Geir Haakon Bjertnæs

Optimal Combinations of Income Tax and Subsidies for Education

Abstract:
Nielsen and Sørensen (1997) find that progressive taxation of labour income is optimal when capital income is taxed. This paper shows that their main result still holds when introducing endogenous choice of occupation, individuals with non- pecuniary preferences for one type of occupation, and tuition fees into the model, provided the subsidy rate for tuition in the high skill occupation is not too low. However, a new result in this paper state that efficiency can be reached when labour income tax is proportional and capital income is taxed, provided that the rates of subsidies for tuition are lower than the labour income tax rate.

Address: Geir Haakon Bjertnæs, Statistics Norway, Research Department.
E-mail: geir.h.bjertnas @ssb.no

1. Introduction

A number of studies analyse what tax and tuition subsidy rates that induce individuals to invest in the optimal amount of education. Different frameworks give different answers to the question. If the only costs of education are forgone earnings, as in Nielsen and Sørensen (1997), and individuals choose how long time to spend on education, proportional taxation of labour income is neutral and therefore optimal. When capital income is taxed, individuals spend too long time on education[1]. The reason is that taking education and hence earning higher labour incomes later in life reduces the need for financial saving and hence reduces capital income taxation. A flat tax on labour income implies no effective taxation on the return to education since both the cost and the gain of the investment are reduced by the same rate. Nielsen and Sørensen (1997) argue in favour of progressive taxation of labour income to tax the gain of education, and hence correct the distortion. In Nerlove et al. (1993), the cost of education is pecuniary, but does not include forgone earnings. In this case a proportional labour income tax will discriminate against human capital investments because only the gain of the investment is taxed. A capital income tax will reduce this distortion.

This paper contributes to the literature by analysing how large the rate of subsidy for tuition should be when the distorting effect of capital income taxation is not corrected by progressive taxation of labour income. However, the mentioned literature analyses tax-distortions on time spent on education. They do not discuss effects on choices of occupation any further. This treatment is not satisfying because different occupations require different time spent on education, and induce different flows of income during life. Taxes have different effect on different flows of income, and hence distort individuals choice of occupation, as well as the time they spend in school. Capital income taxation gives individuals an incentive of spending too long time in school. However, it will also increase the incentives to choose a high skill occupation, because high skilled individuals earn a higher income late in life compared to low skilled individuals. A progressive taxation of labour income will correct the distortion on time spent in school. However, it will also reduce the incentive of choosing a high skill occupation. This effect occurs because high skilled individuals earn higher wages compared to low skilled individuals, and hence pay more labour income taxes. When a reduction in the rate of subsidy for tuition is used to correct the distorting effect of capital income taxation on time spent on education, this also reduces the incentive of choosing a high skill occupation. Since high skilled individuals spend a longer time in school, they also face a larger reduction in subsidies for tuition.

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[1] This is also found in Heckman (1976) and Driffill and Rosen (1983).

This paper analyses tax effects on both choice of occupation and on time spent in school. The following combination of income taxes and subsidies for tuition is found to be optimal. When capital income is not taxed, proportional taxation of labour income is optimal when the rates of subsidies for tuition is equal to the tax rate (This result does not hold when individuals have non-pecuniary preferences for one type of occupation). It is also possible to reach an efficient allocation when labour income taxes are progressive. In this case it is necessary to introduce a high rate of subsidy for tuition for high skilled, to offset the effect from the progressive labour income tax. When capital income is taxed, the main result in Nielsen and Sørensen (1997) still holds, provided the subsidy rate for tuition in the high skill occupation is not too low, i.e. progressive taxation of labour income is optimal when capital income is taxed. Individuals in the low skill occupation receive lower labour income and hence are not affected by the progressive taxation. A low rate of subsidy for education is needed to correct the distortion created by the capital income tax for this group of individuals. When I assume a proportional taxation of labour income, the rate of subsidy for tuition, for all individuals, has to be lower than the labour income tax rate for the allocation to be efficient. In contrast, many countries have subsidy rates way above 100 percent[2], suggesting welfare improving effects of reducing subsidies for tuition. At least for those countries with a proportional income taxation.

Section 2 states the general assumptions. The social planner solution is given in section 3, and the market solution is given in section 4. Optimal tax and subsidy rates are derived in section 5. Section 6 introduces preference for type of occupation. Section 7 concludes, and comments on extensions.

2. General assumptions

Consider a small open economy taking the interest rate in the rest of the world as given. The economy produces one tradable good, which is either invested or consumed. The price is set to unity. Labour is internationally immobile. In each period $\overline{N}$ individuals are born, and all of them live for two periods. At the beginning of the first period individuals choose a high skill or a low skill occupation, how long time to spend in school, and hence how long time to work. In the second period, individuals work. By going to school, individuals become more productive, and can deliver more of their type of skill in the labour market. However, attending school reduces time spent on work, and it requires teaching effort. Each direction of education is connected to one type of skill. These skills are not perfect substitutes in production.

