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Mads Greaker and Lise-Lotte Pade

Optimal CO_2 abatement and technological change

Should emission taxes start high in order to spur R&D?

Abstract:
Many European politicians argue that since technological development is needed to solve the climate problem, the EU should take the lead and set tougher emission targets than what is required by the Kyoto protocol. Moreover, emission trading with other countries outside EU should be limited so as to keep emission quota prices high.

However, the policy of spurring R&D by setting high emission taxes today is not suggested by the literature on climate change and R&D. In this paper we investigate this result further by modeling innovation activity explicitly. In our model both the amount of R&D and the amount of CO2 abatement are decided in a decentralized way by the market as a response to an emission tax. Moreover, we introduce three distinct failures in the market for new innovations; monopolistic pricing behavior, insufficient patent protection and dynamic knowledge spillovers.

Our findings suggest that governments should under some circumstances set a higher carbon tax today if we have technological change driven by R&D than if we have pure exogenous technological change. Based on numerical simulations these circumstances are i) "a standing on shoulders" type of externality in R&D or ii) weak patent protection.

Keywords: Climate policy, technological change, emission tax

JEL classification: Q28, D21, C68

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Address: Mads Greaker, Statistics Norway, Research Department. E-mail: mgr@ssb.no

For printed Discussion Papers contact:

Statistics Norway
Sales- and subscription service
NO-2225 Kongsvinger

Telephone: +47 62 88 55 00
Telefax: +47 62 88 55 95
E-mail: Salg-abonnement@ssb.no

1 Introduction

According to both the latest IPCC report (2007) and the recently published Stern report (2006), atmospheric CO_2 stabilization targets as low as 450 ppm could be needed in order to avoid dangerous anthropogenic interferences with the earths climate system. With reasonable projections of world economic growth, such stabilization targets among others require more than twice as much emission-free power by mid-century than we now derive from fossil fuels (Caldeira et al. 2002).

Clearly, this is a major technological challenge, and many different policy tools are being implemented in order to spur the introduction of carbon free technologies. However, according to among others the Stern report, the current effort is far to weak. The public debate, at least in the European Union, also evolves around different domestic measures going far beyond the obligations of the Kyoto protocol as how to speed up the rate of carbon free technology deployment.

The early economic literature on climate change advocated a more gradual approach to climate change, see for instance the seminal paper by Wigley et al (1996). Since future costs are discounted and a fraction of the carbon emitted today are removed from the atmosphere by natural processes as time passes, we should postpone expensive carbon abatement. On the other hand, in the early literature technological progress was either disregarded or exogenous, and critics claimed that if technological change were made endogenous results would change.

Models with endogenous technological change explains technological progress either by including investments in *knowledge creation* or by assuming that pollution abatement today will lead to more efficient abatement in the future, that is, so-called *learning-by-doing*. Both approaches are treated in Goulder and Mathai (2000). Rather surprisingly, they found that technological change from knowledge creation implied lower carbon taxes both today and in the future, and consequently, also less carbon abatement today.

On the other hand, when endogenous technological change is modelled as learning-by-doing, results correspond better with common intuition. Goulder and Mathai found that learning-by-doing could imply a high tax rate from the beginning, and similar conclusions are also suggested in other contributions, see for instance Grübler and Messner (1998) and Rosendahl (2004).

In Romer (1990) Romer criticizes endogenous growth models based on learning-by-doing. According to Romer these models rely on the assumption that learning is completely nonexcludable. However, in many instances firms are able to protect their new discoveries from being copied by other firms. In this case competition is likely to be imperfect as shown by Dasgupta and Stiglitz (1988), and hence, the simple learning-by-doing set up, applied for instance by Goulder and Mathai, is not appropriate.[1]

In our opinion, some of the criticism of learning-by-doing models also holds with respect to the knowledge creation approach when applied to climate policy. For example, in Goulder and Mathai (2000) innovation of new products and processes is not modelled explicitly, and it is assumed that all market failures connected to knowledge creation are taken care of by other policies than climate policy. Moreover, in their model the government directly decides both the amount of carbon abatement and the rate of knowledge accumulation.

In this paper we look at climate policy with an explicit modelling of the R&D process. Both the decision to invest in R&D and the decision to abate are left to the market. In particular, we adapt the endogenous growth model by Romer (1990) to the issue of carbon abatement. Both the supply and demand for new carbon abatement innovations is then explicitly modelled, and the carbon emitting sectors choose to install new innovations in response to an emission tax.

This also allows us to introduce four sorts of *market failures* or *externalities*: I) Monopolistic pricing of new innovations, II) A dynamic knowledge externality that either is a "standing on shoulders" type or a "fishing out" type, III) A crowding externality in R&D, and IV) Insufficient patent production, which we refer to as a "learning externality". The latter implies that patents may be copied by other firms, and that the incentives for innovation are weakened.

When including decentralized decision making by private agents, numerical simulations indicate that governments should under some circumstances set a higher carbon tax today if we have technological change driven by R&D than if we have pure exogenous technological change. These circumstances are i) "a standing on shoulders" type of externality in R&D and/or ii) weak patent protection.

We derive our results in a fashion that makes the two scenarios easily comparable. That is, we first derive the optimal emission tax path with endogenous technological change. This yields an innovation path which then can be treated as exogenous in another round of simulations in order to find the optimal tax path without endogenous technological change.

While our focus is on the qualitative implications for climate change policy of endogenous technological change, other studies analyze how endogenous technological change is likely to influence the costs of implementing tough climate targets, see for example Buonanno et al. (2003), Goulder and Schnieder (1999) and Gerlagh and Lise (2005). Most of the studies seems to agree that ignoring the existence of endogenous technological change leads to overestimation of the costs of achieving various climate targets, although Popp (2004) warns that some studies might be exaggerating the effects of endogenous technological change by not taking properly into account crowding out of R&D in other sectors. Nordhaus (2002) finds that factor substitution is more impor-

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[1] See also Barro and Sala-i-Martin (2004), page 214.

tant than technological progress, however, Nordhaus (2002) assumes complete crowding out of R&D in other sectors.

