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Abstract:

We reconcile two different strands of the literature: the literature on how new goods impact prices and the literature on productivity growth and firm turnover. To our knowledge, this is the first paper to provide a fully consistent decomposition of aggregate productivity growth that identifies the contribution from new firms producing new varieties. We extend the estimator for the demand elasticity, proposed by Feenstra (1994) and supplemented by Soderbery (2015), in two dimensions: First, we create a two-stage estimation framework that exploits the boundary cases where simultaneity is not an issue, i.e. when supply is elastic or inelastic, to obtain a more efficient estimator. Second, we make it robust towards choice of reference unit. To illustrate the decomposition and estimator, we analyse the case of firm turnover in Norway, using panel data covering the period from 1995 to 2016 for manufacturing firms. Our results indicate that net creation of new varieties from firm turnover contributes by about one half percentage point to annual aggregate productivity growth.

Keywords: Aggregation, Productivity growth, Variety gains, Demand elasticity

JEL classification: C43, E24, O47

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Sammendrag

Vi forener to forskjellige retninger innen litteraturen om produktivitetsmåling: litteraturen om hvordan introduksjon av nye goder i markedet påvirker priser, og litteraturen om hvordan foretaksdynamikk bidrar til produktivitetsvekst. Så vidt vi vet er dette det første arbeidet som gir en konsistent dekomponering av samlet produktivitetsvekst der bidraget fra nye goder er identifisert. For å illustrere vår dekomponering, som involverer en forbedret metode for å estimere etterspørselselastisiteter, analyserer vi paneldata for norske industriforetak i perioden 1995 til 2016. Våre resultater tyder på at introduksjon av nye goder i markedet har bidratt med rundt et halvt prosentpoeng årlig til aggregert produktivitetsvekst i analyseperioden.

1 Introduction

Many new goods have a significant effect on consumer welfare and this impact should be included in a cost-of-living based inflation rate (Groshen et al., 2017). At least two conceptually different ways of doing this have been applied in the literature. For example, the key idea underlying the study by Hausman (1999) was to identify a virtual price for the new good before its appearance (in his case, cellular phones). The virtual price is defined by the price which sets the demand equal to zero. With knowledge about this virtual price, the price decline due to the introduction of a new product can be calculated. An alternative method is to calculate the consumer gain from new varieties directly. Typically, a Constant Elasticity of Substitution (CES) framework is applied. At first sight, the CES framework may look unsuitable to calculate the impact from new varieties since an infinite virtual price is required to set the demand to zero. However, as illustrated by Feenstra (1994), even though the virtual price that drives demand to zero is infinite within a CES framework, the consumer gain from having a new variety available is finite. Within this framework, a new variety will only lower cost-of-living if the new product holds some new characteristics, i.e. it is not perfectly substitutable with existing products. Given an estimate of the elasticity of substitution, the consumer gain from new varieties is easily calculated.

Several papers have applied the Feenstra (1994) framework to calculate consumer gains from new varieties. For example, Broda and Weinstein (2006) use it to analyse the value to U.S. consumers of expanded import product varieties. Harrigan and Barrows (2009) analyse how the end of the multifibre arrangement impacted prices and quality. Broda and Weinstein (2010) found that product turnover lowered a cost of-living index by 0.8 percentage points annually compared with a "fixed goods" price index. The lowering of cost-of-living from new varieties should lead to an equal increase in output, and thus productivity, if these new varieties are produced domestically.

Despite a large literature on reallocation, firm turnover and aggregate productivity growth, this literature has not analysed and decomposed the contribution from new varieties to overall productivity growth, see e.g. Griliches and Regev (1995), Baily et al. (1992), Foster et al. (2001), Foster et al. (2006), Foster et al. (2008) and Acemoglu et al. (2017). All of these studies consider a decomposition which is based on a weighed average of productivity *levels*. When comparing productivity *levels* across firms it is implicitly assumed that the products are perfect substitutes. But, new varieties yield extra welfare to consumers precisely because they hold some new characteristics, i.e. they are not perfectly substitutable with existing products. Two different strands of the literature thus need to be reconciled: the literature on how new goods impact prices and the literature on aggregate productivity growth and firm turnover.

This paper is the first to provide a fully consistent decomposition of aggregate productivity growth that identifies the contribution from new firms producing new varieties. Using the CES approach adopted by Feenstra (1994), we show that the extra impact of firm turnover to aggregate productivity growth from new varieties is approximately given by $(s^N - s^X)/(\sigma - 1)$ where s^N and s^X are the output shares of new and exiting firms, respectively, and σ is the elasticity of substitution between varieties. The decomposition we propose generalises the decomposition used in the literature on firm turnover: if products are perfect substitutes, which is the benchmark case implicitly assumed in the literature, the elasticity of substitution tends to infinity and there is no extra gain from new varieties.

To identify how firm turnover impacts productivity growth requires a good estimator for the elasticity of substitution. In the literature on new goods, following Feenstra (1994), the key idea when estimating the demand elasticity has been to overcome the simultaneity problem in the system of demand and supply equations by utilising the panel structure of the data set. In particular, by using the second order moments of prices and expenditure shares in combination with sign restrictions, the demand elasticity can be identified even when allowing for an upward sloping supply curve. Broda and Weinstein (2006) extended the framework using a grid search of admissible values if the first estimator yields inadmissible estimates, e.g. of the wrong sign. Adding to this literature, Soderbery (2015) created a hybrid estimator (henceforth the Feenstra-Soderbery estimator) combining limited information maximum likelihood (LIML) with a restricted nonlinear LIML routine which was shown to be more robust to data outliers.

Our estimation procedure builds on the Feenstra-Soderbery estimator, but we refine it along two dimensions. The first refinement is that we create a two-stage estimation framework that exploits cases where there are no simultaneity problems, i.e. if supply is elastic or inelastic (as in the case of monopolistic competition), to obtain a more efficient estimator. To be more explicit, it is well known that the demand elasticity σ is finite if and only if $\sigma = 1 - \beta$, where

β is the unique negative solution to $\theta_1\beta^2 + \theta_2\beta - 1 = 0$ and $\theta = (\theta_1,\theta_2)$ is a function of the demand and supply elasticities. In those cases where the first-stage estimate of θ is at the boundary of the parameter space, we switch in the second stage to an estimator that depends on which boundary that is binding in the first stage. The two-stage estimator $\widehat{\sigma}$ is shown to have an asymptotic mixture distribution when (the true) θ is at the boundary of the parameter space, with a closed form expressions for the variance of the estimator. The other refinement of our estimator is to generalise the current practice of choosing a particular reference firm to eliminate fixed effects when generating second order moments of prices and expenditure shares. An unfortunate consequence of the current procedure is that it makes the estimator dependent on the choice of reference firm. We extend current practice by generating a sequence of estimates for each possible reference firm and create a pooled estimator. The pooled estimator is a weighted average of the estimates corresponding to each reference firm.

We illustrate the decomposition of productivity growth and the two-stage estimation procedure using the case of firm turnover in Norway. We have firm-level panel data covering the period from 1995 to 2016 for the manufacturing sector. Estimates of σ range from 2 to 9. Based on these estimates we find that annual aggregate productivity growth has on average been downward biased by about one half percentage point, which is substantial compared to the average productivity growth of almost 2.5 per cent annually.

The rest of this paper is organised as follows. Section 2 outlines the decomposition of aggregate productivity growth and identifies the impact from new varieties. In Section 3, the econometric framework is presented and our proposed two-stage estimator is derived. In Section 4, the data are described and our decomposition is applied empirically. Section 5 provides a conclusion.

2 Decomposition of aggregate productivity growth

Productivity is commonly defined as the ratio of outputs to inputs, both terms measured in volumes. Analytically, a measure of *aggregate productivity growth* may be written as Q_Y/Q_L, where Q_Y represents an index for overall output and Q_L represents an index for overall input usage. This definition of productivity is standard; see Diewert and Nakamura (2003).

The input index, Q_L, may consist of several inputs depending on what measure of productivity is to be analysed. Although the framework we provide below may be generalised to include inputs such as different capital objects, we will proceed with labour as the only input variable. The productivity index will henceforth be a measure of labour productivity.

To understand how firm turnover impacts overall productivity growth, the output and input indices must be decomposed into contributions from continuing, entering and exiting firms. To that end, the following sections outline how both inputs and outputs are aggregated and highlight the link between the literature on firm turnover and productivity growth and the literature on new goods and gains from variety.

2.1 Aggregation of outputs

Our point of departure is the economic approach to index numbers. Within this approach there are at least two ways to interpret the index Q_Y. It can be based on a representative firm producing a single final good where the index Q_Y shows growth in final good production. This is the approach taken in e.g. Hsieh and Klenow (2009). Alternatively, it can be based on a representative consumer maximising utility over the set of goods produced by all firms where the index Q_Y shows growth in utility obtained from consuming those goods. This latter approach is the cost-of-living approach to index theory. It dates back to Konüs (1939) and is applied in e.g. Broda and Weinstein (2006, 2010). Even though both conceptualisations yield the same index, Q_Y, we will follow the latter approach as it provides a clear link from the literature on firm turnover to the established literature on new goods and gains from variety.

Figure 1 illustrates how firm turnover and product innovation may impact the output or utility index Q_Y. The objective of the representative consumer is to maximise utility for a given level of expenditure. The isocost line AA' shows the combination of goods that yields the same expenditure level. In time period $t-1$, only variety Y_2 is available and consumption is at point A. In period t, however, a new firm enters the market and produces a new variety Y_1. The introduction of the new good by the entering firm increases the overall utility for the consumer: the indifference curve shifts outwards and consumption is at point B.

The size of the utility increase depends on the curvature of the indifference curve, or how

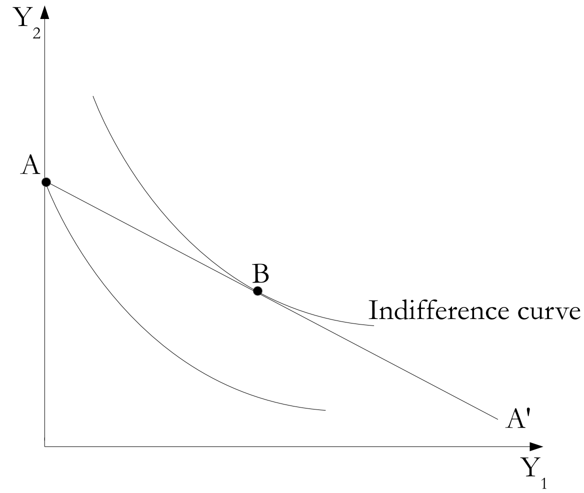

FIGURE 1: Firm turnover, new varieties and consumer welfare

easy it is to substitute one variety for another, as expressed by the elasticity of substitution. When there is some sort of complementarity between varieties, i.e. consumption of one variety stimulates demand for the other variety, the indifference curves will show a curvature as illustrated in Figure 1. However, if varieties are perfect substitutes, the elasticity of substitution tends to infinity and the indifference curves become straight lines. The lower the elasticity of substitution, the higher is the utility gain from having a new variety available.

To analyse how the elasticity of substitution impacts the output index, Q_Y, we follow the lines of Broda and Weinstein (2006) and proceed with a two-level utility function of a representative consumer. The upper level utility, Y_t, is a CES aggregate in a fixed number of composite goods, Y_{it}:

$$Y_t = \left(\sum_{i \in I} \gamma_i Y_{it}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \tag{1}$$

where $\gamma_i > 0$ represents a quality parameter, σ is the elasticity of substitution among the composite goods and I is the set of composite goods. The set of composite goods includes broad categories such as furniture, electronics, clothes etc. Since the purpose of industry classifications is to organise firms into industrial grouping based on similar products and activities, the set I may also be thought of as a set of industries. At the lower level, each composite good is a CES

Table 1: **Classification of firms**

	Continuing	Entering	Exiting
Time period t	■	■	
Time period $t-1$	■		■

■ denotes positive production.

aggregate of different varieties:

$$Y_{it} = \left(\sum_{f \in F_{it}} \gamma_{if} Y_{ift}^{(\sigma_i - 1)/\sigma_i} \right)^{\sigma_i/(\sigma_i - 1)} \tag{2}$$

where Y_{ift} is a variety produced by firm f in industry i, $\gamma_{if} > 0$ represents a quality parameter for each variety, σ_i is the elasticity of substitution among the different varieties in industry i and F_{it} is the set of varieties within the composite good i available at t. Importantly, it is assumed that each firm produces a single variety and all varieties are treated as differentiated across firms. Hence, the set F_{it} can equivalently be interpreted as the set of all firms producing a variety of good i in period t.

