j \geq \alpha_j = \alpha \; \forall j$.

Proposition 1. Assume that condition 1 (exhaustive minimum expenditures) or condition 2 (income elasticities equal to unity) is satisfied. Moreover, assume that condition 3 (joint minimum expenditure) is also satisfied. Then the solution of the linear expenditure system satisfies equality of treatment (ET).

4.2. Equality of outcome

A public authority that has the discretion to determine minimum expenditures may take into account the production functions and abilities of different target groups. Although group-specific expenditure is included as arguments in the utility function, it does not follow that the public authority does not care about the distribution of final output. To see this, assume that a public authority imposes a minimum standard for final output.

Condition 4. *Joint minimum standard for final output*. In this case the government authority requires that $x_j \geq \tau \, \forall j$, where τ is the minimum standard for final output.

Minimum expenditures corresponding to condition 4 can be deduced from the inverse production function

$$
\begin{aligned}
& y_j = f_j^{-1}\left(x_j\right) = g_j\left(x_j\right), \quad \left(j = 1,2,...,r\right), \\
& \varepsilon_j^{-1}\left(x_j\right) = \frac{x_j g_j'\left(x_j\right)}{g_j\left(x_j\right)}, \quad \left(j = 1,2,...,r\right),
\end{aligned} \tag{7}
$$

where $g_j\left(x_j\right)$ is the inverse production function, which in this case is identical to the cost function, and $\varepsilon_j^{-1}\left(x_j\right)$ is consequently the elasticity of the cost function. It follows from the properties of inverse functions that

$$
\varepsilon_j^{-1}\left(x_j\right) = \frac{1}{\varepsilon_j\left(g_j\left(x_j\right)\right)}, \quad \left(j = 1,2,...,r\right). \tag{8}
$$

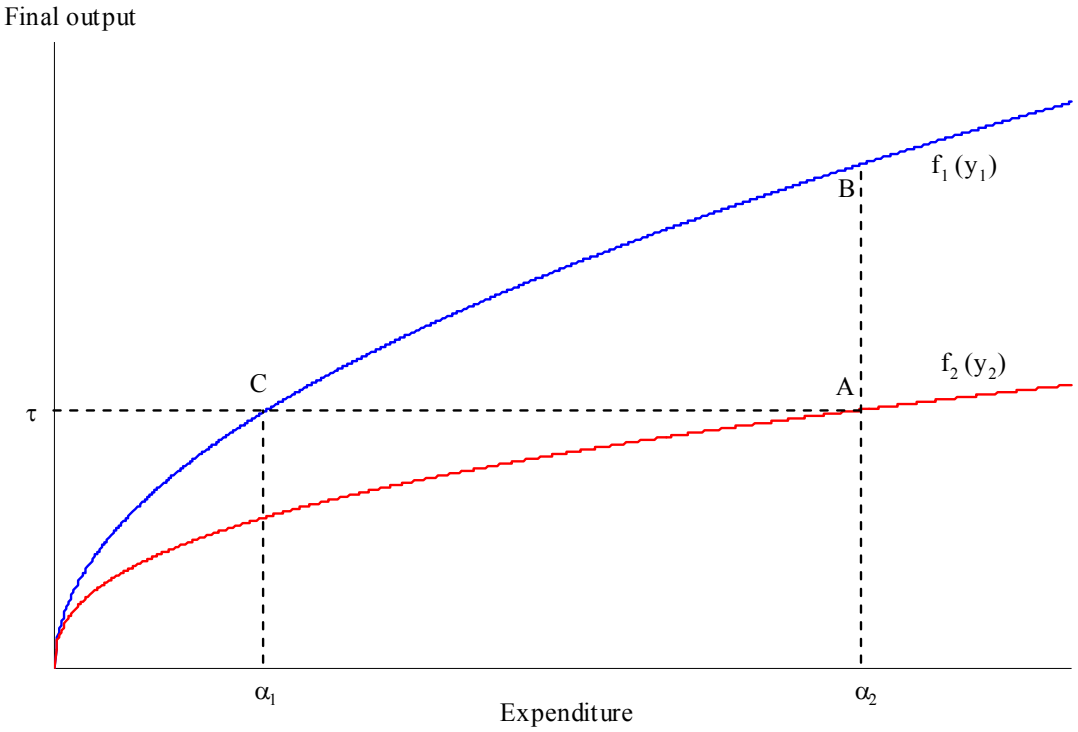
Inserting the minimum standard τ into equation (7) yields the corresponding minimum expenditures

