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Abstract:
This paper analyses the general equilibrium relationship between increases in tax financed public employment and total employment, emphasizing one income effect: Reallocating employment from the private to the public sector reduces non-labour income in the form of profits distributed to workers, since there are no profits in public sectors. This may cause a positive general equilibrium relationship between total employment and tax financed public employment, even if the uncompensated wage elasticity of labour supply is positive. Such a positive relationship is consistent with the stylized facts in generous welfare states such as Norway and Sweden. Precise conditions for a positive general equilibrium relationship are derived within the simplest possible model. It is also shown that if such a positive relationship exists, it will be stronger the higher is public employment. This mechanism turns out to be crucial when explaining the employment effect of an increase in tax financed public employment generated by a realistic CGE model of the Norwegian economy.

Keywords: Taxation, Labour supply, General equilibrium effects

JEL classification: H20, H31

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Sammendrag

Paperet analyserer likevektssammenhengen mellom en gitt økning i skattefinansiert offentlig sysselsetting og total sysselsetting, med hovedvekt på en bestemt inntektseffekt: Når arbeidskraft omallokeres fra privat til offentlig sektor, reduseres den delen av arbeidstilbydernes arbeidsuavhengige inntekt som kommer i form av overskudd, siden det ikke er overskudd i offentlig produksjon. Denne effekten kan føre til at likevektssammenhengen mellom total sysselsetting og skattefinansiert offentlig sysselsetting blir positiv, også i det tilfellet der den ukompenserte arbeidstilbudselastisiteten med hensyn på lønn er positiv. En slik positiv likevektssammenheng er konsistent med det man har observert i generøse velferdsstater som for eksempel Norge og Sverige. Paperet utleder presise betingelser for en positiv likevektssammenheng innenfor en enklest mulig modell. Det viser også at dersom likevektssammenhengen er positiv, vil den være sterkere desto høyere den offentlige sysselsettingen er. Likevektssammenhengen viser seg å være viktig for å forklare sysselsettings-effektene når man bruker en realistisk CGE modell av norsk økonomi til å simulere virkningene av en eksogen økning i skattefinansiert offentlig sysselsetting.

1 Introduction

In highly plausible scenarios for the next decades most OECD countries and several other countries face a strong growth in the demand for health services and old-age care, due to i.a. population ageing, further income growth, and a modest potential for input saving productivity growth in long-term care, see e.g. OECD (2013). In most countries these services are mainly provided by the government sector and tax financed. On average, general taxes and social security contributions financed nearly three-quarter of all health spending in 2008 in the OECD countries, see OECD (2010). Maintaining this high degree of government responsibility, these countries face a substantial acceleration of the growth in public employment, which commands growth in the tax burden.

The subject of this paper is the effect of tax financed growth in public employment on total employment. In addition to being a direct political goal, a high employment rate is the ultimate financial base of public welfare. In particular, the sustainability of welfare states, in which tax rates and and social security benefits are relatively high, relies on high employment rates, see e.g. Andersen (2009). So far the ability of the Scandinavian welfare states to combine high tax rates on labour with high employment ratios, have been relatively successful. Some scholars assess this success as striking, see e.g. Andersen (2009).

The high correlation between tax rates, public welfare and employment in the Scandinavian countries is particularly striking when it is compared with the analysis in Prescott (2004). He concludes that differences in tax rates can account for most of the differences in supply of man hours per capita between the US and some larger European countries. To a large extent Ohanian, Raffo and and Rogerson (2006) support Prescott's conclusion. On the other hand, it has been questioned and critisized by Ljungquist and Sargent (2007) and Andersen (2009). In particular, Prescot relies implicitely on a wage elasticity of labour supply which is much higher than what is typically found in the vast empirical labour supply literature, surveyed in e.g. Evers, De Moij and van Vuuren (2005) and Meghir and Phillips (2008). The majority of empirical studies indicate that a decrease in the after-tax real wage rate has a relatively small negative effect on total labour supply at the intensive margin, reflecting a weak dominance of the substitution effect over the income effect.

Most likely, employment ratios and labour supply are the result of a large number of interacting mechanisms in addition to taxes. For example, Rogerson (2007) stresses that international comparisons of employment ratios and taxes should take into account how the government spends the tax revenues. The negative effect of higher tax rates on labour supply is reinforced if the tax revenue finances transfers that work as subsidies of leisure. This effect has played a major role in the recent wave of reforms of the public old-age pension systems, which aim to strengthen labour supply incentives. On the other hand, the negative labour supply effect of higher tax rates is mitigated, and may be reversed, if the additional tax revenue finances increased provision of public services that are substitutes for household production, see e.g. Lindbeck and Parameswar (1990). Care for children and the elderly are good examples in this respect.

This paper analyses one particular mechanism, which contributes to a positive relationship between tax financed public employment and total employment. More precisely, it accounts for general equilibrium modifications of the income effect on labour supply: Increasing tax financed employment in non-profit government sectors reduces capital income distributed to workers by crowding out private production, which generates normal returns to capital. Together with the standard income effect of raising labour income tax rates, the reduction in this kind of non-labour income stimulates, *cet. par*, labour supply. Specifically, the general equilibrium relationship between total employment and tax financed government employment may be positive, even if the uncompensated wage elasticity of labour supply is positive. To my knowledge this equilibrium effect is not recognized in the literature which analyses the relative strength of the counteracting substitution and income effects on labour supply induced by a higher labour income tax.

This idea is explored within the simplest possible model. The analysis is confined to general equilibria, where changes in total employment equal labour supply responses. Thus, it disregards Keynesian situations in which budget neutral increases in government spending and taxation may stimulate aggregate demand and employment. The relevance of the analysis is therefore likely to be greater for the long-term rather than the short-term effects.