The rest of the paper is laid out as follows: In Section 2 we introduce the model without incorporating the learning externality, that is, imperfect patent protection. In Section 3 we solve the model, and present the most important results. Then in Section 4 we include imperfect patent protection. Finally, in Section 5 we conclude, and suggest directions for further research.

2 The model

We look at a closed economy with constant business as usual emissions. Similar to Romer (1990) we have a research and development sector (R&D sector), an abatement equipment sector corresponding to Romer's intermediate goods sector and an "emission sector" corresponding to the final output sector. As in Romer, the abatement equipment sector (intermediate goods sector) is characterized by imperfect competition, and the R&D sector is characterized by free entry of researchers. Further, in line with Romer, we assume that the emission sector (final good sector) *rents* the abatement equipment from the abatement equipment firms (intermediate goods sector).[2]

2.1 The emission sector

Let business as usual emissions be given by ε^0. Emissions can be reduced by renting CO_2 abatement equipment. At each point in time there are a given number of different abatement technologies available. We assume that each type of abatement technology has a limited potential, and hence, that there are decreasing returns to scale for each technology. Emissions ε_t is then given by:

$$\varepsilon_t = \varepsilon^0 - \sum_{i=1}^{N_t} (u_t^i)^\rho, \tag{1}$$

where u_t^i is the amount of abatement equipment of type i rented at time t, and N_t is the number of different technologies available at time t. The parameter $\rho < 1$ ensures that there are decreasing returns to each type of CO_2 abatement equipment. However, this effect can be circumvented by employing more CO_2 abatement technologies instead of steadily increasing the use of one particular type.[3]

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[2] This may seem unrealistic as most polluting firms own their pollution abatement equipment. On the other hand, in a perfect working capital market the per period rental price of capital equipment will be equal to the per period cost of the capital equipment itself. Thus, given that carbon abatement equipment can be bought and sold at any time, the renting assumption should not influence our result.

[3] One possible type of CO_2 abatement technology is carbon capture and sequestration (CCS) for power production. The design of the CCS equipment will depend on the project that is being considered i.e. steel production or electricity production, with respect to the latter coal or natural gas, available storage technologies etc. Thus, the efficiency of CO_2 abatement is likely to increase with the number of different CCS technologies. See Goeschl and Perino (2007) for a similar modelling of abatement costs.

The emission sector minimizes the sum of emission tax payments and carbon abatement cost:

$$\min_{u_t^i}\left\{\tau_t\left[\varepsilon^0-\sum_{i=1}^{N_t}(u_t^i)^\rho\right]+\sum_{i=1}^{N_t}p_t^i u_t^i\right\}, \tag{2}$$

where τ_t is the carbon tax rate at time t, and p_t^i is the per period rental price of CO_2 abatement equipment of type i. Note that the second term in (2) is the carbon tax payments, and that the third term in (2) is the rental cost of CO_2 abatement equipment, which of course increases in the amount of each particular type of CO_2 abatement equipment rented u_t^i.

From the first order condition for a cost minimum we obtain:

$$p_t^i=\tau_t\rho(u_t^i)^{\rho-1}, \tag{3}$$

that is, the inverse demand function for each type of CO_2 abatement equipment as a function of the tax τ_t and the amount of each type of abatement equipment u_t^i. By rearranging we also have: $u_t^i=\left(\frac{\tau_t\rho}{p_t^i}\right)^{\frac{1}{1-\rho}}$.that is, the demand for each type of CO_2 abatement equipment. Note that the higher the number of available abatement technologies N_t, the lower is the cost of reaching a specific emission target.

2.2 The CO_2 abatement equipment sector

We assume that each equipment supplier produces only one specific kind of CO_2 abatement equipment. Further, each type of CO_2 abatement equipment is unique, and hence, each supplier faces a downward sloping demand curve for its equipment. Thus, each equipment supplier maximizes profit with respect to the amount of equipment to offer:

$$\max_{u_t^i}\pi_t^i=\tau_t\rho(u_t^i)^{\rho-1}u_t^i-b_t^i u_t^i,$$

where we have used (3) to insert for p_t^i, and where b_t^i is the per period cost of providing a standardized piece of CO_2 abatement equipment of type i. From the first-order condition for profit maximum we obtain the supply of abatement equipment at each point in time:

$$u_t^i(\tau)=\left(\frac{\tau_t\rho^2}{b_t^i}\right)^{\frac{1}{1-\rho}}. \tag{4}$$

Further, by inserting, we obtain for the rental price: $p_t^i=\frac{b_t^i}{\rho}$. Note that each CO_2 abatement supplier charges a mark-up over costs. In order to simplify, we assume from now on that all kinds of equipment have the same per period cost, and that this cost is constant over time and equal to b. This implies that all available technologies will be used at each point in time.

Finally, we have for the instantaneous profit of the CO_2 abatement suppliers:

$$\pi_t = \varphi \tau_t^{\frac{1}{1-\rho}}. \tag{5}$$

Note that profits are increasing in the carbon tax rate. (For simplicity we introduce $\varphi = \rho(1-\rho)\left(\frac{\rho^2}{b}\right)^{\frac{\rho}{1-\rho}}$).

The optimal emission tax rate τ_t is a function of time where $t \in [0, \infty\rangle$. From (5) we observe that the future income of each CO_2 abatement supplier will depend on the future path of τ_t. Hence, the discounted profit of a CO_2 abatement firm at time $t > 0$ is equal to:

$$\eta_t = \varphi \int\limits_t^{\infty} \tau(s)^{\frac{1}{1-\rho}} e^{-r(s-t)} ds, \tag{6}$$

where r denotes the market discount rate. We will from now on refer to η_t as the value of a new idea.

2.3 The research and development sector

The CO_2 abatement suppliers buy the right to supply one specific kind of CO_2 abatement equipment from the R&D sector. The licence is infinite, and we denote the licence fee by f_t. Furthermore, in each period the R&D sector offers n_t new technologies for CO_2 abatement. This leads n_t new CO_2 abatement suppliers to enter the CO_2 abatement market by acquiring the right to supply one of the new technologies. Hence, the total number of CO_2 abatement technologies (and CO_2 abatement supplier firms) will accumulate according to:

$$\dot{N}_t = n_t, \tag{7}$$

where $\dot{N}_t$ denotes the time derivative of N_t.

