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Networking Strategy: Cooperate Today in Order to Meet a Cooperator Tomorrow

Abstract:
This paper explains why people might cooperate playing an infinitely recurring prisoners' dilemma in which they change their partner in every period. After two people have played the prisoners' dilemma there is a possibility for networking. Then, two cooperators who played against each other exchange information about cooperators that they have met in previous periods. In the next period, they use this information in order to increase their probability of matching with a cooperator. Whether it is payoff maximizing to cooperate, given that all cooperators network, depends on the share of cooperators in the society. People learn their optimal strategy in a learning process which is represented by a payoff monotonic dynamic. The evolutionary analysis shows that there exist two stable states: One state in which a large share of the people in the society cooperate, and another state in which nobody cooperates.

Keywords: cooperation, evolution, networking, prisoners' dilemma

JEL classification: D11

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1 Introduction

People frequently find themselves in various types of prisoners' dilemma situations. This paper analyses an infinitely recurring prisoners' dilemma in which people change their partner in every period. It explains why a person may choose to cooperate in a prisoners' dilemma type situation even when he knows that he will never play against the same person again. Furthermore, it explains why both cooperators and defectors may exist simultaneously in a society, and why cooperators sometimes are being taken advantage of.

After two people have played the prisoners' dilemma there is a possibility for networking. Then, two cooperators who played against each other exchange information about cooperators that they have met in previous periods. If a cooperator A gets information about another cooperator B in period $t-1$, then A will search for B and try to pre-match with him in period t. If A succeeds in pre-matching B, then A and B will play the prisoners' dilemma against each other in period t. A defector A will never succeed in such pre-matching because a cooperator B will only pre-match with A if A can send a signal by greeting from a cooperator against whom B has played in a previous period. Since a cooperator will never tell a defector about cooperators, a defector will never be able to send such a signal. In each period all people in the society who did not succeed in pre-matching are randomly matched with each other.

By behaving cooperatively people expand their network of cooperative people. The analysis shows that, given that all cooperators network as described above, the probability of matching with a cooperator is larger for a cooperator than for a defector. Whether it is payoff maximizing to cooperate will depend on the share of cooperators in the society. People choose their optimal strategy via a learning process that is represented by a payoff monotonic dynamic from the field of evolutionary game theory. The evolutionary analysis shows that there exist two stable states: one state in which a large share of the people in the society cooperate, and another state in which nobody cooperates.

The paper presupposes that people in each period want to switch partners. People want to switch partner because of gains from diversified trade. As Kandori (1992) argues: ".., the division of labor and specialization are important driving forces of economic progress. Potential gains are larger in diverse transactions with different specialists than with fixed partners".

There have been two different types of models which extend the Folk Theorem to recurring prisoners' dilemma in which people change their partner in every period: Reputation-models (Kandori 1992; and Okuno-Fujiwara and Postlewaite 1995), and contagious-punishment-models (Kandori 1992, Ellison 1994, and Harrington 1995). As apposed to the model presented in this paper, these models derive a cooperative equilibrium in which everybody cooperates and nobody is being taken advantage of. The reputation-models have a cooperative equilibrium in which each person cooperates only if he meets a person who has reputation as a cooperator. He cooperates because defection will lead to loss of his own reputation and hence, future opponents to defect against him. The contagious-punishment-models have a cooperative equilibrium in which each person cooperates as long as he has only met cooperators in previous periods. Each person cooperates because a single defection will eventually cause everybody to defect.

2 The Model

Look at a society consisting of n people. n is a large even number. In every period each person matches with another person in the society with whom he plays the prisoners's dilemma, $\Gamma(\varepsilon)$, given by

$\Gamma(\varepsilon)$:

	defect	cooperate
defect	$0, 0$	$1+\varepsilon, -\varepsilon$
cooperate	$-\varepsilon, 1+\varepsilon$	$1, 1$

Assume that a share α of the people in the society always cooperate, while a share $1-\alpha$ never cooperate. Section 3 derives, for a given α, the probability for a cooperator and the probability for a defector to match with a cooperator. Thereafter, Section 4 determines α endogenously by evolutionary dynamics.

