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Aggregation and Aggregation Biases in Production Functions: A Panel Data Analysis of Translog Models

Abstract:
An applied econometric study of aggregation, based on an unbalanced panel data set for manufacturing plants is presented. Panel data are informative in examining aggregation of variables, parameters, and relationships empirically since they (i) allow estimation at both the micro and the macro level, and (ii) enable comparison of the time series properties of the exactly aggregated micro relationships with those obtained by performing aggregation by analogy. Numerical aggregation of Translog production functions for three manufacturing industries is considered. We show, under linear aggregation of inputs and output, that departures between geometric and arithmetic means of the inputs and correlation between the log-inputs, both their levels and time paths, contribute substantially to aggregation biases in the output volume and instability of the derived input and scale elasticities. Hence, the existence and stability of an approximate "macro Translog production function" over time can be questioned.

Keywords: Aggregation. Panel Data. Translog. Scale elasticity. Input distribution.

JEL classification: C23, C43, D21, E23, L11, L60

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1 Introduction

Many macro economists and macro econometricians who utilize micro based theories for macro-economic purposes rely on a 'representative agent' interpretation of the relationships describing the theory, *i.e.*, they aggregate the relationships informally, 'by analogy'. This way of treating, *e.g.*, production functions, producer factor demand functions, or consumer commodity demand functions is far from being satisfactory. From the formal theory of aggregation[1] it is well known that micro based relationships can only be aggregated to functional macro relationships if certain restrictive assumptions with respect to (i) the form of the micro relationship, (ii) the form of the aggregation procedure for the micro variables and/or (iii) the distribution of the macro variables across micro units are satisfied. One simple such case is linear aggregation of linear relationships with identical coefficients across the micro units. Another case is linear aggregation of linear relationships with varying coefficients across micro units when all micro variables move proportionally over time.

If conditions which ensure the existence of exact aggregate relationships are not satisfied, interesting questions are: Will such relationships hold as more or less good approximations, and will they show acceptable stability over time? If not, which are the most important sources of aggregation bias? The present paper is devoted to these issues and presents an applied econometric study of linear and non-linear aggregation. The data set used is a panel data set for manufacturing plants. Panel data is a very valuable source of information for analyzing aggregation problems empirically, for at least two reasons. First, they allow estimation to be performed both at the micro and at the macro level, the latter for instance by using data constructed as time specific means of the original observations. Second, panel data enable comparison of the time series properties of the exactly aggregated micro relationships and properties derived from them, *e.g.*, certain elasticities, with those obtained by performing aggregation by analogy from the micro level and allow exploration of the various contributions to the aggregation biases. In doing this, the effect of heterogeneity across the micro units can be modeled and investigated. In this paper, however, we do not exploit the full potential of the panel property of our data set.

The specific application we consider is the numerical aggregation of neo-classical production functions by means of a set of unbalanced panel data for manufacturing plants in three industries. The functional form assumed for the micro units is the *Translog function*. Apart from the fact that Translog functions are frequently used in both micro-

[1] On linear aggregation of linear relations see, *e.g.*, the textbook expositions in Allen (1964, chapter 20) and Theil (1971, section 11.3). See also Green (1964, chapters 5 and 12) and Klein (1974, section VIII.3).

and macroeconometrics, and are fairly well-established as representations of the technology of micro units, both in their primal and dual form [see, *e.g.*, Jorgenson (1986)], this functional form is interesting from a more formal point of view, since it combines the aggregation of logarithms with the aggregation of squares and products. To our knowledge, this is the first study exploring aggregation issues with panel data by means of this rather complicated functional form. Our focus is on properties of the distribution of the inputs, in particular the deviation between their geometric and arithmetic means and the correlation patterns of their logarithms. Aggregation studies related to consumer demand [*e.g.*, Muellbauer (1975), Gorman (1981), Lau (1982), Jorgenson, Lau and Stoker (1982), Stoker (1984, 1993), Lewbel (1992), and Hildenbrand and Kneip (1999)], have demonstrated how properties of the distribution of income and demographic characteristics may affect properties of aggregate demand functions. Studies of aggregation related to producer behaviour include Johansen (1972, chapters 3 and 9), Sato (1975), Hildenbrand (1981), Muysken (1987), and Fortin (1991, section 3). A main issue in this literature is the aggregation of input-output coefficients within putty-clay or Leontief micro technologies, with regard paid to the impact on the aggregates of the distribution of the micro coefficients across the firms. We, however, assume neo-classical micro technologies throughout, as is also done, in a Cobb-Douglas context, by, *e.g.*, de Wet (1976), and in a quadratic cost function context, by Koebel (1998).