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[2] Public schools and univerities are free of charge in many countries. This is equivalent to a 100 percent subsidy rate in this analysis. In addition students often receive other direct and indirect subsidies.

The analyses are conducted in an OLG model with individuals living finite lives. A representative consumer model is not sufficient, because individuals choose to become high or low- skilled, and hence are different. However, the model is kept as similar as possible to the representative consumer model in Nielsen and Sørensen (1997), to focus on how their results are affected.

The following notation is followed throughout the paper.

$$N_{age,time}^{direction}$$

Superscript denotes type of occupation (low skill = 1, high skill = 2) for variable N (= number of individuals). Subscript before comma indicates age (y = young, o = old). Subscript after comma indicates time period. Superscript will be ignored for variables not characterised by skill type. Variables that are not characterised by either age or time of involvement in the market will only have one subscript.

Individuals in direction i in cohort t are able to supply $H^i(E_{y,t}^i)$ units of labour skill i when they are working (note that human capital functions are skill specific). $E_{y,t}^i$ is the time they spend in school, which demands $z^i E_{y,t}^i$ units of skill type i for teaching resources. Hence, schools use $N_{y,t}^i z^i E_{y,t}^i$ of skill type i in labour resources, to educate $N_{y,t}^i$ individuals of skill type i (= 1,2).

Gross domestic product Y_t is given by a Cobb-Douglas production function.

(1) $Y_t = \overline{H}_t^{1\,\alpha} \overline{H}_t^{2\,\beta} K_t^{1-\alpha-\beta}$

Physical capital K_t is given by

(2) $K_{t+1} = J_t + (1-\delta)K_t$

where J_t is gross investment and δ denotes rate of depreciation.

Total effective labour input into production of skill type i (=1,2) in period t is given by

(3) $\overline{H}_t^i = N_{y,t}^i H^i(E_{y,t}^i)(1-E_{y,t}^i) + N_{o,t}^i H^i(E_{o,t}^i)f - N_{y,t}^i z^i E_{y,t}^i$

Note that the effort of teachers is subtracted from the arguments in the production function, since they do not participate in production of the good. Working time in the second period of life per individual is fixed and equal f.

The number of young individuals are fixed and divided between the two types of occupations.

(4) $N^1_{y,t} + N^2_{y,t} = \overline{N}$

3. The social planner solution

The efficient solution is given by maximising the present value of consumption, i.e.:

$$Max \sum_{t=0}^{\infty} (\frac{1}{1+r})^t [\overline{H}^{1\alpha}_t \overline{H}^{2\beta}_t K^{1-\alpha-\beta} - J_t]$$

wrt. $N^1_{y,t}, E^1_{y,t}, E^2_{y,t}, J_t$

so that

(3) and (4)

(5) $K_{t+1} = J_t + (1-\delta)K_t$

(6) $N^1_{o,t+1} = N^1_{y,t}$

(7) $E^1_{o,t+1} = E^1_{y,t}$

(8) $E^2_{o,t+1} = E^2_{y,t}$

apply.

In a steady state where $\dfrac{\overline{H}^1_t}{\overline{H}^2_t} = \dfrac{\overline{H}^1_{t+1}}{\overline{H}^2_{t+1}}$, i.e. where different stocks of labour are proportional in the production sector, the foc. becomes (after skipping subscripts)

(9) $H^{1'}_{E^1}(1-E^1) + \dfrac{1}{1+r} H^{1'}_{E^1} f = H^1(E^1) + z^1$

(10) $H^{2'}_{E^2}(1-E^2)+\dfrac{1}{1+r}H^{2'}_{E^2}f=H^2(E^2)+z$

This will determine $E^1_{y,t}$ and $E^2_{y,t}$ respectively.

(11) $\dfrac{\alpha(\dfrac{\overline{H}^1}{\overline{H}^2})^{\frac{-\beta}{\alpha+\beta}}}{\beta(\dfrac{\overline{H}^1}{\overline{H}^2})^{\frac{\alpha}{\alpha+\beta}}}=\dfrac{H^2(E^2)(1-E^2)-z^2E^2+\dfrac{1}{1+r}H^2(E^2)f}{H^1(E^1)(1-E^1)-z^1E^1+\dfrac{1}{1+r}H^1(E^1)f}$

The right hand side in (11) is given when optimal time in school from (9) and (10) is inserted. Then (11) will determine the level of $\dfrac{\overline{H}^1}{\overline{H}^2}$.