Section 2 presents the theoretical model used to analyse the determinants of total employment. In particular, it clarifies the precise conditions for a positive equilibrium relationship between tax financed public employment and total employment. These conditions show that the combination of a positive uncompensated wage elasticity of labour supply and a positive equilibrium effect on labour supply of raising tax financed public employment cannot be ruled out as unrealistic. The analysis also examines the nature of the non-linearity of the employment effect of higher public employment. It is found that if this effect is positive, it will be stronger the higher is the initial public employment. These results are interesting when one tries to explain the combination of high employment and high tax rates in the Scandinavian welfare states, but the theoretical analysis cannot say anything about their empirical relevance. However, Section 3 demonstrates that the analysed mechanisms at least play an important role when one uses a large and realistic CGE model of the Norwegian economy to simulate the employment effects of higher public employment. Section 4 adds some discussion to a summary of the conclusions.

2 Theoretical analysis

2.1 The model

Consider a small open economy in which one competitive private industry use labour (L_P) and capital (K) to produce one good which can be consumed or invested as capital. The product price and the interest rate, r, are exogenously determined in the world markets. The product price is normalized to unity. Abstracting from depreciation and taxes, the capital cost equals r. The technology exhibits constant returns to scale, and the marginal cost function is $c(P_L, r)$, where P_L is the labour cost. Perfect competition implies $1 = c(P_L, r)$, which determines P_L. r (as well as other variables affecting P_L), will be constant in the analysis, so the measure of labour units is chosen so that $P_L = 1$. The time index is suppressed except when dynamics are explicit. Specifically, K_{-1} is the productive capital stock prior to investment in the period we consider. Investments take place instantaneously at the start of each period. Then the optimal input of labour and capital per produced unit will be constant and equal to $a_L = \frac{\partial c(P_L,r)}{\partial P_L}$ and $a_K = \frac{\partial c(P_L,r)}{\partial r}$respectively. The demand for labour and capital from the private industry can then be written

$$L_P = a_L \left(C + K - K_{-1}\right), \tag{2.1}$$

$$K = a_K \left(C + K - K_{-1}\right), \tag{2.2}$$

where C is private consumption, and $K - K_{-1}$ is investment in the period we consider.

Let L_G denote government employment used to produce public services, that do not enter the utility function. The government expenditures equals the wage bill, which is financed on a pay-as-you-go basis by a tax on labour only. More precisely we assume that this tax is a payroll tax, and that t is the payroll tax rate. The government budget constraint is then $(1+t)\,L_G = t\,(L_P + L_G)$, which can be written

$$t = \frac{L_G}{L_P}. \tag{2.3}$$

A representative consumer faces the real wage rate, $W = (1+t)^{-1}$, and an exogenous time endowment T. He maximizes a utility function of C and leisure F, subject to the budget constraint $C = \frac{L}{1+t} + rK$ and the time constraint $T = L + F$, where L is total labour supply. The capital income rK is the only type of non-labour income, I. It is perceived as given by the consumer. We study equilibria where

$$L = L_P + L_G. \tag{2.4}$$

We assume that the utility function takes the CES form. Given the price normalizations, the optimal ratio between consumption and leisure becomes

$$\frac{C}{T-L} = b\,(1+t)^{-\sigma}\,, \tag{2.5}$$

where b is an exogenous preference parameter, and σ is the elasticity of substitution between consumption and leisure. These five equations determine the endogenous varaiables L_P, t, K, L and C. The appendix shows that the implicit equilibrium solution for L can be written in the following reduced form:

$$L - L_G = a_L b \left(1 - \frac{L_G}{L}\right)^{\sigma} (T - L) + a_K \left[L - L_{-1} - (L_G - L_{G,-1})\right] \tag{2.6}$$

This solution for L captures both the income effect of government employment, as well as the substitution effect on labour supply caused by revenue neutral adjustments of the payroll tax rate. A key feature in this model is that it also includes the feedback on capital income resulting from changes in private employment.

2.2 Stationary effects

We shall first confine the discussion of the equilibrium relationship between L_G and L to stationary time paths. All exogenous variable except L_G are constant, whereas L_G is increased to a new permanent level in period 0. As capital does not depreciate, there will be no investment except for period 0, when K_{-1} is predetermined. We will first examine the stationary effects, obtained in period 1, 2, ..., before we characterize the short term effect obtaiend in period 0.Since $K - K_{-1} = L - L_{-1} = 0$ both before and after the shift in period 0, the long run solution for L becomes

$$L - a_L b \left(\frac{L - L_G}{L}\right)^{\sigma} (T - L) = L_G. \tag{2.7}$$

The left hand side may be interpreted as the excess labour supply from the household, i.e. total labour supply less the labour demand needed to produce the household consumption.

It is easily seen that a proportional shift in T and L_G results in the same proportional change in L. Define e_G as the long run elasticity of L with respect to L_G. The Appendix shows that (2.7) implies

$$e_G = \frac{1 - \sigma}{H - \sigma}. \tag{2.8}$$

Here $H = H(L_G, L, T) = \frac{L}{L_G}\left(\frac{T - L_G}{T - L}\right) > \frac{L}{L_G} > 1$ summarizes all information about the initial state of the economy that is relevant for e_G. The main purpose of this paper is to analyze the state dependency of e_G, especially how e_G depends on L_G. To get intuition on the role of the forces working through H, note that H^{-1} equals the the elasticity of L with respect to L_G, contingent on a constant ratio between consumption and leisure $\frac{C}{T-L} = b\left(\frac{L-L_G}{L}\right)^{\sigma}$, see Appendix. Recall from (2.5) that keeping $\frac{C}{T-L}$ constant implies that H^{-1} ignores the substitution effect caused by the change in t. Thus, H comprises the repercussions on the excess labour supply working through non-substitution effects. These are of two kinds. First, raising L by 1 percent, increases this excess labour supply by $\frac{L}{L_G}$ percent. Second, raising L by 1 percent reduces, *cet. par*, leisure by $\frac{L}{T-L}$ percent. Keeping $\frac{C}{T-L}$ fixed implies that consumption and a thereby private employment also decreases by $\frac{L}{T-L}$ percent. *Cet. par*, this raises the household excess labour supply by $\frac{L_P}{L_G}\left(\frac{L}{T-L}\right)$ percent. The total excess supply effect becomes $\frac{L}{L_G} + \frac{L_P}{L_G}\left(\frac{L}{T-L}\right) = \frac{L}{L_G}\left(\frac{T-L_G}{T-L}\right) = H$.