Note that due to *firm turnover* the set F_{it} varies over time. To illustrate, and to introduce notation that will become useful later, let C_{it} denote the set of firms that exists in two consecutive time periods $t-1$ and t. We refer to these as *continuing* firms; see Table 1. *Entering* firms, denoted N_{it}, exist in period t but not in $t-1$. Firms *exiting* in period t, denoted X_{it}, operates in period $t-1$ but not in t. It then follows that the number of firms producing a variety of good i in period t is the union of the set of continuing firms and the set of entering firms: $F_{it} = C_{it} \cup N_{it}$. Correspondingly, the number of firms producing a variety at $t-1$ can be written as the union of the set of continuing and exiting firms in t: $F_{i,t-1} = C_{it} \cup X_{it}$.

To create the aggregate output index, we apply the results of Sato (1976), Vartia (1976a) and Feenstra (1994). Sato (1976) and Vartia (1976a) showed how to calculate a price and a quantum index for a CES aggregator function when the number of goods is constant for different periods. This is useful for the upper tier of aggregation since the number of composite goods is independent of time. Feenstra (1994) generalised the results of Sato (1976) and Vartia (1976a) to handle situations where the number of categories changes over time, which is the case for the set of firms F_{it} producing a variety of good i.

We begin by showing the Sato-Vartia index corresponding to Equation (1). Let P_{it} be the price index of the composite good i and let the volume of the composite good, Y_{it}, be cost-minimising. The output index showing the ratio of utility for two periods, $Q_Y = Y_t/Y_{t-1}$, is then given by a Sato-Vartia index of the composite goods:

$$\ln Q_Y = \sum_{i \in I} w_{it} \Delta \ln Y_{it} \tag{3}$$

with output weight w_{it} equal to:

$$w_{it} = \frac{M(s_{it}, s_{i,t-1})}{\sum_{i \in I} M(s_{it}, s_{i,t-1})}$$

where $s_{it} = V_{it} / \sum_{i \in I} V_{it}$ (the expenditure share of good i) and $M(y, z)$ denotes the logarithmic mean of (non-negative) numbers y and z:

$$M(y, z) = \begin{cases} 0 & \text{if } y = 0 \text{ or } z = 0 \\ y & \text{if } y = z \\ \frac{y - z}{\ln y - \ln z} & \text{otherwise} \end{cases} . \tag{4}$$

A remarkable feature of the Sato-Vartia index is that it is independent of the quality parameters and the elasticity of substitution. Note that in addition to being exact for the CES aggregator function, the Sato-Vartia index also belongs to the complete class of superlative index numbers, as shown by Barnett and Choi (2008). A superlative index is defined as being consistent with a function that approximates a true aggregator function to the second order (Diewert, 1976). The case for using the Sato-Vartia index to aggregate composite goods is thus stronger than its consistence with an underlying CES structure.

To calculate the output index for each composite good we apply the results of Feenstra (1994) to incorporate the impact from firm turnover. He showed that the total index could be decomposed into contributions from a standard Sato-Vartia index across continuous firms and separate contributions from entering and exiting firms. Let s_{it}^N denote the total expenditure share of entering firms within industry i: $s_{it}^{N} = \sum_{f \in N_{it}} V_{ift} / \sum_{f \in F_{it}} V_{ift}$. Also, let $s_{it-1}^{X} = \sum_{f \in X_{it}} V_{if,t-1} / \sum_{f \in F_{i,t-1}} V_{if,t-1}$ denote the total nominal output share in $t-1$ of exiting firms (operating in $t-1$ but not t). Moreover, let

$$w_{ift} = \frac{M(s_{ift}, s_{if,t-1})}{\sum_{f \in C_{it}} M(s_{ift}, s_{if,t-1})}$$

where

$$s_{ift} = \frac{V_{ift}}{\sum_{f \in C_{it}} V_{ift}}.$$

Applying the results of Feenstra (1994) and the product rule, the output index for the composite good can be written:[1]

$$\Delta \ln Y_{it} = \sum_{f \in C_{it}} w_{ift} \Delta \ln Y_{ift} + \left(\frac{\sigma_i}{1-\sigma_i} \right) \ln \left(1 - s_{it}^{N} \right) - \left(\frac{\sigma_i}{1-\sigma_i} \right) \ln \left(1 - s_{i,t-1}^{X} \right). \tag{5}$$

The first term is the standard Sato-Vartia index across continuous firms producing the same composite good. The second and third terms are the contributions from firm turnover. Note that the analytical expressions for entering and disappearing varieties depend on the elasticity of substitution, as illustrated in Figure 1. If all firms are producing the same homogeneous good, the elasticity of substitution tends to infinity and there is no longer any utility gain from firm turnover.

The analytical expression for the aggregate output index follows from inserting Equation (5) into Equation (3), which yields:

$$\ln Q_Y = \sum_{i \in I} w_{it} \left(\sum_{f \in C_{it}} w_{ift} \Delta \ln Y_{ift} + \left(\frac{\sigma_i}{1-\sigma_i} \right) \ln \left(1 - s_{it}^{N} \right) - \left(\frac{\sigma_i}{1-\sigma_i} \right) \ln \left(1 - s_{i,t-1}^{X} \right) \right). \tag{6}$$

Equation (6) represents the complete decomposition of the output index.

[1] The CES approach to calculating welfare gain from new goods is not uncontroversial, see e.g. the comment by Zvi Griliches to Feenstra and Shiells (1996, pp. 273 – 276). Diewert and Feenstra (2017) compare the CES function with an alternative utility function based on a flexible functional form where the reservation price is finite. They find that the CES approach may overstate gains from new varieties, in particular if σ is close to unity.

2.2 Aggregation of inputs

It is common, but not uncontroversial, to aggregate input usage of labour as a simple sum of hours worked across firms. There is a large literature on quality adjustment of labour services dating back to at least Jorgenson and Griliches (1967). Although there are several alternative measures of inputs usage, and since our main contribution is to provide a framework taking account of firm turnover and new varieties, we proceed with the standard approach using the sum of hours worked to derive the index for input usage, Q_L.[2]

Following the procedure of aggregating outputs, we aggregate inputs first across firms in a given industry and then across industries. Let L_t denote the total sum of hours worked across all industries and firms. For our purposes it is useful to write L_t as the sum of hours worked across industries: $L_t = \sum_{i \in I} L_{it}$, where $L_{it} = \sum_{f \in F_{it}} L_{ift}$ is the sum of hours worked in industry i.

Since the Sato-Vartia-Feenstra index is written as log changes it will be useful to rewrite the ratio of sum of hours worked as a weighted average of log changes. To this end, note that the logarithm of the input index, $\ln Q_L \equiv \Delta \ln L_t$, can be *exactly* decomposed as a weighted sum of industry specific contributions:

$$\ln Q_L = \sum_{i \in I} \theta_{it} \Delta \ln L_{it}, \tag{7}$$

where the weights are given by[3]

$$\theta_{it} = \frac{M(L_{it}, L_{i,t-1})}{M(L_t, L_{t-1})}. \tag{8}$$

These weights do not generally add up to unity but their sum is one at the most, see Vartia (1976b, Appendix 4). Moreover, in a particular industry, i, hours worked may be decomposed according to whether firms are continuing, entering or exiting, as follows:

$$\Delta \ln L_{it} = \sum_{f \in C_{it}} \theta_{ift} \Delta \ln L_{ift} - \ln\left(1 - h_{it}^N\right) + \ln\left(1 - h_{i,t-1}^X\right) \tag{9}$$

where h_{it}^N and $h_{i,t-1}^X$ denote the shares of hours worked in entering and exiting firms in industry

[2] The input index we derive in this paper may alternatively be defined within the theory of quality adjustment, see e.g. Brasch et al. (2017).

[3] To see this, note that from the definition of the logarithmic mean in Equation (4), the input index may be written as $\ln(\frac{\sum_{i \in I} L_{it}}{\sum_{i \in I} L_{i,t-1}}) = \frac{\sum_{i \in I} L_{it} - \sum_{i \in I} L_{i,t-1}}{M(\sum_{i \in I} L_{it}, \sum_{i \in I} L_{i,t-1})} = \sum_{i \in I} \left(\frac{M(L_{it}, L_{i,t-1})}{M(\sum_{i \in I} L_{it}, \sum_{i \in I} L_{i,t-1})} \right) \ln \left(\frac{L_{it}}{L_{i,t-1}} \right)$.

i: $h_{it}^N = (\sum_{f \in N_{it}} L_{ft})/(\sum_{f \in F_{it}} L_{ift})$ and $h_{i,t-1}^X = (\sum_{f \in X_{it}} L_{if,t-1})/(\sum_{f \in F_{it-1}} L_{if,t-1})$, and the weights θ_{ift} are defined as follows:

$$\theta_{ift} = \frac{M(L_{ift}, L_{if,t-1})}{M(\sum_{f \in C_{it}} L_{ift}, \sum_{f \in C_{it}} L_{if,t-1})}. \tag{10}$$

Inserting Equation (9) into Equation (7) yields the overall index for input usage:

$$\ln Q_L = \sum_{i \in I} \theta_{it} \left(\sum_{f \in C_{it}} \theta_{ift} \Delta \ln L_{ift} - \ln\left(1 - h_{it}^N\right) + \ln\left(1 - h_{i,t-1}^X\right) \right). \tag{11}$$

Equation (11) decomposes the log change in the total sum of hours worked into contributions from input usage across continuing, entering and exiting firms and represents the complete decomposition of the input index.

2.3 Contribution from product innovation and firm turnover to overall productivity growth

Aggregate productivity growth is defined as the ratio of the output index Q_Y to the input index Q_L. Given the expressions for Q_Y and Q_L in Equation (6) and Equation (11), it follows that aggregate productivity growth can be decomposed as:

$$\begin{aligned} \ln(Q_Y/Q_L) \ &= \sum_{i \in I} w_{it} \Bigg[\sum_{f \in C_{it}} w_{ift} \Delta \ln(Y_{ift}/L_{ift}) - \ln\left(\frac{1 - s_{it}^N}{1 - h_{it}^N}\right) + \ln\left(\frac{1 - s_{i,t-1}^X}{1 - h_{i,t-1}^X}\right) \\ &\qquad - \left(\frac{1}{\sigma_i - 1}\right) \ln\left(\frac{1 - s_{it}^N}{1 - s_{i,t-1}^X}\right) \Bigg] + RWI_t + RBI_t \end{aligned} \tag{12}$$

where

$$RWI_t = \sum_{i \in I} w_{it} \sum_{f \in C_{it}} (w_{ift} - \theta_{ift}) \Delta \ln L_{ift} \tag{13}$$

and

$$RBI_t = \sum_{i \in I} (w_{it} - \theta_{it}) \, \Delta \ln L_{it}. \tag{14}$$

Equations (12)-(14) constitute the complete decomposition of aggregate productivity growth. The first expression inside the square bracket in Equation (12) shows the contribution from productivity growth among continuing firms. The second and third terms represent the contribution from firm turnover in the absence of product innovation. If entering firms have a higher revenue productivity than continuing firms, where revenue productivity is measured as the ratio of revenue to hours worked, entering firms contribute positively to overall productivity growth. Correspondingly, productivity growth will also be higher if exiting firms have a lower revenue productivity than continuing firms.

The fourth term in Equation (12) shows the net effect from creation of new varieties. As illustrated diagrammatically in Figure 1, the overall productivity growth from new varieties depends on the elasticity of substitution. The net contribution can be approximated by $(s_{it}^N - s_{i,t-1}^X)/(\sigma_i - 1)$ when the output and input shares are small.[4] For example, consider the case where the output share of entering firms is $s_{it}^N = 0.07$ and the output share of exiting firms is $s_{it}^N = 0.02$. If $\sigma_i = 2$, the overall contribution to productivity growth from net creation of new varieties is approximately 5 percentage points. The expression also shows that the impact from new varieties depends on the elasticity of substitution in a highly non-linear manner: If $\sigma_i = 3$, the contribution to productivity growth drops to approximately 2.5 percentage points, and to 1.7 percentage points if $\sigma_i = 4$. To identify the contribution from new varieties to overall productivity growth, it is thus crucial to precisely identify the size of the elasticity of substitution. We return to the issue of identifying the elasticity of substitution in the empirical section.

Note that it is not the *number* of entering and exiting firms that drives the overall impact on aggregate productivity growth. Even when the number of entering and exiting firms is equal, if new varieties from entering firms have a higher quality than varieties produced by exiting firms, the output share of entering firms will exceed the output share of exiting firms: $s_{it}^N > s_{i,t-1}^X$. In this case, we get a positive contribution to overall productivity growth from net-creation of new varieties.

