**Discussion Papers No. 229, September 1998**
**Statistics Norway, Research Department**

*John K. Dagsvik, Yu Zhu and Rolf Aaberge*

# A Framework for Empirical Modelling of Consumer Demand with Latent Quality Attributes

**Abstract:**
This paper discusses a particular approach to empirical consumer demand modelling when products are differentiated and the product attributes are unobservable. In contrast to the traditional approach to this problem, see e.g. Epple (1987) and Deaton (1987, 1988), where the product variants are treated as infinitely divisible goods, the present approach assumes that the consumer is making his choice of variant from a set of discrete "packages" of attribute combinations. Subsequently, given the (discrete) choice of variants the corresponding quantities are treated as continuous choices. Thus in this approach the consumer's decision process is formulated as a discrete/continuous choice problem.

The empirical analysis is based on microdata from the Sichuan province in China. We show that in this case the estimation methods work well and yield reasonable results.

**Keywords:** Consumer demand, Differentiated products, Latent product attributes

**JEL classification:** C31, C43, D12

**Acknowledgement:** We are particularly indepted to Bjørn Helge Vatne and Leif Brubakk for programming and estimating different model versions. We thank the State Statistical Bureau of China for providing us with detailed household expenditure data from the Sichuan province.

**Address:** John K. Dagsvik, Statistics Norway, Research Department., P.O.Box 8131 Dep, N-0033 Oslo. E-mail: john.dagsvik@ssb.no

| | |
|---|---|
| **Discussion Papers** | comprises research papers intended for international journals or books. As a preprint a Discussion Paper can be longer and more elaborated than a usual article by including intermediate calculation and background material etc. |

Abstracts with downloadable postscript files of
Discussion Papers are available on the Internet: http://www.ssb.no

For printed Discussion Papers contact:

Statistics Norway
Sales- and subscription service
P.O. Box 1260
N-2201 Kongsvinger

Telephone: +47 62 88 55 00
Telefax: +47 62 88 55 95
E-mail: Salg-abonnement@ssb.no

# 1. Introduction

This paper addresses the problem of specifying and estimating empirical demand systems for differentiated consumption goods, where attributes of the product variants are latent. In standard empirical demand models the problem with unobservable product variants of differentiated products is often ignored or treated rather superficially. Since the seminal theoretical contribution by Rosen (1974), there has been a number of studies in the so-called hedonic price literature that tries to deal with the challenges and problems that arise when consumers have preferences over quantities as well as attributes associated with the product variants.[1] It is, however, the approach developed by Deaton (1987, 1988) which is the closest to the one developed here in the sense that it aims at estimating demand systems in the presence of latent quality attributes. However, the modelling assumptions of Deaton's approach are fundamentally different from ours, as will become apparent below.

Loosely speaking, the difficult modeling and estimation problems stem from the fact that in a rigorous treatment, one needs to take into account that a vide variety of differentiated products are offered in a market with heterogeneous consumers and producers. Consequently, product attributes and associated quantities become choice variables that are determined in market equilibrium. While the respective quantities are continuous, the variants are typically discrete and it seems therefore somewhat artificial to apply standard marginal calculus in this context, as if the variants were infinitely divisible.

Following Lancaster (1979), p. 16; "the problem of analyzing economic systems in which the goods (or many of them) can be infinitely varied in design and specification has always been that of finding a workable framework of analysis." The traditional way of dealing with quality aspects is either simply to increase the number of goods or to apply Hicks aggregation. Unfortunately, in practice it turns out to be difficult to treat each variant as a separate observable commodity category. This is related to the fact that it is problematic to quantify quality precisely. In other words, quality is typically a latent variable which only to a limited extent can be accounted for by classifying variants of the product under consideration into a large number of categories. Although many variants can in principle be classified in observable categories, there will, in practice, be a limit to how many variants one can treat as separate goods in a demand system. To aggregate goods into composite ones is also

[1] See for example, Bartik (1987), Epple (1987), Trajtenberg (1990) and Nerlove (1995) who analyze the econometric problems related to estimating demand and supply models for differentiated products.

problematic. If consumers have heterogeneous preferences the corresponding price indexes will be individual specific and can therefore not readily be implemented in empirical demand analyses.[2]

While many of the contributions in the literature have assumed that variants are infinitely divisible with respect to product attributes, we take a different approach in this paper. Specifically, we argue that the choice setting should rather be formulated as a discrete/continuous one where the discrete dimension corresponds to the choice among product variants. In this setting the consumers problem is to make his choice from a set of feasible discrete "packages" of attribute combinations. To this end we apply a particular econometric framework that allows for a rich flexibility in the representation of the random components of the model. To be more specific, the variants are characterized by prices and a latent quality attributes. The quality attributes reflect the average evaluation of each product variant when price across variants are equal. The set of prices and quality attributes that corresponds to the feasible variants may vary across consumers due to unobserved heterogeneity in the distribution of the locations of the store, and also because different stores may set different prices for the same variant.[3] In addition, consumers may differ in their tastes for a given product variant. To allow for unobserved heterogeneity in preferences and opportunity sets we adapt the general framework developed by Dagsvik (1994). This framework allows the choice set of feasible variants, as well as preferences, to be stochastic. Recall that in this context the randomness is interpreted as stemming from the observing econometrician's lack of information about unobservable heterogeneity in opportunities and tastes. The consumers are, however, assumed to have perfect knowledge about their tastes and opportunities, although other interpretations are possible as we shall discuss below. The particular framework proposed in this paper is convenient for empirical modelling and analysis since it leads to convenient expressions for the distribution of the prices and quality attributes of the chosen variants and the corresponding aggregate demand as a function of the distribution of the preferences and the distribution of the feasible prices and quality attributes in the market. In particular, it is possible to allow for correlation between prices and quality attributes. This is of substantial interest in many cases because it may lead to upward sloping demand curves (see for example Trajtenberg, 1990). The reason for this is that when prices and quality attributes are positively correlated, high prices are to some extent perceived by the customers as a signal of high quality and thus an increase in prices will not necesarily yield a decrease in demand. The problem of commodity aggregation in empirical demand analysis has also been addressed by Anderson (1979).

---

[2] It seems that some authors, such as Chamberlin (1933), p. 79, have thought it impossible to carry through a full formal analysis of variable product design: ".... product variations are in this essence qualitative; they cannot, therefore, be measured along an axis and displayed in a single diagram".

[3] There are several studies that discuss stories that yield equilibrium price dispersion, see for example Burdett and Judd (1983), and references therein.

The paper is organized as follows. Section 2 presents the general choice problem and modelling framework. In Section 3 we discuss the derivation of aggregate relations that follows from the assumptions made in Section 2. These relations include the expenditure system as well as the relationship between the distribution of prices (across unobservable variants and stores) and the distribution of unit values. Also we discuss briefly the equilibrium distribution of prices and attributes. In Section 4 we consider an empirical application on microdata from the Sichuan province of China.

## 2. The modelling framework

In this section we spell out the central elements of the modelling approach. A consumer's choice set consists of a wide variety of different products and stores (locations). Specifically, we assume that we can represent the commodity space by n different types of products (goods), where each product consists of an infinite set of different variants/locations characterized by price and quality attributes. In this paper the n goods refer to the observed commodity categories while the product variants and stores are unobservable to the econometrician. Since the variants and stores are unobservable we may, without loss of generality, treat stores and variants symmetrically in the formalism. Let $Q_j(z)$ be the quantity of observable good j and unobservable location and variant z, and let $T_j^*(z) > 0$ be an unobservable quality/location attribute associated with variant z within commodity category j, $j \le n$. For example, let the commodity type be "bread", available in two stores as the variants, "wheat bread" and "rye bread". Let $z = 1$ represent store A and wheat bread, $z = 2$ store A and rye bread, $z = 3$ store B and wheat bread, and finally $z = 4$ store B and rye bread. These are all possible combinations of locations and variants in this example. The attributes $\left\{T_j^*(z)\right\}$ are consumer specific in the sense that they are subjectively perceived.[4] Thus, $\left\{T_j^*(z)\right\}$ is equivalent to a utility index in the sense that it yields a complete rank ordering of one unit of the product variants. We realize that the above setup is similar to the characteristics approach of Lancaster (1966), where the $T^*$-attribute represents the characteristics dimension. How the prices and $T^*$-attributes are distributed will be discussed below.

Next we state assumptions about the preferences. Let $P_j(z)$ be the price of variant/location z of type j. Evidently, we can represent the vector of product variants and their attributes as the Cartesian product

$$\left(\mathbf{Q}, \mathbf{T}^*\right) = \underset{z}{\times}\left(Q_1(z), T_1^*(z), Q_2(z), T_2^*(z), \ldots, Q_n(z), T_n^*(z)\right).$$

[4] For simplicity, the indexation of consumers and time is suppressed until Section 4.

The consumer is assumed to be perfectly informed about the distribution of product locations, variants and prices. He is assumed to have preferences over variants and associated quantities.

**Assumption A1**

*The utility function U(**Q**,**T***) has the structure*

$$U\left(\boldsymbol{Q},\boldsymbol{T}^{*}\right)=u\left(\sum_{z} S_{1}(z),\sum_{z} S_{2}(z),\ldots,\sum_{z} S_{n}(z)\right),$$

*where*

$$S_{j}(z)=Q_{j}(z)\,T_{j}^{*}(z),$$

*and u is a mapping* $u: R_{+} \rightarrow R_{+}$ *, that is increasing and quasiconcave.*

Assumption A1 implies that within a specific type of good, the different variants are perfect substitutes. This implies that the consumer will only buy one variant of each type of good at a time. This setup is therefore a version of the "Ideal Variety Approach", proposed by Lancaster (1979). The realism of Assumption A1 depends of course on how detailed the observable commodity types are defined. It also depend on the time unit because the consumer specific attributes $\left\{T_{j}^{*}(z)\right\}$ may change from one instant of time to another. If the purchases are made on a daily basis then the perfect substitute assumption might seem rather plausible, while this assumption is quite strong if one assumes that "month" is the proper time unit.

The budget constraint is given by

(2.1) $$\sum_{j=1}^{n} \sum_{z} Q_{j}(z)\,P_{j}(z) \leq y$$

where y is income. Let

(2.2) $$R_{j}(z)=P_{j}(z) / T_{j}^{*}(z).$$

Then it follows that the consumers optimization problem is equivalent to maximizing the utility function (2.1) with respect to $\left\{S_{j}(z), z=1,2,\ldots,\ j=1,2,\ldots,n\right\}$ subject to the budget constraint

(2.3) $$\sum_{j=1}^{n} \sum_{z} S_{j}(z)\,R_{j}(z) \leq y,$$

where $\{R_j(z)\}$ represent "prices". We realize immediately that the problem above is formally equivalent to a conventional consumer optimization problem where $S_j(z), z = 1,2,\ldots,$ are perfect substitutes that enter symmetrically in the model. As mentioned above we realize easily that the consumer will choose only one variant within each observable type of good. Specifically, variant $\hat{z}_j$ will be chosen if

$$R_j(\hat{z}_j) = \min_z R_j(z), \tag{2.4}$$

which means that $\hat{z}_j$ is the variant with the lowest taste-and-quality-adjusted"price".

For notational convenience, let $\hat{R}_j = R_j(\hat{z}_j)$, $\hat{Q}_j = Q_j(\hat{z}_j)$, $\hat{S}_j = S_j(\hat{z}_j)$ and $\hat{P}_j = P_j(\hat{z}_j)$. Let $y_j(\mathbf{r}, y)$, $j = 1,2,\ldots,n,$ be the function that yields expenditure on good of type j that follows from maximizing $u(s_1, s_2, \ldots, s_m)$ subject to $\sum_{j=1}^{n} r_j s_j \le y$, where $\mathbf{r} = (r_1, r_2, \ldots, r_m)$. We realize immediately that the purchased quantity of good j, $\hat{Q}_j$, is given by

$$\hat{Q}_j = \frac{\hat{S}_j \hat{R}_j}{\hat{P}_j} = \frac{y_j(\hat{\mathbf{R}}, y)}{\hat{P}_j} \tag{2.5}$$

where $\hat{\mathbf{R}} = (\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_n)$. Thus, we have expressed the chosen quantities by means of an ordinary and deterministic demand system and $\hat{\mathbf{R}}$. We shall call $\{\hat{R}_j\}$ virtual prices. The effect of unobserved heterogeneity in quality and preferences is thus entirely captured by the virtual prices. The virtual prices as well as the unit prices, $\{\hat{P}_j\}$, are endogeneous because they are associated with the respective chosen product variants/locations. Note that the virtual prices are not observable. They can be interpreted as taste-and-quality-adjusted-prices in the sense that if the virtual prices were known, consumer behavior could be represented by an ordinary deterministic demand system that does not depend on the consumer (within suitable defined population groups) nor on the unobservable product variants. This is so because the "quantities" $S_j(z)$ enter symmetrically in the utility function within each commodity type. Due to this property the virtual prices are in fact latent stochastic price indexes.