There is free entry of researchers into the R&D sector. In particular, we assume that in each period researchers make entry decisions simultaneously, and that all researchers that enter the R&D market develops one idea. The development cost *per idea a* is given by:

$$a(N_t, n_t) = \frac{1}{a_0}(N_t)^{\alpha} n_t, \tag{8}$$

where a_0 is a positive parameter, and we have $-1 < \alpha < 1$.

Note that within each period, the costs of developing an additional technology is increasing in the number of technologies that are made available in the period. Or in other words; the more researchers that enter the R&D sector in each period, the more effort will be required from each researcher for him or her to succeed in developing a unique, and thus, patentable idea.[4]

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[4] A similar convexity assumption is made in Goulder and Mathai with respect to generating additional units of knowledge.

Further, if $\alpha < 0$, the cost is decreasing in the total number of technologies that has been made available historically. In the literature, this case is often coined "standing on shoulders of others", and can be interpreted as a sort of dynamic "learning" externality. The opposite case in which $\alpha > 0$, is coined "fishing out". In this case the costs of developing an additional technology is increasing in the accumulated number of technologies, see for instance Popp (2006) for a discussion of this issue. The development in N_t is external to each researcher in the R&D sector.

In a Nash equilibrium with free-entry of researchers that develop one idea each, the cost per idea must equal the everlasting licence fee i.e. $a(N_t, n_t) = f_t$. Further, due to competitive bidding, the licence fee f_t will be equal to the net present value of profits from the technology η_t. At any time $t > 0$, we then have:

$$\eta_t = \frac{1}{a_0}(N_t)^{\alpha}(n_t). \tag{9}$$

We can then solve for n_t and insert into (7):

$$\dot{N}_t = a_0(N_t)^{-\alpha}\eta_t. \tag{10}$$

Thus, the number of new ideas developed each period will depend on I) The number of ideas developed previously N_t and II) the value of an idea η_t, which again depends on the whole future emission tax path.

Note that the total cost of idea generation in each period is given by $\frac{1}{a_0}(N_t)^{\alpha}(n_t)^2$.Thus, the cost of the *"last idea"* is $\frac{2}{a_0}(N_t)^{\alpha}(n_t) = 2\eta_t$, that is, the costs of the *"last idea"* are higher than the value of the idea along the equilibrium path. This is due to the free entry assumption, and can be looked upon as a sort of *crowding externality.* When entering the research market, each researcher does not take into account that their entry makes it more costly for all other researcher to come up with a new idea. In the growth literature it is thus also referred to as the *"stepping on toes"* effect, see e.g. Jones and Williams (2000).

2.4 Emissions and abatement costs

The object of the government is to set an emission tax path that ensures that the concentration of CO_2 in the atmosphere stabilizes at some future time $\bar{t}$ at a certain level. Let the concentration of CO_2 in the atmosphere at time t be denoted by θ_t. The instantaneous emissions ε_t give rise to the following change in the concentration of CO_2 in the atmosphere:

$$\dot{\theta} = \varepsilon_t - \phi\theta_t. \tag{11}$$

Note that a certain share ϕ of the concentration θ is broken down in the atmosphere. hence, even for $\tau(t) = 0, \forall t$, stabilization of the CO_2 concentration at $\theta = \frac{\varepsilon^0}{\phi}$ will happen due to the constant decay rate. The concentration function (11) is of course simplified, see for instance Farzin and Tahvonen

(1996), but is commonly used in the economic literature, among others in Goulder and Mathai (2000).

When setting the emission tax path, the government minimizes CO_2 abatement costs at each point in time. The costs are of two types; R&D costs and production costs for abatement equipment. Note that R&D costs and production costs for abatement equipment together must be equal to the discounted sum of total abatement outlay for the emission sector subtracted the profits earned on the initial number of ideas. That is, the R&D costs for the initial number of ideas N_0 is already spent, and hence, for these ideas only productions costs should be counted.

3 Solving the model

3.1 The dynamic maximization problem

In order to solve our maximization problem we define η_t as a state variable. For the development in $\eta_t, \forall t > 0$, we have the usual arbritage equation:

$$r\eta = \pi_t + \dot{\eta}, \tag{12}$$

that is, the return you would get from selling the firm and obtain market rent on the asset value must be equal to current profits and the change in value of the firm. Hence, in line with neoclassical capital market theory, we assume that the capital market is working perfect.

Note that $\eta(t)$ must be discontinuous at time $t = 0$. Regardless of its initial value, its value will change at once the tax path is set.[5] We obtain the following optimal controle problem:

$$\min_{\tau_t} \int\limits_0^\infty \left\{ N_t \frac{b}{\rho} u_t - N_0 \pi_t \right\} e^{-rt} dt, \tag{13}$$

where the first term inside the brackets is abatement outlay, and the second term is the profits earned on the initial number of ideas.

The state variables develop according to:

$$\dot{\theta} = \varepsilon^0 - N_t (u_t)^\rho - \phi\theta_t, \tag{14}$$

$$\dot{N}_t = a_0 (N_t)^{-\alpha} \eta_t, \tag{15}$$

$$\dot{\eta}_t = r\eta_t - \pi_t, \tag{16}$$

and:

$$\theta(0) = \theta_0, N(0) = N_0, \theta(\bar{t}) \leq \bar{\theta}, \forall t \geq \bar{t}. \tag{17}$$

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[5] This also implies that time inconsistency could be a problem, see Appendix B.