The last two terms, labeled RWI and RBI, show the contribution from reallocation within and between industries. Reallocation within industries depends on the covariance between firms' input usage and the difference between the output (w_{itf}) and input (θ_{ift}) weights. If more in-

[4]The approximation follows from applying $\ln(1 + z) \approx z$ when $z \approx 0$.

put usage is allocated to firms with higher revenue productivity, reallocation within industries contributes positively to aggregate productivity growth. Correspondingly, reallocation between industries depends on the covariance between input usage at the industry level and the difference between industry output (w_{if}) and input (θ_{it}) weights. Reallocation between industries contributes positively to aggregate productivity growth if more input usage is allocated to industries with higher revenue productivity.

Most of the literature applies a framework based on a weighted average of productivity *levels* to analyse the contribution to overall productivity growth from firm turnover; see e.g. Griliches and Regev (1995), Baily et al. (1992), Foster et al. (2001, 2006, 2008) and Acemoglu et al. (2017). Implicitly these studies assume that products are homogeneous. However, it is only within a framework that allows for non-homogeneous goods that the extra gain in productivity growth from new firms producing new varieties can be identified. The most important difference between the decomposition above and those used in the literature is thus that the above framework allows for products being non-homogeneous, as illustrated by Figure 1. In Appendix B we compare and contrast the decomposition of productivity growth above with the frameworks often used in the literature.

3 Estimation of demand elasticities

In the literature on new goods, the key idea when estimating the demand elasticity has been to overcome the simultaneity problem caused by an upward sloping supply curve by utilising the panel structure of the dataset and reformulating the model in terms of second order moments of prices and expenditure shares. This approach was originally proposed by Feenstra (1994). Broda and Weinstein (2006) and Soderbery (2015) extended this framework along several dimensions. In particular, Soderbery (2015) created a hybrid estimator combining LIML with a restricted nonlinear LIML routine which he showed to be more robust to outliers. In this section, we supplement the Feenstra-Soderbery estimator along two important dimensions: First, we create a two-stage estimation framework that exploits the case when there is no simultaneity problem. Second, we make the routine robust with respect to the choice of reference unit.

3.1 Structural econometric framework

To identify structural parameters in a system of demand and supply equations using panel data on prices and market shares, we follow Broda and Weinstein (2006). The demand share at t of the variety produced by firm f (in industry i), s_{ft}, is assumed to be given by:

$$\ln s_{ft} = \beta \ln p_{ft} + |\beta|(\lambda_t^D + u_f^D + e_{ft}^D),\ \beta \leq 0 \tag{15}$$

where p_{ft} is the price, λ_t^D and u_f^D represent fixed time and firm effects, e_{ft}^D is an error term (with mean zero and finite variance), and $\beta = 1 - \sigma < 0$. The industry subscript i has been dropped for notational convenience. The scaling factor $|\beta|$ ensures well-defined limits when $\beta \to -\infty$ (perfectly elastic demand). We will return to this case below. The inverse supply equation is assumed to be given by:

$$\ln p_{ft} = \alpha \ln s_{ft} + \lambda_t^S + u_f^S + e_{ft}^S \tag{16}$$

where $\alpha = \omega/(1+\omega)$ and $\omega \geq 0$ is the elasticity of supply. Hence $0 \leq \alpha \leq 1$. By differencing Equation (15) and Equation (16):

$$\begin{aligned} \Delta \ln s_{ft} &= \beta \Delta \ln p_{ft} + |\beta|(\Delta\lambda_t^D + \Delta e_{ft}^D) \\ \Delta \ln p_{ft} &= \alpha \Delta \ln s_{ft} + \Delta\lambda_t^S + \Delta e_{ft}^S. \end{aligned}$$

Then define

$$\begin{aligned} \Delta^{(k)} \ln p_{ft} = \Delta \ln p_{ft} - \Delta \ln p_{kt},\ &\Delta^{(k)} \ln s_{ft} = \Delta \ln s_{ft} - \Delta \ln s_{kt} \\ \varepsilon_{fkt}^D = \Delta e_{ft}^D - \Delta e_{kt}^D,\ &\varepsilon_{fkt}^S = \Delta e_{ft}^S - \Delta e_{kt}^S. \end{aligned}$$

It follows that

$$\begin{aligned} \Delta^{(k)} \ln s_{ft} &= \beta \Delta^{(k)} \ln p_{ft} + |\beta| \varepsilon_{fkt}^D \\ \Delta^{(k)} \ln p_{ft} &= \alpha \Delta^{(k)} \ln s_{ft} + \ \varepsilon_{fkt}^S \end{aligned} \tag{17}$$

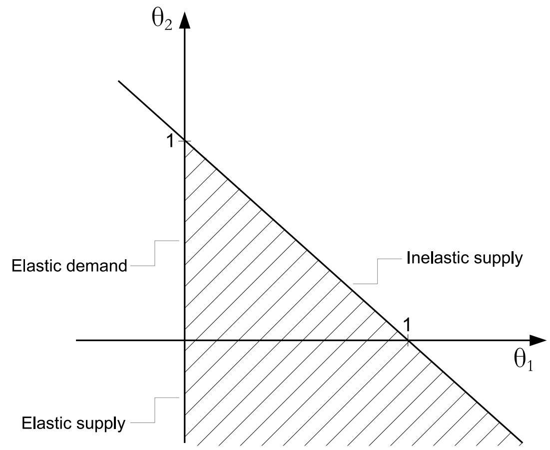

Figure 2: The parameter space. The boundary $\{\theta : \theta_1 > 0 \cap \theta_1 + \theta_2 = 1\}$ corresponds to inelastic supply, $\{\theta : \theta_1 = 0 \cap \theta_2 < 0\}$ to elastic supply and $\{\theta : \theta_1 = 0 \cap 0 \leq \theta_2 \leq 1\}$ to elastic demand.

From Equation (17):

$$(\Delta^{(k)} \ln p_{ft})^2 = \theta_1 (\Delta^{(k)} \ln s_{ft})^2 + \theta_2 (\Delta^{(k)} \ln p_{ft} \Delta^{(k)} \ln s_{ft}) + U_{fkt} \tag{18}$$

where

$$\theta_1 = -\frac{\alpha}{\beta},\ \theta_2 = \frac{1}{\beta} + \alpha \text{ and } U_{fkt} = \varepsilon^D_{fkt} \varepsilon^S_{fkt}$$

Under the identifying assumptions in Feenstra (1994), the idiosyncratic error terms e^D_{ft} and e^S_{ks} are assumed to be independent for *any* (f,t) and (k,s):

$$E(U_{fkt}) = 0.$$

Note that Equation (18) is *not* a valid regression equation for estimating θ, because s_{ft} and p_{ft} depend on ε^D_{fkt} and ε^S_{fkt}, and must therefore be estimated by a method of moments estimator, such as the Feenstra-Soderbery LIML estimator.

TABLE 2: **Parameterisation**

Parameter space of θ	Elasticities (α, σ)	
$\Theta_{int} = \{\theta : \theta_1 > 0 \cap \theta_1 + \theta_2 < 1\}$	$\alpha = \left[\frac{-\theta_2+\sqrt{\theta_2^2+4\theta_1}}{2\theta_1}\right]^{-1}$	$\sigma = 1 + \frac{\theta_2+\sqrt{\theta_2^2+4\theta_1}}{2\theta_1}$
$\Theta_2 = \{\theta : \theta_1 > 0 \cap \theta_1 + \theta_2 = 1\}$	$\alpha = 1$	$\sigma = 1 + \frac{1}{\theta_1}$
$\Theta_3 = \{\theta : \theta_1 = 0 \cap \theta_2 < 0\}$	$\alpha = 0$	$\sigma = 1 - \frac{1}{\theta_2}$
$\Theta_4 = \{\theta : \theta_1 = 0 \cap 0 \leq \theta_2 \leq 1\}$	$\alpha = \theta_2$	$\sigma = \infty$

3.2 Parameter restrictions

The restrictions on the structural parameters α and β: $0 \leq \alpha \leq 1$ and $\beta < 0$ (see above), imply restrictions on θ. First, since $\theta_1 = -\alpha/\beta$, it follows that $\theta_1 \geq 0$, whereas $\alpha \leq 1$ is equivalent to:[5]

$$\theta_1 + \theta_2 \leq 1.$$

Next, assume that $\theta_1 > 0$. Then α^{-1} and β are (real) solutions to $\theta_1 s^2 + \theta_2 s - 1 = 0$. That is

$$\begin{aligned} \alpha^{-1} &= \frac{-\theta_2 + \sqrt{\theta_2^2 + 4\theta_1}}{2\theta_1} > 0 \\ \beta &= \frac{-\theta_2 - \sqrt{\theta_2^2 + 4\theta_1}}{2\theta_1} < 0. \end{aligned} \tag{19}$$

Note that the sign restrictions on β and α are automatically fulfilled since $\sqrt{\theta_2^2 + 4\theta_1} > |\theta_2|$. Finally, assume $\theta_1 = 0$. Then $\alpha = 0$ or $\beta = -\infty$ $(\sigma = \infty)$. If $\alpha = 0$ and $|\beta| < \infty$, $\sigma = 1 - 1/\theta_2$. If $\beta = -\infty$, $\alpha = \theta_2 \geq 0$. Figure 2 illustrates the parameter space and its boundaries. The relation between θ and the elasticities α and β is summed up in Table 2.

Now define

$$g(\theta) = \frac{\theta_2 + \sqrt{\theta_2^2 + 4\theta_1}}{2\theta_1} \text{ for } \theta_1 > 0$$

and

$$\sigma(\theta) = \begin{cases} 1 + g(\theta) & \text{if } \theta_1 > 0 \\ 1 - \frac{1}{\theta_2} & \text{if } \theta \in \Theta_3 \\ \infty & \text{if } \theta \in \Theta_4. \end{cases} \tag{20}$$

[5]To see this: $\alpha \leq 1 \Leftrightarrow \left(-\theta_2 + \sqrt{\theta_2^2 + 4\theta_1}\right)/2\theta_1 \geq 1 \Leftrightarrow \sqrt{\theta_2^2 + 4\theta_1} \geq 2\theta_1 + \theta_2 \Leftrightarrow \theta_2^2 + 4\theta_1 \geq 4\theta_1^2 + \theta_2^2 + 4\theta_1\theta_2 \Leftrightarrow \theta_1 - \theta_1^2 - \theta_1\theta_2 \geq 0 \Leftrightarrow 1 - \theta_1 - \theta_2 \geq 0 \Leftrightarrow \theta_1 + \theta_2 \leq 1$.

Since $g(\theta) = 1/\theta_1$ when $\theta_1 + \theta_2 = 1$ ($\alpha = 1$), $\sigma(\theta)$ expresses σ as a function of θ in accordance with Table 2. By L'Hopital's rule:

$$\lim_{\theta_1 \to 0^+} \sigma(\theta_1, \theta_2) = 1 - \frac{1}{\theta_2} \text{ if } \theta_2 < 0$$

$$\lim_{\theta_1 \to 0^+} \sigma(\theta_1, \theta_2) = \infty \text{ if } \theta_2 \in [0, 1]$$

$$\lim_{\theta_2 \to 0^-} \sigma(0, \theta_2) = \infty$$

Hence $\sigma(\theta)$ is a continuous function of θ for all $\theta \in \Theta$. Note, however, that $\sigma(\theta)$ is not differentiable at $\theta_1 = 0$. Given an estimator $(\widehat{\theta})$ of θ that satisfies all the above parameter constraints, the obvious estimator of σ is $\sigma(\widehat{\theta})$.

Below we first consider the Feenstra-Soderbery estimator of θ and then propose an (asymptotically) more efficient estimator than $\sigma(\widehat{\theta})$ in the case $\widehat{\theta} \in \Theta_2$ (inelastic supply) or $\widehat{\theta} \in \Theta_3$ (elastic supply). Of particular interest is inelastic supply ($\alpha = 1$), since this case corresponds to monopolistic competition. In the existing literature, this fact has been overlooked. For example, in most cases where the *unrestricted* Feenstra-Soderbery estimator $\widehat{\theta}^{(u)} = (\widehat{\theta}_1^{(u)}, \widehat{\theta}_2^{(u)})$ yields $\widehat{\theta}_1^{(u)} + \widehat{\theta}_2^{(u)} > 1$ (in this case $\widehat{\omega}^{(u)} < 0$), the *restricted* Feenstra-Soderbery estimator yields $\widehat{\omega} = 0$ ($\widehat{\alpha} = 0$). This is despite the fact that $\theta_1 + \theta_2 = 1$ implies $(1 - \alpha)/\beta = 1 - \alpha$, with $\alpha = 1$ ($\omega = \infty$) as the only solution. A potential problem seems to be that the (restricted) Feenstra-Soderbery estimation algorithm does not explore solutions at the boundary where $\alpha = 1$ – only the subset where $\theta_1 = 0$ and $\theta_2 \leq 0$ (cf. Figure 1). Also Broda and Weinstein (2006) in their search algorithm restrict ω to be finite, and hence do not examine this boundary. Below we propose a consistent estimator, $\widehat{\sigma}$, of σ that investigates *all* boundary points in Figure 2. As an extension of the existing literature, we provide closed form standard errors of $\widehat{\sigma}$ for any finite σ – including at the boundary.