In addition to cooperate in the prisoners' dilemma, cooperators network in according to the Networking Strategy stated below. At the end of each period two cooperators who played against each other exchange information about possible cooperators they met in the previous period. Then, in the beginning of the next period cooperators try to use this information in order to pre-match with another cooperator. All the people who do not succeed in such pre-matching are randomly matched with each other. The pre-matching at the beginning of each period and the information exchange at the end of each period take place as follows:

Networking Strategy:

- *Information exchange*

  If you played against cooperator A in period $t-1$, and if you played against cooperator B in period t, then you must inform cooperator B that A is also a cooperator at the end of period t.

- *Pre-matching*

  If you received information about cooperator B from cooperator A in period $t-1$, then you must try to pre-match with cooperator B in the beginning of period t. To signal to cooperator B that you are also a cooperator, tell cooperator B that you have heard about him through cooperator A. Furthermore, always agree to pre-match with someone who can signal that he is a cooperator.

In each period a cooperator can pre-match with another cooperator by *being a searcher* or by *being searched for*. A cooperator A is a searcher and a cooperator B is being searched for in period t, if B played against some cooperator C in period $t-2$, and C played against A in period $t-1$. Then A will search for B and try to pre-match with him in the beginning of period t. A will not necessarily succeed in this pre-matching because B might himself, in addition to being searched for, be a searcher in period t. A will fail in pre-matching B if B is himself a searcher. A will, however, succeed in pre-matching B if B is not a searcher.

3 Pre-Matching

Let p_t be the probability that a random cooperator matches with another cooperator either by pre-matching or by random matching in period t. A random cooperator A is searching for some cooperator B in period t if and only if A matched with some cooperator C in period $t-1$, and C matched with B in period $t-2$. There is a probability p_{t-1} that a random cooperator A matched with some cooperator C in period $t-1$, and a probability p_{t-2} that this cooperator C matched with some cooperator B in period $t-2$. Thus, there is a probability $p_{t-1}p_{t-2}$ that a random cooperator is being a searcher in period t. Similar, there is a probability $p_{t-1}p_{t-2}$ that a person being searched for is being a searcher. A searcher will only succeed in pre-matching if he is searching for a person who is not a searcher. Hence, in period t, the probability that a random cooperator succeeds in pre-matching by being a searcher is given by

$$z_t = p_{t-1}p_{t-2}\left(1 - p_{t-1}p_{t-2}\right) \tag{1}$$

This implies that in period t a share $2z_t$ of the cooperators succeed in pre-matching by searching or by being searched for.

In each period all people who did not succeed in pre-matching are randomly matched with each other. A defector will never succeed in pre-matching because a cooperator B will only accept to pre-match with A if A can, as specified by the Networking Strategy, signal to B that he is also a cooperator. Thus, a share $1-2z_t$ of the cooperators and all the defectors are participating in the random matching. Hence, the probability of matching a cooperator in the random matching, s_t, is given by

$$s_t = \frac{\alpha\left(1-2z_t\right)}{\alpha\left(1-2z_t\right)+\left(1-\alpha\right)} = \frac{\alpha - 2\alpha z_t}{1-2\alpha z_t} \tag{2}$$

The probability for a random cooperator to match with another cooperator in period t

is given by

$$p_t = 2z_t + (1 - 2z_t)\, s_t \tag{3}$$

and the probability for a random defector to match with a cooperator in period t is given by

$$q_t = s_t \tag{4}$$

Equation (2), (3) and (4) imply

$$p_t = \frac{\alpha + 2\,(1-2\alpha)\, p_{t-1}p_{t-2}\,(1 - p_{t-1}p_{t-2})}{1 - 2\alpha p_{t-1}p_{t-2}\,(1 - p_{t-1}p_{t-2})} \tag{5}$$

$$q_t = \frac{\alpha\,(1-p_t)}{1-\alpha} \tag{6}$$

Setting $p_t = p_{t-1} = p_{t-2} = p$ in (5) and solve for α determines a unique mapping

$$\alpha\,(p) = \frac{-p + 2p^2 - 2p^4}{-1 + 4p^2 - 2p^3 - 4p^4 + 2p^5} \tag{7}$$

Let the inverse of $\alpha\,(p)$ be denoted by $p\,(\alpha)$. Assume that $p_1 = p_2 = \alpha$. A numerical analysis in the appendix shows that, for a given α, p_t converges to $p\,(\alpha)$. Then, (6) implies that q_t converges to

$$q\,(\alpha) = \frac{\alpha\,(1 - p\,(\alpha))}{1-\alpha} \tag{8}$$

Figure 1 plots $p\,(\alpha)$ and $q\,(\alpha)$. In addition, the probability to play with a cooperator given that there is no networking strategy, $r\,(\alpha)$, is plotted. This figure shows that cooperators succeed in increasing their probability of matching with another cooperator by networking.