# 3. Aggregate relations

## 3.1. The consumers

To obtain aggregate relations that apply to empirical analyses, it is necessary to make further assumptions. To focus on the essential ideas we first present a simplified version of the econometric framework, and postpone the discussion of a more general setting to Section 3.2. To this end we assume in this section that there are only a finite number of feasible variants in the market. Subsequently, this assumption will be relaxed in Section 3.2 so as to allow for a countable number of variants. Without loss of generality we can write $T_j^*(z) = T_j(z)\xi_j(z)$, where $T_j(z)$ represents the mean attribute value of variant z of type j in the population, and $\xi_j(z)$ are taste-shifters that represent the heterogeneity in consumers tastes. Recall that $T_j^*(z)$ and $\xi_j(z)$ are individual specific, while $T_j(z)$ is common to all individuals. According to Lancaster (1966) the attributes $\{T_j(z)\}$ correspond to the notion of *vertical* product differentiation, while the taste-shifters $\{\xi_j(z)\}$ correspond to the notion of *horizontal* product differentiation. We shall in the sequel call $T_j(z)$ the quality attribute associated with variant z.

Let $B_j(p,t)$ denote the set of variants of type j with $P_j(z) = p$ and $T_j(z) = t$, and let $b_j(p,t)$ be the number of variants in $B_j(p,t)$. In this section we assume that there are several variants in $B_j(p,t)$ for each (p,t). This means that there are several variants for which the mean evaluations are equal. In other words, the agents are on average indifferent with respect to variants within $B_j(p,t)$. In Section 3.2 this assumption will be relaxed.

**Assumption A2**

*The variables $\xi_j(z), z = 1,2,\ldots, j = 1,2,\ldots,n,$ are i.i.d. with*

$$P\left(\xi_j(z) \le y\right) = exp\left(-y^{-\alpha_j}\right)$$

*for $y > 0$, where $\alpha_j > 0$ is a constant.*

A useful interpretation of $\alpha_j$ is as

$$\alpha_j^2 = \frac{\pi^2}{6\,\mathrm{Var}\left(\log \xi_j(z)\right)}. \tag{3.1}$$

A possible justification for Assumption A2 is that it is consistent with the notion of "Independence from Irrelevant Alternatives", which we shall discuss further below.

Given the number of variants of each category, $\{b_j(p,t)\}$, the probability, $\hat{g}_j(p,t)$, that a consumer shall choose $\hat{z}_j$ such that $\hat{P}_j = p, \hat{T}_j = t$, given that a variant of type j is purchased, is formally defined by

$$\begin{aligned}\hat{g}_j(p,t) &= P\left(\min_{z\in B_j(p,t)} R_j(z) = \min_{k,r}\ \min_{z\in B_j(r,k)} R_j(z)\right) \\ &= P\left(\max_{z\in B_j(p,t)}\left(\frac{T_j(z)}{P_j(z)}\cdot\xi_j(z)\right) = \max_{k,r}\ \max_{z\in B_j(r,k)}\left(\frac{T_j(z)}{P_j(z)}\cdot\xi_j(z)\right)\right).\end{aligned} \tag{3.2}$$

From Assumption A2 and (3.2) it follows readily that

$$\hat{g}_j(p,t) = \frac{\left(\frac{t}{p}\right)^{\alpha_j} b_j(p,t)}{\sum_{(x,y)\in D_j}\left(\frac{y}{x}\right)^{\alpha_j} b_j(x,y)} \tag{3.3}$$

where $D_j$ is the set of potential variants (combinations of price and quality attributes of type j). The empirical counterpart to $\hat{g}_j(p,t)$ is the number of consumers that purchase variants within $B_j(p,t)$ to the number of consumers that purchase a variant of type j. Let

$$g_j(p,t) = \frac{b_j(p,t)}{\sum_{(x,y)\in D_j} b_j(x,y)} \tag{3.4}$$

be the fraction of supplied variants in $D_j$ which belong to $B_j(p,t)$. When (3.4) is combined with (3.3) we obtain

$$\hat{g}_j(p,t) = \frac{\left(\frac{t}{p}\right)^{\alpha_j} g_j(p,t)}{\sum_{(x,y)\in D_j}\left(\frac{y}{x}\right)^{\alpha_j} g_j(x,y)}. \tag{3.5}$$

The empirical counterpart to $g_j(p,t)$ is the fraction of variants of type j with list price p and quality attribute t that appear in the stores. A possible story on how $g_j(p,t)$ is determined will be discussed briefly in Section 3.3. Due to the consumer's random taste-shifters, $\{\xi_j(z)\}$, a selection effect arises and the distribution of prices (unit values) and quality attributes of the purchased variants will differ from the corresponding distribution of list prices and quality attributes offered in the market. Note that by (3.1) $\alpha_j$ decreases when the variance of $\log \xi_j(z)$ increases, in which case the selection effect in (3.5) decreases. Specifically, the distribution of unit values and market values coincide in the limit.

It follows directly from Assumption A2 that the distribution of $\hat{R}_j$ has the structure

$$P\left(\hat{R}_j \le r\right) = P\left(\max_z\left(\frac{T_j(z)}{P_j(z)}\xi_j(z)\right) \ge \frac{1}{r}\right)$$

$$= 1 - P\left(\max_z\left(\frac{T_j(z)}{P_j(z)}\xi_j(z)\right) \le \frac{1}{r}\right) = 1 - \prod_z P\left(\xi_j(z) \le \frac{1}{r}\frac{P_j(z)}{T_j(z)}\right) = 1 - \exp\left(-r^{\alpha_j}\sum_z\left(\frac{T_j(z)}{P_j(z)}\right)^{\alpha_j}\right)$$

for $r \ge 0$. Hence

(3.6) $$P\left(\hat{R}_j \le r\right) = 1 - \exp\left(-r^{\alpha_j} K_j\right)$$

where

(3.7) $$K_j = \sum_{(x,y)\in D_j}\left(\frac{y}{x}\right)^{\alpha_j} b_j(x,y) = b_j \sum_{(x,y)\in D_j}\left(\frac{y}{x}\right)^{\alpha_j} g_j(x,y)$$

and

(3.8) $$b_j = \sum_{(x,y)\in D_j} b_j(x,y).$$

## 3.2. Extension to random sets of continuously distributed prices and quality attributes

The discrete setting considered above is somewhat unsatisfactory for several reasons. First, it appears to be a rather large variety in product quality, location and service of the stores which makes it difficult to classify variants and stores in a few groups. As a result, the distribution of prices—which may be observed—seem to be nearly continuous. More important, the sets of feasible variants may vary across consumers, due for example to spatial variations in the location of stores. Finally, it is desirable to relax the assumption of Section 3.1 that $B_j(p,t)$ contains more than one variant. Recall that this assumption was made solely for analytic convenience. In this section we shall extend the setting above to allow for unobserved heterogeneity in choice sets as well as continuous distributions of prices and quality attributes.

Let

$$\wp_j = \{(P_j(z), T_j(z), \xi_j(z)), z = 1,2,\ldots\}$$

denote the collection of prices, T-attributes and taste-shifters associated with the product variants within commodity type j. The set $\wp_j$ is specific to each consumer and represents the product variants that are feasible to the consumer. Thus the choice sets are perceived as random by the observer due to unobserved heterogeneity in opportunities. Recall, however, that the consumer is assumed to be perfectly informed about his choice set of feasible variants. For example, some variants may be difficult to obtain in one region while they are readily available in another. Also different consumers may evaluate the variants differently and the tastes of a given consumer may fluctuate over time. The fact that the value of a store to a given consumer may depend on the distance between him and the store is also captured by this framework.

An alternative interpretation of random choice sets is that the set of feasible variants also may be genuinly random to the consumers. By this it is understood that a consumer in fact only considers a subset of the whole set of feasible alternatives in his decision-making process. In each decision "experiment", these subsets may appear to be random although they will, on average, be more or less closely linked to the underlying objective choice set.

**Assumption A2'**

*The vectors in $\wp_j$, $j = 1,2,\ldots,n$, are points of independent inhomogeneous Poisson processes on $R_+^3$ with intensity measure, $M_j(dp, dt, d\varepsilon)$, given by*

$$M_j(dp, dt, d\varepsilon) = b_j\, g_j(p,t)\, \varepsilon^{-\alpha_j - 1}\, dp\, dt\, d\varepsilon$$

*where $b_j > 0$ and $\alpha_j > 0$ are constants, and $g_j(p,t)$ is a bivariate probability density.*

Recall that the Poisson process assumption implies no essential restriction. It simply means that the points in $\wp_j$ are independently distributed. The probability that there is a points in $\wp_j$ for which $P_j(z) \in (p, p+dp)$, $T_j(z) \in (t, t+dt)$, and $\xi_j(z) \in (\varepsilon, \varepsilon + d\varepsilon)$ is equal to $M(dp, dt, d\varepsilon)$.

Recall that $b_j$ can therefore be interpreted as the (mean) number of firms of type j, while $\alpha_j$ is related to the dispersion of the taste-shifters. Specifically, when $\alpha_j$ increases the dispersion of the taste-shifters decreases. Thus, the interpretation of $\alpha_j$ is completely analogous to the one given in Section 3.1.

The interpretation of the particular structure of the intensity measure is as follows: The multiplicative structure of the internsity measure means that the random taste-shifters $\left\{\xi_j(z)\right\}$ are distributed independently of $\left\{P_j(z), T_j(z)\right\}$. In other words, the tastes associated with the feasible variants are not correlated with the corresponding prices and quality attributes. The particular functional form of the intensity measure follows from a version of the "Independence from Irrelevant Alternatives" assumption (IIA). For the sake of making this rationale explicit, let $\hat{G}_j(A;B)$ denote the probability that a consumer shall select a variant within commodity group j with $\left(P_j(z), T_j(z)\right) \in A$, when the choice set is restricted to $\left(P_j(z), T_j(z)\right) \in B$. Here, A and B are Borel sets in $R_+^2$. Then the IIA assumption can be expressed as

$$\hat{G}_j(A;B) = \hat{G}_j(C;B)\,\hat{G}_j(A;C) \tag{3.9}$$

where $A \subset C \subset B$. The interpretation is that the consumer's choice can be viewed as taking place in two stages. In the first stage a subset $C \subset B$ consisting of the most favorable attributes is chosen. In the second stage the most attractive variant with attributes in A is chosen from C. The crucial point here is that in the second stage, the consumer acts as if C is the original choice set, i.e., the attributes in B\C are irrelevant. In this sense IIA may be viewed as an assumption of probabilistic rationality, cf. Luce (1977). Dagsvik (1994) demonstrated that (3.9) implies that provided the set of feasible attributes and prices are distributed according to an inhomogeneous Poisson process it follows that the intensity measure has the particular structure as in Assumption A2´. We can therefore conclude that the behavioral interpretation and justification of Assumptions A2 and A2´ are completely analogous.

**Theorem 1**

*Under Assumptions A1 and A2' the virtual price* $\hat{R}_j$*, for any j, is independent of the set* $\left\{\left(\hat{P}_k, \hat{T}_k\right), k = 1,2,\ldots,n\right\}$*. Furthermore,* $\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_n$*, are independent with cumulative distribution function*

$$P\left(\hat{R}_j \le r\right) = 1 - exp\left(-r^{\alpha_j}\, K_j\right)$$

*for* $r \ge 0$*, where*

$$K_j \equiv b_j\ E\left(\frac{T_j(z)}{P_j(z)}\right)^{\alpha_j} = b_j \int_0^{\infty}\int_0^{\infty}\left(\frac{y}{x}\right)^{\alpha_j} g_j(x,y)\,dx\,dy\,.$$

A proof of Theorem 1 is given in Appendix A.

For the sake of interpretation let us introduce the indexation of the consumers. Thus $\hat{R}_{ij}$, $\hat{P}_{ik}$ and $\hat{T}_{ij}$ are the virtual price, unit price and chosen quality attribute of consumer i, respectively. Thus Theorem 1 states that for given j and k, $\hat{R}_{ij}$ and $\hat{P}_{ik}$, as well as $\hat{R}_{ij}$ and $\hat{T}_{ij}$ are uncorrelated (across consumers). Although the virtual prices $\left\{\hat{R}_j\right\}$, have the surprising property that they are stochastically independent of the set $\left\{\left(\hat{P}_k, \hat{T}_k\right), k = 1,2,\ldots,n\right\}$, i.e., the virtual prices are independent of the unit values and quality attributes of the purchased variants,[5] it follows from Theorem 1 that the distribution of virtual prices and the distribution of unit prices and chosen attributes are functionally related. Also, $\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_n$, are stochastically independent.

[5] This result is a special case of the more general result that the indirect utility is stochastically independent of the choice in the Generalized Extreme Value model, cf. Strauss (1979).

**Corollary 1**

*Under Assumptions A1 and A2'*

$$K_j = \frac{b_j \; E \; T_j(z)^{\alpha_j}}{E \; \hat{P}_j^{\alpha_j}}.$$

A proof of Corollary 1 is given in Appendix A.

## 3.3. The producers

The explicit modelling of producer behavior in the present context will be discussed elsewhere. Here we shall only give a brief outline of the basic idea.