There are three state variables: The concentration of CO_2 in the atmosphere, the number of different abatement technologies, and the value of a new abatement technology innovation. The current value Hamiltonian is given:

$$H = N_t \frac{b}{\rho} u_t - N_0 \pi_t + \lambda_1 \left\{ \varepsilon^0 - N_t (u_t)^\rho - \phi \theta_t \right\}$$

$$+\lambda_2 a_0 (N_t)^{-\alpha} \eta_t + \lambda_3 \left\{ r \eta_t - \pi_t \right\}, \tag{18}$$

The shadow price of carbon emissions λ_1 is clearly positive. That is, given the concentration target, higher emissions in any period, or a higher initial concentration, can only increase abatement costs. On the contrary, the shadow price of new ideas λ_2 is negative. Since the more the emission sector employs one particular idea, the less is the abatement effect, more ideas can only reduce costs (see also Appendix A).

The initial shadow price of the value of an idea λ_3 must be zero, since the initial value of an idea η_0 is essentially free i.e. the government can choose any value at the outset as long as the emission target is reached. Then, due to the *"stepping on toes"* effect, we suspect λ_3 to be positive. That is, if the value of an idea increases such that one more idea emerges, we suspect total abatement cost to increase. The reason is that the cost of the *last idea* is higher than the value of the idea (see Subsection 2.3).

Further, for $t \geq \bar{t}$, we have the following Lagrangian:

$$\mathcal{L}_t = H_t + \mu_t \left(\bar{\theta} - \theta_t \right) \tag{19}$$

As already announced, we will compare two cases; with and without induced technological change. The *with induced technological change* case is identical to the case we have described so far. Furthermore, we will concentrate on the time period from $t = 0$ to $t = \bar{t}$, that is, the time period from now until the atmospheric concentration target is reached.

3.2 The optimal emission tax path

The expression for the maximum principle writes:

$$\frac{\partial H}{\partial \tau_t} = \left[\frac{\tau_t - \lambda_1}{(1-\rho)\tau_t} \right] N_t - \lambda_3 - N_0 = 0. \tag{20}$$

Clearly, we cannot have $\tau_t = \lambda_1$ as long as λ_3 and/or $N_0 \neq 0$. Thus, the standard result that the emission tax rate should be equal to shadow price on emissions do not apply.

The first costate equation is given:

$$\dot{\lambda}_1 = (r + \phi)\lambda_1, \tag{21}$$

From (21) we note the shadow price on emissions grow with the rate $r + \phi$. Since carbon emissions are removed from the atmosphere by a natural process, and since this process is more effective in absolute terms the higher the concentration of carbon, the shadow price on emissions grow with a rate that is higher than the interest rate.

It is hard to characterize all aspects of the emission tax path in the general case, but the following propositions identify some of its properties. We start by looking at the solution when $\alpha = 0$ and $N_0 = 0$. Note that when $N_0 = 0$, (20) will be true for all τ_0, since at $t = 0$, $\lambda_3 = 0$ as well. The following proposition characterizes the path just after $t = 0$ (henceforth 0^+):

Proposition 1 *When there are no ideas initially, no dynamic learning externality in R&D and the patent protection system is perfect, we have* $\tau_t = \frac{1+\rho}{2\rho}\lambda_1$. *That is, the emission tax rate is set higher than the shadow price on emissions, and grows with the same rate as the shadow price on emissions.*

Proof. See Appendix C ■

Note that the extent to which the emission tax rate exceeds the level depends on ρ, that is, the lower the ρ, the higher is the difference. The economic intuition is that the lower the ρ, the higher is the mark-up of each abatement technology supplier and the higher is the deadweight loss from monopoly pricing. Consequently, the emission tax rate should be set higher in order to encourage more usage of each abatement technology.

Note also that the emission tax rate should grow with the same rate as the shadow price on emissions. Since the mark-up in the abatement technology sector is constant, the proportional relationship between the shadow price on emissions and emission tax rate should also be constant.

The proposition implies that $|\lambda_2| = \frac{1}{1+\rho}\eta_t$. That is, the shadow value of a new idea is smaller than the value of a new idea along the equilibrium path. This is due to the *"stepping on toes"* effect mentioned above.

Next, let $N_0 > 0$, $\alpha \neq 0$, and remember that when $\alpha < 0$, we have the "standing on shoulders" case, that is, making new ideas available becomes less costly as the total number of ideas accumulates. Further, when $\alpha > 0$ we have the "fishing out" case, that is, making new ideas available becomes more costly as the number of ideas accumulate. We have the following proposition:

Proposition 2 *When there are some ideas initially and or there are either positive or negative dynamic externalities in R&D, the emission tax rate should not grow with the same rate as the shadow price on emissions.*

Proof. See Appendix C and the numerical simulations. ■

If $N_0 > 0$, our results indicate that it is optimal to set the tax rate somewhat higher initially. The reason is that the initial ideas are free in the sense that R&D cost are not needed before they can be applied. This finding is clearly not in accordance with common intuition, which argues that emission taxes should be high since there are few available technologies!

Further, the simulations presented below suggest that $\alpha < 0$ implies a less steep path, and that $\alpha > 0$ implies a steeper path than when $\alpha = 0$.

That is, "standing on shoulders" yields an initial emission tax rate which is relatively higher compared to the initial emission tax in the "fishing out" case (relatively in the sense; compared to later emission tax rates).

The question is now how endogenous technological change case (ITC case) compares to the exogenous technological change case (ETC case).

4 Comparing the ITC and ETC cases

4.1 The optimal ETC emission tax path

In the ETC case the time path of N_t must be given, that is, for each instant in time, the number of ideas that are made available are given exogenously. The emission tax rate then only affects the usage of each technology.

Since ideas are free in this version of the model, supply can start at once an idea arrives without the CO_2 abatement firms incurring any fixed cost. Further, in order to be able to compare the two versions of the model, we assume that all ideas have constant marginal cost equal to $\frac{b}{\rho}$ instead of b. Thus, with price equal to marginal cost the usage of CO_2 abatement equipment for a given emission tax rate will be unaltered from the model with ITC described above. The current value Hamiltonian in the ETC case is then given by:

$$H^0 = N_t \frac{b}{\rho} u_t + \lambda_1^0 \left\{ \varepsilon^0 - N_t (u_t)^\rho - \phi \theta_t \right\} + \lambda_2^0 \dot{N}_t$$

where $\dot{N}_t$ is exogenous (and identical to n_t above). The superscript "0" on the variables refers to the *ETC case.* For $0 < t \leq \bar{t}$, and after some rearranging, the maximum principle yields:

$$\frac{\partial H^0}{\partial \tau_t^0} = \frac{\tau_t^0 - \lambda_1^0}{(1-\rho)\tau_t} = 0, \tag{22}$$

Clearly, we must have $\tau_t^0 = \lambda_1^0$. This is the standard result, among others found in Nordhaus (1982) and in Goulder and Mathai (2000).

