The SNOW Global Model

Documentation of SNOW Global

Hidemichi Yonezawa

TALL SOM FORTELLER

NOTATER / DOCUMENTS

2024/32

In the series Documents, documentation, method descriptions, model descriptions and standards are published.

Published: 2 September, 2024

ISBN 978-82-587-1011-7 (electronic)
ISSN 2535-7271 (electronic)

Symbols in tables	Symbol
Category not applicable Figures do not exist at this time, because the category was not in use when the figures were collected.	.
Not available Figures have not been entered into our databases or are too unreliable to be published.	..
Confidential Figures are not published to avoid identifying persons or companies.	:
Decimal punctuation mark	.

Preface

This document provides an algebraic description of the SNOW Global model (SNOW-Global), a multi-sector multi-region computable general equilibrium (CGE) model tailored to analyze energy, environmental and climate policies in the world including Norway as a separate region. The development of the SNOW models, including a small open economy version for Norway (SNOW-NO) and an intertemporally dynamic version (SNOW-DYN), have been supported by funding from the Ministry of Finance.

Statistics Norway, 31 August 2024

Linda Nøstbakken

Abstract

In this document, I provide an algebraic description of the SNOW Global model (SNOW-Global), a multi-sector multi-region computable general equilibrium (CGE) model tailored to analyze energy, environmental and climate policies of the world including Norway.

Table of contents

1. Introduction

The Statistics Norway's World (SNOW) models constitute a family of different model variants, including a global version (SNOW-Global), a small open economy version for Norway (SNOW-NO)[1] and an intertemporally dynamic version (SNOW-DYN). This documentation describes the SNOW Global model (SNOW-Global) and includes an explanation of forward calibration for 2030, in addition to presenting an algebraic model description.[2] SNOW-Global is a multi-sector multi-region computable general equilibrium (CGE) model of the global economy (including Norway as a separate region). The model describes market interactions among all agents of the economy (industries, households and government), as well as bilateral trade among regions.

The model is what is called a mixed complementarity system in mathematics (for an explanation, see the technical Appendices in Markusen (2002)). The model includes n commodities (goods and services), which are purchased by households, firms, and the government. Let the commodities be indexed by $g \in G$. The production functions follow constant returns to scale (CRTS).[3] However, note that with the resource factor input, which is sector-specific, in fossil fuel extraction sectors (e.g., crude oil, natural gas, and coal), their supply curves become upward sloping (i.e., effectively decreasing return to scale), and the elasticity of substitution with respect to the resource factor is calibrated such that the supply elasticities are consistent with the literature. Commodities are also classified by their associated region, indexed by $r \in R$ where O indicates own region. The accounts track the incomes of the representative household in each region, decomposed by the primary mobile factors of production as well as sector-specific inputs for fossil fuel extraction sectors and agriculture sector(s).[4]

Table 1.1 summarizes the equilibrium conditions and associated variables, and Tables 1.2 and 1.3 summarizes the parameters. To reduce the notation burden, we consider the perspective from one country so that we can suppress the index of own region r in Table 1.1 and the following equations, although the model is a multi-region model. The non-linear system is formulated in GAMS/MPSGE and solved using the PATH algorithm. I proceed with a description and algebraic representation of each of the conditions itemized in Table 1.1.

[1] Recent analyses using SNOW-NO include Bye et al. (2023), Fæhn et al. (2020), Kaushal et al. (2019), Kaushal and Yonezawa (2022), and Bye et al. (2021). The model documentation is available as Rosnes and Yonezawa (2024).

[2] Recent analyses using SNOW-Global include Fæhn and Yonezawa (2021), Kaushal et al. (2023), and Bye et al. (2022).

[3] As an extension, we could incorporate increasing returns to scale by monopolistic competition under Krugman (1980), firm heterogeneity under Melitz (2003) or foreign direct investment (e.g., Latorre et al. (2020)).

[4] The main economic data of the model is GTAP (Aguiar et al., 2023), and for each project, aggregation of sectors (and regions) is fully flexible to the extent that GTAP data has separate sectors. For example, in climate policy analyses, we tend to aggregate agriculture-related sectors as one agriculture sector, but we could keep separate sectors (e.g., rice and wheat).