3.3 Estimation of θ

In view of the above discussion, we need to impose the constraints $\theta_1 \geq 0$ and $\theta_1 + \theta_2 \leq 1$ when estimating the model. This makes the estimation problem an optimization problem with linear constraints. If the unrestricted Feenstra-Soderbery estimator satisfies $\widehat{\theta}_1^{(u)} \geq 0$

and $\widehat{\theta}_1^{(u)} + \widehat{\theta}_2^{(u)} \leq 1$, all restrictions on $\widehat{\alpha}$ and $\widehat{\beta}$ are automatically fulfilled (replacing θ with $\widehat{\theta}$ in Equation (20)). However, if one or both constraints are violated, we need to identify possible solutions at the boundaries of the parameter space, which is complicated. To simplify the problem, we utilize that the unconstrained Feenstra-Soderbery estimator $\widehat{\theta}^{(u)}$ asymptotically, as the number of firms, n, tends to infinity, has log-density

$$l_n(\theta^0) = -\frac{1}{2}(\widehat{\theta}^{(u)} - \theta^0)' H_n (\widehat{\theta}^{(u)} - \theta^0)$$

(ignoring terms of order $o(n^{-1})$ and normalizing constants), where $H_n = Var(\widehat{\theta}^{(u)})^{-1}$. Now consider the constrained optimum:

$$\widehat{\theta}^{(c)} = \arg\max_{\theta \in \Theta} l_n(\theta) \xrightarrow{P} \theta^0$$

where $\Theta = \{\theta : \theta_1 \geq 0 \cap \theta_1 + \theta_2 \leq 1\}$. The possible boundary solutions are:

$$l^{(r1)} = \max_{\theta} l_n(\theta) \text{ s.t. } \theta_1 + \theta_2 = 1$$

or

$$l^{(r2)} = \max_{\theta} l_n(\theta) \text{ s.t. } \theta_1 = 0 \text{ and } \theta_2 \leq 1.$$

Let the corresponding argmax be denoted $\theta^{(r1)}$ and $\theta^{(r2)}$. Any solution to the first problem must satisfy the first-order condition

$$\frac{\partial l(\theta_1^{(r1)}, 1 - \theta_1^{(r1)})}{\partial \theta_1} = 0.$$

That is, with $H_n = [h_{ij}]_{i,j=1,2}$:

$$\theta_1^{(r1)} = \frac{h_{22} - h_{12}}{h_{11} - 2h_{12} + h_{22}}(1 - \widehat{\theta}_2^{(u)}) + \frac{h_{11} - h_{12}}{h_{11} - 2h_{12} + h_{22}}\widehat{\theta}_1^{(u)}.$$

Note that $\theta_1^{(r1)}$ is a weighted average of $\widehat{\theta_1}^{(u)}$ and $(1-\widehat{\theta_2}^{(u)})$. Next, we consider $l^{(r2)}$ with $\theta^{(r2)} = (0, \theta_2^{(r2)})$ and $\theta_2^{(r2)} \leq 1$. Then, if $\theta_2^{(u)} \leq 1$,

$$\begin{pmatrix} 0 & 1 \end{pmatrix} H_n \begin{pmatrix} \theta_1^{(r1)} - \widehat{\theta_1}^{(u)} \\ \theta_2^{(r1)} - \widehat{\theta_2}^{(u)} \end{pmatrix} = 0$$

which is equivalent to $\theta_2^{(r2)} = \widehat{\theta_2}^{(u)}$. On the other hand, if $\theta_2^{(u)} > 1$, $\widehat{\theta}^{(r2)} = (0, 1)$. Thus

$$\theta^{(r2)} = (0, \min(\widehat{\theta_2}^{(u)}, 1)).$$

Combining all the above cases, we arrive at the following *constrained* estimator:

$$\widehat{\theta}^{(c)} = \begin{cases} \widehat{\theta}^{(u)} & \text{if } \widehat{\theta}^{u} \in \Theta_{int} \\ (\theta_1^{(r1)}, 1-\theta_1^{(r1)}) & \text{if } \widehat{\theta}^{u} \notin \Theta_{int},\ \theta_1^{(r1)} > 0 \text{ and } l(\theta^{(r1)}) > l(\theta_2^{(r2)}) \\ (0, \min(\widehat{\theta_2}^{(u)}, 1)) & \text{otherwise} \end{cases}$$

We will henceforth refer to $\widehat{\theta}^{(c)}$ as the first-stage estimator.

3.4 Estimation of σ when supply is inelastic ($\alpha = 1$)

The estimator $\sigma(\widehat{\theta})$ is not optimal if $\theta_1 + \theta_2 = 1$. To see this, we rewrite the system (15)-(16) on reduced form:

$$\begin{bmatrix} \ln s_{ft} \\ \ln p_{ft} \end{bmatrix} = \begin{bmatrix} \frac{\beta}{1-\alpha\beta}(\lambda_t^S - \lambda_t^D + u_f^S - u_f^D + e_{ft}^S - e_{ft}^D) \\ \frac{-\alpha\beta}{1-\alpha\beta}(\lambda_t^D + u_f^D + e_{ft}^D) + \frac{1}{1-\alpha\beta}(\lambda_t^S + u_f^S + e_{ft}^S) \end{bmatrix} \tag{21}$$

Since $\theta_1 + \theta_2 = 1$ is equivalent to $\alpha = 1$, we obtain:

$$\ln p_{ft} - \ln s_{ft} = \lambda_t^S + u_f^S + e_{ft}^S. \tag{22}$$

Moreover, from Equation (15):

$$\ln p_{ft} = \tau(\ln p_{ft} - \ln s_{ft}) - \frac{|\beta|(\lambda_t^D + u_f^D + e_{ft}^D)}{\beta - 1} \tag{23}$$

where

$$\tau = \begin{cases} \frac{1}{\sigma} & \text{if } \sigma < \infty \\ 0 & \text{if } \sigma = \infty \end{cases}.$$

In this case we estimate the fixed-effects regression:

$$\ln p_{ft} = \tau(\ln p_{ft} - \ln s_{ft}) + \lambda_t + u_f + e_{ft} \text{ s.t. } \tau \geq 0$$

where the unobserved components λ_t, u_f and e_{ft} are defined in accordance with Equation (23). Since the regressor, $\ln p_{ft} - \ln s_{ft}$, is uncorrelated with the error term when $\alpha = 1$ (see Equation (22) and Equation (23)), $\widehat{\tau}^{-1} \xrightarrow{P} \sigma$ if $\sigma < \infty$, and $\widehat{\tau} \xrightarrow{P} 0$ if $\sigma = \infty$.

3.5 Estimation of σ when supply is elastic ($\alpha = 0$)

In this case

$$\ln p_{ft} = \lambda_t^S + u_f^S + e_{ft}^S$$

and we estimate the regression equation:

$$\ln s_{ft} = \psi \ln p_{ft} + \lambda_t + u_f + e_{ft}. \tag{24}$$

Since the regressor, $\ln p_{ft}$, is uncorrelated with the error term when $\alpha = 0$ (see Equation (15) and Equation (24)), $\widehat{\psi} \xrightarrow{P} 1 - \sigma < 0$ if $1 < \sigma < \infty$. In finite samples, it is possible that $\widehat{\psi} \geq 0$, which has no interpretation. In this case, $\widehat{\sigma} = 1 - 1/\widehat{\theta}_2^{(c)}$ is an admissible estimator. Our two-stage estimators of σ and θ is listed in Table 3.

TABLE 3: **Two-stage estimator**

First stage estimator of θ ($\widehat{\theta}^{(c)}$)	Second stage estimator of σ and θ $(\widehat{\sigma}, \widehat{\theta})$	
$\widehat{\theta}^{(c)} \in \Theta_{int}$	$\widehat{\sigma} = 1 + g(\widehat{\theta}^{(c)})$	$\widehat{\theta} = \widehat{\theta}^{(c)}$
$\widehat{\theta}_1^{(c)} + \widehat{\theta}_2^{(c)} = 1$	$\widehat{\sigma} = \begin{cases} \frac{1}{\widehat{\tau}} & \text{if } \widehat{\tau} > 0 \\ \infty & \text{if } \widehat{\tau} = 0 \end{cases}$	$(\widehat{\theta}_1, \widehat{\theta}_2) = \left(\frac{1}{\widehat{\sigma}-1}, 1 - \frac{1}{\widehat{\sigma}-1}\right)$
$\widehat{\theta}_1^{(c)} = 0$ and $\widehat{\theta}_2^{(c)} < 0$	$\widehat{\sigma} = \begin{cases} 1 - \widehat{\psi} & \text{if } \widehat{\psi} < 0 \\ 1 - \frac{1}{\widehat{\theta}_2^{(c)}} & \text{if } \widehat{\psi} \geq 0 \end{cases}$	$\widehat{\theta} = (0, \frac{1}{1-\widehat{\sigma}})$
$\widehat{\theta}_1^{(c)} = 0$ and $0 \leq \widehat{\theta}_2^{(c)} < 1$	$\widehat{\sigma} = \infty$	$\widehat{\theta} = \widehat{\theta}^{(c)}$

3.6 Standard error of estimation

We will now derive expressions for the asymptotic standard error of the two-stage estimator $\widehat{\sigma}$. First, we note that regardless of θ^0 we have, asymptotically:

$$\sqrt{n}(\widehat{\theta}^{(u)} - \theta^0) \overset{D}{\Rightarrow} N(0, \Sigma)$$

with

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}.$$

If $\theta^0 \in \Theta_{int}$, then $Var(\widehat{\sigma})$ follows from a Taylor expansion of $\sigma(\theta)$ around θ^0:

$$\sigma(\widehat{\theta}^u) - \sigma(\theta^0) \overset{D}{\simeq} g(\theta^0)h(\theta^0)'(\widehat{\theta}^u - \theta^0)$$

where $\overset{D}{\simeq}$ means that the error is of order $o_p(n^{-1/2})$ and

$$h(\theta) = \begin{bmatrix} a(\theta) + b(\theta), & b(\theta) \end{bmatrix}'$$

with

$$a(\theta) + b(\theta) = \frac{2\left[\theta_2^2 + 4\theta_1\right]^{-\frac{1}{2}}}{\left(\theta_2 + [\theta_2^2 + 4\theta_1]^{\frac{1}{2}}\right)} - \frac{1}{\theta_1}$$

$$b(\theta) = \frac{1 + \theta_2\left[\theta_2^2 + 4\theta_1\right]^{-\frac{1}{2}}}{\left(\theta_2 + [\theta_2^2 + 4\theta_1]^{\frac{1}{2}}\right)}.$$

Hence

$$Var(\widehat{\sigma}) \simeq \frac{1}{n} g(\theta^0)^2 h(\theta^0)' \Sigma h(\theta^0) \text{ if } \theta^0 \in \Theta_{int}.$$

The formulas for the standard errors of $\widehat{\sigma}$ when θ^0 is located at the boundary of the parameter space are more complicated. First, if $\theta_0 \in \Theta_4$, there are no finite standard errors ($\sigma = \infty$). The results regarding the cases $\theta^0 \in \Theta_2$ and $\theta^0 \in \Theta_3$ are presented in Proposition 1 below.