We are now able to discuss rigorously the properties of e_G. $e_G(\sigma, H)$ is decreasing in σ in the domain $\sigma > 0$, $\sigma \neq H$. $e_G(\sigma, H)$ is a hyperbola with horisontal asymptotes. $\lim_{\sigma \to 0} e_G = H^{-1}$ is the maximum asymptote for $0 < \sigma < H$. $\lim_{\sigma \to \infty} e_G = 1$ is the minimum asymptote for $\sigma > H$. Table 1 provides more information on how e_G depends on σ.

Table 1. Elasticity of employment wrt. public employment

	$\sigma \to H^+$	$\sigma \to H^-$	$\sigma = \frac{H+1}{2}$	$\sigma = 1$	$\sigma \to 0$
$\sigma - 1$	$+$	$+$	$+$	0	$-$
$\sigma - H$	$+$	$+$	$-$	$-$	$-$
e_G	∞	$-\infty$	-1	0	H^{-1}

Table 1 also demonstrates the following result, which can be directly derived from (2.8):

Proposition 1 *A marginal increase in L_G, financed by an increase in the payroll tax rate, will reduce total equilibrium employment when $H > \sigma > 1$, and increase total equilibrium employment when $\sigma > H$ or $0 < \sigma < 1$.*

In order to assess the plausibility of these situations, it is necessary to have intuition on the order of magnitude of H. H is determined once L_G is set, but it depends in a complex way on all exogenous variables and parameters through L, cf. (2.7). This dependency will be taken explicitly into account below. At this stage it is instructive to see that realistic outcomes of the model above suggest that H is much greater than unity. Assume for example that in a period of a day $T = 24 - 10 = 14$ hours can on average be allocated to leisure and work, and that $L_G = 2$. Moreover, assume that the parameters in the model, i.e. a_L, b and σ, imply $L = 7$. This implies $H = 6$ (and $L_P = 5$ and $t = 0.4$.) Consider then alternative situations in which L_G has been raised, combined with a change in model parameters so that L remains equal to 7. For example, raising L_G from 2 to 3.5 then implies $H = 2.1$, $L_P = 3.5$ and $t = 1$, which implies that the consumer wage rate equals 50 percent of the labour cost per hour.) Raising L_G further to $4\frac{2}{3}$ implies $H = 2$, $t = 2$ and $L_P = 2\frac{1}{3}$. An extreme case would be that $t = 3$, which implies $L_G = 5\frac{1}{4}$, $L_P = 1\frac{3}{4}$ and $H = 1\frac{2}{3}$. As pointed out in the introduction, σ is likely to be relatively close to unity. Henceforth, we will therefore confine the discussion to the realistic case where $H > \sigma$ (corresponding to the left arm of the hyperbola of e_G graphed as a function of σ). Then the only possibility for $e_G > 0$ is that $0 < \sigma < 1$, and the upper limit of e_G is $H^{-1} < 1$.

2.3 The link between the employment effect and individual behaviour

This section examines how the equilibrium elasticity e_G relates to the partial elasticities of the individual labour supply. Let $l\,(W, T, I)$ be the individual uncompensated labour supply function derived by maximizing utility with respect to C and F, taking W, T and $I = rK$ as given. Define $l_W\,(W, T, I)$ and $l_I\,(W, T, I)$ as the corresponding labour supply elasticities with respect to W and I. In the present model these elasticities take the form

$$l_W\,(W, T, I) = \left(\frac{T - L}{L}\right) [\theta_C\,(\sigma - 1) + \alpha_I] = \left(\frac{T - L}{L}\right) \theta_C \left(\sigma - \frac{WL}{WL + rK}\right) \tag{2.9}$$

$$l_I\,(W, T, I) = -\left(\frac{T - L}{L}\right) \alpha_I, \tag{2.10}$$

where $\theta_C = \frac{C}{WT+rK} = \frac{WL+rK}{WT+rK}$ is the optimal budget share of consumption in total expenditure, which equals total virtual income $WT + rK$. $\alpha_I = \frac{rK}{WT+rK}$ is the share of non-labour income in total virtual income. The second expression for l_W is derived by utilizing that $1 - \frac{\alpha_I}{\theta_C} = \frac{WL}{WL+rK}$. As is well kniown, the combination of no non-labour income and $\sigma = 1$, i.e. Cobb-Douglas preferences, implies $l_W = 0$, since the substitution effect excactly neutralizes the income effect. The existence of positive non-labour income increases *cet. par* the wage elasticity, because it reduces the weight of the income effect caused by the increase in the wage rate. With positive non-labour income $l_W \geq 0$ requires $\sigma \geq \frac{WL}{WL+rK} < 1$.

The Appendix shows that e_G can be written in terms of the individual labour supply elasticities as follows:

$$e_G = l_W \frac{\partial W}{\partial L_G} \frac{L_G}{W} + l_I \frac{\partial (rK)}{\partial L_G} \frac{L_G}{rK}, \tag{2.11}$$

when one takes into account that $\frac{\partial W}{\partial L_G} \frac{L_G}{W} = \frac{L_G}{L_P} (e_G - 1)$, and $\frac{\partial (rK)}{\partial L_G} \frac{L_G}{rK} = \frac{L}{L_P} e_G - \frac{L_G}{L_P}$ in this model. Thus, there are two reasons why e_G differ from the partial labour supply response to a given change in the wage rate induced by a change in t: Firstly, e_G takes into account that capital income will change by the same proportion as employment in the private industry. Secondly, e_G captures that the change in the wage rate will be endogenous, since the tax base is proportional to the employment in the private industry.