The micro model specifies heterogeneity in the coefficient structure by including a random intercept and some random slope coefficients in the Translog function. The panel property of our data makes estimation and aggregation of this kind of model possible. Random coefficients at the micro level may be interesting in examining properties of aggregates, *inter alia*, because the effect of correlation between coefficients and associated variables can be examined.

The paper is organized as follows. In Section 2 we first give some general remarks on the aggregation framework and next present the specific micro Translog model and characteristics of the distribution of the variables and parameters. The estimation of the parameters from an unbalanced panel data set from Norwegian manufacturing is discussed in Section 3. In Section 4, we describe three ways of aggregating the production function. One performs linear aggregation of logarithms, one performs aggregation of the non-transformed variables, *i.e.*, the antilogs, the last is an intermediate case. We interpret the macro parameters and discuss the aggregation biases in the output volume and in the input and scale elasticities which occur when using the means of the micro coefficients in constructing the aggregates. Throughout, we express, the aggregate relationships not in terms of empirical means of the variables, but in terms of (mathematical) expectations, which is more convenient. When formulating the empirical counterparts to these relationships and investigating aggregation biases by means of panel data, we replace these expectations by the corresponding (empirical) means for each year in the data set. This is the topic of Section 5. Discussion and concluding remarks follow in Section 6.

2 The aggregation problem and model framework

2.1 General remarks on the aggregation problem

Consider a single equation and let Q_{it} and Z_{it} denote its endogenous variable and a vector of exogenous variables, respectively, of micro unit i in period t, and let ψ_i be a unit specific coefficient vector, and u_{it} a disturbance with zero mean and distributed independently of Z_{it} and ψ. Our *micro model* has the form[2]

$$G(Q_{it}) = H[F(Z_{it}), \psi_i] + u_{it}, \qquad i = 1, \ldots, N_t;\ t = 1, \ldots, T, \tag{1}$$

where F is a vector valued, non-linear function, G and H are non-linear functions, N_t is the number of units in period t, and T is the number of periods. The *aggregation problem* can then, somewhat loosely, be formulated as follows: Do there exist aggregation functions $Q_t = g(Q_{1t}, \ldots, Q_{N_t,t})$ and $Z_t = f(Z_{1t}, \ldots, Z_{N_t,t})$, and functions G^*, F^*, and H^*, which are in some sense 'similar to' G, F, and H, such that we from (1) can derive

$$G^*(Q_t) = H^*[F^*(Z_t), \psi] + u_t, \qquad t = 1, \ldots, T, \tag{2}$$

for a coefficient vector ψ depending on the ψ_i's and a disturbance u_t depending on the u_{it}'s, with 'nice' distributional properties? Very often, the answer is 'no' if we want to obtain functions g, f, G^*, F^*, and H^* for which (2) holds.[3] The question of whether we from (1) can derive (2), where u_t has 'not so nice' properties, then naturally arises. In the following, attention will be confined to the case where the functional forms G^* and G as well as F^* and F coincide, *e.g.*, both may be the logarithm function, as is our empirical application.

To make this idea precise, let us consider two cases related to the application we present in this paper. In the first we represent the aggregate $Q_t = g(Q_{1t}, \ldots, Q_{N_t,t})$ by

[2]This formulation assumes that the relationship between Q_{it} and Z_{it} is the same for all t. More generally, we may include t as a shift variable in the H function.

[3]A precise discussion, although with a deterministic formulation of the aggregation problem, is given in Green (1964, chapter 5), see, in particular, Theorem 8.

defining $G(Q_t)$ as the arithmetic mean of the $G(Q_{it})$'s, *i.e.*,[4]

$$(3)\quad G(Q_t) = \frac{1}{N_t}\sum_{i=1}^{N_t} G(Q_{it}) = \frac{1}{N_t}\sum_{i=1}^{N_t} H[F(Z_{it}),\psi_i] + \frac{1}{N_t}\sum_{i=1}^{N_t} u_{it}, \qquad t = 1,\dots,T.$$