The point of departure is an oligopoly setting. Each firm only supplies one variant to the market. Specifically, a producer faces a discrete or countable collection of attribute and price combinations from which he makes his choice of which variant to produce at which price. This set is assumed to be random, which is motivated by bounded rationality, i.e., the producer is perceived as having limited capacity for taking all "objective" opportunities into account when maximizing profits. In addition, the firm's profit function is also random. The randomness of the profit function can be interpreted as stemming from latent variables that are accounted for by the firm but unobserved by the analyst. A contribution to this randomness in the profit function can also arise from bounded rationality in the sense that the firm is unable to perfectly assess the profits that follow from the respective decisions.

A producer is assumed to know the aggregate market demand for each combination of price and variant attributes which he applies to from expected profits. Under particular assumptions about the distributions of the random terms in the profit function and the set of potential attributes and prices it is possible to obtain an explicit expression for the probability distribution of the attributes and prices of the variants supplied in the market. This distribution follows from the assumption that each firm maximizes expected profit with respect to own price and own variant attributes, taking the prices and attributes of the variants produced by other firms as given.

We realize that this setting is similar to Anderson et al. (1992), ch. 6 and 7. However, in contrast to the present model, the model of Anderson et al. assumes a deterministic production technology that corresponds to perfectly rational producers.

## 3.4. The relationship between the mean virtual prices and the distribution of prices

In the context of policy analysis it is necessary to know how changes in the distribution of prices affect the mean virtual prices. To this end it is necessary to know the relationship between the distribution of prices and the virtual prices. This is the topic we shall discuss in this section.

To this end let

(3.10) $$\lambda_j(p) = E\left(T_j(z)^{\alpha_j} \,\middle|\, P_j(z) = p\right).$$

The interpretation of $\lambda_j\left(p^{1/\alpha_j}\right)$ is as the conditional mean of $T_j(z)^{\alpha_j}$ across variants of type j, given $P_j(z)^{\alpha_j} = p$. Thus, the function $\lambda_j(p)$ represents the mean perceived quality across variants with price level p. It follows immediately from (3.10) and the continuous analog to (3.3) that the relationship between the marginal densities of prices, $g_j(p)$, and unit values, $\hat{g}_j(p)$, is given by

(3.11) $$\hat{g}_j(p) = \frac{p^{-\alpha_j}\lambda_j(p)\,g_j(p)}{\int_0^\infty x^{-\alpha_j}\lambda_j(x)\,g_j(x)\,dx}.$$

We realize that if $\lambda_j\left(p^{1/\alpha_j}\right) = w_j\,p$, where $w_j > 0$ is a constant, then the distribution of unit values will coincide with the price distribution.

From Theorem 1 and (3.11) we have that

(3.12) $$K_j = b_j \int_0^\infty x^{-\alpha_j}\lambda_j(x)\,g_j(x)\,dx = b_j\,E\left(P_j(z)^{-\alpha_j}\lambda_j\left(P_j(z)\right)\right).$$

Unfortunately, the functional form of $\lambda_j(\cdot)$ is not known. To restrict the class of plausible functional forms of $\lambda_j(\cdot)$ we shall make an additional assumption.

**Assumption A3**

*A positive scale transform of the prices of the variants, $\{P_j(z), z = 1,2,\ldots,\}$, within each commodity group does not affect the conditional distribution of unit values given that a variant is purchased.*

Assumption A3 seems reasonable since only changes in relative prices should matter due to the fact that the conditional density of unit values $\hat{g}_j(\cdot)$ is independent of income.

**Theorem 2**

*Assume that $\lambda_j(\cdot)$ is continuous and that Assumptions A1, A2' and A3 hold. Then $\lambda_j(\cdot)$ is a power function.*

A proof of this result is given in Appendix A.

Thus Theorem 2 provides support to a theoretical justification for the specification

$$\lambda_j\left(p^{1/\alpha_j}\right) = A_j\, p^{\kappa_j}, \tag{3.13}$$

where $A_j > 0$ and $\kappa_j$ are constants. From (3.13) we obtain that $A_j$ has the interpretation

$$A_j = \frac{E\left(T_j(z)^{\alpha_j}\right)}{E\left(P_j(z)^{\alpha_j \kappa_j}\right)}. \tag{3.14}$$

From (3.13) we realize that $\lambda_j\left(p^{1/\alpha_j}\right)$ is convex when $\kappa_j > 1$ and concave when $\kappa_j < 1$. This means that when $\kappa_j > 1$, increasing prices do not reduce the attractiveness of the product variants as much as when $\kappa_j < 1$, because high prices are perceived as a strong indication of high quality, and vice versa. When $\kappa_j > 1$, for example, this relationship between prices and quality is strengthened as the price level increases.

From (3.13), (3.14) and (3.12) we obtain that

$$K_j = b_j\, E\, T_j(z)^{\alpha_j}\, \frac{E\, P_j(z)^{\alpha_j \kappa_j - \alpha_j}}{E\, P_j(z)^{\alpha_j \kappa_j}}. \tag{3.15}$$

From the fact that the virtual prices are Weibull distributed it follows readily (see for example Johnson and Kotz, 1972) that

$$E\,\hat{R}_j = \Gamma\left(1 + \frac{1}{\alpha_j}\right) K_j^{-1/\alpha_j}. \tag{3.16}$$

If we assume that $b_j\, E\, T_j(z)^{\alpha_j}$ is constant then it follows from (3.16) after a suitable normalisation that

(3.17) $$E\,\hat{R}_j = \frac{\left(E\,P_j(z)^{\alpha_j\kappa_j}\right)^{1/\alpha_j}}{\left(E\,P_j(z)^{\alpha_j\kappa_j-\alpha_j}\right)^{1/\alpha_j}}.$$

For the sake of clarification let us consider the interpretation of this index in the case where $\kappa_j$ is close to zero, equal to one and two, respectively. When $\kappa_j \approx 0$, this expression reduces to the generalized harmonic mean

$$\left(E\,P_j(z)^{-\alpha_j}\right)^{-1/\alpha_j}.$$

Note that this expression is little affected by the right tail of price distribution but is homogeneous of degree one such that a proportional increase in all prices increases the index by the same factor. This means that since quality in this case is not correlated with price, high prices will have a small effect on the price index simply because consumers will not buy from stores with high prices (or variants with high prices). In the "reference case" with $\kappa_j = 1$, the index above reduces to the generalized harmonic mean

$$\left(E\,P_j(z)^{\alpha_j}\right)^{1/\alpha_j}.$$

This reference case means that relative changes in prices yield the same relative changes in mean perceived quality. In this case we realize that high prices will be much more important that in the previous case, unless $\alpha_j$ is very small. Recall that a small $\alpha_j$ means large heterogeneity in tastes, and consequently the effect of the price dispersion will be reduced. This conforms with the intuition that since consumers value the product variants differently, the influence on demand of a specific price distribution will to some extent vary across consumers in an unpredictable manner. Consider finally the case when $\kappa_j = 2$. Then the index above has the form

$$\left(\frac{E\,P_j(z)^{2\alpha_j}}{E\,P_j(z)^{\alpha_j}}\right)^{1/\alpha_j}.$$

Since $\left(E\,P_j(z)^{2\alpha_j}\right)^{1/\alpha_j}$ is a factor in the formulae above the effect of the right tail of the price distribution will be larger than the previous cases. This is intuitively plausible since in this case high prices are perceived as signals of high quality.

Finally, let us consider the case with very large population heterogeneity in tastes, i.e., when $\alpha_j \to 0$. By using l'Hôpital's rule we get from (3.17) that

(3.18) $$\lim_{\alpha_j \to 0} E\,\hat{R}_j = \exp\left(E \log P_j(z)\right).$$

Let $P_j(z_r)$, $r = 1,2,\ldots,M,$ be a set of randomly selected prices. Then an estimate of the right hand side of (3.18) is

$$\exp\left(\frac{1}{M}\sum_{r=1}^{M} \log P_j(z_r)\right) = \left(\prod_{r=1}^{M} P_j(z_r)\right)^{1/M}$$

which we recognize as the geometric mean of the prices. Notice that in this case the parameter $\kappa_j$ vanishes in the index formulae.

# 4. An empirical application on microdata from China

## 4.1. A brief sketch of the urban economic development in Sichuan

China's economy has since liberation in 1949 been dominated by a high degree of central planning. However, during the recent decade a series of market-oriented economic reforms have been introduced in order to increase productivity and improve the level of living of Chinese households. The market-oriented reforms began in the late seventies and were then mainly aimed at the rural economic system. At the end of 1984 the government decided to introduce significant urban economic reforms. Important aspects of the reforms were to decentralize decisions to the local government level or even to the firm level and allow firms to retain a larger fraction of profits and to make use of performance-linked bonus payment. Altogether, the economic reforms resulted in a considerable increase in productivity and output and on average in the level of living.

Although the urban reform of 1984 concerns all provinces, the south eastern coastal provinces seem to have been in the forefront of the reform process. This is probably due to the establishment of special economic zones and the introduction of an urban price reform in a few eastern coastal provinces. The broadness and complexity of the reform package of 1984 makes it, however, hard to distinguish between the other provinces with respect to how they have succeeded in emplementing the reforms.

Sichuan is the largest province in China with about 110 million people or 10 per cent of the total population in 1990. The degree of urbanization is, however, low. Only two of the cities has a population above 1 million people, while the remaining 24 cities are medium-sized or small; most of them with fewer than 200 000 residents. The relatively low degree of urbanization in Sichuan is mainly due to the economic history of the region. The industrial production was very low before 1949.

In the years following the First Five-Year Plan, however, development of the industry production in this region was given high priority. Later, in the sixties and seventies, Sichuan was heavily affected by the "Third Front" policy which aimed at developing the military industry as a response to an expected attack from Soviet and USA. During this period as much as 40 to 60 per cent of the total industrial investments was yearly absorbed by the "Third Front" policy. It seems likely that most of these investments went to urban areas and may explain why urban Sichuan on average experienced a considerable increase in level of living between 1965 and 1975. Although Sichuan benefitted largely from the "Third Front" policy this region has, however, not been in the frontline of the urban economic reforms and export-oriented development strategy that were initiated in 1984.

However, the economic reforms have encouraged the establishment of a free market for trade of consumer goods. It has been noted that the enlarged choice opportunities have affected the consumption of free market goods versus goods from state stores even though major food items have been subsidized by the Chinese Government. The price differences are especially significant for items like coarse grain, fresh and dried vegetables, fruits and seafood. The higher prices in the free market are mainly due to better product quality but are to a certain extent also related to less waiting time.

## 4.2. The data set

This study is based on individual household data from the State Statistical Bureau's Urban Household Survey (UHS) for Sichuan province during the 1988-1991 period. A household is defined to include all persons living in the same dwelling and having common board. A particular attractive feature of the UHS is its continuity in recording the income and consumption data. Each household is keeping daily records of its cash income and its consumption quantities and expenditures for monthly collection by survey officials. The 1990-data have previously been applied by Aaberge et al. (1992) to examine the distributional structure of income and consumption in 1990, by Bjerkholt and Zhu (1993) to study the living conditions in 1990, and savings by Aaberge and Zhu (1998).

The data used in this paper is a four-year rotational monthly panel. Since 1988, the UHS has rotated by changing one third of the sample each year. Therefore the length of the panel ranges from 12 to 36 months for our sample.

In spite of the wide range of data available, we choose to focus on food in this study. Information on household monthly total expenditure and physical quantities, as well as the corresponding figures for purchases conducted on the so-called "free-market trade" are available for 39 foods items. The free market trade typically takes place in open air stalls on pavements or squares. Until late 1990, there was an overwhelming dominance of state-owned shops on the market for grain and edible vegetable oil, which were still heavily subsidised by the government. In the case of tobacco, liquor, tea and sugar, there was even a state monopoly. However, for the non-staple food items, free market trade clearly dominated. In this paper, we shall not attempt to distinguish between state-shop purchases and free market trade.

Data on six groups of food commodities were constructed out of the original data sources. These are as follows: (1) "Grain", comprising coarse grain, flour and rice; (2) "Fresh vegetables, melons and fruits"; (3) "Meats (comprising pork, beef and mutton), and fish"; (4) "Poultry and eggs"; (5) "Other foods"; (6) "Beverages, tobacco and tea". These expenditure categories accounted for approximately 50 per cent of total expenditure in this period.

According to a most reliable source of information inside the State Statistical Bureau, the tight price control on grain was released in the summer of 1991 in most part of China, including Sichuan province. Due to panic purchases and even speculations, the prices of grain went sky-rocketing before they stabilized towards the end of the year. Indeed, the mean unit value for grain was about 70 per cent higher in June than that of the previous month in our sample. In this paper, we do not attempt to study the impact of change of price regimes. Accordingly, observations of the last seven months will not be used.