Proposition 2 *When non-labour income is positively related to labour income through equilibrium effects, and $\sigma = 1$, there is no equilibrium effect on total employment of an increase in tax financed government employment, whereas the individual uncompensated labour supply response to the partial decrease in the conumer real wage rate is negative.*

Proof. It follows directly from (2.9) that $l_W|_{\sigma=1} = \left(\frac{T-L}{L}\right) \alpha_I > e_O|_{\sigma=1} = \frac{\sigma-1}{\sigma-H} = 0$. □

2.4 Non-linearity and state dependence

This section analyses how e_G depends on the initial L_G. The insights from this analysis is important in order to see how L will respond to non-marginal increments in L_G. We shall confine the analysis to the most relevant parameter combinations, i.e. $0 < \sigma < H$.

Let $El_{L_G} e_G(L_G)$ be the elasticity of e_G with respect to L_G. Under this assumption we shall prove the following proposition:

Proposition 3 *$El_{L_G} e_G(L_G) > 0$, and sign $\left(\frac{\partial e_G}{\partial L_G}\right) = $ sign (e_G). Consequently, if $e_G > 0$, then L is progressively increasing in L_G, even if L_G is financed by a tax on labour income. Conversely, if $e_G < 0$, an increase in L_G further reduces e_G. Raising L_G increases the upper limit for e_G when $0 < \sigma < H$.*

Proof. $El_{L_O} e_o(L_O)$ can be written

$$El_{L_O} e_o(L_O) = -\left(\frac{H}{H-\sigma}\right) h_O, \tag{2.12}$$

where $h_o \equiv El_{L_O} H$ is the total elasticity of $H(L, L_O, T)$ with respect to L_O. h_O becomes

$$h_o = (e_O - 1) + \left(\frac{-L_O}{T - L_O}\right) - \left(\frac{-L}{T-L}\right) e_O = \left(\frac{T}{T-L}\right)\left(e_O - \frac{L}{L_O} H^{-1}\right). \tag{2.13}$$

The first statement in the proposition follows from the following implications: $0 < \sigma < H \Rightarrow e_O < H^{-1} < \frac{L}{L_O} H^{-1} \Rightarrow h_o < 0 \Rightarrow El_{L_O} e_o(L_O) > 0$. The sign of $\frac{\partial e_O}{\partial L_O}$then follows from the expression:

$$\frac{\partial e_O}{\partial L_O} = e_O \frac{El_{L_O} e_o(L_O)}{L_O}. \tag{2.14}$$

□

Thus, $|e_G|$ is increasing in L_G, and the effect is self-reinforcing. Recall that $e_G > 0$ when $0 < \sigma < 1$, and $e_G < 0$ when $1 < \sigma < H$. $h_G < 0$ also implies that H^{-1}, the upper limit for e_G when $0 < \sigma < H$, increases when L_G increases. However, it is not obvious that an equilibrium exists for any choice of L_G. Numerical experiments based on a plausibly parameterized version of the model above, demonstrate for example that a general equilibrium may not exist when L_G is raised beyond 50 percent of total employment when $\sigma = 1,5$. The reason is that the necessary tax rate then becomes so high that the representative individual will not demand - for leisure or for production of private consumption - the hours which has not employed in the public sector. The

Appendix describes the assumptions underlying this numerical experiment.

Intuition on the equilibrium relationship between public and total employment can be obtained through the following reasoning: For a given L, an exogenous rise in L_G equal to 1 percent disturbs the labour market equilibrium through two channels: 1) the increase in L_G creates an equally large excess labour demand; 2) the higher tax rate needed to finance the higher public employment induces a negative substitution effect on private consumption, which causes a fall labour demand from the private sector equal to σ percent. Consequently, prior to adjustment of L, raising L_G by 1 percent, increases the excess demand for labour from 0 to $1-\sigma$ percent of the initial L_G-level. If labour supply were increased so that L increased by 1 percent, would affect this imbalance by $H-\sigma$ percent of the initial L_G-level. Recall that H measures the net excess labour supply effect of a proportional decrease in leisure and private consumption. Ex post the increase in L_G, the additional labour supply will be employed in the private sector, thus contributing to reduce the tax rate. For concreteness, assume that $\sigma < 1 < H$. Then the labour market equilibrium can be restored by a sufficiently large increase in L, i.e. $e_G > 0$. The smaller is $H-\sigma$, the weaker is the fall in excess labour demand for a given increase in L, and the necessary employment effect has to be stronger.

Recall that $\sigma < H \Rightarrow h_G < 0$, because a higher L_G implies that both a given percentage increase in L and the reduction of leisure (proportional to the reduction of private consumption) represents a relatively smaller contribution to rebalancing the labour market. Measured in percent of the initial L_G-level, these effects are, respectively, $\frac{L}{L_G}$ and $\frac{L_P}{L_G}\left(\frac{L}{T-L}\right)$. They add up to H: $\frac{L}{L_G}+\frac{L_P}{L_G}\left(\frac{L}{T-L}\right)=\frac{L}{L_G}\left(\frac{T-L_G}{T-L}\right)=H$. This explains why a given increase in L_G reduces H, which in turn commands a greater increase in L in order to restore labour market equilibrium than in the case with a smaller initial L_G.

2.5 Short run effects

Since K_{-1} is predetermined in period 0 when L_G increases, investments will temporarily deviate from 0 in this period. Consequently, the employment effect in this period differs from the stationary effect dicussed above. We confine the analysis to marginal changes based on a linearization of the equilibrium condition for the labour market (2.6) around a stationary equilibrium in period -1. We have $L_0 - L_{-1} = dL_0$, $L_{G,0} - L_{G,-1} = dL_{G,0}$. Let $f(L, L_G) \equiv a_L b \left(1 - \frac{L_G}{L}\right)^{\sigma} (T - L)$ denote the non-linear term in (2.6). A linear approximation of the change in L between the periods -1 and 0 yields $df_0 = \frac{\partial f_{-1}}{\partial L} dL_0 + \frac{\partial f_{-1}}{\partial L_G} dL_{G,0}$. Here $f_{-1} = f(L_{-1}, L_{G,-1}) = L_{-1} - L_{G,-1}$, and the partial derivatives are $\frac{\partial f_{-1}}{\partial L} = \left[\sigma\left(\frac{L_{G,-1}}{L_{-1}-L_{G,-1}}\right) - \left(\frac{L_{-1}}{T-L_{-1}}\right)\right]\frac{f_{-1}}{L_{-1}}$ and $\frac{\partial f_{-1}}{\partial L_G} = -\sigma\left(\frac{L_{G,-1}}{L_{-1}-L_{G,-1}}\right)\frac{f_{-1}}{L_{G,-1}}$. The Appendix shows how these expressions can be used to obtain the short run equilibrium relationship between marginal changes in L and L_G respectively:

$$\left(\frac{L_{G,-1}}{L_{-1}}\sigma - \frac{T - L_{G,-1}}{T - L_{-1}} + a_K\right) dL_0 = (\sigma - 1 + a_K)\, dL_{G,0}. \tag{2.15}$$