Proposition 1 *Consider an infinitely recurring prisoners' dilemma $\Gamma\,(\varepsilon)$ in which people change their partner in every period. Assume that both cooperators and defectors are present in the society. If all cooperators follow the Networking Strategy, then the probability of matching a cooperator is larger for a cooperator than for a defector.*

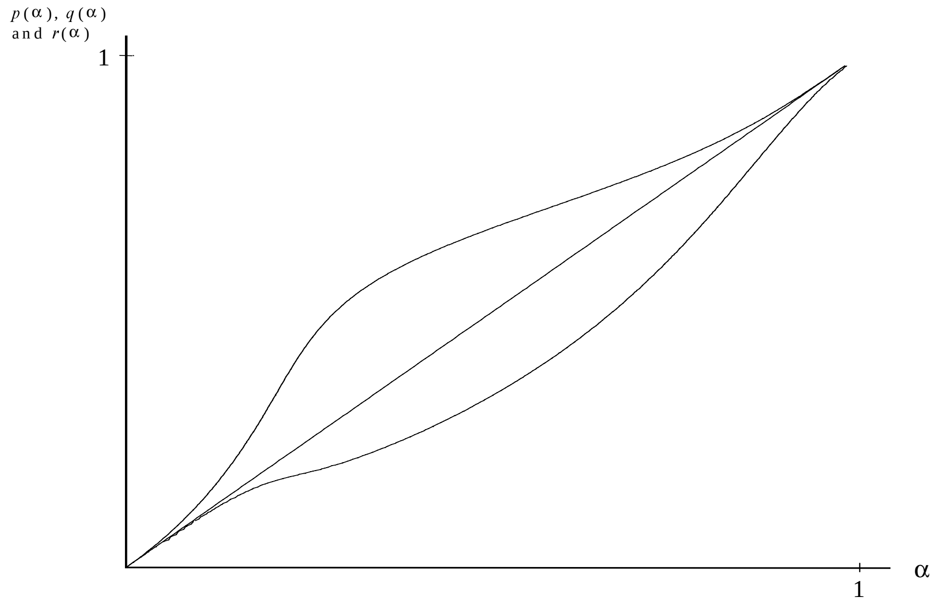

Figure 1: The probability of matching a cooperator. $p(\alpha)$, $r(\alpha)$, and $q(\alpha)$ are the upper, middle and lower graph respectively.

4 Evolutionary Dynamics

Refer to a period, in which a share of the people revise their strategy, as a revision period. Between two revision periods, let the average periodical payoff of a cooperator and a defector be denoted by $\pi_c(\alpha)$ and $\pi_{nc}(\alpha)$ respectively. Assume that a large number of periods take place between two revision periods such that the payoff in the periods immediately after a revision period, in which p_t and q_t have not yet converged to $p(\alpha)$ and $q(\alpha)$, have a negligible effect on $\pi_c(\alpha)$ and $\pi_{nc}(\alpha)$. Then the payoff matrix, $\Gamma(\varepsilon)$, implies that

$$\pi_c(\alpha) = p(\alpha) - (1 - p(\alpha))\varepsilon \quad (9)$$

$$\pi_{nc}(\alpha) = q(\alpha)(1 + \varepsilon) \quad (10)$$

Thus,

$$\begin{aligned} \Delta\pi(\alpha) &= \pi_c(\alpha) - \pi_{nc}(\alpha) \\ &= (p(\alpha) - q(\alpha))(1 + \varepsilon) - \varepsilon \quad (11) \end{aligned}$$

People do not at the outset possess all the information needed in order to calculate which strategy gives them highest payoff. They learn how to revise their strategy through imitating the successful ones or through reinforcement learning. It has been shown that such learning processes can be represented by the replicator dynamics (Gale, Binmore and Samuelson 1995; Björnerstedt and Weibull 1996; Börgers and Sarin 1997; and Schlag 1998). The replicator dynamics says: the growth rate of the population share using a certain strategy equals the difference between the strategy's current payoff and the current average payoff in the population (Weibull 1995, p. 73). A payoff monotonic dynamic is a more general dynamic of which the replicator dynamic is a specific case. A payoff monotone dynamic says: if a strategy i has higher current payoff than a strategy j, then the growth rate of the population share using strategy i is larger than the growth rate of the population share using strategy j (Fudenberg and Levine 1998).