(12) $(1-\alpha-\beta)\overline{H}^{1\alpha}\overline{H}^{2\beta}K^{-\alpha-\beta}=r+\delta$

(See appendix for the calculation).

(9) and (10) states that a marginal increase in the time spent in school by any individual do not change present value of consumption. (11) states that consumption possibilities do not change if any individual change their choice of occupation. (12) states that the marginal product of capital is equal to the user cost of capital.

Note that it is possible to reach steady state in period zero. To see this, insert (3) into $\dfrac{\overline{H}^1_{-,t}}{\overline{H}^2_{-,t}}$. This forms a simple difference equation. Both $N^1_{t,t}$ and $N^2_{t,t}$ will oscillate towards constant values when f is not too large (i.e. when individuals do not work too much in their last period of life). The intuition is that keeping $\dfrac{\overline{H}^1_{-,t}}{\overline{H}^2_{-,t}}$ constant requires the allocation of young individuals in each direction of education to adjust so that the labour supply into production of one skill type relative to the other skill type is constant. If initially there are many low skill individuals in the old generation, many individuals in the young generation have to take the high skill occupation. In the next period many old individuals will be high skilled. Then many of the young generation have to take the low skill type of education, and so on.

4. The market solution

In the market solution there is free competition in all markets. The individuals can lend and borrow freely to the given interest rate. When tax on capital income is introduced, individuals are taxed with the same rate on foreign and domestic- source income. The public sector collects taxes, transfers subsidies, and distributes the net tax revenue in equal sized lump sum transfers to all individuals in all generations, after compensating individuals in the old generation that suffer a loss when taxes are changed. These transfers will ensure that no individual is losing when tax reforms that lead to the efficient solution is introduced. The transfers do not affect the choice of education for any individual. Tax and subsidy changes are introduced at the beginning of the first period in the model, so the young generation living in the first period can make their choice of education based on the new rates, while the old one can not. I want to keep the condition that capital is chosen optimally in every period (including the first one), to simplify the analysis. This means that producers adjust real capital freely, and will not be surprised by changes in the stocks of high and low skilled workers due to tax changes. Individuals expect unchanging wage rates. These expectations turn out to be rational.

4.1. Individuals

Individuals that choose the high skill occupation maximise a standard neo classical utility function $u(c_{y,t}^2, c_{o,t+1}^2)$ w.r.t. $c_{y,t}^2, c_{o,t+1}^2, E_{y,t}^2$ given the following budget constraint

$$(13) \quad c_{y,t}^2 + \frac{1}{1+r(1-\tau_r)} c_{o,t+1}^2 \leq$$

$$(1-\tau_1)W_t^2 x(1-E_{y,t}^2) + (1-\tau_2)[W_t^2 H^2(E_{y,t}^2) - W_t^2 x](1-E_{y,t}^2) - (1-s^2)W_t^2 z^2 E_{y,t}^2$$

$$+ \frac{1}{1+r(1-\tau_r)}[(1-\tau_1)W_{t+1}^{2e} xf + (1-\tau_2)(W_{t+1}^{2e} H^2(E_{y,t}^2) - W_{t+1}^{2e} x)f] + transfers \equiv A_t^{2e}$$

when $H^2(E_{y,t}^2) > x$

W_t^2 is the wage rate for high skill labour in period t. τ_1 is the tax rate for labour income below $W_t^2 x$, and τ_2 is the tax rate above this level. x is the maximal amount of high skill labour a individual can supply, and still face τ_1 as labour income tax rate. The tax system is specified this way, because it

simplifies the analyses. τ_r is the tax rate on capital income. s^2 is the subsidy rate of tuition for high skilled. Expected variables are marked with superscript e.

This problem can be separated. The individuals can maximise A_t^{2e} w.r.t. $E_{y,t}^2$, (and then the utility w.r.t. $c_{y,t}^2$ and $c_{o,t+1}^2$). The first order condition for this problem is (after skipping subscripts)

$$(14) \quad (1-\tau_2)W_t^2 H^2{}'_{E^2}(1-E^2)+\frac{1}{1+r(1-\tau_r)}(1-\tau_2)W^{2e}H^2{}'_{E^2}\cdot f$$

$$= (1-\tau_1)W^2 x+(1-\tau_2)(W^2H^2(E^2)-W^2 x)+(1-s^2)W^2 z^2$$

The first two terms on the r.h.s. is forgone labour income of increasing time spent in school, and the last term is the increase in tuition of staying longer in school, so the r.h.s. is the cost of increasing time in school. The l.h.s. is the gain of increasing time spent in school. It consists of getting more productive, and hence getting better paid.