Table 1.1 General equilibrium conditions

Equilibrium Conditions	(Equation)		Associated Variables
Dual representation of preferences and technologies:			
Armington-like unit-cost functions	(2.1)	$\forall g \in G$	A^g : Armington-like activity
Input-output technologies for non primary energy	(2.7)	$\forall g \in G$ but $g = xe$	Y^g: Production level
Input-output technologies for primary energy	(2.10)		Y^{xe}: Production level
Unit expenditure function	(2.12)		U: Household utility index
Unit cost of public purchase	(2.13)		PUB: Government activity
Unit cost of investment	(2.14)		INV: Investment activity
Market clearance conditions:			
Composite goods and services	(3.1)	$\forall g \in G$	PA^g : Composite price indexes
Markets for output	(3.5)	$\forall g \in G$	PY^g: Output prices
Markets for imports	(3.6)	$\forall g \in G$ and $r \neq O$	PM^g_r: Import prices
Factor markets	(3.7)	$\forall f \in F$	PF_f: Factor prices
Mobile labor	(3.8)		PL_{mob}: Mobile labor price
Sector specific labor	(3.10)	$\forall g \in G$	PL_g: Sector-specific labor price
Fixed real investment	(3.11)		$PINV$: Unit cost of investment
Fixed real public spending	(3.12)		PG: Unit cost of public good
Nominal utility equals Income	(3.13)		PC: Unit expenditure index
Balance of payments	(3.14)		PFX: Price of foreign exchange
Income balance:			
Domestic agent income	(4.1)		RA_h: Household Income
Government budget	(4.2)		$GOVT$: Government spending
Auxiliary Conditions:			
Fixed real public spending	(5.1)		T: Index on direct taxes
Capital in steady state	(5.2)		S_{CAP}: Capital stock
Investment in the steady state	(5.3)		INV: Investment activity
Unemployment rate	(5.4)		UNE: Unemployment

Table 1.2 Model parameters

Symbol	Description
Elasticity of substitution parameters	
σ_{mat}	Material vs. rest
σ_{vae}	Value-added vs. energy composite
σ_{ele}	Electricity vs. fossil fuel composite
σ_{fe}	Among fossil fuel products
σ_{lab}	Mobile labor vs. sector specific labor
σ_{va}^{g}	Value added composite
σ_{DM}^{k}	Armington elasticity of substitution (Domestic vs. foreign)
σ_{MM}^{k}	Armington elasticity of substitution (among foreign)
σ_{xe}	Resource factor vs. rest
Other parameters	
$\overline{sav}$	Reference saving
$\overline{pub}$	Reference level of government spending
$\overline{dtax}$	Reference level of direct tax from household to government
$\overline{ftrn}$	Reference capital account surplus
ϕ_{r}^{g}	Share of domestic or import component in Armington activity
θ^{mat}	Share of material cost in marginal cost
θ^{ML}	Share of mobile labor component in labor composite
θ^{fe}	Share of aggregate fossil fuel products
θ^{mat}	Share of aggregate material intermediate inputs
θ^{va}	Share of value-added cost in value-added plus energy cost bundle
θ^{f}	Share of each factor cost in value added cost bundle
θ^{i}	Share of each intermediate input i
θ^{res}	Share of resource factor input
μ_{C}^{g}	Expenditure share commodity g in private consumption
μ_{G}^{g}	Expenditure share commodity g in public consumption
μ_{INV}^{g}	Expenditure share commodity g in investment consumption

Table 1.3 Parameter values for elasticities

Parameter	Description	Value
σ_{mat}	Material vs. rest	0.5
σ_{vae}	Value-added vs. energy composite	0.5
σ_{ele}	Electricity vs. fossil fuel composite	0.5
σ_{fe}	Among fossil fuel products	0.5
σ_{lab}	Mobile labor vs. sector specific labor	2
σ_{va}^{g}	Value-added bundle	GTAP values
σ_{DM}^{g}	Armington elasticity (Domestic vs. foreign)	GTAP values
σ_{MM}^{g}	Armington elasticity (among foreign)	GTAP values
σ_{xe}	Resource factor vs. rest	Calibrated
s_{xe}	Price elasticity of primary energy production	See notes
ϵ_{emp}	Elasticity of unemployment with respect to real wage	0.1

Notes: For each project (and each sector in each region), if necessary, we adjust them. Nevertheless, here are the central values. The supply elasticites are set to 4 for coal and 1 for crude oil and natural gas sectors. For Armington elasticities, see Hummels (2001) and Hertel et al. (2007).