**Table 1. Summary statistics based on monthly observations for the period January 1, 1988 - May 31, 1991. The number of observations equals 6969**

| Variable | Monthly mean | Standard deviation | Min | Max |
|---|---|---|---|---|
| Income | 440.01 | 223.70 | 0 | 4237.86 |
| Total expenditure | 348.73 | 187.84 | 43.38 | 4458.21 |
| Sum of expenditure (Group 1-6) | 181.88 | 76.37 | 20.09 | 743.28 |
| Budget shares (incl. zero expenditure): | | | | |
| Grain | 0.1111 | 0.0611 | 0.0011 | 0.5792 |
| Fresh vegetables, melons and fruits | 0.2047 | 0.0850 | 0.0034 | 0.6112 |
| Meats and fish | 0.2769 | 0.1115 | 0.0080 | 0.7903 |
| Poultry and eggs | 0.1364 | 0.0818 | 0.0032 | 0.5554 |
| Other food | 0.1298 | 0.0847 | 0.0008 | 0.7972 |
| Beverages, tobacco and tea | 0.1411 | 0.1095 | 0.0007 | 0.7485 |
| Expenditure (excl. zero expenditure): | | | | |
| Grain | 18.90 | 11.35 | 0.12 | 142.90 |
| Fresh vegetables, melons and fruits | 36.29 | 19.93 | 0.20 | 247.97 |
| Meats and fish | 51.25 | 32.63 | 0.78 | 359.96 |
| Poultry and eggs | 24.36 | 17.76 | 0.80 | 148.40 |
| Other food | 24.17 | 20.63 | 0.10 | 366.61 |
| Beverages, tobacco and tea | 26.91 | 28.74 | 0.12 | 330.49 |
| Unit values: | | | | |
| Grain | 0.6590 | 0.5870 | 0.2141 | 7.6600 |
| Fresh vegetables, melons and fruits | 0.7319 | 0.2773 | 0.0228 | 3.8121 |
| Meats and fish | 4.9325 | 0.8622 | 0.5538 | 23.3333 |
| Poultry and eggs | 5.5795 | 1.8026 | 0.4000 | 24.5455 |
| Other food | 3.4085 | 1.6551 | 0.2166 | 49.0000 |
| Beverages, tobacco and tea | 3.7457 | 5.4700 | 0.0806 | 97.5000 |

Note:
Total expenditure = Total living expenditure - expenditure on major consumer durables
Budget share = expenditure/total expenditure

## 4.3. Estimation strategies and empirical results for the linear expenditure model

We shall apply the framework developed above to estimate a demand model for selected types of food. We shall call the basic time unit, "purchase day". This is a suitable selected time unit so as to make the assumption of variants being perfect substitutes (within each type of good) realistic. Let $y_{ijtd}$ denote the budget expenditure on good j for household i in month t and purchase day d, and let $\hat{R}_{ijtd}$ be the corresponding virtual price. We assume that the taste-shifters are independent over time, which implies that the virtual prices are independent over time. Let $y_{itd}$ be the total consumption expenditure for household i in month t on all the six goods in month t, purchase day d.[6]

As a demand model we choose the *linear expenditure system* (LES) given by

[6] The symbols $T_{jtd}(z)$ and $b_{jt}$ which will appear below are defined analogously.

(4.1) $$y_{ijtd}=\left(\gamma_j^*+\eta_{ij}\right)\hat{R}_{ijtd}+\beta_j\left(y_{itd}-\sum_{k=1}^{n}\left(\gamma_k^*+\eta_{ik}\right)\hat{R}_{iktd}\right)$$

for $j=1,2,\ldots,n\,(n=6)$, where $\eta_{ij}$ are zero mean random effects that are constant over time and uncorrelated across goods, $\{\beta_j\}$ and $\{\gamma_j^*\}$ are unknown parameters with $\beta_k>0$ and

(4.2) $$\sum_{k=1}^{n}\beta_k=1.$$

From the fact that the virtual prices are Weibull distributed it follows readily (see for example Johnson and Kotz, 1972) that

(4.3) $$E\,\hat{R}_{ijtd}=\Gamma\left(1+\frac{1}{\alpha_j}\right)K_{jt}^{-1/\alpha_j}.$$

In this application we shall assume that the mean quality and the number of producers change slowly over time such that $b_{jt}E\left(T_{jtd}(z)^{\alpha_j}\right)$ can be considered constant. By (4.3) and Corollary 1 we therefore get

(4.4) $$E\,\hat{R}_{ijtd}=c_j\left(E\,\hat{P}_{ijtd}^{\alpha_j}\right)^{1/\alpha_j}$$

where $\hat{P}_{ijtd}$ is the unit value of good j in month t, purchase day d and

$$c_j=b_{jt}^{-1/\alpha_j}\,\Gamma\left(1+1/\alpha_j\right)\left(E\,T_{jt}(z)^{\alpha_j}\right)^{-1/\alpha_j}.$$

Unfortunately, the available microdata are monthly aggregates. The problem with "month" as time unit is that the assumption that the variants are (on average) perfect substitute may be unrealistic. However, the available data forces us to use month as time unit. In the application below we approximate the mean $E\,\hat{P}_{ijtd}^{\alpha_j}$ by the corresponding empirical mean, i.e.,

(4.5) $$E\,\hat{P}_{ijtd}^{\alpha_j}\cong\frac{1}{N_t}\sum_{i=1}^{N_t}\hat{P}_{ijt}^{\alpha_j}$$

where $N_t$ is the number of households in the sample with positive consumption on good j in month t and $\{\hat{P}_{ijt}\}$ are the "monthly" unit values.

Since the virtual prices are not directly observable we apply the relationships (4.4) and (4.5), which yield that

$$(4.6)\qquad \sum_{d=1}^{D} \hat{R}_{ijtd} = c_j D\left(\frac{1}{N_t}\sum_{i=1}^{N_t} \hat{P}_{ijt}^{\alpha_j}\right)^{1/\alpha_j} + v_{ijt}^*$$

where $v_{ijt}^*$ is a random variable with zero mean and D is the number of purchase days per month. When (4.6) is inserted into (4.1) we get the following model for the monthly aggregates;

$$(4.7)\qquad y_{ijt} = \gamma_j\left(\frac{1}{N_t}\sum_{i=1}^{N_t} \hat{P}_{ijt}^{\alpha_j}\right)^{1/\alpha_j} + \beta_j\left(y_{it} - \sum_{k=1}^{6} \gamma_k\left(\frac{1}{N_t}\sum_{i=1}^{N_t} \hat{P}_{ikt}^{\alpha_k}\right)^{1/\alpha_k}\right) + v_{ijt}$$

where $y_{ijt}$ is the expenditure of household i on good j in month t, $y_{it}$ is the total expenditure in month t, $\gamma_j = \gamma_j^* c_j D$, and $v_{ijt}$ is a zero mean random term that is uncorrelated with the regressors.

The total expenditure per month may be endogenous. Similarly to the linear extended expenditure system developed by Lluch (1973), cf. Deaton (1986), p. 1779, one may think of the consumption behavior within a month or a year, as a dynamic choice problem. As a result, $y_{it}$ will be determined by some kind of Euler conditions that would depend on the interest rate, the time discount factor, and possibly the virtual prices, or their expectations (subjective). Here, we shall simply ignore this problem.

In principle, we could now estimate $\{\alpha_j\}$, $\{\gamma_j\}$ and $\{\beta_j\}$ by nonlinear least squares. Unfortunately, the estimation procedure suggested above does not work well in our case. The estimates of $\{\alpha_j\}$ are either very imprecise or are negative. We shall therefore turn to another procedure that exploits the theoretical relationship between the means and the variances of the virtual prices. To this end let

$$(4.8)\qquad V_{ijrt} = \frac{y_{ijt}}{\beta_j} - \frac{y_{irt}}{\beta_r}$$

and $a_j = \gamma_j^* / \beta_j$ , $\tilde{\eta}_{ij} = \eta_{ij} / \beta_j$ . It follows from (4.1) and (4.8) that

$$(4.9)\qquad V_{ijrt} = \left(a_j + \tilde{\eta}_{ij}\right)\sum_{d=1}^{D} \hat{R}_{ijtd} - \left(a_r + \tilde{\eta}_{ir}\right)\sum_{d=1}^{D} \hat{R}_{irtd} .$$

From (4.9) it follows readily that for $r \neq j, k \neq j,$ and $k \neq r$;

(4.10) $\mathrm{Cov}\left(V_{ijrt}, V_{ijkt}\right) = \mathrm{Var}\left(\left(a_j + \tilde{\eta}_{ij}\right)\sum_{d=1}^{D} \hat{R}_{ijtd}\right) = \left(1+\tau_j^2\right) a_j^2 \, D \, \mathrm{Var}\, \hat{R}_{ijtd} + \tau_j^2 \, a_j^2 \, D^2 \left(E\, \hat{R}_{ijtd}\right)^2,$

where $\tau_j^2 \, a_j^2 = \mathrm{Var}\, \tilde{\eta}_{ij}$. Similarly, we get

$$\mathrm{Cov}\left(V_{ijrt}, V_{ijk,t-1}\right) = \tau_j^2 \, a_j^2 \, D^2 \, E\, \hat{R}_{ijtd} \, E\, \hat{R}_{ij,t-1,d} \, . \tag{4.11}$$

From the fact that the virtual prices are Weibull distributed one gets, similarly to (4.3), that

$$\mathrm{Var}\, \hat{R}_{ijtd} = \theta_j \left(E\, \hat{R}_{ijtd}\right)^2 \tag{4.12}$$

where

$$1+\theta_j = \frac{\Gamma\left(1+\frac{2}{\alpha_j}\right)}{\Gamma\left(1+\frac{1}{\alpha_j}\right)^2}. \tag{4.13}$$

When (4.12) is inserted into (4.10) we obtain that

$$\mathrm{Cov}\left(V_{ijrt}, V_{ijkt}\right) = m_j^{-2} \, a_j^2 \left(E\, \hat{R}_{ijtd}\right)^2 D^2 \tag{4.14}$$

where

$$m_j^{-2} = \frac{\theta_j \left(1+\tau_j^2\right)}{D} + \tau_j^2 \, . \tag{4.15}$$

Next, define

$$S_{jt}^2 = \frac{1}{(n-1)(n-2)} \sum_{k \neq j} \sum_{\substack{r \neq k \\ k \neq j}} \frac{1}{N_t} \sum_{i=1}^{N_t} \left(V_{ijrt} - \overline{V}_{jrt}\right)\left(V_{ijkt} - \overline{V}_{jkt}\right) \tag{4.16}$$

and

$$M_{jt} = \frac{1}{(n-1)(n-2)} \sum_{k \neq j} \sum_{\substack{r \neq k \\ k \neq j}} \frac{1}{N_t} \sum_{i=1}^{N_t} \left(V_{ijrt} - \overline{V}_{jrt}\right)\left(V_{ijk,t-1} - \overline{V}_{jk,t-1}\right) \tag{4.17}$$

where $\overline{V}_{jkt}$ denotes the mean of $V_{ijkt}$ across households. We realize from (4.11), (4.14), (4.16) and (4.17) that

$$\text{(4.18)} \qquad E\,S_{jt}^2 = \operatorname{Cov}\left(V_{ijrt}, V_{ijkt}\right) = m_j^{-2}\, a_j^2\, D^2 \left(E\,\hat{R}_{ijtd}\right)^2$$

and

$$\text{(4.19)} \qquad E\,M_{jt} = \operatorname{Cov}\left(V_{ijrt}, V_{ijk,t-1}\right) = \tau_j^2\, a_j^2\, D^2\, E\,\hat{R}_{ijtd}\, E\,\hat{R}_{ij\,t-1,d}\,.$$

Due to (4.18) and (4.19) we have that

$$\text{(4.20)} \qquad \left|a_j\right| D\,E\,\hat{R}_{ijtd} \approx m_j\, S_{jt}$$

and

$$\text{(4.21)} \qquad \tau_j^2 \approx \frac{M_{jt}}{m_j^2\, S_{jt}\, S_{j,t-1}}\,.$$

Furthermore, when (4.15) and (4.21) are combined we obtain an estimator $\theta_j^*$ for $\theta_j$, given by

$$\text{(4.22)} \qquad \theta_j^* = \frac{D \sum_{t\geq 2} \left(S_{jt}\, S_{j,t-1} - M_{jt}\right)}{\sum_{t\geq 2} \left(m_j^2\, S_{jt}\, S_{j,t-1} + M_{jt}\right)}$$

provided $m_j$ and $\left\{\beta_j\right\}$ are known. Finally, when we insert (4.20) into (4.1) and sum over purchase days we get

$$\text{(4.23)} \qquad y_{ijt} = \beta_j\, m_j\, S_{jt} + \beta_j \left( y_{it} - \sum_{k=1}^{n} \beta_k\, m_k\, S_{kt} \right) + u_{ijt}$$

where $u_{ijt}$ is a random term with the property that $E\,u_{ijt} \approx 0$ when the number of households is large. We can now apply the results above to estimate the model in four stages.

*Stage one*: Estimate $\left\{\beta_j\right\}$ by OLS from the Engel functions,

$$y_{ijt} = s_{jt} + \beta_j\, y_{it} + \varepsilon_{ijt}$$

where $\left\{\varepsilon_{ijt}\right\}$ are random terms.

*Stage two*: Use the estimates $\{\hat{\beta}_j\}$ from stage one to calculate estimates $\hat{S}_{jt}$ and $\hat{M}_{jt}$ for $S_{jt}$ and $M_{jt}$ given by (4.16) and (4.17). Estimate $\{m_j\}$ by maximum likelihood or GLS by means of

(4.24) $$\bar{y}_{jt} - \hat{\beta}_j \bar{y}_t = m_j \hat{\beta}_j \hat{S}_{jt} - \hat{\beta}_j \sum_{k=1}^{n} m_k \hat{\beta}_k \hat{S}_{kt} + \bar{u}_{jt}$$

with $\hat{\beta}_j \hat{S}_{jt}$, $j = 1,2,\dots,n$, $t = 1,2,\dots,$ as explanatory variables, where $\bar{y}_{jt}$, $\bar{y}_t$ and $\bar{u}_{jt}$ are the respective population means of $\{y_{ijt}\}$, $\{y_{it}\}$ and $\{u_{ijt}\}$.