Equation (11) imply that cooperators and deviators have the same payoff if and only if

$$p(\alpha) - q(\alpha) = \frac{\varepsilon}{1+\varepsilon} \tag{12}$$

By plotting $p(\alpha) - q(\alpha)$ Figure 2 illustrates the payoff monotone dynamic of the game represented by (9) and (10). Note from the figure that there exist an $\varepsilon^{\max}(\approx 0.34)$ such that if $0 < \varepsilon < \varepsilon^{\max}$, then (12) has two solutions, α' and α'', such that $0 < \alpha' < \alpha'' < 1$. Furthermore, $p(\alpha) - q(\alpha) > \frac{\varepsilon}{1+\varepsilon}$ if $\alpha \in (\alpha', \alpha'')$ and $p(\alpha) - q(\alpha) < \frac{\varepsilon}{1+\varepsilon}$ if $\alpha \in (0, \alpha') \cup (\alpha', 1)$. Thus, the game has two asymptotically stable states. One state, $\alpha = \alpha''$, in which a large share of the population cooperate, and another state, $\alpha = 0$, in which nobody cooperates. The difference in individual payoff between the asymptotically stable state in which a share α'' cooperate and the asymptotically stable state in which nobody cooperate is given by $q(\alpha'')(1+\varepsilon)$. Hence, the asymptotically stable state $\alpha'' > 0$ Pareto dominates the asymptotically stable state $\alpha = 0$ since $q(\alpha'') > 0$ if $\alpha'' > 0$.

Proposition 2 *Consider an infinitely recurring prisoners' dilemma $\Gamma(\varepsilon)$ in which people change their partner in every period. Assume that all cooperators play the Networking Strategy, and that people learn whether to be a cooperator or a defector in a process which can be*

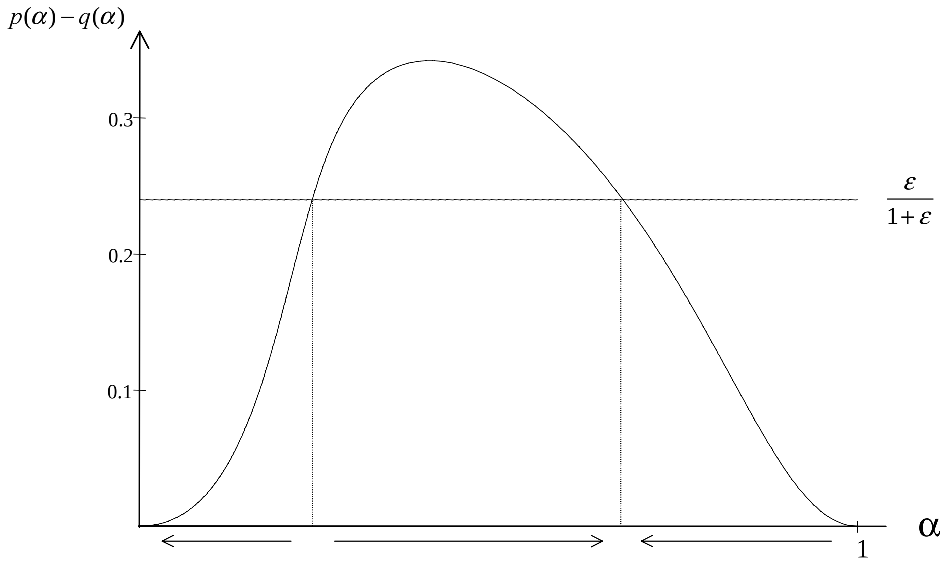

Figure 2: The payoff monotone dynamic.

represented by a payoff monotonic dynamic. Then, there exist an $\varepsilon^{\max}$ *such that if and only if* $0 < \varepsilon < \varepsilon^{\max}$, *the game has two asymptotically stable states: one state* α'', $0 < \alpha'' < 1$, *in which a large share of the population cooperate, and another state,* $\alpha = 0$, *in which nobody cooperates. The former state Pareto dominates the latter. Furthermore, the game has two unstable stationary states* α', $0 < \alpha' < \alpha''$, *and* $\alpha = 1$. *If* $\alpha \in \{(0, \alpha'), (\alpha'', 1)\}$, *then more and more cooperators will become defectors, and if* $\alpha \in (\alpha', \alpha'')$, *then more and more defectors will become cooperators.*