$$
\alpha_j = g_j(\tau), \quad \left(j = 1,2,...,r\right). \tag{9}
$$

Note that positive marginal productivities imply that marginal costs are positive $\left(g_j'(\cdot) > 0 \forall j\right)$, and consequently that the minimum expenditures α_j increase as a function of τ. Moreover, a low-ability group needs more resources than a high-ability group to produce a given level of final output. Therefore minimum expenditures derived from a joint final output standard are decreasing as a function of target group ability. This result is displayed in Figure 3 in the case of two different target groups with production functions $f_1\left(y_1\right)$ and $f_2\left(y_2\right)$. It is assumed that target group 1 has higher

ability than target group 2. In order to meet the joint standard for final output, the derived minimum expenditure for group 2 is larger than for group 1 $(\alpha_2 > \alpha_1)$. Thus group 2 is compensated for its lower ability to produce final output.

Figure 3. Minimum expenditures derived from a joint minimum standard for final output



By inserting equation (9) into (6) and differentiating with respect to τ, the derivative of target group *j*'s share of expenditure is given by

(10) $$\frac{ds_j}{d\tau} = \frac{s_j}{\tau}\left[\varepsilon_j^{-1}(\tau) - \sum_{j=1}^{r} s_j \varepsilon_j^{-1}(\tau)\right], \quad (j = 1,2,...,r).$$

According to equation (10) a marginal increase in τ will benefit target groups with higher than average cost function elasticities evaluated at the minimum standard τ. An interesting question is under what conditions a marginal increase in τ will result in a redistribution of expenditure from high-ability groups to low-ability groups.

Proposition 2. Assume that minimum expenditures are derived from a joint minimum standard for final output τ, and that condition 2 (income elasticities equal to unity) is satisfied. Then an increase in τ will induce a redistribution of expenditure towards low-ability groups provided that

(i) The relationships between different production functions satisfy strong technological dominance, and

(ii) the elasticities of all production functions are non-increasing as a function of expenditure.

Referring to Figure 3 it is relevant to compare elasticities in points A, B and C. From the strong technological dominance condition it follows that $\varepsilon_2(\alpha_2) < \varepsilon_1(\alpha_2)$. From the non-increasing elasticity condition it follows that $\varepsilon_1(\alpha_2) \le \varepsilon_1(\alpha_1)$. Consequently, $\varepsilon_2(\alpha_2) < \varepsilon_1(\alpha_1)$ and $\varepsilon_2^{-1}(\tau) > \varepsilon_1^{-1}(\tau)$. Conditions (i) and (ii) imply that the elasticity of the cost function evaluated at τ is decreasing with target group ability. Thus one can infer that a marginal increase in τ will benefit low-ability groups at the expense of high-ability groups.

Another interesting result for minimum expenditures that are derived from a joint minimum standard for final output as in equation (9) is that one may obtain different EO allocations.

Proposition 3. Assume that minimum expenditures are derived from a joint minimum standard for final output τ, and that condition 1 (exhaustive minimum expenditures) is satisfied. Then the solution of the linear expenditure system satisfies equality of outcome (EO).

By combining equation (9) and condition 1 it follows that τ depends on *y* as defined by the relationship $\sum_{j=1}^{r} z_j g_j\left(\tau^{EME}\right) = y$, where τ^{EME} is the level of the minimum standard for final output that is compatible with exhaustive minimum expenditures. By implicit differentiation one may show that τ^{EME} is increasing in *y*.

4.3. Equality of marginal cost

Public authorities may also determine minimum expenditures to obtain different EMC solutions. To see this, assume that a standard η is imposed for the first derivative of the cost function.

Condition 5. *Joint minimum standard for marginal cost*. In this case the government authority requires that $g_j'\left(x_j\right) \geq \eta \forall j$, which is equivalent to $f_j'(y_j) \leq 1/\eta \,\forall j$, where η is the minimum standard for marginal cost.

Minimum expenditures corresponding to condition 5 can be deduced from the inverse marginal productivity function

$$y_j = h_j\left(f_j'\left(y_j\right)\right) = f_j'^{-1}\left(f_j'\left(y_j\right)\right), \quad \left(j = 1,2,...,r\right), \tag{11}$$

Where $h_j(\cdot)$ is the inverted function of the marginal productivity of target group *j*. Since the second derivative of the production function is negative, it follows that the first derivative is a one-to-one function, and consequently the inverse functions in equation (11) exist. Inserting the standard η into equation (11) yields the corresponding minimum expenditures

$$\alpha_j = h_j\left(\frac{1}{\eta}\right), \quad \left(j = 1,2,...,r\right). \tag{12}$$

Note that concave production functions imply that $h_j'(\cdot) < 0$, and consequently that minimum expenditures α_j are increasing as a function of η. Moreover, technological dominance and concave production functions imply that the minimum expenditures in equation (12) are increasing with target group ability.