Individuals choosing the low skill occupation will face a similar problem. However, different human capital functions in each direction induce individuals in the low skill direction to stay a shorter time in school, earn less per unit of time, and hence never face the tax rate τ_2. Solving a similar problem as the one above gives the first order condition (after skipping subscripts)

$$(15) \quad W^1(1-\tau_1)H^1{}'_{E^1}(1-E^1)+\frac{1}{1+r(1-\tau_r)}W^{1e}(1-\tau_1)H^1{}'_{E^1}\cdot f = W^1(1-\tau_1)H^1(E^1)+(1-s^1)W^1 z^1$$

The interpretation is similar to the one above. Time spent in school will be determined in (14) and (15), as functions of the tax and subsidy rates.

I will assume an inner solution, i.e. that $0 < N_{y,t}^1 < \overline{N}$. Then

$$(16) \quad A_t^{1e} = A_t^{2e}$$

apply in each period. If not, all individuals would choose the direction that gave highest expected lifetime income. This condition together with the solutions for time spent in school will determine the relationship between the wage rates $\frac{W_t^1}{W_t^2}$.

4.2. Producers

The individuals that are saving from the first period of life to increase their consumption in the last period of life own shares of the companies, or have invested it abroad. When tax on capital income are introduced it does not matter what kind of ownership the individuals have chosen. They have to pay tax on the interest rates gained from the investment. To match the rate of interest from abroad, investment in domestic companies have to give the same return as investments abroad. Companies maximise present value of profit, taking factor prices as given.

The problem is

$$max \sum_{t=0}^{\infty} (\frac{1}{1+r})^t [\overline{H}_t^{1\alpha} \overline{H}_t^{2\beta} K_t^{1-\alpha-\beta} - W_t^1 \overline{H}_t^1 - W_t^2 \overline{H}_t^2 - (r+\delta)K_t]$$

w.r.t. $\overline{H}_t^1, \overline{H}_t^2, K_t$

The first order conditions are

(17) $\alpha \overline{H}_t^{1\alpha-1} \overline{H}_t^{2\beta} K_t^{1-\alpha-\beta} = W_t^1$

(18) $\beta \overline{H}_t^{1\alpha} \overline{H}_t^{2\beta-1} K_t^{1-\alpha-\beta} = W_t^2$

(19) $(1-\alpha-\beta) \overline{H}_t^{1\alpha} \overline{H}_t^{2\beta} K_t^{-\alpha-\beta} = r+\delta$

4.3. The schools

The schools will supply the amount of education that the individuals are demanding, given that the individuals are covering salary for the teachers minus subsidies for education.

5. The optimal combination of tax and subsidy rates

To close the model I impose equilibrium conditions for the labour market in each direction of education. (16), (17), (18), (19) and the definitions of lifetime income give the expression (after skipping subscripts)

$$(20) \quad \frac{\alpha(\dfrac{\overline{H}^1}{\overline{H}^2})^{\frac{-\beta}{\alpha+\beta}}}{\beta(\dfrac{\overline{H}^1}{\overline{H}^2})^{\frac{\alpha}{\alpha+\beta}}} =$$

$$\frac{(1-\tau_1)x(1-E^2)+(1-\tau_2)(H^2(E^2)-x)(1-E^2)-(1-s^2)z^2E^2+\dfrac{1}{1+r(1-\tau_r)}[(1-\tau_1)xf+(1-\tau_2)(H^2(E^2)-x)f]}{(1-\tau_1)H^1(E^1)(1-E^1)-(1-s^1)z^1E^1+\dfrac{1}{1+r(1-\tau_r)}(1-\tau_1)H^1(E^1)f}$$

To find out if the allocation in the market solution is the same as in the social planner solution, I need to compare (20) with (11), (14) with (10) and (15) with (9). The government determines more than three policy variables to satisfy three equations. Hence, several combinations of policy variables give an efficient solution, and the government can determine the level of tax revenue (in most cases).

5.1. Optimal combinations of tax and subsidy rates when $\tau_r = 0$

1. $\tau_1 = \tau_2 = s^1 = s^2 \geq 0$. This combination is optimal because within both occupations the cost of increasing time spent in school is reduced by the same magnitude as the gain of increasing time spent in school. Also (11) and (20) are identical. The interpretation is that the tax/ subsidy system does not reduce the present value of lifetime income (for consumption) more for individuals choosing one type of occupation, compared to choosing the other (see appendix).

2. If $\tau_1 = s^1$ then time spent in school is chosen optimally for individuals in the low skill occupation. If $\tau_2 > \tau_1$ is fixed, then s^2 and x can be chosen so that time spent in school for high skilled, and the skill formation, is chosen optimally. In this case $\tau_2 < s^2$, i.e. with progressive taxation of labour income it is necessary to subsidise tuition for the high skilled with a rate higher than the tax rates used for labour income.
The extra tax paid because of progressive taxation is equal to the extra subsidy received because $s^2 > s^1$, for individuals in the high skilled direction.