The corresponding short run elasticity of L with respect to L_G, contingent on a budget neutral payroll tax rate adjustment, can be written:

$$e_{G,} = \frac{dL_0}{dL_{G,0}}\frac{L_{G,-1}}{L_{-1}} = \frac{\sigma - 1 + a_K}{\sigma - H_{-1} + \frac{L_{-1}}{L_{G,-1}}a_K}, \tag{2.16}$$

where $H_{-1} = \frac{L_{-1}}{L_{G,-1}}\left(\frac{T-L_{G,-1}}{T-L_{-1}}\right)$. $e_{G,0}$ degenerates to the corresponding stationary elasticity, e_G, when $a_K = 0$, which is equivalent with no investments, when the initial H in (2.8) equals H_{-1}. Compared with the corresponding long run elasticity, e_G, the numerator in $e_{G,0}$ is increased by the input share of capital, a_K, whereas the denominator is increased by $\frac{L_{-1}}{L_{G,-1}}a_K$. In order to compare the short run and the long run elasticities, we calculate

$$e_{G,0} - e_G = \frac{\sigma - 1 + a_K}{\sigma - H_{-1} + \frac{L_{-1}}{L_{G,-1}}a_K} - \frac{\sigma - 1}{\sigma - H_{-1}} = \frac{a_K\left(1 - e_0\frac{L_{-1}}{L_{G,-1}}\right)}{\sigma - H_{-1} + \frac{L_{-1}}{L_{G,-1}}a_K} \tag{2.17}$$

Thus, $e_{G,0} > e_G$ if $e_G < \frac{L_G}{L} \wedge \sigma - H_{-1} + \frac{L_{-1}}{L_{G,-1}}a_K > 0$or if $\sigma - H_{-1} + \frac{L_{-1}}{L_{G,-1}}a_K < 0 \wedge e_G > \frac{L_G}{L}$. Is this likely? Confining the discussion to the most realistic case where $0 < \sigma < H$. In this case it was shown above that $e_G < H^{-1} = \frac{L_G}{L}\left(\frac{T-L}{T-L_G}\right) < \frac{L_G}{L}$, so the numerator in (2.17) is positive. Thus, in this case $e_{G,0} > e_G$ if the capital share a_K and/or the employment share of the private sector $\frac{L}{L_G} = 1 + \frac{L_P}{L_G}$sufficiently large to make

$\sigma - H_{-1} + \frac{L_{-1}}{L_{G,-1}} a_K > 0$. Note that in this case the short run elasticity may be positive whereas the long run elasticity is negative. This is the case if $\sigma - 1 + a_K > 0 > \sigma - 1$. Moreover, unless $\frac{L_{-1}}{L_{G,-1}} a_K$ is "too large", the denominator in $e_{G,0}$ will be closer to 0 than the negative denominator in e_G, and the short run employment response may be much greater in absolute value than the long run response.

The expression for $e_{G,0}$ implies the following proposition:

Proposition 4 *A marginal increase in a_K from 0 implies $\left.\frac{\partial e_{G,0}}{\partial a_K}\right|_{a_K=0} = 1 - \frac{L_{-1}}{L_{G,-1}} e_{G,0} = 1 - \frac{L_{-1}}{L_{G,-1}} e_G$. Thus, if $e_G < 0 \Leftrightarrow H > \sigma > 1$, then $\left.\frac{\partial e_{G,0}}{\partial a_K}\right|_{a_K=0} > 1$. Therefore $e_G < e_{G,0} < 0$ for at least small values of a_K, i.e. the decline in L is smaller in the short run than in the long run. If $\sigma = 1 \Leftrightarrow e_G = 0$, $e_{G,0} = \frac{a_K}{\frac{L_{-1}}{L_{G,-1}}\left(a_K - \frac{T-L_{G,-1}}{T-L_{-1}}\right)} = 0$, and $\left.\frac{\partial e_{G,0}}{\partial a_K}\right|_{a_K=0} = 1$. When $a_K > 0$, the sign of $e_{G,0}$ depends on the sign of $\frac{T-L_{G,-1}}{T-L_{-1}} - a_K$. $|e_{G,0}| \to \infty$ when $\frac{T-L_{G,-1}}{T-L_{-1}} \to a_K$. If $0 < e_G < 1 \Leftrightarrow 1 > \sigma > 0$, $e_{G,0}$ may become negative with a high absolute value, since a higher a_K reduces the absolute value of the initially negative denominator.*

Proof.

$$\left.\frac{\partial e_{O,0}}{\partial a_K}\right|_{a_K=0} = \frac{\sigma - H_{-1} - (\sigma - 1)\frac{L_{-1}}{L_{O,-1}}}{\sigma - H_{-1}} = 1 - \frac{L_{-1}}{L_{O,-1}}\left(\frac{1-\sigma}{H_{-1} - \sigma}\right) = 1 - \frac{L_{-1}}{L_{O,-1}} e_{O,0}, \quad (2.18)$$

since $e_{O,0} = \frac{1-\sigma}{H_{-1}-\sigma}$ when $a_K = 0$. It follows that $\left.\frac{\partial e_{O,0}}{\partial a_K}\right|_{a_K=0} > 1$ when $e_{O,0} < 0$, and $\left.\frac{\partial e_{O,0}}{\partial a_K}\right|_{a_K=0} = 1$ when $e_{O,0} = 0$. This proves the first two parts of the proposition. The last part follows follows directly from the ambiguity of the sign of $\left.\frac{\partial e_{O,0}}{\partial a_K}\right|_{a_K=0}$ when $e_{O,0} > 0$. □