2. Dual representation of technologies and preferences

Technologies and preferences are represented through value functions that embed the optimizing behavior of agents. Any linearly-homogeneous transformation of inputs into outputs is fully characterized by a unit-cost (or expenditure) function. Generally, setting the output price equal to optimized unit cost yields the equilibrium condition for the activity level of the transformation. That is, a competitive constant-returns activity will increase up to the point that marginal benefit (unit revenue) equals marginal cost. In general, we will use the convention of setting unit revenues (left-hand side) equal to unit cost (right-hand side) and associating this equilibrium condition with a transformation activity level.

Agents in each region wishing to purchase a particular good or service g face an aggregate price PA^g. In constructing the aggregate prices, we will rely on the following notation for the component prices:

PY^g Price of output for sector g,
PM^g Price of import composite of good g.

Assuming a Constant Elasticity of Substitution (CES) aggregation of the components we equate the prices to the CES unit-cost functions:

$$PA^g \quad = \quad \left(\phi_D^g (PY^g)^{1-\sigma_{DM}^g} + \phi_M^g (PM^g)^{1-\sigma_{DM}^g}\right)^{1/(1-\sigma_{DM}^g)}, \tag{2.1}$$

where σ_{DM}^g is the Armington elasticity of substitution, and PM^g is the composite of imports from all other countries by using the Armington elasticities σ_{MM}^g. The arguments of these functions are the component prices. The ϕ parameters are CES distribution parameters that indicate scale and weighting of the arguments. These are calibrated to the social accounts such that the accounts are replicated in the benchmark equilibrium.

We have domestic production in accordance with the input output data. The exact nesting structures of production functions can be flexible for each project (also for each sector within each project), as the scope and focus is different among projects. Here we describe one illustrative setting as an example. The technology includes an upstream CES value-added nest which then combines business services and ultimately then this composite combines with other intermediates in fixed proportions. Let PF_f indicate the price of primary factor of production $f \in F$ and let P_g^{vas} be the value-added business-services composite price for sector g. Note that we could specify that all the labor is mobile among sectors, or alternatively we could assume that some share of labor is mobile among sectors, while the rest of labor is sector specific.[5] In the setting of some sector specific labor, PF_{Lab} is the price of the composite of mobile labor and sector-specific labor, and thus it is sector-specific, although PF_{Lab} does not include g index for reducing the notation burden. Let PL^g and PL_{mob} indicate the price of the sector specific labor and mobile labor, respectively. The composite of value added and energy, P_g^{vae}, is the CES aggregate of value added and energy (P_g^{va} and P_g^e) as follows:

$$P_g^{vae} \quad = \quad \left[(1-\theta_g^{va})(P_g^e)^{1-\sigma_{vae}} + \theta_g^{va}(P_g^{va})^{1-\sigma_{vae}}\right]^{1/(1-\sigma_{vae})}, \tag{2.2}$$

[5] Here we write a general setting where some share of labor is sector specific, but we can think of the special case of zero share of sector specific labor as 100% mobile labor case.

$$P_g^{va} \quad = \quad \left(\sum_f \theta_g^f [(1+t_{fg})PF_f]^{1-\sigma_{va}^g} \right)^{1/(1-\sigma_{va}^g)}, \tag{2.3}$$

$$PF_{Lab} \quad = \quad \left[(1-\theta_g^{ML})(PL^g)^{1-\sigma_{lab}} + \theta_g^{ML}(PL_{mob})^{1-\sigma_{lab}} \right]^{1/(1-\sigma_{lab})}. \tag{2.4}$$

The energy composite, P_g^e, is the composite of electricity and the composite of fossil fuels (i.e., coal, natural gas and refined oil products):

$$P_g^e \quad = \quad \left[(1-\theta_g^{ele})(P_g^{fe})^{1-\sigma_{ele}} + \theta_g^{ele}[(1+t_{ele,g}^{int})PA_{ele}]^{1-\sigma_{ele}} \right]^{1/(1-\sigma_{ele})}, \tag{2.5}$$

$$P_g^{fe} \quad = \quad \left(\sum_{fe} \theta_g^{fe} [(1+t_{fe,g}^{int})PA_{fe}]^{1-\sigma_{fe}} \right)^{1/(1-\sigma_{fe})}. \tag{2.6}$$