*Stage three*: Use the estimates $\{\hat{m}_j\}$ obtained from stage two to calculate $\{\hat{\theta}_j\}$ using (4.22). Then use (4.13) to obtain estimates for $\{\alpha_j\}$. In the application below we have chosen $D = 8$ which corresponds to about two purchase days per week.

*Stage four*: Insert the estimates of $\{\alpha_j\}$ into (4.27) and estimate.

In the estimation we do not use observations with zero expenditure on any of the six commodities.

A closer look at the data reveals that there are, apart from “Grain”, substantial seasonal variations in consumption. For “Meats and fish” there is a peak consumption in December and January, for “Poultry and eggs” the peak consumptions occurs in February. For “Other goods” peak consumption occurs in January and February, and for “Beverages, tobacco and tea” it occurs in February. For “Fresh vegetables, melons and fruit”, there is a peak consumption in July and August. Except for the last group, and “Grain” the peak consumption of the other commodities is related to celebration of the new year. To account for these seasonal effect in preferences we have allowed the parameters $\{m_j\}$ and $\{\gamma_j\}$ to depend on time as follows:

(4.25) $$m_{jt} = m_{0j}\left(1 - \delta_{jt}\right) + \delta_{jt}\, m_{1j}$$

and

(4.26) $$\gamma_{jt} = \gamma_{0j}\,(1 - \delta_{jt}) + \delta_{jt}\,\gamma_{1j}$$

where $\{\delta_{jt}\}$ is equal to one in the respective months the consumption is at a peak for commodity j, and zero otherwise. For the reader's convenience we restate the final aggregate empirical model below:

(4.27) $$\bar{y}_{jt} = \gamma_{jt}\left(\frac{1}{N_t}\sum_{i=1}^{N_t} \hat{P}_{ijt}^{\alpha_j}\right)^{1/\alpha_j} + \beta_j\left(\bar{y}_t - \sum_{k=1}^{6} \gamma_{kt}\left(\frac{1}{N_t}\sum_{i=1}^{N_t} \hat{P}_{ikt}^{\alpha_k}\right)^{1/\alpha_k}\right) + \bar{v}_{jt}\,.$$

We have estimated $\{m_{0j}, m_{1j}\}$ from (4.24) and $\{\gamma_{0j}, \gamma_{1j}\}$ from (4.27) by the method of maximum likelihood, based on additive multinormally distributed disturbances. The structure of the disturbances in (4.1) implies a particular structure of the error terms in (4.24) and (4.27). However, for simplicity we have not taken the theoretical structure of the error terms explicitely into account.

**Table 2. Parameter estimates of the modified Linear Expenditure System***

| | Parameters | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | Goodness of fit ($R^2$) | |
| Type of good | $\beta$ | $\gamma_{0j}$ | $\gamma_{1j}$ | $\alpha$ | Auto-correlation | Aggregate model | Household model |
| Grain | 0.061<br>(17.8) | 18.43<br>(12.2) | 18.43<br>(12.2) | 0.39 | | 0.22 | 0.15 |
| Fresh vegetables, melons and fruit | 0.156<br>(42.7) | 21.00<br>(5.7) | 37.35<br>(8.4) | 0.50 | | 0.88 | 0.45 |
| Meats and fish | 0.300<br>(87.3) | 1.73<br>(1.5) | 7.77<br>(8.6) | 0.30 | 0.13<br>(1.4) | 0.88 | 0.57 |
| Poultry and eggs | 0.115<br>(32.0) | 1.65<br>(4.9) | 2.76<br>(6.3) | 0.36 | | 0.75 | 0.26 |
| Other food | 0.153<br>(42.6) | 0.87<br>(0.7) | 3.39<br>(3.6) | 0.33 | | 0.82 | 0.33 |
| Beverages, tobacco and tea | 0.215 | -0.55<br>(1.3) | 2.17<br>(1.9) | 0.59 | | 0.79 | 0.34 |

* The t-values are given in parenthesis. The t-values given here do not account for statistical errors in the $\alpha$-estimates. Recall also that by assumption, $\gamma_{01} = \gamma_{11}$. The loglikelihood is equal to -450.6.

We have also imposed an AR(1) serial correlation structure with a common autocorrelation coefficient. As demonstrated by Berndt and Savin (1975), the common autocorrelation parameter is implied by the "adding up" restrictions of a demand system.

From Table 2 we notice that the $\alpha$-estimate is highest for "Beverages, tobacco and tea". The group "Fresh vegetables, melons and fruit" has the second largest $\alpha$-estimate. The group "Meats and fish" has the lowest $\alpha$-estimate and thus is the group with highest population heterogeneity in tastes.

From the estimates of $\gamma_{1j}$ we realize that there are very strong seasonal effects. It is also noteworthy that the minimum levels of consumption are substantially higher for the groups "Grain", and "Fresh vegetables, melons and fruit", than for the remaining groups. For "Beverages, tobacco and tea", the off-season $\gamma$-value is slightly negative (although not significantly different from zero). Recall that a negative $\gamma$-value would imply that a corner solution is possible. Note that the t-values for $\{\hat{\gamma}_{0j}\}$ and $\{\hat{\gamma}_{1j}\}$ given in Table 2 may overestimate the true values because the errors in the estimates of $\{\alpha_j\}$ and $\{\beta_j\}$ are not accounted for. In Table 2 we have reported two goodness of fit measures ($R^2$). The first one corresponds to the aggregate model (4.27). The second $R^2$ corresponds to the underlying micromodel given in (4.7). More detailed estimation results are reported in the appendix. From Figures 1 to 6 in the Appendix we see how well $\{S_{jt}\}$, which by (4.20) can be interpreted as a price index, predicts the fluctuations in mean virtual prices.

To check how sensitive the estimates of $\{\gamma_{0j}, \gamma_{1j}\}$ are with respect to variations in the $\alpha_j$-parameters we have estimated the model with $\alpha_j = 0.5$ for all j. The results are given in Table 3. By comparing Table 2 and Table 3 we realize that the parameter estimates are practically the same in these two cases and also the goodness of fit measure is equal for the two specifications.

**Table 3. Parameter estimates of the modified Linear Expenditure System***

| Type of good | Parameters | | | | | |
|---|---|---|---|---|---|---|
| | $\beta$ | $\gamma_{0j}$ | $\gamma_{1j}$ | $\alpha$ | Auto-correlation | Goodness of fit ($R^2$) Aggregate model |
| Grain | 0.061<br>(17.8) | 18.82<br>(12.3) | 18.82<br>(12.3) | 0.50 | | 0.24 |
| Fresh vegetables, melons and fruit | 0.156<br>(42.7) | 21.05<br>(5.8) | 37.43<br>(4.4) | 0.50 | | 0.88 |
| Meats and fish | 0.300<br>(87.3) | 1.76<br>(1.5) | 7.83<br>(8.6) | 0.50 | 0.12<br>(0.9) | 0.88 |
| Poultry and eggs | 0.115<br>(32.0) | 1.66<br>(5.0) | 2.78<br>(6.4) | 0.50 | | 0.75 |
| Other food | 0.153<br>(42.6) | 0.90<br>(1.2) | 3.46<br>(3.6) | 0.50 | | 0.82 |
| Beverages, tobacco and tea | 0.215 | -0.51<br>(-0.4) | 2.10<br>(2.0) | 0.50 | | |

* The t-values are given in parenthesis. The t-values given here do not account for statistical errors in the $\alpha$-estimates. Recall also that by assumption, $\gamma_{01} = \gamma_{11}$. Loglikelihood is equal to -449.8.

As discussed above we can apply these results to carry out policy simulations when the list prices are increased—or decreased—by a common scale transform. We can, however, not change the "shape" of the price distribution, $g_j(p)$, because we are unable to assess the implications for the price index, $E\hat{R}_j$, due to the fact that $\kappa_j$ is unknown.

## 5. Conclusions

In this paper we have discussed a particular modelling approach which explicitly accounts for the fact that consumer goods differ by quality and location, and that these characteristics are usually unobservable to the analyst. Specifically, we have developed an empirical model for consumer expenditure that can be viewed as a modified Linear Expenditure System. We have also developed a particular estimation procedure for this model. The empirical application is based on a microdata from the Sichuan province in China. For this particular application the estimation procedure works quite well. However, due to the fact that we do not have access to data on market prices, we are unable to estimate key structural parameters of the model. The implication of this is that we can only perform policy simulation experiments in which the prices within the respective commodity groups are subject to scale transformations.

# References

Aaberge, R., X. Chen, J. Li and X. Li (1992): The Structure of Economic Inequality among Households Living in Urban Sichuan and Liaoning, 1990, Discussion Papers, No. 70, Statistics Norway.

Aaberge, R. and Y. Zhu (1998): The Pattern of Household Savings During Hyperinflation. Discussion Papers no. 217, Statistics Norway.

Aczél, J. (1996): *Lectures on Functional Equations and their Applications*. Academic Press, New York.

Anderson, R.W. (1979): Perfect price aggregation and empirical demand anaysis. *Econometrica*, **47**, 1209-1230.

Anderson, S.P., A. de Palma and J.F. Thisse (1992): *Discrete Choice Theory of Product Differentiation*. MIT Press, Cambridge.

Bartik, T.J. (1987): The Estimation of Demand Parameters in Hedonic Price Models. *Journal of Political Economy*, **95**, 81-88.

Berndt, E.R. and N.E. Savin (1975): Estimation and Hypothesis Testing in Singular Equation System with Autoregressive Disturbances. *Econometrica*, **43**, 937-957.

Bjerkholt, O. and Y. Zhu (1993): Living Conditions of Urban Chinese Households around 1990. Discussion Papers, No. 87, Statistics Norway.

Burdett, K. and K.L. Judd (1983): Equilibrium Price Dispersion. *Econometrica*, **51**, 955-969.

Chamberlin, E. (1933): *The Theory of Monopolistic Competition*. Harvard University Press, Cambridge.

Dagsvik, J.K. (1994): Discrete and Continuous Choice, Max-stable Processes and Independence from Irrelevant Attributes. *Econometrica*, **62**, 1179-1205.

Deaton, A.S. (1986): Demand Analysis. In Z. Griliches and M.D. Intriligator (eds.), *Handbook of Econometrics, Vol. III*, North-Holland, New York.

Deaton, A.S. (1987): Estimation of Own- and Cross-Price Elasticities from Household Survey Data. *Journal of Econometrics*, **36**, 7-30.

Deaton, A.S. (1988): Quality, Quantity, and Spatial Variation of Price. *American Economic Review*, **78**, 418-430.

Epple, D. (1987): Hedonic Prices and Implicit Markets: Estimating Demand and Supply Functions for Differentiated Products. *Journal of Political Economy*, **95**, 59-80.

Johnson, N.L. and S. Kotz (1970): *Continuous Univariate Distributions*. Houghton Mifflin Company, Boston.

Lancaster, K.J. (1966): A New Approach to Consumer Theory. *Journal of Political Economy*, **74**, 132-157.

Lancaster, K.J. (1979): *Variety, Equity and Efficiency*. Columbia University Press, New York.

Lluch, C. (1973): The Extended Linear Expenditure System. *European Economic Review*, **4**, 21-32.

Luce, R.D. (1977): The Choice Axiom after Twenty Years. *Journal of Mathematical Psychology*, **15**, 215-233.

Nerlove, M. (1995): Hedonic Price Functions and the Measurement of Preferences: The Case of Swedish Wine Consumers. *European Economic Review*, **39**, 1697-1716.

Rosen, S. (1974): Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition. *Journal of Political Economy*, **82**, 35-55.

Strauss, D. (1979): Some Results on Random Utility Models. *Journal of Mathematical Psychology*, **20**, 35-52.

Trajtenberg, M. (1990): Product Innovations, Price Indices, and (Miss) Measurement of Economic Performance. Working Paper, No. 3261, NBER, Cambridge, MA.