3 Empirical relevance

One way to check the empirical relevance of the mechanisms discussed above is to simulate the effect on total employment of increasing public employment on a Computational General Equilibrium (CGE) model, which provides a realistic description of an actual economy, including the forces that affect total employment. There are numerous such CGE experiments. Here, we shall consider some of the results for the Norwegian economy presented in Holmøy, Kravik, Nielsen and Strøm (2009) (hereafter HKNS). HKNS simulate the effects of increasing the employment in the government Health and Care sectors on government finances and macroeconomic aggregates. All these simulation experiments were budget neutral through endogenous adjustments of the payroll tax rate. To this end HKNS employed a version of the so-called MSG6 model, which is a large scale CGE model of the Norwegian economy.

The MSG6 model is described in e.g. Heide, Holmøy, Lerskau and Solli (2004) and Holmøy and Strøm (2011). In the present context the most important features are: Supply and demand are balanced in all specified markets, including the labour market, producers and consumers behave rationally according to microeconomic theory, the government revenues and expenditures are described in detail, and there is an annual government budget constraint. The uncompensated wage elasticity of total labour supply is 0.1, which is in line with microeconometric estimates. Thus, the substitution effect slightly dominates the income effect of a partial change in the consumer real wage rate.

The CGE simulations show, see Figures 1 and 2:

1. An increase in public employment has a positive long run effect on total employment.

2. A stronger increase in public employment, magnifies the long run effect on total employment.

3. An increase in public employment has a negative short run effect on total employment.

[*Remark: Figure 1 and 2 about here.*]

In a rich model such as MSG6, there are several complex and interacting forces driving these results. However, systematic studies of the MSG6 reveal the crucial role is played by the profits from private industries and other types of endogenous non-labour income received by households, which is positively related to employment in the private sector. It turns out that the working of the MSG6 model correspond to the case discussed above, in which the uncompensated labour supply elasticity, l_W, is positive, whereas $\sigma < 1$. The theoretical analysis showed that that the long run equilibrium effect on total employment is positive in this case, and that the strength of this effect increases when L_G grows. It also showed why the short run employment effect can be negative, also when the long run effect is positive, i.e. the third simulated result listed above.

4 Discussion and conclusions

This paper has analysed one particular mechanism, which - to my knowledge - has not previously been recognized in the discussion of how total employment is affected by taxes used to finance publi emloyment. The premise for the basic idea is that production in government sectors typically does not generate profits, whereas private production earns profits in order to provide a competitive return to capital. Thus, when the increase in public employment crowds out employment in private firms, capital income received by households fall to the extent that capital and labour complementary inputs. This income effect adds to the income effect associated with a higher tax rate on labour income, and contributes to raise labour supply. The presence of this income effect allows a positive equilibrium relationship between public employment, the labour income tax rate and total employment. The paper has rigorously clarified the conditions for such a situations. It cannot be ruled out as unrealistic for theoretical reasons. The theoretical discussion also showed that a positive long run employment effect of a given increase in public employment can be progressive, i.e. self-reinforcing. These results describe the long run employment effect. The paper also shows that the short run effect may be much stronger and have a different sign due to the temporary effects on consumption caused by adjustments of the the capital stock.

The empirical relevance of these mechanisms remain unclear. However, they turn out to be crucial when explaining how increasing tax financed public employment affect total employment in a realistic CGE model of the Norwegian economy.

In order to have a sharp focus this paper has ignored several potentially relevant effects influencing how total employment depends on public employment. When discussing the employment effect og increasing the provision of public services, it is important to account for how these services enter the utility function. If they affect private utility, variation in these services translate into another effect on non-labour income. However, the credo of this paper is that this discussion can be separated from the one presented here.

Perhaps a more severe shortcoming of the discussion above is that it treats all consumers as identical, also with respect to capital income. In the real world capital income is much more unevenly distributed than labour income. In the extreme case where all capital income is received by pensioners or other non-working consumers, the mechanism discussed above has no empirical relevance. This would also be the case if all the capital income variation affected only those who have a completly inelastic labour supply behaviour. The importance of these points can only be clarified by further research. To the extent that they are relevant, it has important implications for disaggregation in empirical modeling of the determinats of aggregate employment in the long run.

Appendix: Analytical derivations

1. Eq. (2.6): The equilibrium solution of L

Inserting (2.3) and (2.5) into (2.1) yields: $L_P = a_L b \left(1 + \frac{L_G}{L_P}\right)^{-\sigma} (T - L) + a_L (K - K_{-1})$. (2.2) and (2.1) implies $K = \frac{a_K}{a_L} L_P$. The implicit reduced form solution for L follows by inserting these two expressions into (2.4):

$$
\begin{aligned}
L &= L_P + L_G \Leftrightarrow && (4.1)\\
L &= a_L (C + K - K_{-1}) + L_G \Leftrightarrow \\
L - L_G &= a_L \left(b (1+t)^{-\sigma} (T - L) + K - K_{-1}\right) \Leftrightarrow \\
L - L_G &= a_L b \left(1 + \frac{L_G}{L_P}\right)^{-\sigma} (T - L) + a_L \frac{a_K}{a_L} (L_P - L_{P-1}) \Leftrightarrow \\
L - L_G &= a_L b \left(\frac{L - L_G}{L}\right)^{\sigma} (T - L) + a_K \left[L - L_{-1} - (L_G - L_{G,-1})\right] && (4.2)
\end{aligned}
$$