The composite of "material intermediate inputs" (i.e., non-energy intermediate inputs) and value added plus energy, P_g^{vae}, is the CES aggregate of two CES aggregates (P_g^{mat} and P_g^{vae}) as follows:

$$PY^g \quad = \quad \left[(1-\theta_g^{mat})(P_g^{vae})^{1-\sigma_{mat}} + \theta_g^{mat}(P_g^{mat})^{1-\sigma_{mat}} \right]^{1/(1-\sigma_{mat})}, \tag{2.7}$$

$$P_g^{mat} \quad = \quad \sum_{i \neq E} \theta_g^i (1+t_{ig}^{int}) PA_i, \tag{2.8}$$

where the set E is energy goods (electricity and fossil fuel products). t_{ig}^{int} is the tax in sector g on purchases of good i and t_{fg} is the factor tax. The substitution elasticity between value added and energy composite is given by σ_{vae}, whereas the substitution between factors are given by σ_{va}, and σ_{lab} is the substitution elasticity between mobile and sector specific labor. The substitution elasticity between electricity and fossil fuel products is given by σ_{ele}, whereas the substitution between fossil fuel products are given by σ_{fe}. The θ is share parameters determined in the calibration to the input-output accounts.

Regarding the primary energy production sectors (i.e., coal, natural gas, and crude oil), it includes the resource factor that is sector-specific, and thus this sector is subject to decreasing returns to scale. We calibrate the elasticity of substitution betweeen the resource factor and the rest of inputs to match the given price elasticities of supply, denoted s_{xe}. As Rutherford (2002) shows, the calibrated substitution elasticitiy σ_{xe} is given by

$$\sigma_{xe} = s_{xe} \frac{\theta_{res}}{1-\theta_{res}}, \tag{2.9}$$

where θ_{res} is the value share of resource factor input. Then, instead of equation (2.7), the top-level unit cost function of the primary energy production sector becomes

$$PY^{xe} = \left[\theta^{res}(PF_{res})^{1-\sigma_{xe}} + (1-\theta^{res})(P_{xe}^{oth})^{1-\sigma_{xe}} \right]^{1/(1-\sigma_{xe})}, \tag{2.10}$$

$$P^{oth}_{xe} = \left[(1-\theta^{mat}_{xe})(P^{vae}_{xe})^{1-\sigma_{mat}} + \theta^{mat}_{xe}(P^{mat}_{xe})^{1-\sigma_{mat}}\right]^{1/(1-\sigma_{mat})}. \tag{2.11}$$

Final demand includes three categories: household demand, government demand, and investment. The representative agents for each household h are assumed to have identical Cobb-Douglas preferences over the aggregated goods and services. The preferences are specified via a unit expenditure function associated with an economy-wide utility index (U). Let PC be the true-cost-of-living index indicated by the following unit expenditure function:

$$PC = \prod_g [(1+t^{cons}_g)PA^g]^{\mu^g_C}, \tag{2.12}$$

where the μ are value shares. The government faces a Leontief price index, PG, for government purchases:

$$PG = \sum_g \mu^g_G(1+t^{gov}_g)PA^g. \tag{2.13}$$

Similarly the price of investment, $PINV$ is a Leontief aggregation of commodity purchases:

$$PINV = \sum_g \mu^g_{INV}(1+t^{inv}_g)PA^g. \tag{2.14}$$

Equations (1) through (2.14) define all of the transformation technologies for the model. Next we turn to a specification of the market clearance conditions for each price.

3. Market clearance conditions

For each good or service there is a market, and, for any non-zero equilibrium price, supply will equal demand. We will use the convention of equating supply, on the left-hand side, to demand, on the right-hand side. The unit-value functions presented above are quite useful in deriving the appropriate compensated demand functions, by the envelope theorem (Shephard's Lemma).

Supply of the composite goods and services, trading at PA^g, is given by the activity level, A^g, and demand is derived from each production or final demand activity that uses the good or service. The market clearance condition is given by

$$A^g \quad = \quad \sum_i h_{gi}(Y^i, \boldsymbol{p}) + \mu_C^g U \frac{PC}{(1+t_g^{cons})PA^g} + \mu_G^g PUB + \mu_{INV}^g INV, \tag{3.1}$$

where $h_{gi}(Y^i, \boldsymbol{p})$ are the conditional input demands (as a function of output and the price vector). These are found by taking the partial derivative of the unit cost function for sector i with respect to the gross of tax price of input g. For material intermediate inputs:

$$h_{gi}(Y^i, \boldsymbol{p}) \quad = \quad \theta_i^g \theta_i^{mat} Y^i \left(\frac{PY^i}{P_i^{mat}}\right)^{\sigma_{mat}} \tag{3.2}$$

where P_i^{mat} is the composite price of material inputs as defined in equation (2.8).