The costate equation with respect to the atmospheric concentration of emissions is also the same as in Nordhaus and in Goulder and Mathai:

$$\dot{\lambda}_1^o = (r + \phi)\lambda_1^o. \tag{23}$$

Thus, in the ETC case the emission tax rate should grow with the rate $r + \phi$. We can then compare the ITC and ETC cases:

4.2 Without knowledge externality or initial ideas

The development in the number of ideas is identical in the two cases. Moreover, when $\alpha = 0$ and $N_0 = 0$ we know from Proposition 1 that the emission tax rate in the ITC case should grow with the same rate as in the ETC case. Thus, the following solution is consistent with the maximum principle: $\tau = \tau^0$ i.e. the tax rates are identical in the two cases. If the ITC tax rate had started higher, it would always be higher, and since the development in the number of ideas is identical in two cases, we would have had too much abatement.

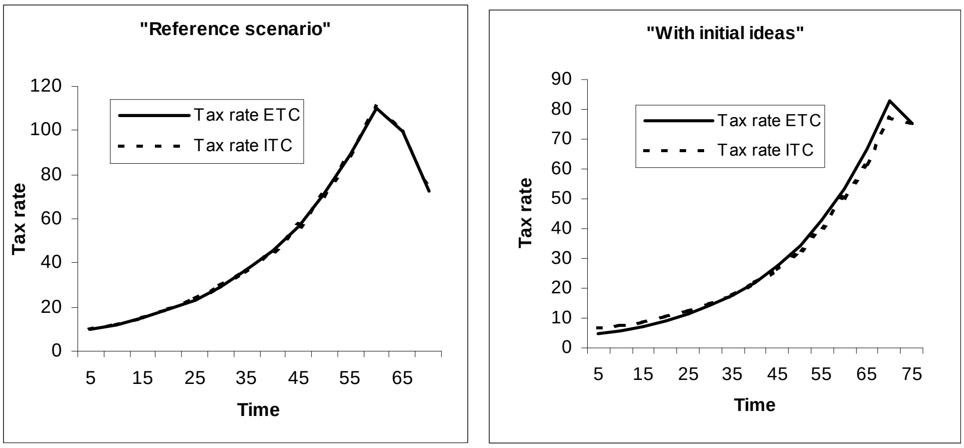
Likewise, if the ITC tax rate had started lower, we would have had too little abatement.

4.3 With knowledge externality or initial ideas

For both cases it is hard to provide analytical results, and instead, we have run numerical simulations on the model. The simulation model has 40 periods of 5 years each. Concentration starts at 380 and is not allowed to exceed 500. The model is calibrated such that in the base case i.e. when $\alpha = 0, N_0 = 0$, the emission tax rate starts at just below \$10, and peaks at about \$100. With these tax rates the target is reached in 70 years. After that the emission tax falls due to a steady inflow of more technologies, which makes it easier and easier to keep emissions ε_t equal to the decay $\phi\theta_t$ (see equation (11)).

From the optimal ITC tax path, we obtain the endogenous development in the variables n_t and N_t. The same development is then implemented as an exogenous development, and the model is solved over again for the optimal taxation path in the ETC case. The results are presented in the following figures, and we focus on the first 70 years:

Figure 1 "The effect of having initial ideas"

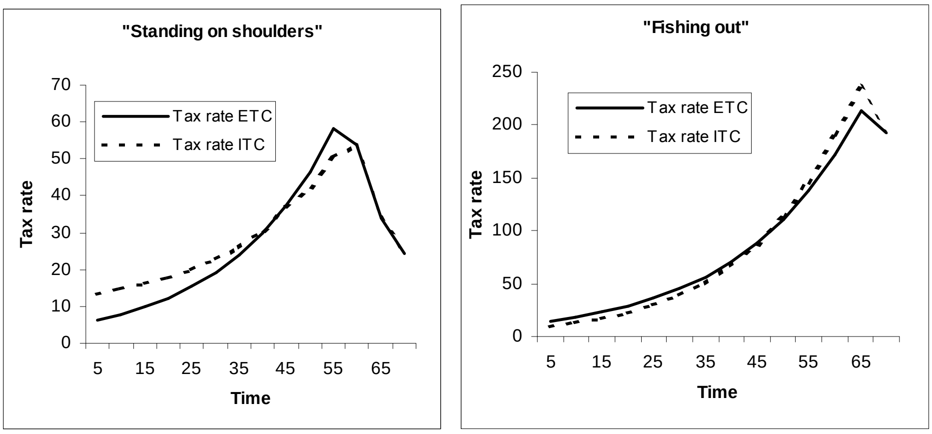


In the "reference" scenario there are no spill-overs and no initial ideas. In line with Proposition 1 the ITC and ETC emission tax paths are then identical.

In the "with initial ideas" scenario approximately 25% of the ideas was assumed to be available from the start. As seen from the two figures, future emission tax rates can then be much lower since it becomes a lot less costly to reach the target. Moreover, as seen from the figure at the right, the ITC path is less steep than the ETC path. The reason is obvious: In the ETC case all ideas cost $\frac{b}{\rho}$, while in the ITC case the initial ideas only cost b since the R&D cost connected to the development of these ideas is sunk.

Next we look at the effect of knowledge spillovers. The size of the spillover α, is set such that in the "standing on shoulders" case total discounted costs are half of the "reference" scenario, and in the "fishing out" case they double compared to the "reference" scenario. This amount to α being equal to ± 0.5. In order to isolate the effects we have set $N_0 = 0$.