5.2. Optimal combination of tax and subsidy rates when $\tau_r > 0$

Production and consumption is now adapted to different effective interest rates, so Fishers separation theorem does not hold. However, I will argue that maximising total present value of income discounted with the interest rate from abroad, still is part of the social planner solution.

When individuals adapt their consumption, they behave as if decreasing consumption in one period by one unit will increase consumption in the next period by $1 + r(1 - \tau_r)$ units. However, saving one unit in one period will generate $r\tau_r$ units of extra tax revenue in the next period. When all individuals save one unit in one period, this generates an increase in tax revenues and hence in lump-sum transfers of $r\tau_r$ per individual[3]. The actual increase in consumption the following period becomes $1 + r$ per individual. Taking account of this effect, I will assume that utility of individuals increases with lifetime income discounted with the interest rate from abroad. In this case maximising present value of income discounted with the interest rate from abroad is part of the social planner solution. Hence I can still use the social planner solution in section 3 to evaluate efficiency[4].

The effect of only taxing capital income is that individuals put more weight on income earned in the second period of life, compared to the first period of life. Then it is profitable for individuals to increase their time spent in school in both directions of education. In the market solution too many individuals choose to become high skilled, because individuals in this direction have higher income in the second period of life compared to individuals in the low skill direction. This follows from $A_t^{1e} = A_t^{2e}$ and that individuals in the high skill occupation choose to stay a longer time in school compared to individuals in the low skill occupation. The other tax and subsidy rates have to neutralise this effect, for the allocation to be efficient. I will go through the tax and subsidy rates that lead to the same allocation as in the social planner solution.

The only way to neutralise the effect from capital income taxation on time spent in school for low skill individuals is by setting $\tau_1 > s^1$. Labour income tax will affect the entire gain, and part of the cost, of increasing time spent in school. The rate of subsidy (which affect the other part of the cost of increasing time spent in school), has to be lower than the tax rate, to reduce incentives of spending time in school. There are several ways to neutralise the effect from capital income taxation on time

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[3] Individuals have rational expectations about lump sum transfers .

[4] If the government can choose the capital income tax rate, it would be optimal not to tax capital income within this modell, since capital income distort the allocation of consumption.

spent in school for individuals in the high skill direction. When time spent in school is chosen optimal, and $\tau_1 = \tau_2$, then $\tau_2 > s^2$. In this case a low subsidy rate for education have to neutralise the effect from the capital income tax. If time spent in school is chosen optimal, and $\tau_1 \leq s^2$ or $\tau_2 \leq s^2$, then $\tau_2 > \tau_1$. This follows from (10) and (14). Intuitively, when subsidies for education are not too low for high skill individuals, and hence do not reduce the incentive of staying longer in school, labour income above some level have to be taxed at a higher rate than labour income below this level. This is the only way to reduce the incentive of staying longer in school induced by the capital income tax.

To check if the tax and subsidy rates that are consistent with optimal allocation of time spent in school, also is consistent with optimal allocation of individuals in each direction of education, I compare (11) with (20). All combinations are consistent with an optimal allocation. In the case where $\tau_1 = \tau_2$ (and $\tau_2 > s^2$), the government do not have the possibility to set the level of tax rates, and hence control the level of tax revenue. The tax rates are all used to secure an efficient solution. In all other cases at least one tax rate can be chosen freely by the government.

6. Preference for type of skill

Assume that all individuals prefer one type of occupation. The social planner solution is found by adding $N_{y,t}^2 a$ (in each period) into the maximisation problem in section 3. a is the extra utility of choosing direction 2 measured in consumer gods per individual.

In the market solution I replace $A_t^{1e} = A_t^{2e}$ with $A_t^{1e} = A_t^{2e} + a$. The tax and subsidy rates affect the time individuals spend in school in exactly the same way as in the analyses above. The effect on the allocation of individuals in each direction of education is changed, because the utility part a in the high skill direction is not taxable. The above results will be affected in the following way:

When $\tau_r = 0$ it is no longer optimal to set $\tau_1 = \tau_2 = s^1 = s^2 > 0$. Since the utility part in the high skill direction is not taxed, individuals choosing the low skill direction are taxed more heavily. This induces to many individuals to choose the high skill occupation. If all rates are set equal to zero, the market solution is efficient. Then there would be no tax revenue generated to the public sector, so this solution is not interesting.

If $\tau_1 = s^1$ then time spent in school is optimal in the low skill direction. If $\tau_2 > \tau_1$ then s^2 and x can be chosen so that time spent in school in the high skill direction, and the allocation of individuals in

the two directions, are identical to the optimal solution. In this case $\tau_2 < s^2$, which is identical to the analyses above. However for this to be a optimal combination, extra tax paid because of progressive taxation have to be larger than extra subsidies received because of a high subsidy rate, for individuals in the high skilled direction.