Similarly, the electricity input demand is:

$$\begin{aligned} h_{ele,i}(Y^i, \boldsymbol{p}) \quad = \quad & \theta_i^{ele}(1-\theta_i^{va})(1-\theta_i^{mat})Y^i \left(\frac{PY^i}{P_i^{vae}}\right)^{\sigma_{mat}} \left(\frac{P_i^{vae}}{P_i^e}\right)^{\sigma_{vae}} \\ & \left(\frac{P_i^e}{(1+t_{ele,i}^{int})PA_{ele}}\right)^{\sigma_{ele}} \end{aligned} \tag{3.3}$$

The demand of fossil fuel products are:

$$\begin{aligned} h_{fe,i}(Y^i, \boldsymbol{p}) \quad = \quad & \theta_i^{ele}(1-\theta_i^{va})(1-\theta_i^{mat})Y^i \left(\frac{PY^i}{P_i^{vae}}\right)^{\sigma_{mat}} \left(\frac{P_i^{vae}}{P_i^e}\right)^{\sigma_{vae}} \left(\frac{P_i^e}{P_i^{fe}}\right)^{\sigma_{ele}} \\ & \left(\frac{P_i^{fe}}{(1+t_{fe,i}^{int})PA_{fe}}\right)^{\sigma_{fe}} \end{aligned} \tag{3.4}$$

Market clearance for the output depends on supply (simply given as an activity of production) and domestic and foreign demand from the Armington activity:

$$Y^g \quad = \quad \phi_D^g A^g \left(\frac{PA^g}{PY^g}\right)^{\sigma_{DM}^g} + \sum_r FORIM_r^g. \tag{3.5}$$

Import demand is derived from the Armington activities. For $r \neq O$, we have the following:

$$IM_r^g \quad = \quad \phi_r^g A^g \left(\frac{PA^g}{PM_r^g}\right)^{\sigma_{DM}^g}. \tag{3.6}$$

Factor markets clear, where factor supply is given by the exogenous endowments to households, denoted $\overline{S}_f$, and input demands are derived from the cost functions:

$$\overline{S}_f = \sum_i \theta_f^i \theta_i^{va}(1-\theta_i^{mat})Y^i \left(\frac{PY^i}{P_i^{vae}}\right)^{\sigma_{mat}} \left(\frac{P_i^{vae}}{P_i^{va}}\right)^{\sigma_{vae}} \left(\frac{P_i^{va}}{(1+t_{fi})PF_f}\right), \tag{3.7}$$

where P_i^{va} is the composite value-added price as defined in equaiton (2.3). Regarding the labor endowment, $\overline{S}_{Lab}$ is the total labor endowment including both mobile and sector specific labor. Denoting the endowment of mobile labor $\overline{SL}^{mob}$, we have

$$\overline{SL}^{mob} = \sum_i \theta_i^{ML} DL_i^{com} \left(\frac{PF_{Lab}}{PL_{mob}}\right)^{\sigma_{lab}}, \tag{3.8}$$

where DL_i^{com} is the sectoral demand of composite labor, and it is specified as following:

$$DL_i^{com} = \theta_{Lab}^i \theta_i^{va}(1-\theta_i^{mat})Y^i \left(\frac{PY^i}{P_i^{vae}}\right)^{\sigma_{mat}} \left(\frac{P_i^{vae}}{P_i^{va}}\right)^{\sigma_{vae}} \left(\frac{P_i^{va}}{(1+t_{Lab,i})PF_{Lab}}\right). \tag{3.9}$$

Denoting the endowment of sector specific labor $\overline{SL}_i^{sec}$, we have

$$\overline{SL}_i^{sec} = (1-\theta_i^{ML})DL_i^{com} \left(\frac{PF_{Lab}}{PL^i}\right)^{\sigma_{lab}}. \tag{3.10}$$

Real investment equals real savings by households:

$$INV = \overline{sav}. \tag{3.11}$$

Real government purchases equal the nominal government budget scaled by the government price index:

$$PUB = \frac{GOVT}{PG}. \tag{3.12}$$

Household utility (U) equals nominal income across households scaled by the true-cost-of-living index. That is, in each region we have an aggregate activity U, which supplies utility to the representative household of that region, and its nominal income is RA. The corresponding market clearance condition is thus

$$U = \frac{RA}{PC}. \tag{3.13}$$

The final market clearance condition reconciles the balance of payments. The supply of foreign exchange includes its generation in the export activities and net borrowing from the rest of the world (net capital account surpluses). The real capital account surplus is held fixed at the exogenous benchmark observation, denoted $\overline{ftrn}$. Foreign exchange is demanded for direct import purchases as well as the payments to foreign agents for their contribution to production.