If N_t is not too small, we may exploit a law of the large numbers and associate the arithmetic mean with the expectation [cf. Fortin (1991, section 2), Stoker (1993, section 3), and Hildenbrand (1998, section 2)], which simplifies the argument somewhat. We then get

$$(4)\qquad G(Q_t) = \mathsf{E}[G(Q_{it})] = \mathsf{E}\left[H[F(Z_{it}),\psi_i]\right], \qquad t = 1,\dots,T,$$

since u_{it} has zero mean. The expression on the right hand side of (4) is then the correct expression for the expectation of $G(Q_{it})$, conditional on period t. It depends, in addition to the functional forms F and H, on the joint distribution of Z_{it} and ψ_i, but is independent of the distribution of u_{it}. Eq. (4) is not, however, in general an aggregate function, since no aggregate of the Z_{it}'s of the form $Z_t = f(Z_{1t},\dots,Z_{N_t,t})$, or $\mathsf{E}(Z_{it})$, occurs on its right hand side.

A second way of aggregating (1) is to invert G before taking the mean and define Q_t as the arithmetic mean of the Q_{it}'s. Instead of (3) and (4) we then obtain

$$(5)\qquad Q_t = \frac{1}{N_t}\sum_{i=1}^{N_t} Q_{it} = \frac{1}{N_t}\sum_{i=1}^{N_t} G^{-1}[H[F(Z_{it}),\psi_i] + u_{it}], \qquad t = 1,\dots,T,$$

and

$$(6)\qquad Q_t = \mathsf{E}(Q_{it}) = \mathsf{E}\left[G^{-1}\{H[F(Z_{it}),\psi_i] + u_{it}\}\right], \qquad t = 1,\dots,T.$$

The expression on the right hand side of (6) is then the correct expression for the expectation of Q_{it}, conditional on period t. It depends, in addition to the functional forms F, G, and H, on the joint distribution of Z_{it} and ψ_i, as well as on the distribution of u_{it}, when G is non-linear. Neither is (6), in general, an aggregate function, since no aggregate of the Z_{it}'s of the form $Z_t = f(Z_{1t},\dots,Z_{N_t,t})$, or $\mathsf{E}(Z_{it})$, occurs on its right hand side.

Let ψ be a mean value of the ψ_i's, defined and estimated in some way. In general, the expressions on the right hand side of eqs. (4) and (6) will be different from

$$(7)\qquad [\mathsf{E}[G(Q_{it})]]^* = H[\mathsf{E}(F(Z_{it})),\psi], \qquad t = 1,\dots,T,$$

where we associate $\mathsf{E}[F(Z_{it})]$ with $F^*(Z_t)$ in (2), and

$$(8)\qquad [\mathsf{E}(Q_{it})]^{**} = G^{-1}\{H[F(\mathsf{E}(Z_{it})),\psi]\}, \qquad t = 1,\dots,T,$$

[4]In the following discussion, the macro variables are averages, but the entire analysis can be easily translated to the case where they are sums.

where we associate $F[\mathsf{E}(Z_{it})]$ with $F^*(Z_t)$ in (2), respectively. We interpret $[\mathsf{E}[G(Q_{it})]]^*$ as an *aggregate by analogy* corresponding to $G^*(Q_t)$ in (2) and $[\mathsf{E}(Q_{it})]^{**}$ as an aggregate by analogy corresponding to Q_t in (2).

2.2 The micro model framework

We next elaborate our micro model framework. Let Y be output and X_j the j'th input $(j = 1, \dots, n)$, which correspond to the scalar variable Q and to the vector variable Z, respectively, in the general description above. The G and F functions are the logarithm function, and H is a linear function plus a quadratic form. We suppress, for the moment, the observation number subscripts (i, t) on the variables and write the basic one output, n input Translog production function as

$$y = \alpha + \sum_j \beta_j x_j + \tfrac{1}{2} \sum_j \sum_k \gamma_{jk} x_j x_k + u, \tag{9}$$

where $y = \ln(Y)$, $x_j = \ln(X_j)$, α, β_j, and γ_{jk} are parameters corresponding to ψ, and u is a disturbance. This equation exemplifies (1). We use the following notation:

$$\begin{gathered} \bar{\alpha} = \mathsf{E}(\alpha), \quad \bar{\beta}_j = \mathsf{E}(\beta_j), \quad \rho_{jk} = \operatorname{corr}(x_j, x_k), \quad \lambda_{jj} = \operatorname{corr}(x_j, \beta_j), \\ \nu_j = \frac{\operatorname{std}(x_j)}{\mathsf{E}(x_j)}, \quad \mu_j = \frac{\operatorname{std}(\beta_j)}{\mathsf{E}(\beta_j)}. \end{gathered} \tag{10}$$