When I repeat the analyses with the constraint $\tau_r > 0$, combinations of tax and subsidy rates that are consistent with optimal time spent in school are unchanged. These rates are still consistent with the condition for optimal skill formation.

7. Conclusion and possible extensions

This paper extends the model in Nielsen and Sørensen (1997) by introducing choice of occupation, individuals with non- pecuniary preferences for a specific type of occupation, and tuition fees. Their main result hold provided subsidies for tuition is not to low in the high skilled direction, i.e. progressive taxation of labour income is optimal to correct the distorting effect of capital income taxation on human capital investments. However, I find that the distortion might also be corrected when labour income taxation is proportional by subsidising tuition with a lower rate than the rate used for labour income taxation. These results hold when individuals have non- pecuniary preferences for a specific occupation.

The analyses do not cover all aspects of income taxation and subsidies for tuition. Introducing endogenous labour supply and/ or liquidity constraints are interesting extensions. The result in Nielsen and Sørensen (1997) holds with some modifications when these extensions are introduced into their analysis. Another interesting aspect is how the allocation of high and low skilled labour is affecting the growth rate. Romer (1990) argues that wage rates for high skill labour are lower than their productivity because of positive external effects from employing them in the research sector. Hence, in a second best solution where the government has no direct means to affect the allocation of high skilled in the research sector, the education of high skilled individuals should be subsidised.

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$J_0(K_t, N_{o,t}^1, E_{o,t}^1, E_{o,t}^2)$ is the value function for the problem in section 3, so

$$J_0(K_t, N_{o,t}^1, E_{o,t}^1, E_{o,t}^2) = max[(N_{y,t}^1(H^1(E_{y,t}^1)(1 - E_{y,t}^1) - z^1 E_{y,t}^1) + N_{o,t}^1 H^1(E_{o,t}^1)f)^\alpha$$

$$((\overline{N} - N_{y,t}^1)(H^2(E_{y,t}^2)(1 - E_{y,t}^2) - z^2 E_{y,t}^2) + (\overline{N} - N_{o,t}^1)H^2(E_{o,t}^2)f)^\beta K_t^{1-\alpha-\beta} - J_t$$

$$+ (\frac{1}{1+r})J_0(J_t + (1-\delta)K_t, N_{y,t}^1, E_{y,t}^1, E_{y,t}^2)]$$

wrt. $N_{y,t}^1, E_{y,t}^1, E_{y,t}^2, J_t$

foc:

(1) $\dfrac{\partial[]}{\partial N_{y,t}^1} = \alpha \overline{H}_t^{1\,\alpha-1} \overline{H}_t^{2\,\beta} K_t^{1-\alpha-\beta} (H^1(E_{y,t}^1)(1 - E_{y,t}^1) - z^1 E_{y,t}^1)$

$$- \beta \overline{H}_t^{1\,\alpha} \overline{H}_t^{2\,\beta-1} K_t^{1-\alpha-\beta} (H^2(E_{y,t}^2)(1 - E_{y,t}^2) - z^2 E_{y,t}^2) + (\frac{1}{1+r})\frac{\partial J_0(J_t + (1-\delta)K_t, N_{y,t}^1, E_{y,t}^1, E_{y,t}^2)}{\partial N_{o,t+1}^1} = 0$$

(2) $\dfrac{\partial[]}{\partial E_{y,t}^1} = \alpha \overline{H}_t^{1\,\alpha-1} \overline{H}_t^{2\,\beta} K_t^{1-\alpha-\beta} N_{y,t}^1 (H^{1'}_{E_{y,t}^1}(1 - E_{y,t}^1) - H^1(E_{y,t}^1) - z^1)$

$$+ (\frac{1}{1+r})\frac{\partial J_0(J_t + (1-\delta)K_t, N_{y,t}^1, E_{y,t}^1, E_{y,t}^2)}{\partial E_{o,t+1}^1} = 0$$

$$(3)\ \frac{\partial[]}{\partial E_{y,t}^2} = \beta \overline{H}_t^{1\,\alpha} \overline{H}_t^{2\,\beta-1} K_t^{1-\alpha-\beta} (\overline{N} - N_{y,t}^1)(H^2{'}_{E_{y,t}^2} (1 - E_{y,t}^2) - H^2(E_{y,t}^2) - z^2)$$

$$+ \left(\frac{1}{1+r}\right) \frac{\partial J_0(J_t + (1-\delta)K_t, N_{y,t}^1, E_{y,t}^1, E_{y,t}^2)}{\partial E_{o,t+1}^2} = 0$$

$$(4)\ \frac{\partial[]}{\partial J_t} = -1 + \left(\frac{1}{1+r}\right) \frac{\partial J_0(J_t + (1-\delta)K_t, N_{y,t}^1, E_{y,t}^1, E_{y,t}^2)}{\partial K_{t+1}} = 0$$