Proposition 1. *If $\theta^0 \in \Theta_2$, the asymptotic mean and variance of $\widehat{\sigma}$ are given by*

$$E(\widehat{\sigma}) = \sigma - \frac{1}{\sqrt{2n\pi}} g(\theta^0)\left[a(\theta^0)\frac{\sigma_{11} + \sigma_{12}}{\sigma_{11} + \sigma_{22} + 2\sigma_{12}} + b(\theta^0)\right]\sqrt{\sigma_{11} + \sigma_{22} + 2\sigma_{12}} + o(n^{-1/2})$$

and

$$\begin{aligned} Var(\widehat{\sigma}) = \frac{g(\theta^0)^2}{2n} & \left\{ a(\theta^0)^2 \left[\sigma_{11} - \frac{(\sigma_{11} + \sigma_{12})^2}{\sigma_{11} + \sigma_{22} + 2\sigma_{12}}\right] \right. \\ & \left. + \left[a(\theta^0)\frac{\sigma_{11} + \sigma_{12}}{\sigma_{11} + \sigma_{22} + 2\sigma_{12}} + b(\theta^0)\right]^2 (\sigma_{11} + \sigma_{22} + 2\sigma_{12}) \left(1 - \frac{1}{\pi}\right) \right\} + \frac{Var(\widehat{\tau}^{-1})}{2} + o(n^{-1}) \end{aligned}$$

If $\theta^0 \in \Theta_3$, define

$$\theta_1^* \equiv E(\widehat{\theta}_1^{(u)} | \widehat{\theta}_1^{(u)} > 0) = n^{-1/2}\sqrt{\frac{2\sigma_{11}}{\pi}} + o(n^{-1/2})$$

and

$$\theta_2^* \equiv E(\widehat{\theta}_2^{(u)} | \widehat{\theta}_1^{(u)} > 0) = \theta_2^0 + n^{-1/2}\sigma_{12}\sqrt{\frac{2}{\pi\sigma_{11}}} + o(n^{-1/2})$$

Then

$$E(\widehat{\sigma}) = \sigma + \frac{1}{2}\left[g(\theta^*) + \frac{1}{\theta_2^0}\right] + o(n^{-1/2})$$

and

$$Var(\widehat{\sigma}) = \frac{g(\theta^*)^2}{2n}\left\{b(\theta^*)^2\left(\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}\right) + \left[a(\theta^*) + b(\theta^*)(1 + \frac{\sigma_{12}}{\sigma_{11}})\right]^2 \sigma_{11}(1 - \frac{2}{\pi})\right\}$$
$$+ \frac{Var(\widehat{\psi})}{2} + \frac{1}{4}\left[g(\theta^*) + \frac{1}{\theta_2^0}\right]^2 + o(n^{-1})$$

See Appendix A for a proof. Note that $\lim \theta^* = \theta^0$ and $\lim g(\theta^*) = -1/\theta_2^0$ (although $g(\theta^0)$ is not defined when $\theta_1^0 = 0$). Hence plim $\widehat{\sigma} = \sigma$.

3.7 Pooling of estimates across reference firms

The Feenstra-Soderbery estimator requires that a fixed firm is chosen as the reference firm. This makes the estimator dependent on this ad hoc choice. A simple modification would be to generate a sequence of unrestricted Feenstra-Soderbery estimators, $\{\widehat{\theta}^{(s)}\}_{s=1}^N$, for each possible reference firm, s, and then choose as a final estimator some pooled estimator, $\widehat{\theta}^{(P)}$. The simplest approach is to choose $\widehat{\theta}^{(P)}$ as the arithmetic mean. Then

$$\widehat{\theta}^{(P)} = \frac{1}{N}\sum_{s=1}^{N}\widehat{\theta}^{(u,s)}.$$

To estimate $Var(\widehat{\theta}^{(P)})$ we use that:

$$Var(\widehat{\theta}^{(P)}) = \frac{1}{N^2}\sum_{j=1}^{N}\sum_{k=1}^{N} Cov(\widehat{\theta}^{(u,j)}, \widehat{\theta}^{(u,k)})$$

where, for random vectors x and y, $Cov(x, y) = E(xy') - E(x)E(y)'$ and $Var(x) = Cov(x, x)$. While estimates of $H_{ns} = Var(\widehat{\theta}^{(u,s)})^{-1}$ are simple by-products of the Fenstra-Soderbery estimator (see above), the challenge is to estimate $Cov(\widehat{\theta}^{(u,j)}, \widehat{\theta}^{(u,k)})$ for $j \neq k$. Obviously, the estimates of θ using different reference firms from the same sample are *not* independent. Fortunately, resampling methods can be applied. Due to its computational simplicity and because it treats all possible reference firms symmetrically, we use the jackknife (resampling without replacement). Our method is the following: Let $\widehat{\theta}_{-i}^{(u,s)}$ denote the unconstrained Fenstra-Soderbery estimate of θ in i' the resample, i.e. when firm $i \in \{1, ..., n\}$ is excluded from the sample and s is the

Table 4: **Number of firm-year observations**

Industry	NACE	Continuing firm	Entering firms	Exiting firms	Total
Food prod.	10	4888	344	324	5566
Wood prod.	16	3400	207	185	3808
Mineral prod.	23	1876	86	87	2072
Metal prod.	25	5119	338	284	5766
Machinery	28	3128	185	186	3527
Other	32	1681	99	100	1912
All industries		20092	1259	1166	22517

reference firm. Then define

$$\widehat{\theta}_{\cdot}^{(u,s)} = \frac{1}{n-1} \sum_{i \notin s} \widehat{\theta}_{-i}^{(u,s)}.$$

Our jackknife estimator of $Cov(\widehat{\theta}^{(u,j)}, \widehat{\theta}^{(u,k)})$ is then:

$$\widehat{Cov}(\widehat{\theta}^{(u,j)}, \widehat{\theta}^{(u,k)}) = \frac{1}{n-2} \sum_{i \notin \{j,k\}} (\widehat{\theta}_{-i}^{(u,j)} - \widehat{\theta}_{\cdot}^{(u,j)})(\widehat{\theta}_{-i}^{(u,k)} - \widehat{\theta}_{\cdot}^{(u,k)})' \text{ for all } j,k \in \{1,...,N\}.$$

When N is large, it may be necessary to estimate $Var(\widehat{\theta}^{(P)})$ based on a (randomized) subset of the N reference firms. In our sample, where N is less than 100, the above estimator is computationally feasible.

4 Empirical application

4.1 Data and operationalisations

Our population is limited to incorporated firms (including publicly owned) in the six largest manufacturing industries observed during 1995–2016. A list of the industries, with the number of firm-year observations per industry, is given in Table 4.

We define (labour) productivity as value added per employee in real prices. Value added is defined as gross value of production minus the value of intermediate inputs. Intermediate input is not directly available in the statistics, but is calculated residually as total operating costs minus the sum of labor costs and capital costs (including depreciation). Value added can be interpreted as the contribution of labor and capital inputs to operating income (before taxes) during the year. Our data source regarding labour input is Statistics Norway's Employer-Employee Register, which is a matched employer-employee dataset.

We deflate value added in current prices using firm-specific price indices of value added. Data on firm specific prices are taken from the Producer Price Index (PPI[6]). The PPI measures the price development of first hand sales of products to the Norwegian market and to export markets. The sample in the PPI consists of about 630 commodity groups.[7]

Our general model does not specify the time unit, but refers to t as *period* t. In practice, the shortest possible periodicity is one *year*. To reduce timeliness problems, we consider a periodicity of 3 years and "aggregate" variables within each period as described below. Timeliness problems may be particularly important for start-up firms, since a newly registered firm is typically only partially active during its first year of operation, which may even be later than its year of registration. If s denotes the calendar year ($s = 1995$ is the first observation year) and t is the period number, the relation between them is as follows:

$$t = \left[\frac{s - 1995}{3}\right], \; s = 1995, 1996, ..., 2016$$

where $[x]$ is the integer value of x (e.g. $[1.5] = 1$). The set of continuing, exiting and entering firms in period t are defined as follows:

C_t: Firms operative in year $1995 + 3(t-1)$ *and* $1995 + 3t$

E_t: Firms operative in year $1995 + 3t$ but *not* $1995 + 3(t-1)$

X_t: Firms operative in $1995 + 3(t-1)$ but *not* $1995 + 3t$

This means that a firm is entering in period t if its first date of operation is during the interval $(1995 + 3(t-1), 1995 + 3t)]$, it is exiting in period t if its last date of operation is during

[6]See `https://www.ssb.no/en/ppi/`.

[7]Thanks to Marina Rybalka for making these data available.

Table 5: **Estimates of parameters**

Industry	First-stage		Second-stage					Feenstra-Soderbery est. Unrestricted		Restricted
	Two-stage estimator[a]									
	$\widehat{\theta}_1^P$	$\widehat{\theta}_2^P$	$\widehat{\theta}_1$	$\widehat{\theta}_2$	$\widehat{\sigma}$	SE($\widehat{\sigma}$)	95% CI[b]	$\widehat{\theta}_1^{(u)}$	$\widehat{\theta}_2^{(u)}$	$\widehat{\sigma}$
10	0.19	0.90	0.14	0.86	7.93	1.54	$[5.5, 11.8]$	0.16	0.90	7.55
16	-0.17	1.08	0.22	0.78	5.56	0.69	$[4.4, 7.2]$	-0.25	1.42	1.66
23	-1.08	2.08	0.78	0.22	2.28	0.36	$[1.6, 3.1]$	0.28	0.76	4.69
25	0.35	0.42	0.35	0.42	3.39	0.24	$[3.0, 3.8]$	-0.32	-1.20	1.33
28	-0.49	1.50	0.17	0.87	8.92	4.12	$[4.2, 12.4]$	0.35	1.81	6.67
32	-0.46	1.70	0.27	0.73	4.67	0.91	$[3.2, 7.0]$	-0.64	1.72	1.74

[a] See Table 3 for definition of the two stages
[b] Transformed from symmetric confidence interval (CI) of $\widehat{u}$ where $\widehat{u} \equiv \ln(\widehat{\sigma} - 1)$ and $SE(\widehat{u}) \simeq SE(\widehat{\sigma})/(\widehat{\sigma} - 1)$

$[1995 + 3(t-1), 1995 + 3t)$, and it is continuing in period t if it is operating throughout $[1995 + 3(t-1), 1995 + 3t]$.

4.2 Empirical results

The first two columns of Table 5 show the first-stage estimator $\widehat{\theta}^{(P)}$ (i.e. after pooling $\widehat{\theta}^{(u,s)}$ across all possible reference firms, as explained in Section 3). We see that the first-stage (pooled) estimator satisfies the parameter constraints only for NACE 25. In all other industries, $\widehat{\theta}_1^{(P)} < 0$ or $\widehat{\theta}_1^{(P)} + \widehat{\theta}_2^{(P)} > 1$. The unrestricted estimates of θ using the Feenstra-Soderbery estimator are also shown in Table 5 (any differences between these estimates and our first-stage estimates are due to pooling). In three of the industries (NACE 23, 25 and 28), the estimates differ significantly. These results are clear evidence that the Feenstra-Soderbery estimator is not robust with respect to the choice of reference firm.

Our second-stage estimates of θ are depicted in columns 4 and 5. We see that in all cases where $\widehat{\theta}^{(P)}$ is an inadmissible value, the second-stage estimator satisfies $\widehat{\theta}_1 + \widehat{\theta}_2 = 1$. In these cases, $\widehat{\sigma} = 1/\widehat{\tau}$, where $\widehat{\tau}$ is the positive fixed effects regression estimate of τ from Equation (23). The estimates of σ depicted in Table 5 lie in the range of $2.3 - 8.9$. It is interesting to compare our

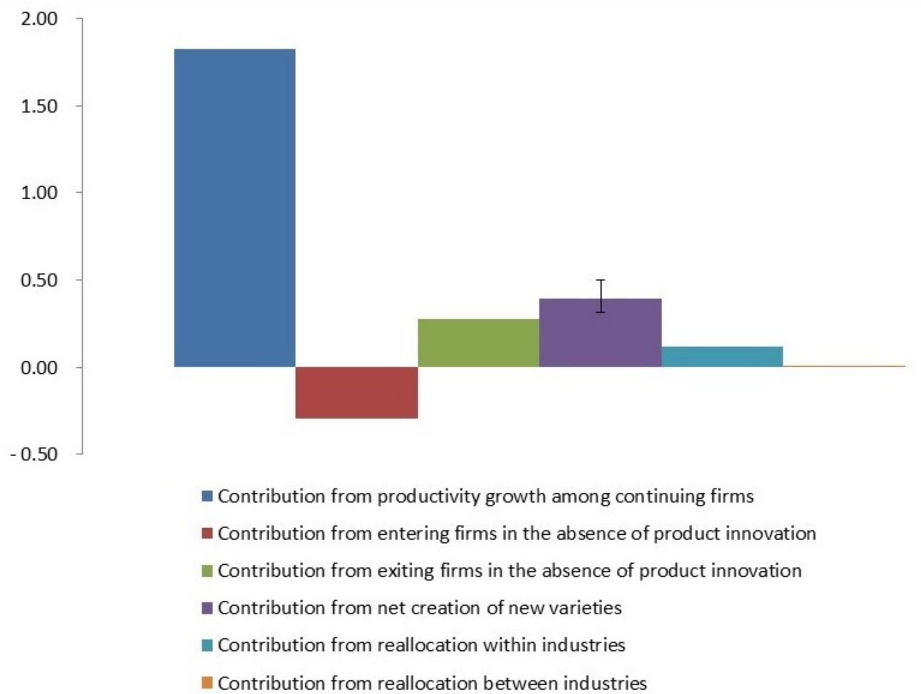

FIGURE 3: Contributions to aggregate productivity growth. Average annual growth rate in per cent across the sample 1996-2016

σ-estimates with $\widehat{\sigma}$ from the restricted Feenstra-Soderbery estimator depicted in the last column of Table 5 (this estimator does not provide standard errors of $\widehat{\sigma}$ when $\widehat{\theta}^{(u)}$ is at the boundary of the parameter space). The two sets of σ-estimates differ significantly: in only one industry does $\widehat{\sigma}$ from the Feenstra-Soderbery method lie within the 95 per cent confidence interval of our method. The most striking difference is that while the three lowest Feenstra-Soderbery estimates of σ are well below 2 in three industries (NACE 16, 25 and 32), implying a large impact of new varieties on the output growth index, the lowest estimate with our method is 2.3 (in NACE 23). The standard errors and confidence intervals in Table 5 show that σ is precisely estimated when $\widehat{\sigma}$ is small or moderate (less than 6), but more imprecisely estimated for higher values of $\widehat{\sigma}$.