Figure 2 "The effect of knowledge spillovers"



In the "standing on shoulders" case the ITC emission tax path starts higher than the ETC emission tax path, and after a while becomes lower until year 70. Thus, the growth rate of the emission tax rate is lower than the growth rate of the shadow price of emissions.

On the contrary, in the "fishing out" case the ITC emission tax path starts lower and becomes higher from year 50 on. Note also that the ITC path under "fishing out" are steeper than the ITC path under "standing on shoulders" reflecting that generating ideas is more costly in this case.

Observe also that the initial tax rates do not differ much compared to how much they differ about the time the concentration ceiling is reached. The tax rates in the period from 5 to 10 are 9.7, 13.1 and 9.4 in the "reference", "standing on shoulders" and "fishing out scenario", respectively. In period 65-70 they are however 99.6, 33.9 and 237.2, respectively.

The simulations have been repeated with parameter values of α in the range $[-1, 1]$. Moreover, we have run several sensitivity analyses varying the curvature of the emission abatement function ρ, the discount rate r, the decay rate ϕ, and the cost parameter b and a_0. All the time we get the same distinction between the "standing on shoulders case" and the "fishing out case".

5 Imperfect patent protection

In reality patents are not infinitely lived. When we in spite of this fact have chosen to model patents as infinitely lived, it is because we believe that firms holding patents often become technology leaders within their field, and hence, succeed in continuously updating their patents so as to keep some degree of market power. However, firms do not always succeed in doing this, and technologies may become generic. This is for instance implicitly assumed when modelling technological change by industry wide learning curves.

A convenient way to introduce a learning externality in our model is to assume that patents may be copied without cost at any point in time. Patents that are copied will be supplied at marginal cost b_t^i for ever after.[6] In particular, we assume that the probability of still holding a patent obtained at $t = t_0$, at time time t is equal to $e^{-\chi(t-t_0)}$. Hence, the value of an idea changes to:

$$\eta_t = \varphi \int_t^{\infty} \tau(s)^{\frac{1}{1-\rho}} e^{-(r+\chi)(s-t)} ds, \tag{24}$$

where $(r+\chi)$ is the new discount rate when future profit is conditioned on the future state in which you still hold the patent. The new arbitage equation is given by:

$$r\eta = \pi_t + \dot{\eta} - \chi\eta, \tag{25}$$

that is, the return you would get from selling the firm and obtain market rent on the asset value must be equal to current profits and the change in value of the firm subtracted the risk of loosing the whole firm.

Let M_t denote the accumulated number of ideas at time t which no longer is protected by a patent. The development in N_t and M_t can then be expressed by:

$$\dot{N}_t = a_0(N_t)^{-\alpha}\eta_t - \chi N_t, \tag{26}$$

$$\dot{M}_t = \chi N_t. \tag{27}$$

Thus, at any moment in time there will be two different markets for carbon abatement equipment; one market in which price exceeds marginal cost and one in which price equals marginal cost. We denote the supply of each type j in the latter market by z^j. Emissions is then given by: $\varepsilon_t = \varepsilon^0 - \sum_{i=1}^{N_t}(u_t^i)^\rho - \sum_{j=1}^{M_t}(z_t^j)^\rho$. At each instant in time the emission sector solves:

$$\min_{u_t^i} \left\{ \tau_t \left[\varepsilon^0 - \sum_{i=1}^{N_t}(u_t^i)^\rho - \sum_{j=1}^{M_t}(z_t^j)^\rho \right] + \sum_{i=1}^{N_t} p_t^i u_t^i + \sum_{j=1}^{M_t} b_t^j z_t^j \right\}, \rho < 1, \tag{28}$$

where b_t^j is the marginal cost of equipment of type j.

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[6] See for instance Barro and Sala-i-Martin (2004), Section 6.2, Erosion of monopoly power", page 305.

As before we assume that all types of equipment have the same marginal cost, and that the cost is time invariant. We then have that $p_t^i = \frac{b}{\rho}\forall i$, and $b_t^j = b\forall j$. It is then easy to solve for z_t^j. The expression for discounted stream of abatement costs also includes the number of ideas that no longer is protected by patents, M_t, and we have one more state variable M_t. In order to simplify we only look at the case in which $N_0 = 0$. The object of the government is then:

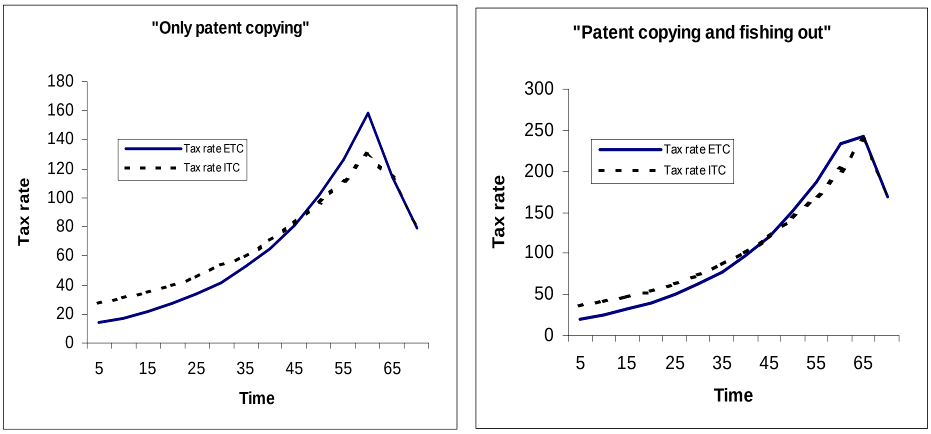
$$\min_{\tau_t} \int_0^\infty \left\{ N_t \frac{b}{\rho} u_t + M_t b z_t \right\} e^{-rt} dt \tag{29}$$

given (14) and (17) from Section 3.1 together with (25), (26) and (27) from above. Note that $z_t > u_t$ due to marginal cost pricing when ideas get copied.