By differentiating $J_0(K_t, N_{o,t}^1, E_{o,t}^1, E_{o,t}^2)$ and inserting from the constraints of the maximisation problem I get:

$$(5)\ \frac{\partial J_0(J_t + (1-\delta)K_t, N_{y,t}^1, E_{y,t}^1, E_{y,t}^2)}{\partial N_{o,t+1}^1} = \alpha \overline{H}_{t+1}^{1\,\alpha-1} \overline{H}_{t+1}^{2\,\beta} K_{t+1}^{1-\alpha-\beta} H^1(E_{y,t}^1) f$$

$$- \beta \overline{H}_{t+1}^{1\,\alpha} \overline{H}_{t+1}^{2\,\beta-1} K_{t+1}^{1-\alpha-\beta} H^2(E_{y,t}^2) f$$

$$(6)\ \frac{\partial J_0(J_t + (1-\delta)K_t, N_{y,t}^1, E_{y,t}^1, E_{y,t}^2)}{\partial E_{o,t+1}^1} = \alpha \overline{H}_{t+1}^{1\,\alpha-1} \overline{H}_{t+1}^{2\,\beta} K_{t+1}^{1-\alpha-\beta} N_{y,t}^1 H^1{'}_{E_{y,t}^1} f$$

$$(7)\ \frac{\partial J_0(J_t + (1-\delta)K_t, N_{y,t}^1, E_{y,t}^1, E_{y,t}^2)}{\partial E_{o,t+1}^2} = \beta \overline{H}_{t+1}^{1\,\alpha} \overline{H}_{t+1}^{2\,\beta-1} K_{t+1}^{1-\alpha-\beta} (\overline{N} - N_{y,t}^1) H^2{'}_{E_{y,t}^2} f$$

(8) $\dfrac{\partial J_0(J_t + (1-\delta)K_t, N^1_{y,t}, E^1_{y,t}, E^2_{y,t})}{\partial K_{t+1}} = (1-\alpha-\beta)\overline{H}^{1\ \alpha}_{t+1}\overline{H}^{2\ \beta}_{t+1}K^{-\alpha-\beta}_{t+1} + 1 - \delta$

(8) into (4) give

(9) $(1-\alpha-\beta)\overline{H}^{1\ \alpha}_{t+1}\overline{H}^{2\ \beta}_{t+1}K^{-\alpha-\beta}_{t+1} = r + \delta$

(5), (9) and (1) give

(10) $\alpha\left(\dfrac{\overline{H}^1_t}{\overline{H}^1_t}\right)^{\frac{-\beta}{\alpha+\beta}}(H^1(E^1_{y,t})(1-E^1_{y,t}) - z^1 E^1_{y,t}) - \beta\left(\dfrac{\overline{H}^1_t}{\overline{H}^2_t}\right)^{\frac{\alpha}{\alpha+\beta}}(H^2(E^2_{y,t})(1-E^2_{y,t}) - z^2 E^2_{y,t})$

$= -\left(\dfrac{1}{1+r}\right)\left[\alpha\left(\dfrac{\overline{H}^1_{t+1}}{\overline{H}^2_{t+1}}\right)^{\frac{-\beta}{\alpha+\beta}}H^1(E^1_{y,t})f - \beta\left(\dfrac{\overline{H}^1_{t+1}}{\overline{H}^1_{t+1}}\right)^{\frac{\alpha}{\alpha+\beta}}H^2(E^2_{y,t})f\right]$

(6), (9) and (2) give

(11) $\alpha\left(\dfrac{\overline{H}^1_t}{\overline{H}^2_t}\right)^{\frac{-\beta}{\alpha+\beta}}(H^{1'}_{E^1_{y,t}}(1-E^1_{y,t}) - H^1(E^1_{y,t}) - z^1) + \left(\dfrac{1}{1+r}\right)\alpha\left(\dfrac{\overline{H}^1_{t+1}}{\overline{H}^2_{t+1}}\right)^{\frac{-\beta}{\alpha+\beta}}H^{1'}_{E^1_{y,t}}f = 0$

(7), (9) and (3) give

(12) $\beta\left(\dfrac{\overline{H}^1_t}{\overline{H}^2_t}\right)^{\frac{\alpha}{\alpha+\beta}}(H^{2'}_{E^2_{y,t}}(1-E^2_{y,t}) - H^2(E^2_{y,t}) - z^2) + \left(\dfrac{1}{1+r}\right)\beta\left(\dfrac{\overline{H}^1_{t+1}}{\overline{H}^2_{t+1}}\right)^{\frac{\alpha}{\alpha+\beta}}H^{2'}_{E^2_{y,t}}f = 0$