$$\sum_{r\neq O}\sum_g FORIM_r^g + \overline{ftrn} = \sum_{r\neq O}\sum_g IM_r^g. \tag{3.14}$$

4. Income Balance Conditions

The representative agent (household) earns income from factor endowments, but disposable income nets out savings and a direct tax transfer to the government. Real savings is held fixed (by the coefficient $\overline{sav}_h$). We also hold fixed the real level of government spending, but this requires an adjustment in direct taxes on households. Removal of tariffs, for example, impact the government budget and the shortfall is made up for by an endogenous increase in the direct taxes on households. We use the auxiliary variable T to scale the direct taxes appropriately. In addition, the household is assumed to hold any benchmark net international capital flows. The household's budget is given by

$$\begin{aligned} RA &= \sum_f PF_f \overline{S}_f \\ &- \overline{sav} PINV \\ &- \overline{dtax} PG \times T \\ &+ \overline{ftrn} PFX \end{aligned} \tag{4.1}$$

The government budget is given by net direct and indirect taxes on domestic and international transactions. The full nominal government budget is

$$\begin{aligned} GOVT &= \overline{dtax} PG \times T \\ &+ \sum_g t_g^{cons} PA^g \mu_C^g U \frac{PC}{(1+t_g^{cons})PA^g} \\ &+ \sum_g t_g^{inv} PA^g \mu_{INV}^g INV \\ &+ \sum_g t_g^{gov} PA^g \mu_G^g PUB \\ &+ \sum_i \sum_g t_{gi}^{int} PA_g h_{gi}(Y^i, \boldsymbol{p}) \\ &+ \sum_i \sum_f t_{fi} PF_f \theta_f^i \theta_i^{va} (1-\theta_i^{mat}) Y^i \left(\frac{PY^i}{P_i^{vae}}\right)^{\sigma_{mat}} \left(\frac{P_i^{vae}}{P_i^{va}}\right)^{\sigma_{vae}} \left(\frac{P_i^{va}}{(1+t_{fi})PF_f}\right) \\ &+ \sum_{r \neq O} \sum_g t_{gr}^{imp} (PFX) IM_r^g \\ &+ \sum_{r \neq O} \sum_g t_g^{exp} PX_r^g FORIM_r^g \end{aligned} \tag{4.2}$$

Again, the index T is adjusted endogenously to hold the real level of public spending fixed.

5. Auxiliary Conditions

In addition to the three sets of standard conditions presented above, we use an auxiliary condition to fix the real size of the government. Specifically, we need to determine the index which scales direct taxes on households. Associated with the variable T is the following condition:

$$PUB = \overline{pub}. \tag{5.1}$$

Second, we could use auxiliary conditions to consider the steady state simulation instead of the static simulation. While the capital stock is exogenous with the endogenous capital return in the static simulation, the capital stock is endogenous with the exogenous capital return (demanded by investors) in the steady state simulation as modelled in Balistreri et al. (2009). The capital return is determined by the following equation with the associate variable of capital stock S_{CAP}:

$$PF_{CAP} = PINV. \tag{5.2}$$

In this steady state setting, we assume that the real investment is decreased as the capital stock shrinks. Specifically, we assume that the percentage change in investment equals the percentage change in capital supply as Francois et al. (2013) does.

$$\frac{\Delta INV}{INV_0} = \frac{\Delta K}{K_0}, \tag{5.3}$$

where INV_0 and K_0 are the benchmark value of investment and capital supply, respectively. In the steady state simulation, we replace the capital market clearance condition of (3.7) with the equation (5.2), and we also replace the fixed investment condition (3.11) with endogenous investment condition (5.3).

Lastly, we can also use the following auxiliary condition to include the endogenous unemployment. As suggested by Blanchflower and Oswald (1994), we characterize the unemployment by a wage curve. With the unemployment feature in the model, we modify the labor endowment by following the equation:

$$log \frac{PF_{LAB}}{PC} = \epsilon_{emp} \, log \frac{UNE}{UNE_0}, \tag{5.4}$$

where UNE is the unemployment rate in the counterfactual simulation, UNE_0 is the unemployment rate at the benchmark, and ϵ_{emp} is the elasticity of the unemployment with respect to real wage.