An interesting result for minimum expenditures that are derived from a joint minimum standard for marginal cost as in equation (12) is that one may obtain different EMC allocations.

Proposition 4. Assume that minimum expenditures are derived from a joint minimum standard for marginal cost η, and that condition 1 (exhaustive minimum expenditures) is satisfied. Then the solution of the linear expenditure system satisfies equality of marginal cost (EMC).

By combining equation (12) and condition 1 it follows that η depends on *y* as defined by the relationship $\sum_{j=1}^{r} z_j h_j\left(1/\eta^{EME}\right) = y$, where η^{EME} is the level of the minimum standard for marginal cost that is compatible with exhaustive minimum expenditures. By implicit differentiation one may show that η^{EME} is increasing in *y*.

4.4. Compromises between allocation principles

So far, the discussion has demonstrated that ET, EO as well as EMC allocations can be obtained by changing the distribution of minimum expenditures. Maximization of distributional preferences for a given budget constraint includes the three benchmark allocation principles as special cases. Thus, the community preference model provides a flexible framework for analyzing different types of distributional preferences. Moreover, minimum expenditures could be determined as a compromise between any of the three benchmark solutions. For instance, assume that minimum expenditures are determined by

$$\alpha_j = \lambda g_j\left(\tau^{EME}\right) + (1-\lambda) h_j\left(\frac{1}{\eta^{EME}}\right), \quad (j = 1,2,...,r), \tag{13}$$

where λ is the weight that is put on equity in the equity-productivity tradeoff $(0 \le \lambda \le 1)$. The EO solution is obtained if $\lambda = 1$, and the EMC solution is obtained if $\lambda = 0$. By inserting equation (13) into equation (6) and differentiating with respect to λ, the derivative of target group j's budget share is given by

$$\frac{ds_j}{d\lambda} = s_j\left[\frac{g_j\left(\tau^{EME}\right) - h_j\left(1/\eta^{EME}\right)}{\lambda g_j\left(\tau^{EME}\right) + (1-\lambda) h_j\left(1/\eta^{EME}\right)} - \sum_{j=1}^{r} s_j \frac{g_j\left(\tau^{EME}\right) - h_j\left(1/\eta^{EME}\right)}{\lambda g_j\left(\tau^{EME}\right) + (1-\lambda) h_j\left(1/\eta^{EME}\right)}\right], \tag{14}$$
$$(j = 1,2,...,r).$$

Under the assumption of technological dominance $g_j\left(\tau^{EME}\right)$ is decreasing in ability and $h_j\left(1/\eta^{EME}\right)$ is increasing in ability. Consequently a marginal increase in λ will result in a redistribution of resources from high-ability groups to low-ability groups.

4.5. Empirical applicability

The problem of estimating different target group production functions may face difficulties due to data limitations. The data that are required include not only final outputs (or service outcomes) on the level of target groups or individual recipients, but also measurement of how expenditures are allocated on target groups by the public service producer.

Since output data are not available on a regular basis, one might use the model discussed above to provide empirical evidence on governmental priorities across ability types. To estimate the linear expenditure system it suffices to have access to expenditure data and the population distribution on target groups. Moreover, when the service producing agent is responsible for more than one type of

service, it is not required to know the distribution of expenditures on target groups. The linear expenditure system extended to account for spending on several service sectors is given by

$$
\begin{aligned}
& z_j y_{ij} = \alpha_{ij} z_j + \beta_{ij}\left(y - \sum_{i=1}^{k}\sum_{j=1}^{r} \alpha_{ij} z_j \right), \quad (i = 1,2,...,k), \quad (j = 1,2,...,r), \\
& \sum_{i=1}^{k}\sum_{j=1}^{r} \beta_{ij} = 1,
\end{aligned} \tag{15}
$$

where α_{ij} is minimum expenditure per person in target group *j* in service sector *i*, β_{ij} is the marginal budget share for target group *j* in sector *i*, and y_{ij} is expenditure per person in target group *j* provided by service sector *i*. Assume that y_{ij} is not reported in accounting data. Nevertheless, minimum expenditures can be identified by imposing the following multiplicative structure on the marginal budget shares

$$
\begin{aligned}
& \beta_{ij} = \beta_i \theta_{ij}, \quad (i = 1,2,...,k), \quad (j = 1,2,...,r), \\
& \sum_{i=1}^{k} \beta_i = 1, \\
& \sum_{j=1}^{r} \theta_{ij} = 1, \quad (i = 1,2,...,k),
\end{aligned} \tag{16}
$$