Table 6 depicts average annual productivity growth rates over the observation period 1996-2016 for the six industries, and for aggregate manufacturing (all six industries). When showing the results, we have split the whole observation period into seven intervals ($t = 1, 2, ..., 7$), each covering three years, as explained above. For each 3-year interval we present average *annual* growth rates in per cent corresponding to $\sigma = \infty$ (no impact of new varieties) and $\sigma = \widehat{\sigma}$ (the

TABLE 6: **Productivity growth rates. By industry and aggregate for all industries, in per cent**

NACE	σ[a]	Period (t)							
		96-98[b]	99-01	02-04	05-07	08-10	11-13	14-16	Mean
10	∞	2.1	-1.4	4.7	3.0	3.1	0.5	0.6	1.8
	$\widehat{\sigma}$	3.2	-0.7	5.2	3.3	3.4	0.7	0.4	2.2
16	∞	1.0	0.7	0.5	0.3	0.2	0.2	-0.2	0.4
	$\widehat{\sigma}$	-0.5	-0.6	6.2	-0.5	6.3	-0.8	3.5	1.9
23	∞	-1.7	-3.5	2.7	5.0	1.2	3.7	3.3	1.5
	$\widehat{\sigma}$	-1.2	-2.1	3.7	5.5	1.4	4.1	2.9	2.0
25	∞	8.2	-1.6	2.4	4.6	2.0	2.8	-3.4	2.1
	$\widehat{\sigma}$	10.8	-0.6	3.2	5.1	2.7	3.2	-3.7	3.0
28	∞	5.2	-4.3	6.3	4.8	3.7	-2.6	-4.2	1.3
	$\widehat{\sigma}$	5.4	-4.2	6.7	5.1	3.8	-2.5	-4.2	1.4
32	∞	4.0	4.1	6.5	-3.0	9.9	1.1	5.4	4.0
	$\widehat{\sigma}$	4.4	5.8	6.9	-2.7	10.0	1.3	5.1	4.4
All industries	∞	2.8	-1.2	4.2	3.1	3.5	1.1	0.1	1.9
	$\widehat{\sigma}$	3.8	-0.5	4.7	3.4	3.7	1.2	-0.1	2.3

[a] $\widehat{\sigma}$ refers to the estimated value of σ in Table 5
[b] Average annual rates during 3-year interval

estimated demand elasticity for each industry). The results for all industries then involve a weighted average of all the industry-specific estimates of σ.

We first note that there are two distinct 3-year periods marked by negative or near-zero aggregate productivity growth: 1999-2001 and 2014-2016, respectively, and one 3-year period with very high productivity growth: 2002-2004. From 2002 to 2010 productivity growth was persistently high: between 3.4–4.7 per cent annually. This period overlaps with the height of the oil-fueled boom-period that lasted from 2001 until the financial crisis of 2008 (when oil prices surged from $20 to more than $100 per barrel). This period is followed by a clear downward trend i productivity growth after 2010: Annual productivity growth, including the contribution

TABLE 7: **Sources of aggregate productivity growth.**[a] **Growth rates in per cent**

Source	96-98	99-01	02-04	05-07	08-10	11-13	14-16	Mean
Continuing firms	2.7	-1.3	5.1	3.0	3.2	0.8	-0.8	1.8
Entering firms ($\sigma = \infty$)	-0.6	-0.2	-0.5	-0.2	-0.2	-0.4	-0.0	-0.3
Exiting firms ($\sigma = \infty$)	0.0	0.1	0.1	0.1	0.1	0.1	1.4	0.3
RWI	0.8	0.2	-0.5	-0.0	0.3	0.4	-0.4	0.1
RBI	-0.1	-0.0	0.0	0.2	0.1	0.1	-0.2	0.0
New varieties ($\sigma = \widehat{\sigma}$)[b]	1.0	0.7	0.5	0.3	0.2	0.2	-0.2	0.4
	(0.08)	(0.08)	(0.06)	(0.04)	(0.03)	(0.03)	(0.03)	(0.05)
Total productivity growth	3.8	-0.5	4.7	3.4	3.7	1.2	-0.1	2.3

[a] Decomposed according to Equation (14)

[b] Standard error in the estimated contribution from new varieties (due to $\widehat{\sigma}$) in parentheses

from net creation of new varieties, was 3.7 per cent during 2008-2010, it fell to 1.2 per cent during 2011-2013, and then further to practically zero (0.1 per cent) during 2014-2016.

New varieties contribute significantly to total productivity growth. If we average across the whole observation period, annual productivity growth is 1.9 per cent with no allowance for new varieties and 2.3 per cent when new varieties are taken into account. Thus, on average, the estimated contribution form new varieties to total productivity growth is 0.4 percentage points annually, which is economically significant.

Table 7 depicts the results of the decomposition of aggregate productivity growth into six sources: i) within-firm productivity growth, ii) entering firms when all products are assumed perfect substitutes ($\sigma = \infty$), iii) exiting firms when $\sigma = \infty$, iv) reallocation between continuing firms in the same industries (RWI), v) reallocation between continuing firms in different industries (RBI), and vi) net creation of new varieties (with $\sigma = \widehat{\sigma}$ – the estimated industry-specific demand elasticities). The terms i)-iii) correspond to the first three terms within the squared bracket in Equation (12), whereas iv) corresponds to the last term in the squared bracket (which is zero if $\sigma = \infty$). The decomposition of labor productivity growth into its various sources – and over time — is also depicted in Figures 3 – 4, with 95 per cent confidence intervals for the contribution

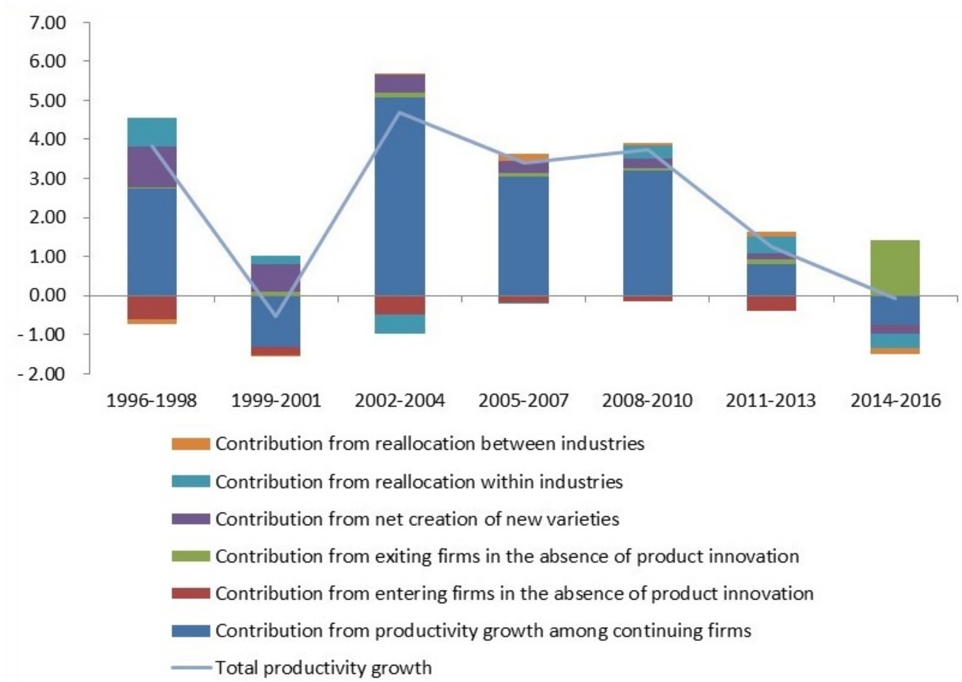

Figure 4: Contributions to aggregate productivity growth during 1996 – 2016. Average annual growth rate in per cent

from net creation of new varieties indicated by markers.

We see that the within-firm productivity growth among continuing firms is the most important source of aggregate productivity growth (contributing with 1.8 percentage points annually to the total annual average productivity growth of 2.3 per cent). Net creation of new varieties contributes 0.4 percentage point to the average productivity growth. This contribution is statistically highly significant, as seen from the estimated standard error of 0.05 and the indicated 95 per cent confidence interval (see Figure 3). Disregarding the impact of new varieties, exiting firms contributed positively to productivity growth (0.3 percentage point). This reflects the fact that exiting firms on average have lower productivity levels than survivors. This effect is exceptionally strong in 2014-2016, where closures contribute 1.4 percentage point annually during a period where total productivity growth was almost zero. Figure 3 shows that reallocation of labor within and – in particular – between industries is of minor importance to aggregate productivity growth.

The finding that entry of new firms contributes negatively to productivity growth (disre-

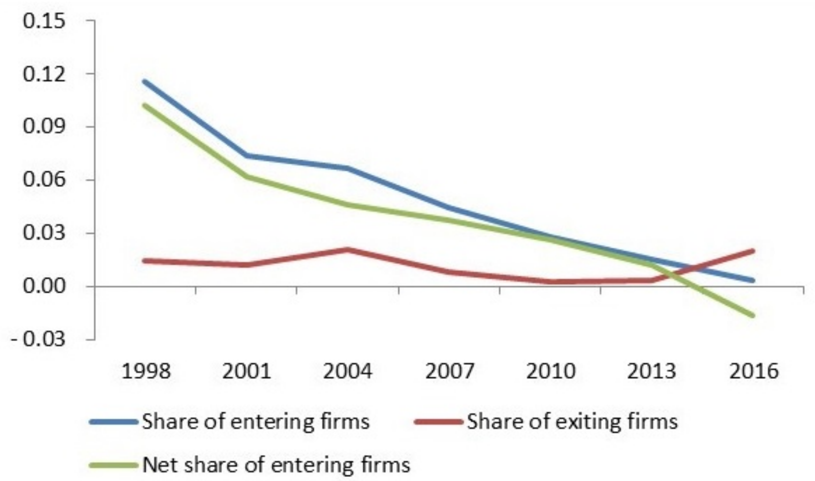

FIGURE 5: Output shares of entering and exiting firms

garding the impact from new varieties) may seem to contradict conventional wisdom that entry- and exit dynamics contributes to "creative destruction", whereby inefficient old firms are replaced by new and more efficient firms. However, our finding is not surprising in view of the high exit rates among young firms, and is consistent with the results in Golombek and Raknerud (2015), who document strong selection based on productivity among start-up firms. Moreover, our results are in line with conventional decompositions of aggregate productivity growth for the whole mainland Norwegian economy (Iancu and Raknerud, 2017).

Figure 5 shows that the (gross) output share of entering firms and the net share of entering firms (share of entering firms less that of exiting firms), have been monotonically decreasing since 1998. The net share even became negative in 2016, leading to negative (value-weighted) net creation of new varieties during 2014–2016. The strong increase in the output share of exiting firms from 2014 to 2016 explains the exceptionally high positive contribution to productivity growth from closures in this period (as commented on above), whereas all other components contributed negatively, almost exactly offsetting the first effect.

5 Conclusion

The contribution of this paper is twofold. First, based on the economic approach to index numbers, we have provided a fully consistent decomposition of aggregate productivity growth that identifies the contribution from new firms producing new varieties. The novelty of this decomposition lies in the way we have reconciled the literature on how new goods impact prices and the literature on aggregate productivity growth and firm turnover. The decomposition provided in this paper encompasses many of the frameworks currently adopted in the literature.