From (9) and (10), using the relationship between centered and non-centered second order moments, it follows that the expected log-output can be expressed in terms of expectations of logarithms of the inputs and the distributional properties of the micro variables and coefficients as

$$\begin{aligned} \mathsf{E}(y) &= \mathsf{E}(\alpha) + \sum_j \mathsf{E}(\beta_j x_j) + \tfrac{1}{2} \sum_j \sum_k \gamma_{jk} \mathsf{E}(x_j x_k) \\ &= \mathsf{E}(\alpha) + \sum_j [\mathsf{E}(\beta_j)\mathsf{E}(x_j) + \operatorname{cov}(\beta_j, x_j)] + \tfrac{1}{2} \sum_j \sum_k \gamma_{jk} [\mathsf{E}(x_j)\mathsf{E}(x_k) + \operatorname{cov}(x_j, x_k)], \end{aligned}$$

which can be written as

$$\mathsf{E}(y) = \bar{\alpha} + \sum_j (1 + a_j) \bar{\beta}_j \mathsf{E}(x_j) + \tfrac{1}{2} \sum_j \sum_k (1 + c_{jk}) \gamma_{jk} \mathsf{E}(x_j) \mathsf{E}(x_k), \tag{11}$$

where

$$a_j = \frac{\operatorname{cov}(x_j, \beta_j)}{\mathsf{E}(x_j)\mathsf{E}(\beta_j)} = \lambda_{jj} \mu_j \nu_j, \qquad c_{jk} = \frac{\operatorname{cov}(x_j, x_k)}{\mathsf{E}(x_j)\mathsf{E}(x_k)} = \rho_{jk} \nu_j \nu_k. \tag{12}$$

Eq. (11) is basic in describing the aggregation procedures and investigating the time series properties of the aggregation biases. We do not incorporate optimizing conditions (*e.g.*, derived from complete or partial profit maximization or cost minimization) in our framework, as has been done by, *e.g.*, Klein (1946a), Green (1964, chapter 6), and Koebel (1998). Our aim is to obtain approximate aggregate production functions whose existence does not rely on specific behavioural assumptions. "The aggregate production function should not depend upon profit maximization, but purely on technological factors" [Klein (1946b, p. 303)].

3 Econometric specification, data, and estimation

We now describe the parametrization of (9), the data, and the estimation procedure within an unbalanced panel data context with one output (Y) and four inputs ($n = 4$), capital (K), labour (L), energy (E) and materials (M). We also include a deterministic quadratic trend (t) intended to capture the level of the technology and let the trend interact with the inputs.[5] This application draws on the estimation results in Biørn, Lindquist and Skjerpen (2002). Let subscripts i and t denote the plant and the year of observation, respectively, and consider the following parametrization of (9) with random intercept and random first order slope coefficients

$$(13) \qquad y_{it} = \alpha_i^* + \kappa_1 t + \tfrac{1}{2}\kappa_2 t^2 + \sum_j \beta_{ji} x_{jit} + \tfrac{1}{2}\sum_j \sum_k \gamma_{jk} x_{jit} x_{kit} + \sum_j \delta_j t x_{jit} + u_{it}, \quad j,k = K,L,E,M,$$

where $y_{it} = \ln(Y_{it})$ and $x_{jit} = \ln(X_{jit})$ $(j = K,L,E,M)$. Here α_i^* and β_{ji} $(j = K,L,E,M)$ are *random coefficients* specific to plant i, whereas κ_1, κ_2, δ_j and γ_{jk} $(j,k = K,L,E,M)$ are plant invariant. The genuine disturbance u_{it} has zero expectation and variance σ_{uu}. We let $x_{it} = (x_{Kit}, x_{Lit}, x_{Eit}, x_{Mit})'$, collect all the random coefficients for plant i in the vector

$$\psi_i = (\alpha_i^*, \beta_{Ki}, \beta_{Li}, \beta_{Ei}, \beta_{Mi})'$$

and describe the heterogeneity of the coefficient structure as follows: All x_{it}, u_{it}, and ψ_i are independently distributed, with

$$E(\psi_i) = \psi = (\bar{\alpha}^*, \bar{\beta}_K, \bar{\beta}_L, \bar{\beta}_E, \bar{\beta}_M)', \qquad E[(\psi_i - \psi)(\psi_i - \psi)'] = \Omega,$$

where Ω is a symmetric, but otherwise unrestricted matrix.