## Appendix A

**Proof of Theorem 1**

Recall that $\left(\hat{P}_j, \hat{T}_j\right)$ is defined by $\hat{P}_j = P_j\left(\hat{z}_j\right)$, $\hat{T}_j = T_j\left(\hat{z}_j\right)$, where

(A.1) $$\hat{z}_j = \arg\max_z \left( \alpha_j \log\left(\frac{T_j(z)}{P_j(z)}\right) + \xi_j^*(z) \right)$$

and $\xi_j^*(z) = \alpha_j \log \xi_j(z)$. Let A be a Borel set in $R_+^2$, $\overline{A}$ the complement of A, and define

(A.2) $$L_j(A) = \max_{z \atop \left(P_j(z), T_j(z)\right) \in A} \left( \alpha_j \log\left(\frac{T_j(z)}{P_j(z)}\right) + \xi_j^*(z) \right)$$

and

(A.3) $$L_j(\overline{A}) = \max_{z \atop \left(P_j(z), T_j(z)\right) \notin A} \left( \alpha_j \log\left(\frac{T_j(z)}{P_j(z)}\right) + \xi_j^*(z) \right).$$

It follows that $L_j(A)$ and $L_j(\overline{A})$ are stochastically independent because maximum is taken over Poisson points in disjoint sets, A and $\overline{A}$. Moreover, it can be demonstrated that $L_j(A)$ and $L_j(\overline{A})$ are extreme value distributed (type III), i.e.,

(A.4) $$P\left(L_j(C) \le x\right) = \exp\left(-D_j(C)\, e^{-x}\right)$$

for C equal to A and $\overline{A}$, respectively, where

(A.5) $$D_j(C) = b_j \iint_{(x,y) \in C} \left(\frac{y}{x}\right)^{\alpha_j} g_j(x,y)\, dx\, dy.$$

(See Dagsvik (1994), for a proof of this.) We have

(A.6) $$\begin{aligned} &P\left(\left(\hat{P}_j, \hat{T}_j\right) \in A, \hat{R}_j > r\right) \\ &= P\left(L_j(A) > L_j\left(\overline{A}\right), \max\left(L_j(A), L_j\left(\overline{A}\right)\right) < r^{-\alpha_j}\right) \\ &= P\left(L_j(A) > L_j\left(\overline{A}\right), L_j(A) < r^{-\alpha_j}\right). \end{aligned}$$

Now by straight forward calculus, (A.4) and (A.6) yield

$$
\begin{aligned}
&P\left(\left(\hat{P}_j, \hat{T}_j\right) \in A, \hat{R}_j > r\right) \\
&= \frac{D_j(A)}{D_j(A) + D_j\left(\overline{A}\right)} \exp\left(-r^{-\alpha_j}\left(D_j(A) + D_j\left(\overline{A}\right)\right)\right) \\
&= P\left(L_j(A) > L_j\left(\overline{A}\right)\right) P\left(\max\left(L_j(A), L_j\left(\overline{A}\right)\right) < r^{-\alpha_j}\right) \\
&= P\left(\left(\hat{P}_j, \hat{T}_j\right) \in A\right) P\left(\hat{R}_j > r\right),
\end{aligned} \tag{A.7}
$$

which proves that $\hat{R}_j$ and $\left(\hat{P}_j, \hat{T}_j\right)$ are independent. Moreover, (A.7) also implies that

$$
P\left(\hat{R}_j > r\right) = \exp\left(-r^{-\alpha_j}\left(D_j(A) + D_j\left(\overline{A}\right)\right)\right). \tag{A.8}
$$

Since

$$
D_j(A) + D_j\left(\overline{A}\right) = D_j\left(A \cup \overline{A}\right) = D_j\left(R_+^2\right) = K_j,
$$

the proof of Theorem 1 is complete.

Q.E.D.

**Proof of Corollary 1**

From Theorem 1 and the continuous analogue of (3.5) it follows that

$$
K_j \hat{g}_j(p,t) = b_j \left(\frac{t}{p}\right)^{\alpha_j} g_j(p,t) \tag{A.9}
$$

which is equivalent to

$$
K_j p^{\alpha_j} \hat{g}_j(p,t) = b_j t^{\alpha_j} g_j(p,t). \tag{A.10}
$$

Since

$$
E \hat{P}_j^{\alpha_j} = \iint p^{\alpha_j} \hat{g}_j(p,t)\, dt
$$

and

$$
E T_j(z)^{\alpha_j} = \iint t^{\alpha_j} g_j(p,t)\, dp\, dt,
$$

(A.10) implies that

(A.11) $$K_j \, E \, \hat{P}_j^{\alpha_j} = b_j \, E \, T_j^{\alpha_j} \, .$$

Q.E.D.

**Proof of Theorem 2**

Let $\hat{g}_{j\theta}(p)$ denote the density of unit values within commodity group j after the prices have been multiplied by a common positive scale θ. The corresponding density of prices equals $g_j\left(p/\theta\right)/\theta$. Hence, by (3.11)

(A.12) $$\hat{g}_{j\theta}(p) = \frac{p^{-\alpha_j} \lambda_j(p) g_j\left(p\theta^{-1}\right)}{\int_0^\infty x^{-\alpha_j} \lambda_j(x) g_j\left(x\theta^{-1}\right) dx} .$$

By change of variable in the integral in the denominator of (A.12) we get

(A.13) $$\hat{g}_{j\theta}(p\theta) = \frac{p^{-\alpha_j} \lambda_j(\theta p) g_j(p)}{\theta \int_0^\infty x^{-\alpha_j} \lambda_j(\theta x) g_j(x) dx} .$$

Under Assumption A3 it follows readily that

$$\hat{g}_{j\theta}(p) = \hat{g}_{j1}\left(p\theta^{-1}\right)\theta^{-1}$$

which implies that for all $p \in (0, \infty)$

(A.14) $$\frac{\hat{g}_{j\theta}(p\theta)}{\hat{g}_{j\theta}(\theta)} = \frac{\hat{g}_{j1}(p)}{\hat{g}_{j1}(1)} .$$

When (A.13) and (A.14) are combined we obtain that

(A.15) $$\frac{\lambda_j(\theta p)}{\lambda_j(\theta)} = \frac{\lambda_j(p)}{\lambda_j(1)} .$$

Let $f_j(p) = \lambda_j(p)/\lambda_j(1)$. Then (A.15) yields

(A.16) $$f_j(\theta p) = f_j(p) \, f_j(\theta) .$$

Eq. (A.16) is a Cauchy type functional equation which only continuous solution is $f_j(p) = p^{\beta_j}$ . Hence $\lambda_j(p) = a_j\, p^{\beta_j}$ , where $a_j > 0$ and $\beta_j$ are constants.

Q.E.D.

## Appendix B

**Table B1. Intermediate estimation results**

| Type of good | Parameters | | | |
|---|---|---|---|---|
| | Estimates* | | θ | |
| | $m_{0j}$ | $m_{1j}$ | Estimates | Goodness of fit ($R^2$) |
| Grain | 1.18<br>(22.3) | 1.18<br>(22.3) | 11.19 | 0.47 |
| Fresh vegetables, melons and fruit | 1.25<br>(14.8) | 1.52<br>(17.2) | 5.05 | 0.80 |
| Meats and fish | 0.72<br>(4.6) | 1.12<br>(16.1) | 29.17 | 0.95 |
| Poultry and eggs | 0.70<br>(10.8) | 0.95<br>(6.9) | 14.67 | 0.88 |
| Other food | 0.39<br>(5.3) | 0.60<br>(6.3) | 19.07 | 0.78 |
| Beverages, tobacco and tea | 0.08<br>(1.2) | 0.35<br>(3.5) | 3.27 | |

* t-values are given in parenteses.

The estimation results given in Table B1 are based on (4.24) with AR(1) error terms (with a common autocorrelation parameter). The estimated autocorrelation was found to be negligible.