2. Eq. (2.8): The long run elasticity of L with respect to L_G

Logarithmic differentiation of (2.7) and setting $\frac{dL_G}{L_G} = 1$, implies

$$
\begin{aligned}
\frac{L}{L_G}e_G - \frac{L_P}{L_G}\left[\sigma\left(\frac{L}{L_P}e_G - \frac{L_G}{L_P} - e_G\right) + \left(\frac{-L}{T-L}\right)e_G\right] &= 1 \Leftrightarrow \qquad (4.3)\\
\frac{L}{L_G}e_G - \sigma\left(e_G - 1\right) + \frac{L_P}{L_G}\left(\frac{L}{T-L}\right)e_G &= 1 \Leftrightarrow\\
\left[\frac{L}{L_G}\left(1 + \frac{L_P}{T-L}\right) - \sigma\right]e_G &= 1-\sigma \Leftrightarrow\\
e_G &= \frac{1-\sigma}{\frac{L}{L_G}\left(\frac{T-L_G}{T-L}\right) - \sigma} = \frac{1-\sigma}{H-\sigma}.
\end{aligned}
$$

3. The interpretation of $H^{-1} = \frac{L_G}{L}\left(\frac{T-L}{T-L_G}\right)$

Logarithmic differentiation of $L - L_G = k\left(T-L\right)$, where k is the constant ratio between consumption and leisure $\frac{C}{T-L}$, implies when setting $\frac{dL_G}{L_G} = 1$: $El_{L_G}L|_{C=kF} = \frac{L_G}{L(1+k)} = \frac{L_G}{L}\frac{1}{1+\frac{L-L_G}{T-L}} = \frac{L_G}{L}\left(\frac{T-L}{T-L_G}\right) = H^{-1}$. Thus, H^{-1} is the elasticity of L w.r.t. L_G, contingent on a constant ratio between consumption and leisure.

4. Numerical experiments

The numerical version of the theoretical model is based on the following assumptions: All prices, or rather price indices, have been normalised to unity in the initial equilibrium. For transparency variables are measured per capita per day. $T = 24 - 10 = 14$ after deduction of 10 hours per day which cannot be freely allocated. In the base case $\sigma = 1,1$,which is in line with economtric estimates of aggregate labour supply in Norway. Moreover, in the initial equilibrium $L = F = 7$, $L_G = 2$, and $L_P = 5$. Then $t = 2/5$. $a_L = 3/4$ and $a_K = 1 - a_L = 1/4$. The sum of private consumption and net investments is set to 1000 NOK per capita per day, of which $P_L L_P = 750$ and $rK = 250$. It follows that $P_L = 150$. Given these assumptions, a stationary equilibrium with $C = 1000$ is consistent with $b = 4/3$.

Solving this numerical model for different assumptions about tax financed L_G and σ confirm the theoretical results: When $\sigma = 1$, L, F/L_P, and accordingly F/C, is invariant to L_G. When $\sigma < 1$, L is progressively increasing in L_G, and the effect is stronger the lower is σ. When $\sigma > 1$, an increase in L_G progressively reduces L, and the reduction is stronger the higher is σ. The limits of this domain are demonstrated by setting $\sigma = 1,5$.

Then no equilibrium exists when L_G is doubled from 2 to 4, because the necessary tax rate then becomes so high that the representative individual will not demand - for leisure or for production of private consumption - the hours which has not employed in the public sector.

5. Eq. (2.15): The short run effect

Assume all exogenous variables but L_G constant in period 0. The equilibrium condition for the labour market (2.6), linearized around the equilibrium in period -1, can be written:

$$\begin{aligned} df_0 &= \left[\left[\sigma\left(\frac{L_{G,-1}}{L_{-1}-L_{G,-1}}\right)-\left(\frac{L_{-1}}{T-L_{-1}}\right)\right]\frac{dL_0}{L_{-1}}-\sigma\left(\frac{L_{G,-1}}{L_{-1}-L_{G,-1}}\right)\frac{dL_{G,0}}{L_{G,-1}}\right] f_{-1} \qquad (4.4)\\ &= \left(\sigma\frac{L_{G,-1}}{L_{-1}}-\frac{L_{-1}-L_{G,-1}}{T-L_{-1}}\right)dL_0-\sigma dL_{G,0}. \end{aligned}$$

Inserting the expressions for df_0, dL and dL_G yields the expresion in Eq. (2.15) of the relationship between dL and dL_G in period 0 :

$$\begin{aligned} L_0-L_{G,0} &= f\left(L_0,L_{G,0}\right)+a_K\left[L_0-L_{-1}-\left(L_{G,0}-L_{G,-1}\right)\right]\\ &\Leftrightarrow\ L_{-1}+dL_0-\left(L_{G,-1}+dL_{G,0}\right)=f\left(L_{-1},L_{G,-1}\right)+df_0+a_K\left(dL_0-dL_{G,0}\right)\\ &\Leftrightarrow\ L_{-1}+dL_0-\left(L_{G,-1}+dL_{G,0}\right)=L_{-1}-L_{G,-1}+df_0+a_K\left(dL_0-dL_{G,0}\right)\\ &\Leftrightarrow\ dL_0-dL_{G,0}=\left(\sigma\frac{L_{G,-1}}{L_{-1}}-\frac{L_{-1}-L_{G,-1}}{T-L_{-1}}\right)dL_0-\sigma dL_{G,0}+a_K\left(dL_0-dL_{G,0}\right)\\ &\Leftrightarrow\ \left(\frac{L_{G,-1}}{L_{-1}}\sigma-\frac{T-L_{G,-1}}{T-L_{-1}}+a_K\right)dL_0=\left(\sigma-1+a_K\right)dL_{G,0}. \end{aligned}$$

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Figure 1. Percentage deviations in public and total employment from a reference scenario