where β_i is the marginal budget share for service sector *i*, and θ_{ij} is the share of sector-specific discretionary incomes in service sector *i* that is allocated to target group *j*. Inserting equation (16) into equation (15) and aggregating across target groups within each service sector leads to the linear expenditure system

$$
y_i = \sum_{j=1}^{r} \alpha_{ij} z_j + \beta_i \left(y - \sum_{i=1}^{k}\sum_{j=1}^{r} \alpha_{ij} z_j \right), \quad (i = 1,2,...,k), \tag{17}
$$

where $y_i = \sum_{j=1}^{r} z_j y_{ij}$ is expenditure per person in the population provided by service sector *i*, which is supposed to be reported in accounting data. Due to the additive properties of the linear expenditure system, it is thus possible to estimate minimum expenditures for different target groups and in different service sectors. Aaberge and Langørgen (2003) used this approach to estimate the distribution of minimum expenditures on the basis of expenditure data for Norwegian local governments. For instance, they found that mentally disabled children receive a fairly high minimum expenditure in primary schools, and moreover that the mentally disabled above school-age receive a rather high minimum standard in care for the elderly and disabled. In this case it is plausible to assume

that the mentally disabled are technologically dominated by other target groups. Consequently the results may suggest that equality of outcome is given a significant positive weight in the equity-productivity trade-off. However, this weight may as well turn out to fall considerably below 1, as is indicated by higher service outcomes in high-ability groups.[6] Thus the present paper provides an improved foundation for interpreting minimum expenditures.

Another interesting observation is that central governments frequently impose minimum standards on local governments that are pertaining to specific target groups. For instance, in Norway a national reform in the early 1990s introduced extended legal rights for the mentally disabled that receive local public services. Another example is that children below 3 years of age are entitled to be twice as much staffed as older children in local public day-care centers. Such regulations may suggest that the central government wants to influence the equity-productivity trade-off in the production of local public services. Moreover, since regulations are frequently imposed in the form of minimum standards, this also justifies that the linear expenditure system provides an attractive framework for interpreting the behavior of government decision-makers.

A limitation of the model in equation (17) is that marginal budget share parameters for different target groups (θ_{ij} -parameters) are not identified. Corresponding to condition 2 and equation (6) one can assume that $\theta_{ij} = \alpha_{ij} z_j \Big/ \sum_{j=1}^{r} \alpha_{ij} z_j$. In this case the allocation of discretionary incomes on target groups is proportional to target group shares of minimum expenditures.[7] Moreover, the minimum expenditures express the general priority that is given to different target groups.

However, if the central government is determining minimum expenditures through extensive use of regulations and minimum standards, it is possible that minimum expenditures and discretionary incomes are allocated differently on target groups. For instance, local governments may have distributional preferences that differ from the distribution of minimum expenditures imposed by the central government. Consequently, local governments may use discretionary incomes to depart from the distributional policy imposed as central government regulations. In that case, the estimated minimum expenditures provide information on central government priorities, while the spending priorities resulting from local government decisions may differ from central government priorities, especially in municipalities with large per capita discretionary incomes. To test such hypotheses it is

[6] For instance, test scores of children in Norway are positively related to the education level of parents.

[7] Note that the specification in equation (17) does not restrict income elasticities to equal one on the level of service sectors.

required to observe the distribution of spending on target groups, which would allow identification of models like equation (5) and equation (15). Provided that this type of data is available, this may become an issue for future research.

5. Conclusion

This paper discusses the allocation of public services on target groups. It is demonstrated that production functions for the transformation of public services into service outcomes can be used to compare the production process for different target groups. Thus the paper introduces production functions that are target group specific. Production of final output (or service outcomes) depends on how much expenditure each target group receives and on the ability of each target group to produce final output. The comparison between different production functions highlights that there exists a trade-off between equity and productivity, provided that the conditions for technological dominance are fulfilled. In that case equity considerations imply a high priority of low-ability groups, whereas productivity considerations imply a high priority of high-ability groups.

Evidence on governmental priorities across target groups can be obtained from estimation of a linear expenditure system. If governmental preferences are guided by equality of treatment (ET), the estimated minimum expenditures for different target groups should not differ significantly. The interpretation of unequal treatment depends on whether or not technological dominance can be assumed. If technological dominance is satisfied, one would expect that estimated minimum expenditures decrease with target group ability, provided that government preferences are guided by equality of outcome (EO). By contrast, one would expect that estimated minimum expenditures increase with target group ability, provided that government preferences are guided by equality of marginal cost (EMC). If technological dominance is not satisfied, public authorities are expected to give a high priority to target groups with a low initial endowment and a high marginal productivity in the production of service outcomes.

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