**Figure 1.Plot of $S_{1t}$ and $\left(E\hat{P}_{i1t}^{\alpha_1}\right)^{1/\alpha_1}$**

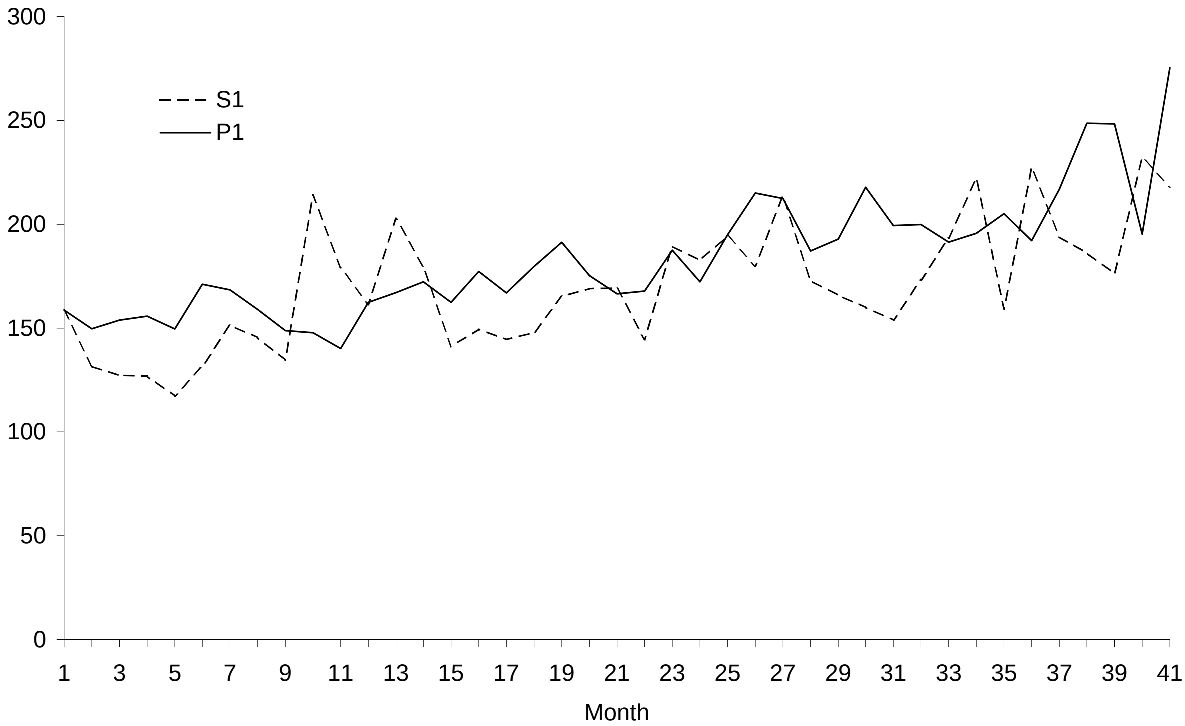


**Figure 2.Plot of $S_{2t}$ and $\left(E\hat{P}_{i2t}^{\alpha_2}\right)^{1/\alpha_2}$**

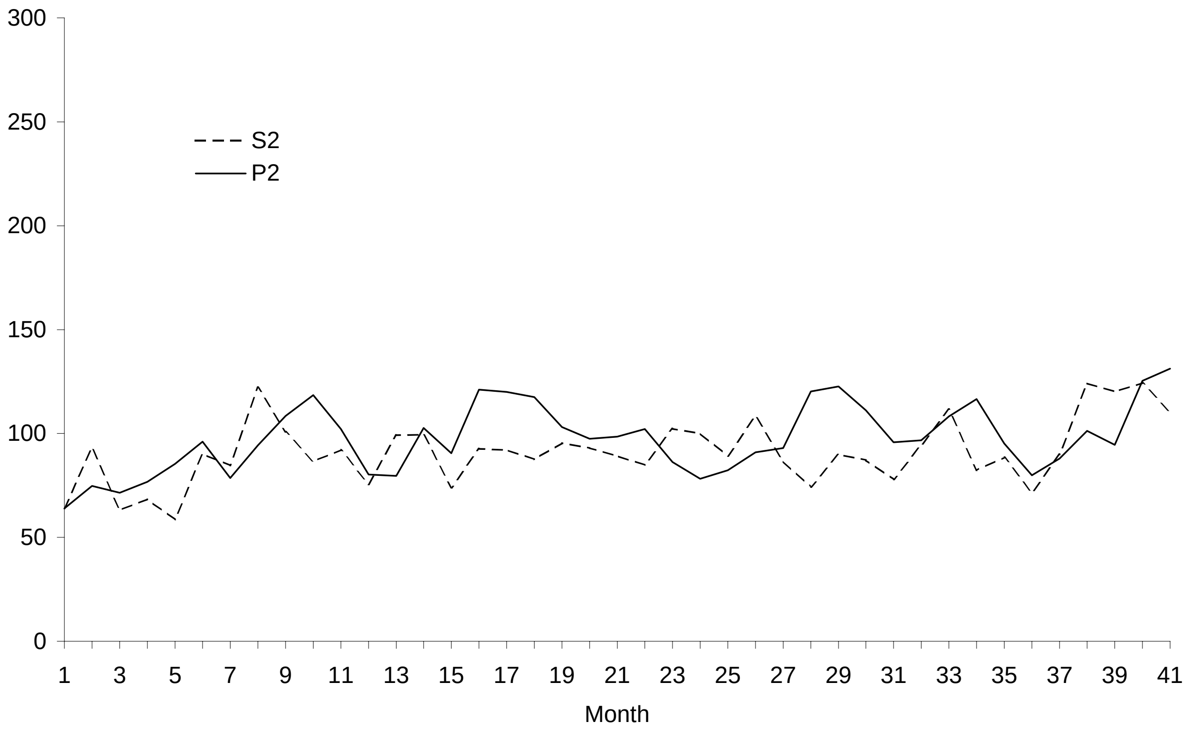

**Figure 3.Plot of $\mathbf{S_{3t}}$ and $\left(\mathbf{E\hat{P}_{i3t}^{\alpha_3}}\right)^{1/\alpha_3}$**

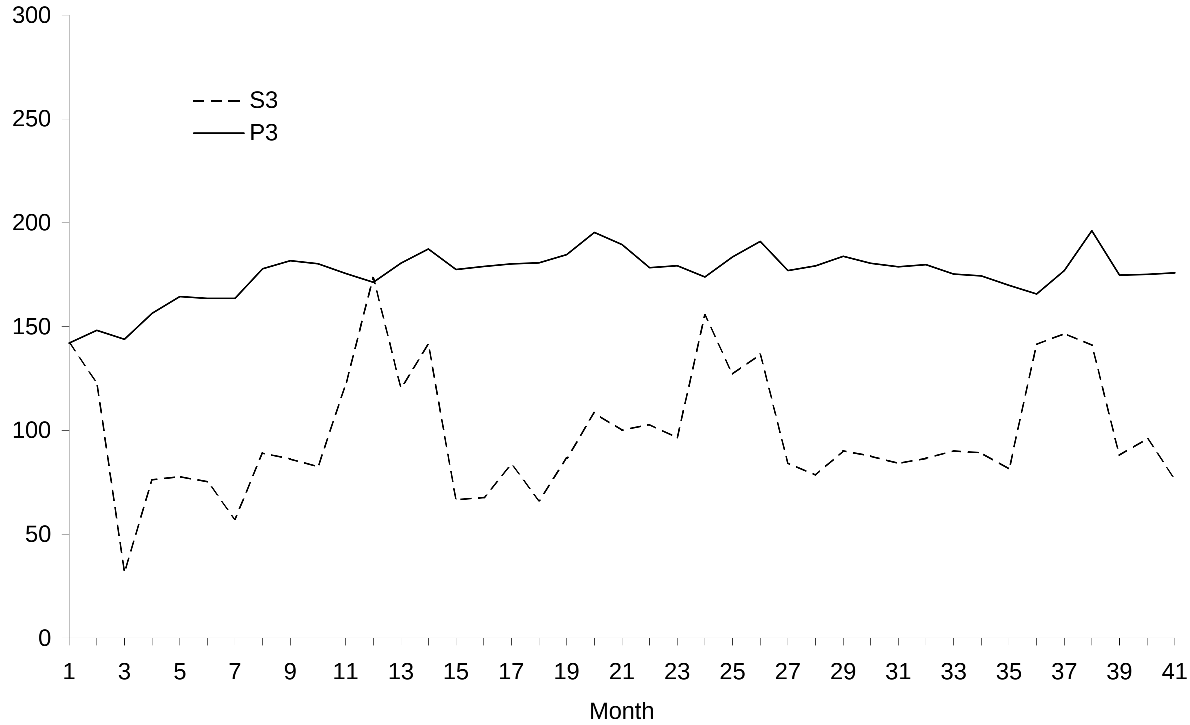


**Figure 4.Plot of $\mathbf{S_{4t}}$ and $\left(\mathbf{E\hat{P}_{i4t}^{\alpha_4}}\right)^{1/\alpha_4}$**

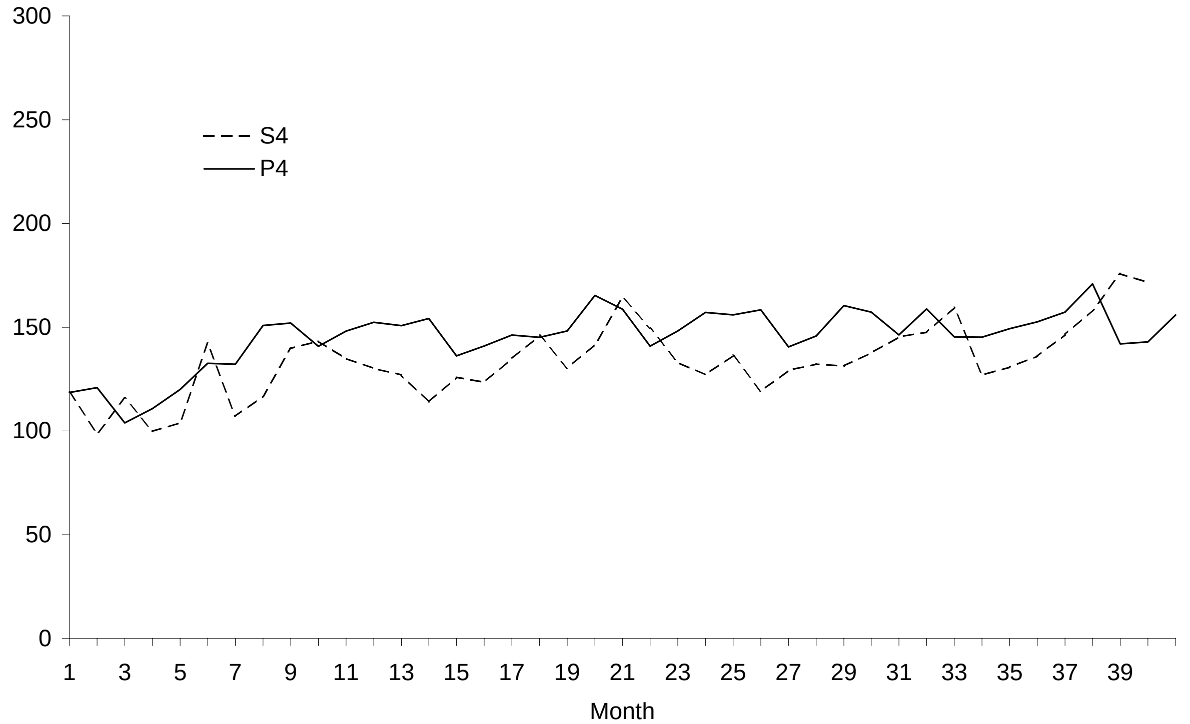

**Figure 5.Plot of $S_{5t}$ and $\left(E\hat{P}_{i5t}^{\alpha_5}\right)^{1/\alpha_5}$**

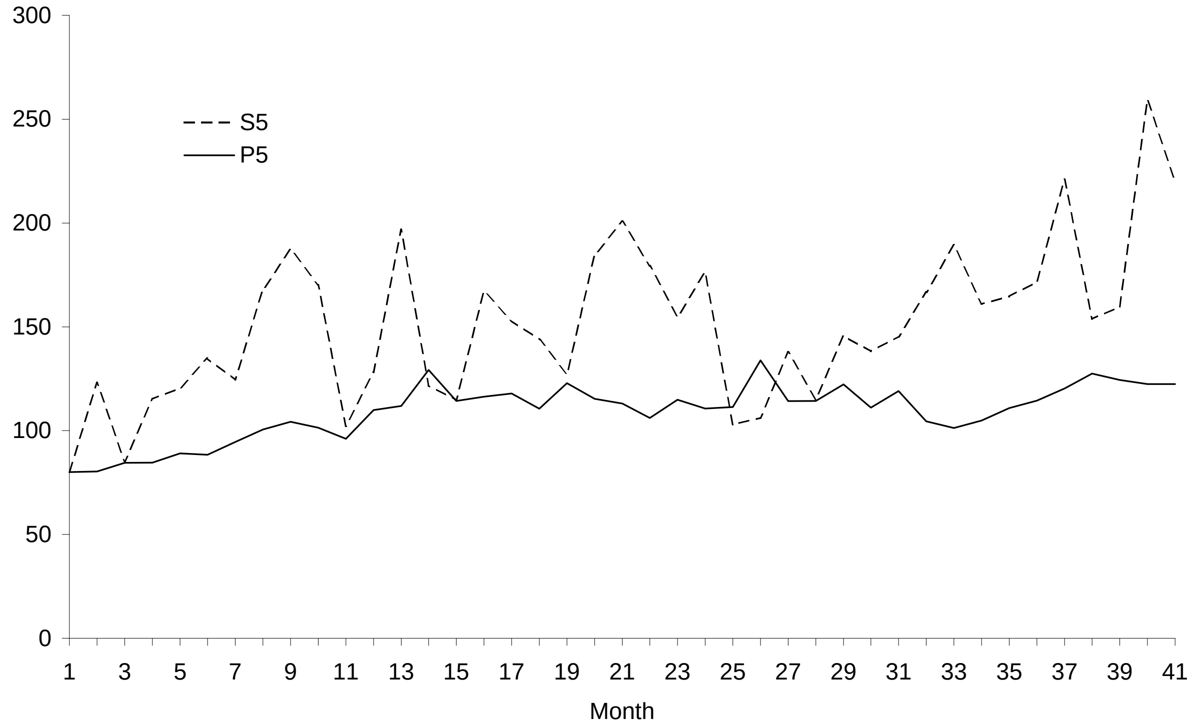


**Figure 6.Plot of $S_{6t}$ and $\left(E\hat{P}_{i6t}^{\alpha_6}\right)^{1/\alpha_6}$**

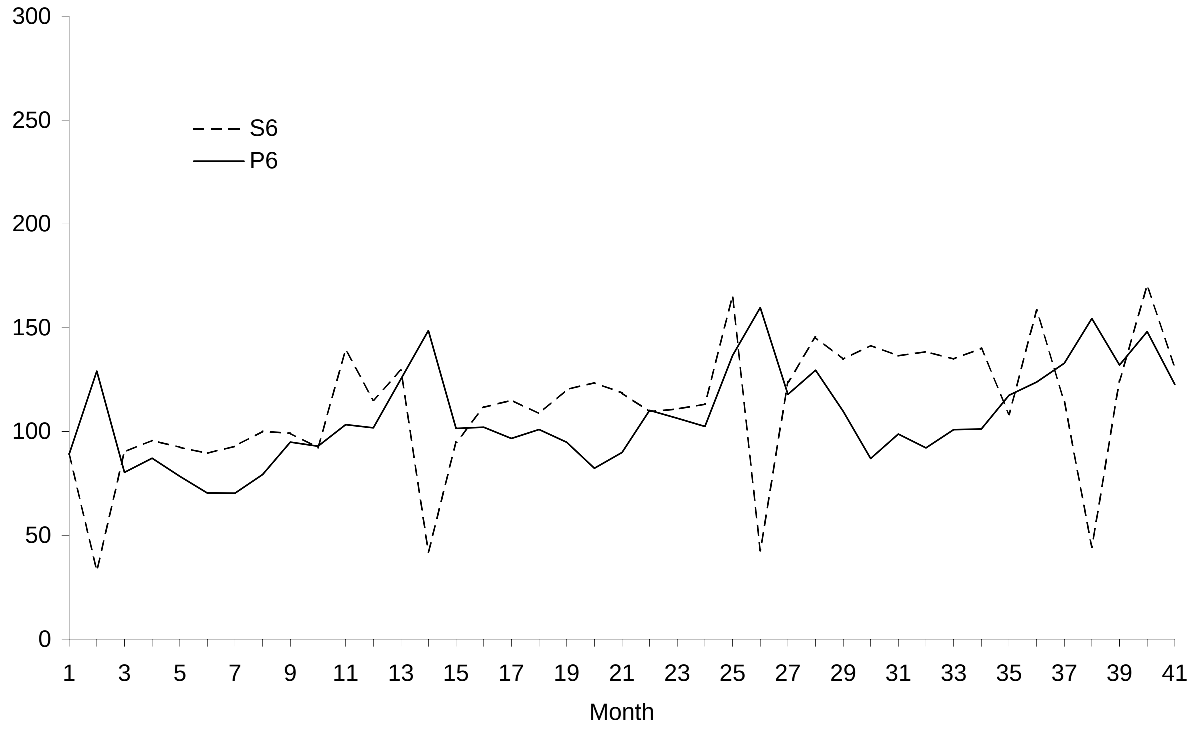

**Figure 7.Plot of $\overline{\mathbf{y}}_{1t}$ and $\left(\mathbf{E}\hat{\mathbf{P}}_{i1t}^{\alpha_1}\right)^{1/\alpha_1}$**

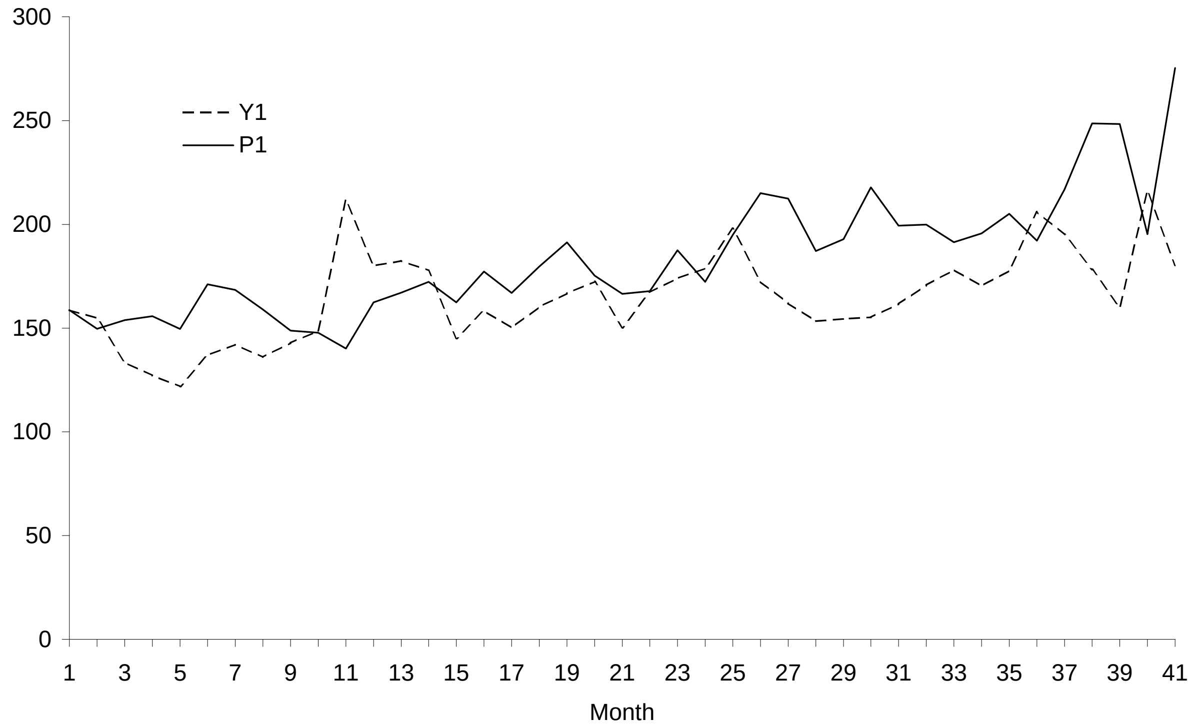


**Figure 8.Plot of $\overline{\mathbf{y}}_{2t}$ and $\left(\mathbf{E}\hat{\mathbf{P}}_{i2t}^{\alpha_2}\right)^{1/\alpha_2}$**