Second, we have extended the Limited Information Maximum Likelihood (LIML) estimator of Soderbery (2015), which is a refinement of Feenstra (1994) and Broda and Weinstein (2006), and is currently the most advanced method in the literature to estimate demand elasticities. To overcome the simultaneity problem, this literature uses the second order moments of prices and expenditure shares in combination with sign restrictions to identify demand elasticities. We have created a two-stage estimation framework that exploits the cases when there are no simultaneity problems, i.e. when supply is elastic or inelastic. Elastic or inelastic supply occur at the boundary of the parameter space. Hence, if the first-stage estimate is located at the boundary of the parameter space, we switch in the second stage to an estimator that utilises first order moments of prices and expenditure shares to improve efficiency. In particular, we have shown that the case of inelastic supply is both empirically and theoretically important. We also derive closed form variance formulas of the two-stage estimator. Another refinement in our estimation procedure relates to the choice of reference unit. Instead of choosing one particular firm as a reference firm, which involves ad hoc elements and raises robustness issues, we extend current practice by generating a sequence of estimates for each possible reference firm and create a "pooled" estimator across all possible choices.

Our results indicate that the effect from new varieties on aggregate productivity growth is highly significant and amounts to about one-half percentage point annually for some major manufacturing industries in Norway during the period from 1996 to 2016. This result is based on estimates of the demand elasticity ranging from 2.3 to about 9. Our estimates of demand elasticities at the industry level differ in many cases significantly from those of the Feenstra-Soderbery estimator, and are typically higher. Moreover, we have demonstrated that the choice of reference firm is of importance.

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A Proof of Proposition 1

Below we will expand $g(\widehat{\theta}^u)$ around $g(\theta^*)$ for two different values of θ^* satisfying:

$$g(\widehat{\theta}^u) - g(\theta^*) \overset{D}{\simeq} g(\theta^*) \left\{ \left[\frac{2\left[\theta_2^{*2} + 4\theta_1^*\right]^{-\frac{1}{2}}}{\left(\theta_2^* + \left[\theta_2^{*2} + 4\theta_1^*\right]^{\frac{1}{2}}\right)} - \frac{1}{\theta_1^*} \right] (\widehat{\theta}_1^u - \theta_1^*) + \left[\frac{1 + \theta_2^*\left[\theta_2^{*2} + 4\theta_1^*\right]^{-\frac{1}{2}}}{\left(\theta_2^* + \left[\theta_2^{*2} + 4\theta_1^*\right]^{\frac{1}{2}}\right)} \right] (\widehat{\theta}_2^u - \theta_2^*) \right\}$$

$$= g(\theta^*) \left\{ (a(\theta^*) + b(\theta^*))(\widehat{\theta}_1^u - \theta_1^*) + b(\theta^*)(\widehat{\theta}_2^u - \theta_2^*) \right\}$$

(see Section 3.6 for explanations of notation).

Case 1: $\theta^0 \in \Theta_2$**.** Here we set

$$\theta^* = \theta^0$$

Asymptotically, with probability one, either $\widehat{\theta} = \theta^{r1}$ or $\widehat{\theta} = \widehat{\theta}^u \in \Phi_{int}$. To examine the behavior of $\widehat{\theta}^u$ given that $\widehat{\theta}^u \in \Phi_{int}$, define

$$\begin{aligned} \Delta &= \widehat{\theta}_1^{(u)} - \theta_1^0 + \widehat{\theta}_2^{(u)} - \theta_2^0 \\ &= \widehat{\theta}_1^{(u)} + \widehat{\theta}_2^{(u)} - 1 \end{aligned}$$

Then $\widehat{\theta} \in \Phi_{int}$ is equivalent to $\widehat{\theta}_1^{(u)} > 0$ and $\Delta < 0$ and $\widehat{\theta} = \theta^{r1} \in \Phi_2$ is equivalent to $\widehat{\theta}_1^{(u)} > 0$ and $\Delta \geq 0$. Moreover,

$$\Delta \overset{D}{\simeq} n^{-1/2}\sigma_\Delta Z, \text{ where } \sigma_\Delta = \sqrt{\sigma_{11} + \sigma_{22} + 2\sigma_{12}} \text{ and } Z \sim N(0,1).$$

Furthermore

$$\widehat{\theta}_2^{\,u} - \theta_2^0 = \Delta - (\widehat{\theta}_1^{(u)} - \theta_1^0)$$

where

$$\widehat{\theta}_1^{(u)} - \theta_1^0 \overset{D}{\simeq} \chi\Delta + \varepsilon$$

with

$$\chi = \frac{Cov(\Delta, \widehat{\theta}_1^{(u)})}{Var(\Delta)} \simeq \frac{\sigma_{11} + \sigma_{12}}{\sigma_\Delta^2}$$

and

$$\varepsilon \stackrel{D}{=} N(0, \sigma_\varepsilon^2).$$

Then ε is conditionally independent of Δ with

$$\sigma_\varepsilon^2 = n^{-1}\left[\sigma_{11} - \frac{(\sigma_{11}+\sigma_{12})^2}{\sigma_\Delta^2}\right] = n^{-1}\left[\sigma_{11} - \frac{(\sigma_{11}+\sigma_{12})^2}{\sigma_{11}+\sigma_{22}+2\sigma_{12}}\right]$$

A Taylor expansion $\widehat{\theta}^u$ around $\theta^* = \theta^0$ gives:

$$\begin{aligned} g(\widehat{\theta}^u) - g(\theta^0) &\stackrel{D}{\simeq} g(\theta^0)\left\{\left(a(\theta^0)+b(\theta^0)\right)(\widehat{\theta}_1^u - \theta_1^0) + b(\theta^*)(\widehat{\theta}_2^u - \theta_2^0)\right\} \\ &= g(\theta^0)\left\{a(\theta^0)\varepsilon + \left[a(\theta^0)\chi + b(\theta^0)\right]\Delta\right\} \end{aligned}$$

It follows that

$$\begin{aligned} E(g(\widehat{\theta}^u)|\Delta<0) &\simeq g(\theta^0) + g(\theta^0)\left[a(\theta^0)\chi + b(\theta^0)\right]E(\Delta|\Delta<0) \\ Var(g(\widehat{\theta}^u)|\Delta<0) &\simeq g(\theta^0)^2\left\{a(\theta^0)^2\sigma_\varepsilon^2 + \left[a(\theta^0)\chi + b(\theta^0)\right]^2 Var(\Delta|\Delta<0)\right\} \end{aligned}$$

The well-known expressions for $E(Z|Z>0)$ and $Var(Z|Z>0)$ are:

$$Var(Z|Z>0) = 1 - \psi(0)^2$$

and

$$E(Z|Z>0) = \psi(0)$$

where $\psi(\cdot)$ is the inverse Mills ratio:

$$\psi(0) = \phi(0)/\Phi(0) = 2\phi(0) = \frac{2}{\sqrt{2\pi}} = \sqrt{\frac{2}{\pi}}.$$

Since $\Delta \overset{D}{\simeq} n^{-1/2}\sigma_\Delta Z$:

$$E(\Delta|\Delta < 0) = -E(-\Delta| - \Delta > 0) \simeq -n^{-1/2}\sigma_\Delta E(Z|Z > 0) = -n^{-1/2}\sigma_\Delta \psi(0)$$
$$Var(\Delta|\Delta < 0) \simeq n^{-1}\sigma_\Delta^2 Var(Z|Z > 0) = n^{-1}\sigma_\Delta^2(1 - \psi(0)^2)$$

Hence

$$E(g(\widehat{\theta}^u)|\Delta < 0) \simeq g(\theta^*) - g(\theta^*)\left[a(\theta^*)\chi + b(\theta^*)\right] n^{-1/2}\sigma_\Delta \psi(0)$$
$$Var(g(\widehat{\theta}^u)|\Delta < 0) \simeq g(\theta^*)^2 \left\{a(\theta^*)^2\sigma_\varepsilon^2 + \left[a(\theta^*)\chi + b(\theta^*)\right]^2 n^{-1}\sigma_\Delta^2(1 - \psi(0)^2)\right\}$$

Thus, if $\theta^0 \in \Theta_2$, $\widehat{\sigma}$ has an asymptotic mixture distribution

$$\widehat{\sigma} - \sigma \overset{D}{\simeq} 1(\Delta < 0)(g(\widehat{\theta}^u) - g(\theta_0)) + (1 - 1(\Delta < 0))(\widehat{\tau}^{-1} - \sigma)$$

where

$$\Pr(\Delta < 0) \simeq \Pr(n^{-1/2}\sigma_\Delta Z < 0) = \frac{1}{2}$$

Let D be a binary variable with $\Pr(D = 1) = P$ and $Y = DY_1 + (1 - D)Y_0$. By the rules of double expectation and total variance:

$$E(Y) = PE(Y_1|D = 1) + (1 - P)E(Y_0|D = 0)$$

and

$$Var(Y) = PVar(Y_1|D = 1) + (1 - P)Var(Y_0|D = 0)$$
$$+ P(1 - P)\left[E(Y_1|D = 1) - E(Y_0|D = 0)\right]^2$$

Hence

$$
\begin{aligned}
E(\widehat{\sigma}) &\simeq \sigma + \frac{1}{2}(E(g(\widehat{\theta}^u)|\Delta < 0) - g(\theta^0)) + \frac{1}{2}E(\widehat{\tau}^{-1} - \sigma) \\
&= \sigma + \frac{1}{2}(E(g(\widehat{\theta}^u)|\Delta < 0) - g(\theta^0)) \\
Var(\widehat{\sigma}) &\simeq \frac{1}{2}Var(g(\widehat{\theta}^u)|\Delta < 0) + \frac{1}{2}Var(\widehat{\tau}^{-1}) \\
&+ \frac{1}{4}\left(E(g(\widehat{\theta}^u)|\Delta < 0) - g(\theta^0)\right)^2
\end{aligned}
$$

That is:

$$
\begin{aligned}
E(\widehat{\sigma}) &\simeq \sigma - \frac{1}{2}g(\theta^0)\left[a(\theta^0)\chi + b(\theta^0)\right]n^{-1/2}\sigma_\Delta\psi(0) \\
&= \sigma - \frac{1}{\sqrt{2n\pi}}g(\theta^0)\left[a(\theta^0)\frac{\sigma_{11} + \sigma_{12}}{\sigma_{11} + \sigma_{22} + 2\sigma_{12}} + b(\theta^0)\right]\sqrt{\sigma_{11} + \sigma_{22} + 2\sigma_{12}}
\end{aligned}
$$

and

$$
\begin{aligned}
Var(\widehat{\sigma}) &\simeq \frac{1}{2}g(\theta^0)^2\left\{a(\theta^0)^2\sigma_\varepsilon^2 + \left[a(\theta^0)\chi + b(\theta^0)\right]^2 n^{-1}\sigma_\Delta^2(1 - \psi(0)^2)\right\} + \frac{1}{2}Var(\widehat{\tau}^{-1}) \\
&+ \frac{1}{4}\left[g(\theta^0)\left[a(\theta^0)\chi + b(\theta^0)\right]n^{-1/2}\sigma_\Delta\psi(0)\right]^2 \\
&= g(\theta^0)^2\left\{\frac{1}{2}a(\theta^0)^2\sigma_\varepsilon^2 + \frac{1}{2}\left[a(\theta^0)\chi + b(\theta^0)\right]^2 n^{-1}\sigma_\Delta^2(1 - \psi(0)^2) + \right. \\
&\left.\frac{1}{4}n^{-1}\left[a(\theta^0)\chi + b(\theta^0)\right]^2\sigma_\Delta^2\psi(0)^2\right\} + \frac{1}{2}Var(\widehat{\tau}^{-1}) \\
&= \frac{g(\theta^0)^2}{2n}\left\{a(\theta^0)^2\left(\sigma_{11} - \frac{(\sigma_{11} + \sigma_{12})^2}{\sigma_{11} + \sigma_{22} + 2\sigma_{12}}\right) + \right. \\
&= \left.\left[a(\theta^0)\frac{\sigma_{11} + \sigma_{12}}{\sigma_{11} + \sigma_{22} + 2\sigma_{12}} + b(\theta^0)\right]^2(\sigma_{11} + \sigma_{22} + 2\sigma_{12})\left(1 - \frac{1}{\pi}\right)\right\} + \frac{Var(\widehat{\tau}^{-1})}{2}
\end{aligned}
$$

Case 2: $\theta^0 \in \Theta_3$. Here $\sigma = 1 - (\theta_2^0)^{-1}$. We define

$$
\begin{aligned}
\theta_1^* &= E(\widehat{\theta}_1^{(u)}|\widehat{\theta}_1^{(u)} > 0) \\
\theta_2^* &= E(\widehat{\theta}_2^{(u)}|\widehat{\theta}_1^{(u)} > 0)
\end{aligned}
$$

Asymptotically, with probability one, $\widehat{\theta}_2 = \widehat{\theta}_2^{\,u} < 0$ and either $\widehat{\theta}_1 = \widehat{\theta}_1^{\,u} > 0$ ($\widehat{\theta} = \widehat{\theta}^u \in \Theta_{int}$) or $\widehat{\theta}_1^{\,u} \le 0$ ($\widehat{\theta} \in \Theta_3$).