The intuition behind the evolutionary dynamic is as follows: For $0 < \alpha < \alpha'$ a defector has higher payoff than a cooperator because the probability of meeting a cooperator is only slightly higher for cooperators than for defectors. Cooperators eventually learn this disadvantage of their strategy and become defectors, i.e. $\dot{\alpha} < 0$. For $\alpha' < \alpha < \alpha''$ a cooperator has higher payoff than a defector because the probability of meeting a cooperator is now substantial higher for cooperators than for defectors. Defectors will eventually learn this disadvantage of their strategy and become cooperators, i.e. $\dot{\alpha} > 0$. For $\alpha > \alpha''$ a defector has again higher payoff than a cooperator because the probability of meeting a cooperator is only slightly higher for cooperators than for defectors. At this point there are so many cooperators in the society that the Networking Strategy becomes less efficient. A searcher often fails to pre-match and has to participate in random matching because the person he is searching for is himself busy searching. Cooperators will eventually learn this disadvantage of their strategy and become defectors, i.e. $\dot{\alpha} < 0$.

5 Conclusion

This paper studied an infinitely recurring prisoners' dilemma, $\Gamma(\varepsilon)$, in which people change their partner in each period. If cooperators follow the Networking Strategy, and if ε is below a critical value, then this game has two asymptotically stable states: one state in which nobody cooperates, and one state in which a large share of the population cooperate. In the cooperative stable state cooperators cooperate in order to increases their future likelihood to meet cooperators. This gives a cooperator incentives to continue to cooperate even after he has been taken advantage of.

Appendix: Numerical Analysis

Numeric analysis applied to (5) assuming that $p_1 = p_2 = \alpha$:

α	0,00	0,10	0,20	0,30	0,40	0,50	0,60	0,70	0,80	0,90	1,00
p_1	0,00	0,10	0,20	0,30	0,40	0,50	0,60	0,70	0,80	0,90	1,00
p_2	0,00	0,10	0,20	0,30	0,40	0,50	0,60	0,70	0,80	0,90	1,00
p_3	0,00	0,10	0,20	0,30	0,40	0,50	0,60	0,70	0,80	0,90	1,00
p_4	0,00	0,12	0,25	0,38	0,51	0,62	0,70	0,77	0,83	0,90	1,00
p_5	0,00	0,12	0,26	0,41	0,53	0,64	0,71	0,77	0,83	0,90	1,00
p_6	0,00	0,12	0,28	0,44	0,57	0,66	0,71	0,77	0,83	0,90	1,00
p_7	0,00	0,12	0,29	0,46	0,58	0,66	0,71	0,77	0,83	0,90	1,00
p_8	0,00	0,12	0,30	0,47	0,59	0,66	0,71	0,77	0,83	0,90	1,00
p_9	0,00	0,12	0,30	0,49	0,60	0,66	0,71	0,77	0,83	0,90	1,00
p_{10}	0,00	0,12	0,31	0,49	0,60	0,66	0,71	0,77	0,83	0,90	1,00
p_{11}	0,00	0,12	0,31	0,50	0,60	0,66	0,71	0,77	0,83	0,90	1,00
p_{12}	0,00	0,12	0,32	0,51	0,60	0,66	0,71	0,77	0,83	0,90	1,00
p_{13}	0,00	0,12	0,32	0,51	0,60	0,66	0,71	0,77	0,83	0,90	1,00
p_{14}	0,00	0,12	0,32	0,51	0,61	0,66	0,71	0,77	0,83	0,90	1,00
p_{15}	0,00	0,12	0,32	0,51	0,61	0,66	0,71	0,77	0,83	0,90	1,00
p_{16}	0,00	0,12	0,32	0,51	0,61	0,66	0,71	0,77	0,83	0,90	1,00
p_{17}	0,00	0,12	0,32	0,52	0,61	0,66	0,71	0,77	0,83	0,90	1,00
p_{18}	0,00	0,12	0,32	0,52	0,61	0,66	0,71	0,77	0,83	0,90	1,00
p_{19}	0,00	0,12	0,32	0,52	0,61	0,66	0,71	0,77	0,83	0,90	1,00
p_{20}	0,00	0,12	0,33	0,52	0,61	0,66	0,71	0,77	0,83	0,90	1,00
$p(\alpha)$	0,00	0,12	0,33	0,52	0,61	0,66	0,71	0,77	0,83	0,90	1,00

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