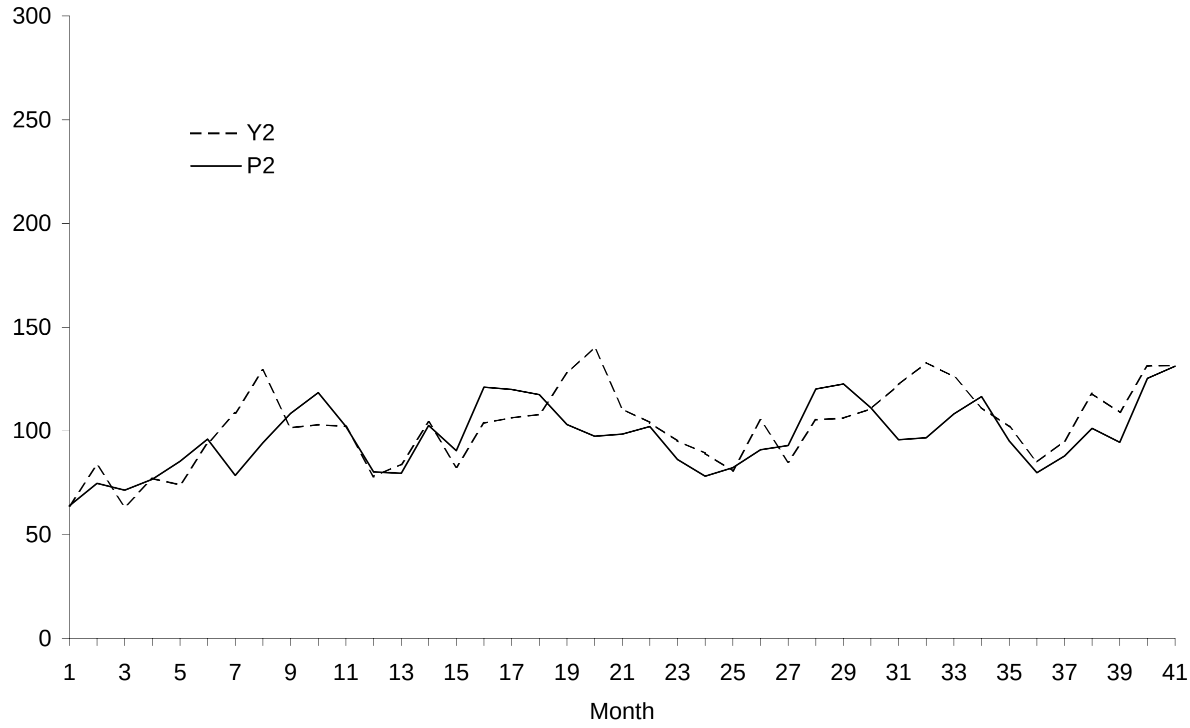

**Figure 9.Plot of $\bar{y}_{3t}$ and $\left(E\hat{P}_{i3t}^{\alpha_3}\right)^{1/\alpha_3}$**

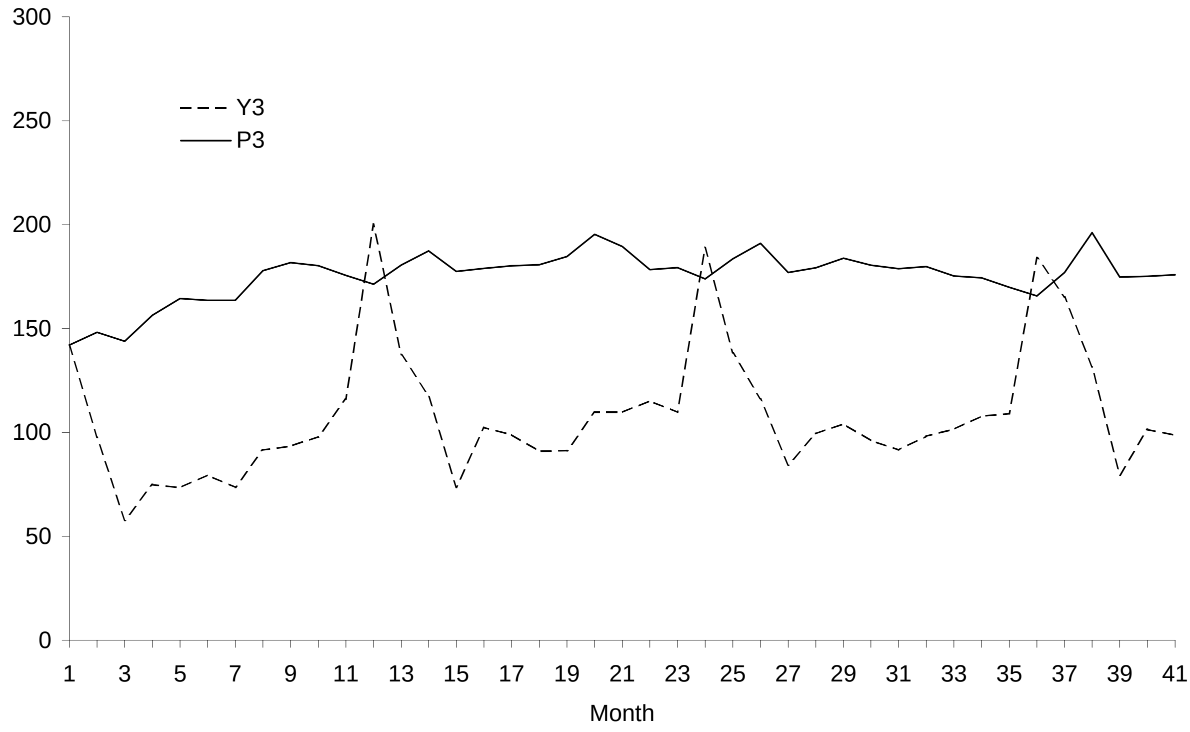


**Figure 10. Plot of $\bar{y}_{4t}$ and $\left(E\hat{P}_{i4t}^{\alpha_4}\right)^{1/\alpha_4}$**

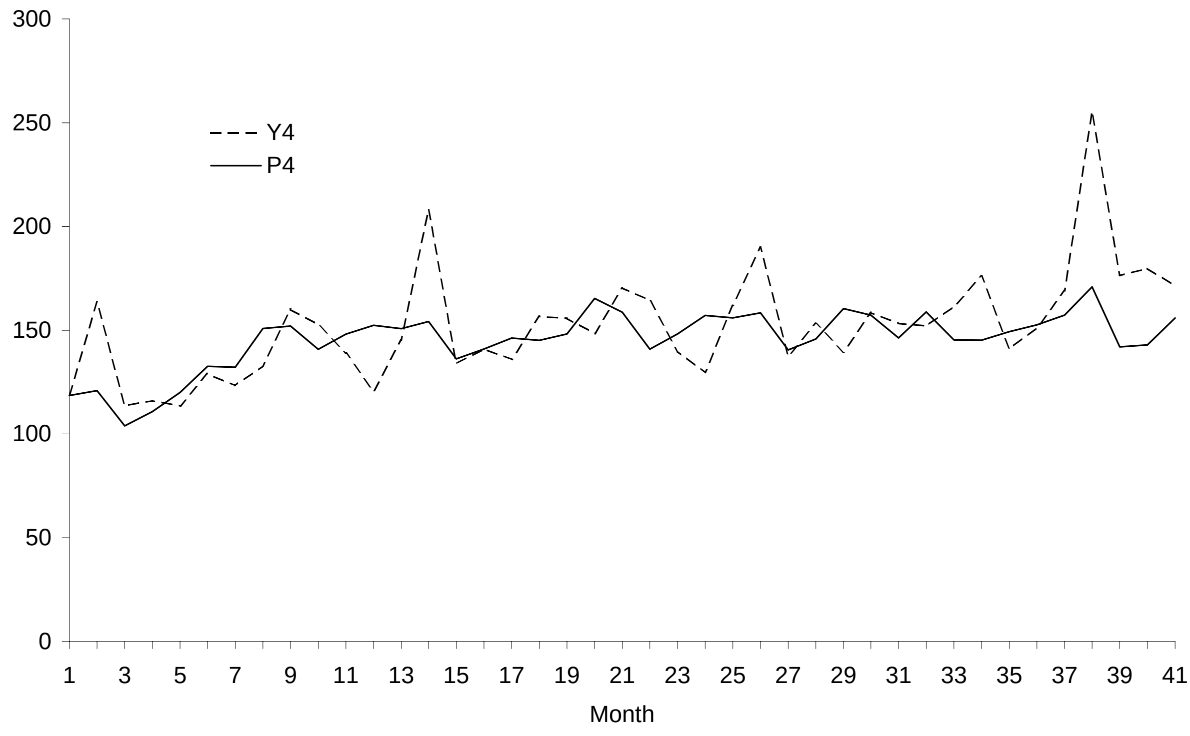

**Figure 11. Plot of $\overline{y}_{5t}$ and $\left(E\hat{P}_{i5t}^{\alpha_5}\right)^{1/\alpha_5}$**

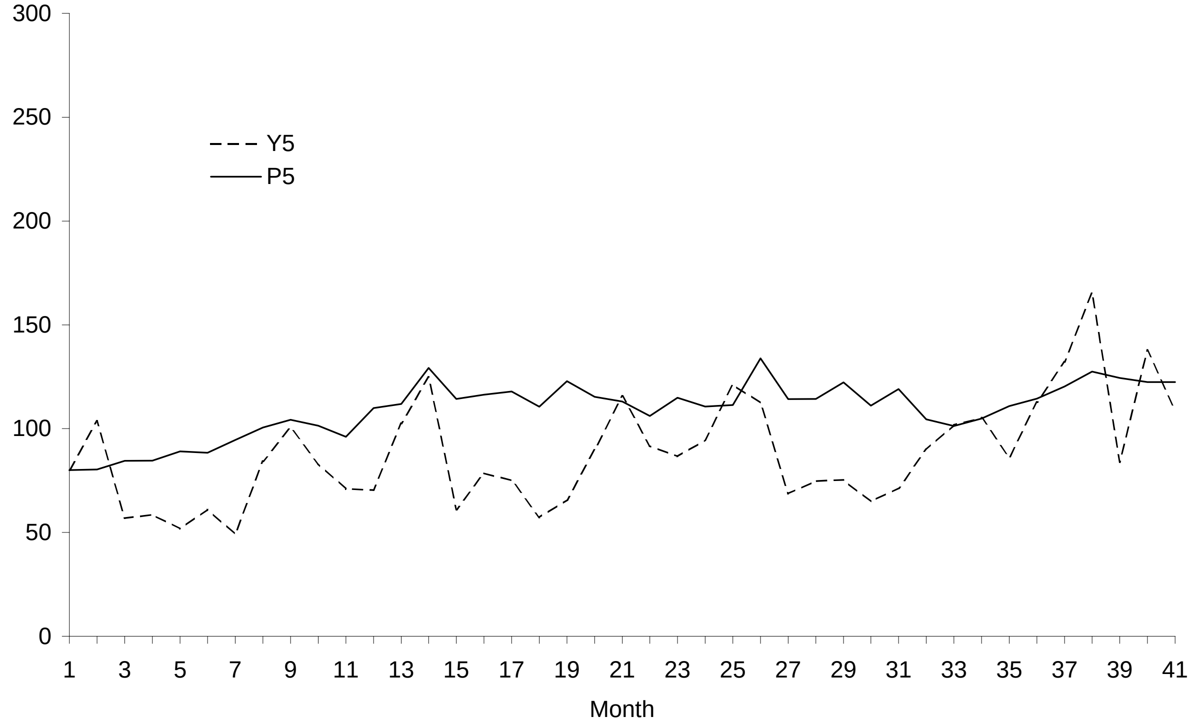


**Figure 12. Plot of $\overline{y}_{6t}$ and $\left(E\hat{P}_{i6t}^{\alpha_6}\right)^{1/\alpha_6}$**