In the first case, $\sigma = 1 + g(\widehat{\theta}^u)$. We can write

$$\widehat{\theta_2}^{(u)} - \theta_2^0 = \Pi\widehat{\theta}_1^{(u)} + \eta$$

with

$$\Pi = \frac{Cov(\widehat{\theta_2}^{(u)}, \widehat{\theta_1}^{(u)})}{Var(\widehat{\theta_1}^{(u)})} \simeq \frac{\sigma_{12}}{\sigma_{11}},$$

and

$$\eta \stackrel{D}{=} N(0, \sigma_\eta^2).$$

where η is conditionally independent of $\widehat{\theta}_1^{(u)}$ with

$$\sigma_\eta^2 = n^{-1}\left[\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}\right]$$

If follows that

$$\theta_1^* = E(\widehat{\theta}_1^{(u)}|\widehat{\theta}_1^{(u)} > 0) \simeq n^{-1/2}\sqrt{\sigma_{11}}\psi(0) = n^{-1/2}\sqrt{\frac{2\sigma_{11}}{\pi}}$$
$$\theta_2^* = \theta_2^0 + \Pi E(\widehat{\theta}_1^{(u)}|\widehat{\theta}_1^{(u)} > 0) \simeq \theta_2^0 + \Pi n^{-1/2}\sqrt{\sigma_{11}}\psi(0) = \theta_2^0 + n^{-1/2}\sigma_{12}\sqrt{\frac{2}{\pi\sigma_{11}}}$$

and

$$\widehat{\theta}_2^u - \theta_2^* = \Pi(\widehat{\theta}_1^{(u)} - \theta_1^*) + \eta$$

We then get

$$\begin{aligned}
g(\widehat{\theta}^u) - g(\theta^*) &\stackrel{D}{\simeq} g(\theta^*)\left\{(a(\theta^*) + b(\theta^*))\,(\widehat{\theta}_1^u - \theta_1^*) + b(\theta^*)(\widehat{\theta}_2^u - \theta_2^*)\right\} \\
&= g(\theta^*)\left\{(a(\theta^*) + b(\theta^*))\,(\widehat{\theta}_1^u - \theta_1^*) + b(\theta^*)(\Pi(\widehat{\theta}_1^{(u)} - \theta_1^*) + \eta)\right\} \\
&= g(\theta^*)\left\{b(\theta^*)\eta + [a(\theta^*) + b(\theta^*)(1 + \Pi)]\,(\widehat{\theta}_1^{(u)} - \theta_1^*)\right\}
\end{aligned}$$

Hence

$$
\begin{aligned}
E(g(\widehat{\theta}^u)|\widehat{\theta}_1^{(u)} > 0) &\simeq g(\theta^*) \\
Var(g(\widehat{\theta}^u)|\widehat{\theta}_1^{(u)} > 0) &\simeq g(\theta^*)^2 \left\{ b(\theta^*)^2 \sigma_\eta^2 + \left[a(\theta^*) + b(\theta^*)(1+\Pi)\right]^2 n^{-1} \sigma_{11} (1 - \psi(0)^2) \right\} \\
&= \frac{g(\theta^*)^2}{n} \left\{ b(\theta^*)^2 \left[\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}} \right] + \left[a(\theta^*) + b(\theta^*)(1 + \frac{\sigma_{12}}{\sigma_{11}}) \right]^2 \sigma_{11} (1 - \frac{2}{\pi}) \right\}
\end{aligned}
$$

where we used that

$$
Var(\widehat{\theta}_1^{(u)}|\widehat{\theta}_1^{(u)} > 0) \simeq n^{-1} \sigma_{11} (1 - \psi(0)^2)
$$

Now consider $\widehat{\theta_1}^u < 0$. Then, asymptotically with probability one, $\widehat{\theta}_1 = 0$ and $\widehat{\sigma} = 1 - \widehat{\psi}$. Hence

$$
\widehat{\sigma} - \sigma = -\widehat{\psi} + (\theta_2^0)^{-1}
$$

and

$$
\begin{aligned}
E(\widehat{\sigma}|\widehat{\theta_1}^u < 0) &\simeq \sigma \\
Var(\widehat{\sigma}|\widehat{\theta_1}^u < 0) &\simeq Var(\widehat{\psi})
\end{aligned}
$$

Combining the two cases: If $\theta^0 \in \Theta_3$, $\widehat{\sigma}$ is asymptotically distributed as

$$
\widehat{\sigma} - \sigma \overset{D}{\simeq} 1(\widehat{\theta}_1^u > 0)(g(\widehat{\theta}^u) + (\theta_2^0)^{-1}) + 1(\widehat{\theta}_1^u < 0)(-\widehat{\psi} + (\theta_2^0)^{-1})]
$$

where

$$
\Pr(\widehat{\theta}_1^u > 0) \simeq \Pr(n^{-1/2}\sqrt{\sigma_{11}} Z > 0) = \frac{1}{2}
$$

Hence

$$
E(\widehat{\sigma}) \simeq \sigma + \frac{1}{2}\left[g(\theta^*) + (\theta_2^0)^{-1}\right]
$$

and

$$
\begin{aligned}
Var(\widehat{\sigma}) &\simeq \frac{1}{2} Var(g(\widehat{\theta}^u | \widehat{\theta}_1^u > 0) + \frac{1}{2} Var(\widehat{\sigma} | \widehat{\theta_1}^u < 0) \\
&+ \frac{1}{4} \left[g(\theta^*) + (\theta_2^0)^{-1} \right]^2 \\
&= \frac{1}{2n} g(\theta^*)^2 \left\{ b(\theta^*)^2 \left[\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}} \right] + \left[a(\theta^*) + b(\theta^*)(1 + \frac{\sigma_{12}}{\sigma_{11}}) \right]^2 \sigma_{11}(1 - \frac{2}{\pi}) \right\} \\
&+ \frac{1}{2} Var(\widehat{\psi}) + \frac{1}{4} \left[g(\theta^*) + (\theta_2^0)^{-1} \right]^2
\end{aligned}
$$

B Relation to existing literature

Much of the literature analyses the contribution to overall productivity growth from firm turnover by applying a framework based on a weighted average of productivity levels; see e.g., Griliches and Regev (1995); Baily et al. (1992); Foster et al. (2001, 2006) and Foster et al. (2008). In the following we point out similarities and differences between the standard frameworks used in the literature and the novel framework outlined in Equation (12). In particular, we show how the framework outlined in Equation (12) generalises the frameworks typically used in the literature.

Following the notation used in the main text, the *level* of productivity in a firm is defined as the ratio of outputs to inputs in real terms Y_{ift}/L_{ift}. A weighted arithmetic average productivity level across all firms can then be written

$$\Pi_{it} = \sum_{f \in F_{it}} \pi_{ift}(Y_{ift}/L_{ift}), \tag{25}$$

where the weights π_{ift} sum to unity and F_{it} denotes the set of all firms producing a variety of good b. For this average to have a meaningful interpretation, all firms must be producing identical or homogeneous products. For example, if one firm is producing 50 tablets per man hour and another firm is producing 40 tablets per man hour, the average number of tablets produced per hour is 45, if weights are equal across the two firms. If firms are not producing homogeneous products, one is comparing apples and oranges when taking the average in Equation (25). For example, if one firm is producing 1000 cellular phones per man hour and another firm is producing 50 tablets per man hour, what is the average productivity across those two firms? Since the two firms are producing two different products it is not meaningful to compare productivity levels across firms. This basic insight relates to the basic index number problem and it illustrates the restrictiveness in using Equation (25) as a starting point for decomposing aggregate productivity growth.

The assumption of homogeneous products implicitly underlying Equation (25) can be made explicit in terms of the framework outlined in Section 2.3. Consider the aggregation of varieties in Equation (2): $Y_{it} = \left(\sum_{f \in F_{it}} \gamma_{if} Y_{ift}^{(\sigma_i - 1)/\sigma_i}\right)^{\sigma_i/(\sigma_i-1)}$. All of the varieties are homogeneous if the following assumptions hold

$$\gamma_{if} = 1 \text{ and } \sigma_i \to \infty \text{ for all } f \in F_{it},\ i \in I.$$

Given these assumptions, aggregation of output is then reduced to a summation across the homogeneous products, i.e. $Y_{it} = \sum_{f \in F_{it}} Y_{ift}$. One way to measure the average productivity level in Equation (25) is by the ratio of outputs to inputs

$$\Pi_{it} = \frac{Y_{it}}{L_{it}} = \frac{\sum_{f \in F_{it}} Y_{ift}}{\sum_{f \in F_{it}} L_{ift}} = \sum_{f \in F_{it}} \pi_{ift}(Y_{ift}/L_{ift})$$

where the weights now are defined as input shares: $\pi_{ift} = \frac{L_{ift}}{\sum_{f \in F_{it}} L_{ift}}$. For example, Iancu and Raknerud (2017) apply this weighting scheme. It is however most common to base the weights π_{ift} on output shares.[8] After comparing productivity levels by industry (or product) one may average the results across industries, i.e.

$$\Pi_t = \sum_{i \in I} \pi_{it} \Pi_{it}, \tag{26}$$

where aggregate output shares typically are used as weights. The change in average productivity, as defined by Equation (25) and Equation (26), may be decomposed as

$$\begin{aligned} \Delta \Pi_t = \sum_{i \in I} \pi_{it} \Bigg[\sum_{f \in C_{it}} \overline{\pi_{ift}} \Delta (Y_{ift}/L_{ift}) + \sum_{f \in N_{it}} \pi_{ift} \left(\overline{\frac{Y_{ift}}{L_{ift}}} - \overline{\Pi}_i \right) \\ - \sum_{f \in X_{it}} \pi_{if,t-1} \left(\overline{\frac{Y_{if,t-1}}{L_{if,t-1}}} - \overline{\Pi}_i \right) \Bigg] + \widetilde{RWI}_t + \widetilde{RBI}_t. \end{aligned} \tag{27}$$

The first term within the square brackets represents a within component showing the weighted average of productivity *growth* across continuing firms. The last two terms inside the square bracket represent the contribution from entering and exiting plants, respectively. Note that the impact from firm turnover on productivity growth depends on the productivity levels of entering and exiting firms relative to the average productivity level: aggregate productivity increases if

[8] A rationale for doing this is to take the reciprocal of the aggregate inverse productivity measure, see Diewert and Fox (2010).

either entering firms are more productive than the average or exiting firms are less productive than the average.

The two last terms represent reallocation effects within and between industries, respectively, and they are given by

$$\widetilde{RWI}_t = \sum_{i \in I} \pi_{it} \sum_{f \in C_{it}} \left(\overline{\frac{Y_{ift}}{L_{ift}}} - \overline{\Pi}_i \right) \Delta\pi_{ift} \tag{28}$$

$$\widetilde{RBI}_t = \sum_{i \in I} \overline{\Pi}_i \Delta\pi_{it}. \tag{29}$$

Reallocation within industries ($\widetilde{RWI}$) contributes positively to aggregate productivity if the weight of high productivity firms increases. Reallocation between industries ($\widetilde{RBI}$) contributes positively to aggregate productivity if the weight of highly productive industries increases.

The framework outlined in Equation (27) is conceptually very similar to what is typically found in the literature. For example, Foster et al. (2008) also starts out with a weighted average of productivity levels across firms in the first stage of aggregation (Equation (25)). However, instead of applying a weighted average of productivity levels at the second stage of aggregation (Equation (26)), Foster et al. (2008) calculate a weighted average of changes in productivity levels at the industry/product level, i.e. $\Delta\Pi_t = \sum_{i \in I} \pi_{it}\Delta\Pi_{it}$. The only difference between that approach and Equation (27), is that Equation (27) also holds the impact from reallocation between industries. Importantly, the underlying assumption that products are homogeneous within industries is common to the decompositions typically applied in the literature and Equation (27).

There are both similarities and differences between the decomposition in Equation (27) and the framework outlined in Equation (12). First, while Equation (12) decomposes the productivity growth, measured by the difference between the log change of the output and the input index, Equation (27) shows the absolute change in the weighted average of productivity levels. Second, the weighting scheme may be somewhat different between the two decompositions depending on how the weights π_{it} and π_{ift} are defined.

The most important difference between the two decompositions is that Equation (12) generalises the framework underlying the decomposition in Equation (27), i.e. it allows for products being non-homogeneous. When products are non-homogeneous the entry of a new firm increases the number of varieties and the overall level of output. In Equation (12), the net impact from new varieties on aggregate productivity growth is given by the term

$$\left(\frac{1}{1-\sigma_i}\right)\ln\left(\frac{1-s_{it}^{N}}{1-s_{i,t-1}^{X}}\right).$$

This effect on aggregate productivity growth is absent in Equation (27).

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