The analytical solution to the model in the case when patent protection is imperfect is hard to interpret. Thus, we have instead ran numerical simulations on the model. First, we found the optimal emission tax path with ITC as before. Next, we assumed that N_t developed exogenously, and we found the optimal emission tax path with ETC. The development in M_t follows automatically from (27) in both cases. In order to be able to compare the two cases, we set marginal cost equal to $\frac{b}{\rho}$ when the technology has a patent, and b when not. Moreover, we set price equal to marginal cost. The usage of the two categories of technologies is then identical in the two cases.

As above we present two simulations of the model. As mentioned the simulation model has 40 periods of 5 years each, and the stabilization of the concentration is reached 70 years into the future. We use a loss rate of 2.5% each year. The results are presented in the following figure, and we focus on the first 70 years:

Figure 3 "Optimal tax paths with imperfect patent protection"



We present the result with and without the "fishing out" knowledge externality. In the figure to the left there are only patent copying. Note that the ITC emission tax path starts higher, becomes equal, and finally lower in about year 50. The emission ceiling is reached in year 70.

When we introduce "fishing out" this effect is reduced, but it does not vanish. Even though the "fishing out" effect is as strong as in the former simulation ($\alpha = 0.5$), the government should still start with a higher tax rate in the ITC case than in the ETC case. The simulations have been repeated with parameter values on χ in the range $[0.001, 0.5]$, the latter amounting to 50% loss rate each year. Moreover, we have run several sensitivity analyses varying the knowledge spillover α, the curvature of the emission abatement function ρ, the discount rate r, the decay rate ϕ, and the cost parameter b and a_0. As long as $\alpha \leq 0$, patent copying implies a higher tax rate from the start. In most cases, it is also so when $\alpha > 0$, but not necessarily.

6 Discussion and Conclusion

Unlike in Goulder and Mathai we find that governments could have reasons to set a higher carbon tax today if we have technological change driven by R&D than if we have pure exogenous technological change. The result is *dependent* of the direction and size of the knowledge externality and the extent to which there is a learning externality. In particular, if we have either a "standing on shoulders" type of knowledge externality or weak patent protection or both, our findings suggest that a high emission tax rate from the start may be warranted. The robustness of this result has also been tested in several simulations.

On the other hand, we consider a case without any form of R&D subsidies or deployment subsidies for new technologies. Hence, our results are only valid if such subsidies are not available. Clearly, future research should also look more into the use of various forms of subsidies, like in van der Swaan et al.(2002). The several externalities suggest using more instruments, and both R&D subsidies and abatement deployment subsidies is easy to incorporate in the model.

A high carbon tax today would lead to more usage of the few technologies available today. Thus, our result has some resemblance with the findings in the learning by doing literature. Learning by doing implies that abatement costs *only* declines as a response to actual use of abatement technology. On the other hand, the learning by doing strand of literature seems to be assuming that firms can not at all appropriate the technological improvements created by their own learning.

If firms could appropriate parts of their own learning, firms might be willing to supply carbon abatement to prices below marginal cost given that future emission tax rates were going to be high. Thus, the need for a high initial tax rate would likely be weakened (see Spence, 1981, for a general analysis of the implications of learning).

Our results are in line with Gerlagh et al. (2008) and Hart (2008). Gerlagh et al. (2008) model patents with a finite lifetime, and thus, investments in R&D is below the social optimum. They then find that the emission tax rate should be set above the shadow price on emissions in a second best world where only emission taxes and R&D subsidies are available to the policy maker. Hart (2008) also arrives at similar result, but unlike this paper and Gerlagh et al. (2008), he also considers crowding out of investments in production technology.

A caveat is that climate policies may be time inconsistent if decisions about R&D and abatement are not taken by the government, but by independent agents. To the extent that governments find it hard to commit to a emission tax path, this could pose a serious problem. Future contributions should clearly look more into this topic.

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A The sign on the shadow prices

Given the concentration target, the optimal emission tax path $\tau^*(t)$ minimizes the discounted sum of R&D costs and abatement costs. The optimal emission tax path $\tau^*(t)$ corresponds to a certain emission level $\bar{\varepsilon}_t^*$ at each point in time t. The optimal usage of each abatement technology at each point in time then amounts to: $u_t^* = (\frac{\varepsilon^0 - \bar{\varepsilon}_t^*}{N_t})^{\frac{1}{\rho}}$, and the abatement cost of reaching the target

$\bar{\varepsilon}_t^*$ at each point in time t equals: $b(\varepsilon^0 - \bar{\varepsilon}_t^*)^{\frac{1}{\rho}} N_t^{-(\frac{1-\rho}{\rho})}$. Hence, at each point in time the cost of reaching the target $\bar{\varepsilon}_t^*$ *decreases* in N_t. Note also that emissions is a bad, that is, at each point in time the lower the target $\bar{\varepsilon}_t^*$, the higher the costs. We therefore conjecture that the shadow value of N_t must be positive for all t. Moreover, that the shadow value of the concentration must be negative for all t since a higher concentration will translate into a lower emission level $\bar{\varepsilon}_t^*$ at least for one point in time.

B Time consistency

It is easy to check that the optimal controls are time consistent in the model of Goulder and Mathai since the maximization problem is time invariant and the two state variables follows smooth paths. However, with decentralized decisions by private agents, as in our model set up, time consistency could constitute a problem.

Let τ_t^* be the optimal emission tax path, and let θ_t^*, N_t^* and η_t^* be the associated paths of the control variables. Suppose that the optimal plan has been followed until time $t = \hat{t} > 0$. Imagine a new decision maker who minimizes the discounted sum of abatement costs from time $\hat{t}$ onward, that is:

$$\min_{\tau(t)} \int_{\hat{t}}^{\infty} \left\{ N_t \frac{b}{\rho} u_t - N_{\hat{t}} \pi_t \right\} e^{-rt} dt,$$

subject to (14) to (16), but with the initial values of the state variables given by $\theta^*(\hat{t})$ and $N^*(\hat{t})$. The question is whether the new decision maker would choose the original controle τ_t^* from $\hat{t}$ and onwards?