(11) ((12)) defines $E^1_{y,t}$ ($E^2_{y,t}$) as a function of $\dfrac{\overline{H}^1_t}{\overline{H}^2_t}$ and $\dfrac{\overline{H}^1_{t+1}}{\overline{H}^2_{t+1}}$. inserting these into (10) will give

$\dfrac{\overline{H}^1_{t+1}}{\overline{H}^2_{t+1}}$ as a function of $\dfrac{\overline{H}^1_t}{\overline{H}^2_t}$. Steady state is defined as $\dfrac{\overline{H}^1_{t+1}}{\overline{H}^2_{t+1}} = \dfrac{\overline{H}^1_t}{\overline{H}^2_t}$. Inserting this into (10), (11) and

(12) give:

$$(13) \quad \frac{\alpha(\dfrac{\overline{H}_t^1}{\overline{H}_t^2})^{\frac{-\beta}{\alpha+\beta}}}{\beta(\dfrac{\overline{H}_t^1}{\overline{H}_t^2})^{\frac{\alpha}{\alpha+\beta}}} = \frac{H^2(E_{y,t}^2)(1-E_{y,t}^2)-z^2E_{y,t}^2+\dfrac{1}{1+r}H^2(E_{y,t}^2)f}{H^1(E_{y,t}^1)(1-E_{y,t}^1)-z^1E_{y,t}^1+\dfrac{1}{1+r}H^1(E_{y,t}^1)f}$$

$$(14) \quad H^{1'}{}_{E_{y,t}^1}(1-E_{y,t}^1)+(\frac{1}{1+r})H^{1'}{}_{E_{y,t}^1}f = H^1(E_{y,t}^1)+z^1$$

$$(15) \quad H^{2'}{}_{E_{y,t}^2}(1-E_{y,t}^2)+(\frac{1}{1+r})H^{2'}{}_{E_{y,t}^2}f = H^2(E_{y,t}^2)+z^2$$

respectively.

To find out what characterises an efficient allocation I manipulate the condition $A_t^{1e} = A_t^{2e}$ to get:

$$(1+r)[W_t^1 H^1(E_{y,t}^1)(1-E_{y,t}^1)-W_t^1 z^1 E_{y,t}^1]+W_{t+1}^1 H^1(E_{o,t+1}^1)f$$

$$= (1+r)[W_t^2 H^2(E_{y,t}^2)(1-E_{y,t}^2)-W_t^2 z^2 E_{y,t}^2]+W_{t+1}^2 H^2(E_{o,t+1}^2)f$$

$$+ r\tau_r[W_t^1(1-\tau_1)H^1(E_{y,t}^1)(1-E_{y,t}^1)-W_t^1(1-s^1)z^1 E_{y,t}^1-c_{y,t}^1-[W_t^2(1-\tau_1)x(1-E_{y,t}^2)$$

$$+W_t^2(1-\tau_2)(H^2(E_{y,t}^2)-x)(1-E_{y,t}^2)-W_t^2(1-s^2)z^2 E_{y,t}^2-c_{y,t}^2]]$$

$$+ (1+r)[\tau_1 W_t^1 H^1(E_{y,t}^1)(1-E_{y,t}^1)-s^1 W_t^1 z^1 E_{y,t}^1-[\tau_1 W_t^2 x(1-E_{y,t}^2)$$

$$+\tau_2 W_t^2(H^2(E_{y,t}^2)-x)(1-E_{y,t}^2)-s^2 W_t^2 z^2 E_{y,t}^2]]$$

$$+ \tau_1 W_{t+1}^1 H^1(E_{o,t+1}^1)f -[\tau_1 W_{t+1}^2 xf + \tau_2 W_{t+1}^2(H^2(E_{o,t+1}^2)-x)f]$$

I have added and subtracted $c_{y,t}^1 (= c_{y,t}^2)$ in the third fraction on the r.h.s.

Economic interpretation: If $\alpha \overline{H}_t^{1\,\alpha-1} \overline{H}_t^{2\,\beta} K_t^{1-\alpha-\beta} = W_t^1$ is inserted into the first and second fraction on the l.h.s, and $\beta \overline{H}_t^{1\,\alpha} \overline{H}_t^{2\,\beta-1} K_t^{1-\alpha-\beta} = W_t^2$ on the r.h.s, then this expression is identical to (10) if fraction 3.- 6. are zero and the arguments in the production function are equal. The economic interpretation of this is that individual`s in high and low skill occupation pay the same amount of taxes (net of subsidies for education) under optimal taxation.