6. Greenhouse Gas Emissions

Greenhouse gas emissions include CO_2, CH_4, N_2O, and fluorinated gases. The main data of GTAP includes CO_2 from the combustion of fossil fuels (i.e., coal, natural gas, and oil products) for each fossil fuel consumption of each sector or final demand in each region.[6] These CO_2 from the combustion of fossil fuels are combined with the consumption of fossil fuels in a fixed proportion (i.e., Leontief function).

GTAP Satellite Data of "Complementary Greenhouse Gas emissions" includes non-energy combustion CO_2 and non-CO_2 greenhouse gases (i.e., CH_4, N_2O, and fluorinated gases). One simplified and common way to include these greenhouse gas emissions (for those that are appropriate to be considered in this way) is by combining them in a fixed proportion with the activity level or production level. Since some of these emission data include the association with endowment by industries and input use by industries, using those information should be possible as well, which we have not explored yet.

[6]The emission intensity of the "same" fossil fuel product (e.g., refined oil products) is heterogeneous depending on sectors and regions. This reflects the heterogeneity of types of even for the "same" refined oil products (e.g., gasoline vs. jet fuel) and prices.

7. Climate Policies

One main area of the simulation analysis of this model is climate policies. These include carbon pricing policies (through carbon tax or emission trading systems), which have long been advocated by economists, and technology policies (i.e., technology mandates and performance standards), which are commonly implemented in many countries. Also, facing the concern of carbon leakage, border carbon adjustments (e.g., the EU Carbon Border Adjustment Mechanism) have been commonly implemented in this type of simulation model.

Regarding carbon pricing polices, we set the benchmark level of emissions as the endowment for the government (or households). If we set a smaller endowment (e.g., 90%) as a counterfactual shock, then the model solves for the endogenous emission price to achieve the specified emission reduction. Similarly, for a carbon tax, we can set the level of the endowment of emissions as endogenous such that the emission price is set to a given carbon tax.

It is straightforward to consider different emission pricing among sectors. Specifically, it is fully flexible, ranging from no trading (each sector in each region has own emission price) to a uniform price among some sectors in some regions (e.g., EU ETS), to a global uniform emission price. We can simply set up the emission trading market based on sectors, regions, and types of greenhouse gas emissions as well. In other words, we could also allow emission trading among different greenhouse gases, if we want.

Another common aspect explored in this type of models is how the extra revenues from carbon pricing are used. In the literature (e.g., Rausch et al. (2011) and Beck et al. (2015)), it is common to consider three ways (or some combinations of them): lump-sum return to households, lowering wage tax, and lowering capital tax.

Technology policies are commonly implemented as a combination of subsidies on green technology and tax on non-green technology (or all technology) to achieve a certain share of green technology (e.g., Holland et al. (2009) and Rausch and Yonezawa (2023)). In this case, it is not about how to use the extra revenues, but instead how to finance the subsides. The tax on non-green technology or all technology is possible. Alternatively, we can use a more general tax outside of the corresponding sector.

8. Forward Calibration

We often use a forward calibration technique to create a hypothetical future economy (e.g., 2030). In that case, We follow the methodology described by Böhringer et al. (2009). We use forecasted data of GDP, energy demand, and energy prices. There are two widely used data sources: the International Energy Outlook data from the US EIA (Energy Information Administration) and the World Energy Outlook data from the IEA (International Energy Agency).

However, we must be careful about how we interpret this hypothetical future economy, which is supposed to be a reference economy against which we implement counterfactual policies, such as emission reduction policies. If the forecasted data already reflects some future climate policies, the counterfactual climate policies will not fully capture the impact of the entire climate policies because some parts are already embedded into the reference future economy. For example, if the forecasted data reflects a future economy where half of the emission reduction target is already achieved, then the counterfactual simulation (in the model) to achieve the emission target captures only the remaining half of the policies.

References

Aguiar, A., Chepeliev, M., Corong, E., and van der Mensbrugghe, D. (2023). The global trade analysis project (gtap) data base: Version 11. *Journal of Global Economic Analysis*, 7.

Balistreri, E. J., Rutherford, T. F., and Tarr, D. G. (2009). Modeling Services Liberalization: The Case of Kenya. *Economic Modelling*, 26(3):668–679.

Beck, M., Rivers, N., Wigle, R., and Yonezawa, H. (2015). Carbon tax and revenue recycling: Impacts on households in british columbia. *Resource and Energy Economics*, 41:40–69.