The original problem faced by the decision maker at time $t = 0$ can be transformed as follows:

$$\min_{\tau(t)} \int_{0}^{\infty} \left\{ N_t \frac{b}{\rho} u_t - N_0 \pi_t \right\} e^{-rt} dt = \min_{\tau(t)} \int_{0}^{\hat{t}} \left\{ N_t \frac{b}{\rho} u_t - N_0 \pi_t \right\} e^{-rt} dt$$

$$+ \min_{\tau(t)} e^{-r\hat{t}} \int_{\hat{t}}^{\infty} \left\{ N_t \frac{b}{\rho} u_t - N_{\hat{t}} \pi_t \right\} e^{-r(t-\hat{t})} dt,$$

where $e^{-r\hat{t}}$ is a constant.

If $\theta^*(\hat{t})$, $N^*(\hat{t})$ and $\eta^*(\hat{t})$ were common, note that $\min_{\tau(t)} \int_{\hat{t}}^{\infty} \left\{ N_t \frac{b}{\rho} u_t - N_{\hat{t}} \pi_t \right\} e^{-r(t-\hat{t})} dt$ subject to (14) to (16) must yield the same control as

$\min_{\tau(t)} e^{-r\hat{t}} \int\limits_{\hat{t}}^{\infty} \left\{ N_t \frac{b}{\rho} u_t - N_{\hat{t}} \pi_t \right\} e^{-r(t-\hat{t})} dt$ subject to (14) to (16). Thus, we have time consistency as long as $\theta^*(\hat{t})$, $N^*(\hat{t})$ and $\eta^*(\hat{t})$ follow from history, and cannot be chosen freely. On the other hand, in principle, $\eta^*(\hat{t})$ is again free. That is, if no additional constraints apply, the new decision makes can choose any value for $\eta(\hat{t})$ since it only depends on the future emission tax path (which is the controle to be chosen).

C Proof of Propositions

In the following we will show that $\dot{\tau}_t = (r + \phi)\, \tau_t$ is consistent with the maximum principle when $\alpha = 0$ and $N_0 = 0$, and not if $\alpha \neq 0$ and/or and $N_0 \neq 0$ The first order condition, the three costate equations, and two of the three equations of motion are respectively given by:

$$1 - \frac{\lambda_1}{\tau_t} = (1 - \rho) \frac{(\lambda_3 + N_0)}{N_t}, \tag{30}$$

$$\dot{\lambda}_1 = (r + \phi) \lambda_1, \tag{31}$$

$$\dot{\lambda}_2 = r\lambda_2 - [\rho\tau_t - \lambda_1] (u_t)^\rho + \alpha\lambda_2 \frac{\dot{N}_t}{N_t}, \tag{32}$$

$$\dot{\lambda}_3 = -\lambda_2 a_0 (N_t)^{-\alpha}. \tag{33}$$

$$\dot{N}_t = \eta_t a_0 (N_t)^{-\alpha}, \tag{34}$$

$$\dot{\eta}_t = r\eta_t - \pi_t, \tag{35}$$

When $\dot{\tau}_t = (r + \phi)\, \tau_t$, the fraction $\frac{\lambda_1}{\tau_t}$ is constant. From (30), this implies that $\frac{(\lambda_3 + N_0)}{N_t}$ must be constant. By taking the time derivative of $\frac{(\lambda_3 + N_0)}{N_t}$ and using (33) and (34) we then have:

$$-\lambda_2 = \frac{(\lambda_3 + N_0)}{N_t} \eta_t,$$

that is, the shadow price of a new idea is a constant fraction of the value of a new idea. By (32) and (35) we finally have:

$$\left[\frac{\lambda_1}{\tau_t} - \frac{2\rho}{(1+\rho)} \right] \frac{1+\rho}{\alpha\rho} \frac{N_t}{\lambda_3 + N_0} = \frac{\eta_t}{p u_t} \frac{n_t}{N_t} \tag{36}$$

As long as $\alpha \neq 0$, this cannot be true. The left hand side of (36) is constant, while both fractions on the right hand side will typically change in the same direction. That is, $\frac{n_t}{N_t}$ will decrease as N_t picks up and $\frac{\eta_t}{\pi_t}$ will decrease as pu_t picks up due to the increasing use of each abatement technology u_t (η_t

is the discounted sum of future π_t, and hence it will always increase slower, if increasing at all).

Then, we consider the case when $\alpha = 0$. The equation (36) can then be reduced to: $\frac{\lambda_1}{\tau_t} = \frac{2\rho}{(1+\rho)}$, and thus since $\rho < 1$, $\tau_t > \lambda_1$. And further, we have:

$$\lambda_3 + N_0 = \frac{1}{1+\rho} N_t$$

This cannot be true since when $t = 0$, $\lambda_3 = 0$ and we have: $N_0 = \frac{1}{1+\rho} N_0$! We are thus left with case $N_0 = \alpha = 0$.

This implies: $\frac{\lambda_3}{N_t} = \frac{1}{1+\rho}$ and $-\lambda_2 = \frac{1}{1+\rho}\eta_t$.That is, the shadow value of a new idea is smaller than the value of a new idea along the equilibrium path. This is due to the crowding externality.

D About the numerical illustrations

We have obtained the numerical illustrations by using Excel's "solver" tool. The business as usual emissions ε^0 is set to 25, and the initial concentration to 430. Furthermore, the model is programmed as a discrete time model in which N_t accumulates according to: $N_t = N_{t-1} + n_t$. The total number of periods is 40, and each period is 5 years. We assume that ideas developed in one period are not ready for sale before in the next period. The model is solved such that CO_2 concentration target is not allowed to exceed 550 at any time. Thus, in the basic set up we do not allow overshooting, however, when we allow it, results are not changed. The simulations showed in the figures used the following parameter values: $N_0 = 0$ and 5, $\rho = 0.15, b = 15$, $a_0 = 15$, $\alpha = -0.5/0.5$, $r = 0.04$ (the yearly discount rate), $\chi = 0.025$ (the yearly copy rate in the case with learning externalities) and $\varphi = 0.025$. The different model versions can of course be obtained from the authors upon request.