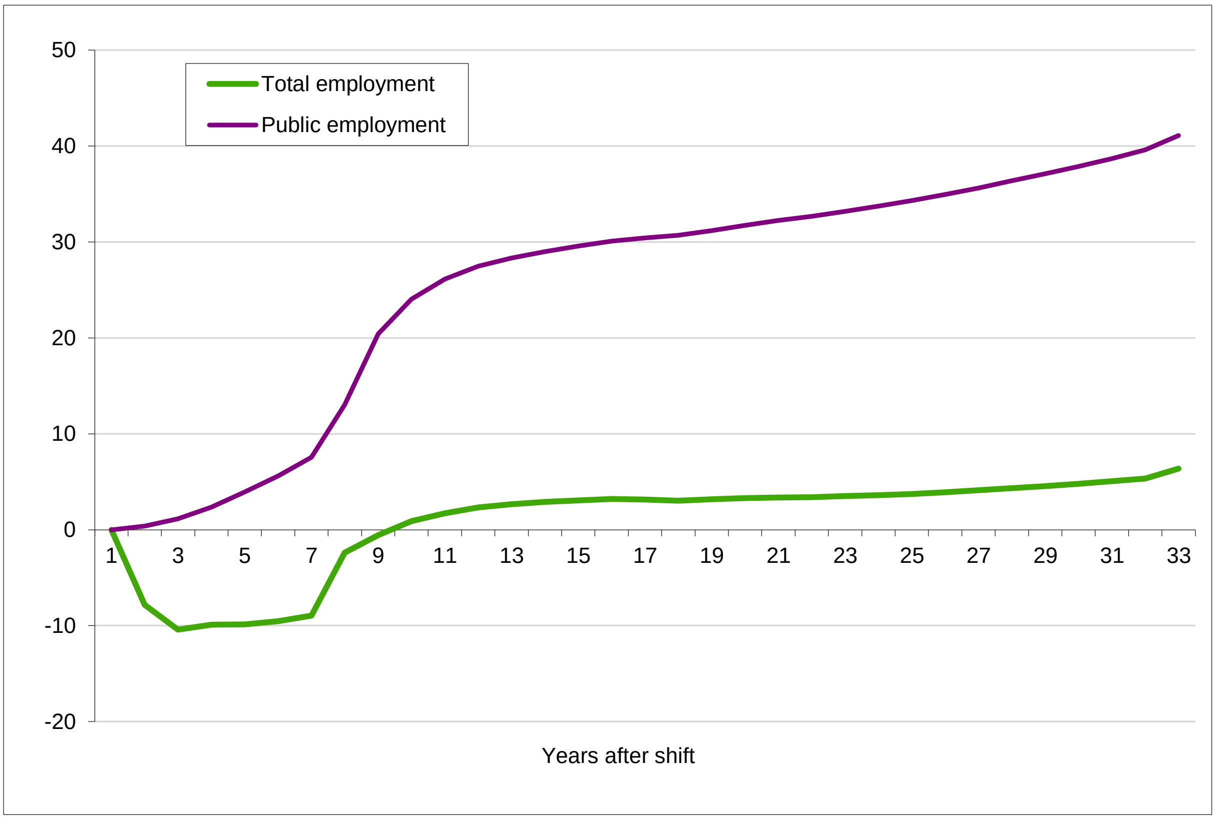
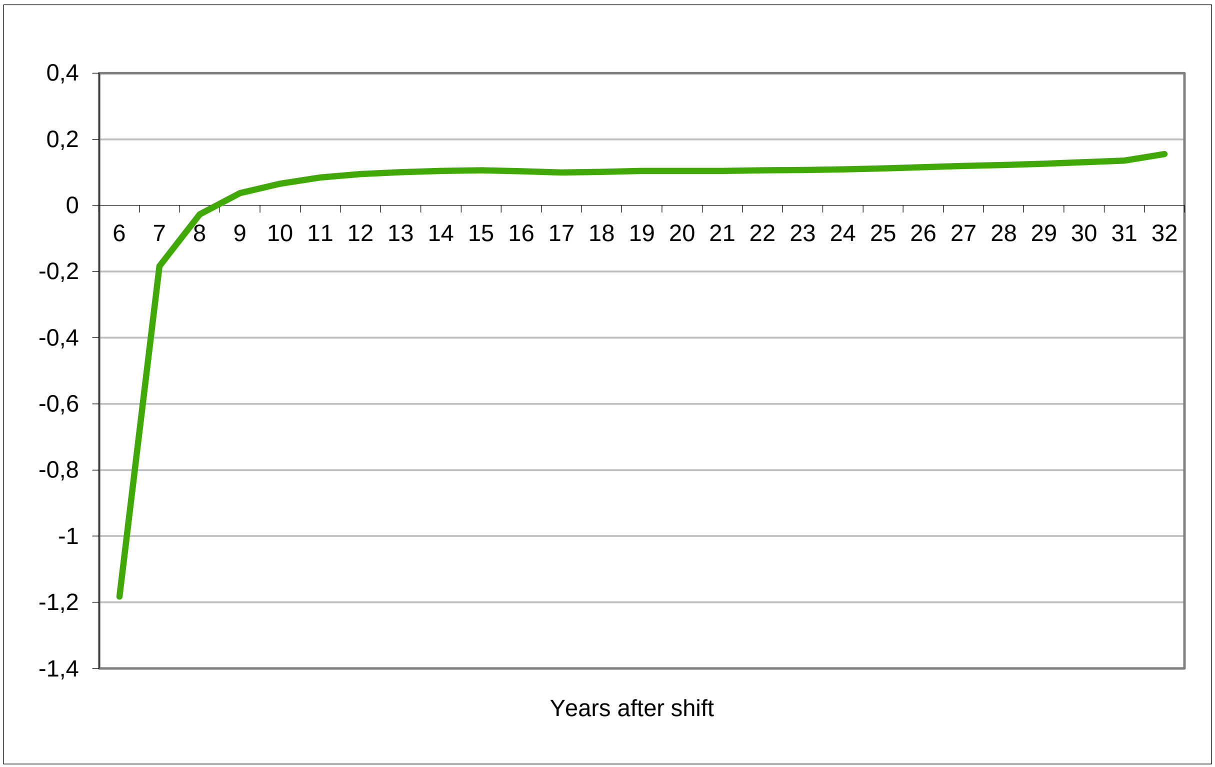


Figure 2. Elasticity of total employment with respect to public employment



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