Blanchflower, D. G. and Oswald, A. J. (1994). Estimating a Wage Curve for Britain 1973-90. *The Economic Journal*, 104(426):pp. 1025–1043.

Bye, B., Fæhn, T., Kaushal, K. R., Storrøsten, H. B., and Yonezawa, H. (2021). Politikk på politikk - derfor koster klimapolitikken. *Samfunnsøkonomen*, 135(2):45–56.

Bye, B., Kaushal, K. R., Rosnes, O., Turner, K., and Yonezawa, H. (2023). The Road to a Low Emission Society: Costs of Interacting Climate Regulations. *Environmental and Resource Economics*, 86:565–603.

Bye, B., Kaushal, K. R., and Storrøsten, H. B. (2022). EU's suggested carbon border adjustment mechanisms: Impact on Norwegian industries. Statistics norway (ssb) report no. 2022/48.

Böhringer, C., Löschel, A., Moslener, U., and Rutherford, T. F. (2009). Eu climate policy up to 2020: An economic impact assessment. *Energy Economics*, 31:S295–S305. International, U.S. and E.U. Climate Change Control Scenarios: Results from EMF 22.

Francois, J., Manchin, M., Norberg, H., Pindyuk, O., and Tomberger, P. (2013). Reducing Transatlantic Barriers to Trade and Investment: An Economic Assessment. IIDE Discussion Papers 20130401, Institue for International and Development Economics.

Fæhn, T., Kaushal, K. R., Storrøsten, H. B., Yonezawa, H., and Bye, B. (2020). Abating greenhouse gases in the Norwegian non-ETS sector by 50 per cent by 2030. Statistics norway (ssb) report no. 2020/23.

Fæhn, T. and Yonezawa, H. (2021). Emission targets and coalition options for a small, ambitious country: An analysis of welfare costs and distributional impacts for norway. *Energy Economics*, 103:105607.

Hertel, T., Hummels, D., Ivanic, M., and Keeney, R. (2007). How Confident Can We Be of CGE-based Assessments of Free Trade Agreements? . *Economic Modelling*, 24(4):611 – 635.

Holland, S. P., Hughes, J. E., and Knittel, C. R. (2009). Greenhouse gas reductions under low carbon fuel standards? *American Economic Journal: Economic Policy*, 1(1):106–46.

Hummels, D. (2001). Toward a Geography of Trade Costs. mimeo, purdue university. http://www.krannert.purdue.edu/faculty/hummelsd/research/toward/tgtc.pdf.

Kaushal, K. R., Lindholt, L., and Yonezawa, H. (2019). Effects of changes in electricity prices on the power-intensive industries and other sectors in Norway towards 2030. Statistics norway (ssb) report no. 2019/43.

Kaushal, K. R., Lindholt, L., and Yonezawa, H. (2023). Emission pricing and CO_2 compensation in the EU: The optimal compensation to the power-intensive and trade-exposed industries for increased electricity pricesEffects of changes in electricity prices on the power-intensive industries and other sectors in Norway towards 2030. Statistics norway discussion paper no. 1008.

Kaushal, K. R. and Yonezawa, H. (2022). Increasing the CO_2 tax towards 2030 - Impacts on the Norwegian economy and CO_2 emissions. Statistics norway (ssb) report no. 2022/43.

Krugman, P. (1980). Scale Economies, Product Differentiation, and the Pattern of Trade. *American Economic Review*, 70(5):950–59.

Latorre, M. C., Olekseyuk, Z., and Yonezawa, H. (2020). Trade and foreign direct investment-related impacts of brexit. *The World Economy*, 43(1):2–32.

Markusen, J. R. (2002). *Multinational Firms and the Theory of International Trade*. Cambridge MA: MIT Press.

Melitz, M. (2003). The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica*, 71(6):1695–1725.

Rausch, S., Metcalf, G. E., and Reilly, J. M. (2011). Distributional impacts of carbon pricing: A general equilibrium approach with micro-data for households. *Energy Economics*, 33:S20–S33. Supplemental Issue: Fourth Atlantic Workshop in Energy and Environmental Economics.

Rausch, S. and Yonezawa, H. (2023). Green technology policies versus carbon pricing: An intergenerational perspective. *European Economic Review*, 154:104435.

Rosnes, O. and Yonezawa, H. (2024). The SNOW Model for Norway: Documentation of SNOW-NO. Statistics norway (ssb) document no. 2024/16.

Rutherford, T. (2002). Lecture notes on constant elasticity functions. mimeo.