Since our focus will be on aggregation biases on a yearly basis it is convenient to rewrite (13) as

$$(14) \qquad y_{it} = \alpha_{it} + \sum_j \beta_{jit} x_{jit} + \tfrac{1}{2}\sum_j \sum_k \gamma_{jk} x_{jit} x_{kit} + u_{it}, \quad j,k = K,L,E,M,$$

[5]This may be associated with adding t as a separate argument in H in (1).

where $\alpha_{it} = \alpha_i^* + \kappa_1 t + \frac{1}{2}\kappa_2 t^2$ and $\beta_{jit} = \beta_{ji} + \delta_j t$, satisfying

$$E(\alpha_{it}) = \bar{\alpha}_t = \bar{\alpha}^* + \kappa_1 t + \frac{1}{2}\kappa_2 t^2, \quad E(\beta_{jit}) = \bar{\beta}_j + \delta_j t.$$

In the following we sometimes suppress the indices for plant and year and write (14) as (9) with $j, k = K, L, E, M$.

TABLE 1. NUMBER OF PLANTS CLASSIFIED BY NUMBER OF REPLICATIONS
p = no. of observations per plant, N_p = no. of plants observed p times,

Industry	*Pulp & paper*		*Chemicals*		*Basic metals*	
p	N_p	$N_p p$	N_p	$N_p p$	N_p	$N_p p$
22	60	1320	29	638	44	968
21	9	189	0	0	2	42
20	5	100	3	60	4	80
19	3	57	0	0	5	95
18	1	18	2	36	2	36
17	4	68	4	68	5	85
16	6	96	9	144	5	80
15	4	60	6	90	4	60
14	3	42	1	14	5	70
13	4	52	3	39	3	39
12	7	84	1	12	10	120
11	10	110	2	22	7	77
10	12	120	3	30	6	60
09	10	90	2	18	5	45
08	7	56	2	16	2	16
07	15	105	2	14	13	91
06	11	66	3	18	4	24
05	14	70	3	15	5	25
04	9	36	2	8	6	24
03	18	54	3	9	3	9
02	5	10	3	6	6	12
01	20	20	7	7	20	20
Sum	237	2823	90	1264	166	2078

The unknown parameters are estimated by Maximum Likelihood, using the PROC MIXED procedure in the SAS/STAT software [see Littell *et al.* (1996)]. Positive definiteness of Ω is imposed as an *a priori* restriction. Details on the application of the Maximum Likelihood procedure in the present context is given in Biørn, Lindquist and Skjerpen (2002, Appendix A). The data are sets of unbalanced panel data for the years 1972 – 1993 from three Norwegian manufacturing industries, *Pulp and paper*, *Chemicals*, and *Basic metals*. A further description is given in Appendix A). Tables 1 and 2 describe the unbalance. Table 1 classifies the observations by the number of years (maximum 22, minimum 1). Table 2, in which the plants are sorted by the calendar year in which they are observed, shows that there is a negative trend in the number of plants for all industries, most strongly for Pulp and paper.

TABLE 2. NUMBER OF PLANTS BY CALENDAR YEAR

Year	*Pulp & paper*	*Chemicals*	*Basic metals*
1972	171	57	102
1973	171	59	105
1974	179	62	105
1975	175	64	110
1976	172	66	109
1977	158	65	111
1978	155	63	109
1979	146	63	102
1980	144	62	100
1981	137	61	100
1982	129	61	99
1983	111	58	95
1984	108	59	87
1985	106	60	89
1986	104	60	84
1987	102	51	87
1988	100	50	85
1989	97	49	83
1990	99	48	81
1991	95	53	81
1992	83	47	71
1993	81	46	83
Sum	2823	1264	2078

4 Aggregation procedures and aggregation biases

In this section, we consider three ways of aggregating the Translog production function exactly (subsections 4.1 and 4.2), and compare them with the incorrect aggregation by analogy (subsection 4.3).

4.1 Aggregation in terms of expectations of logarithms

Eq. (11) is a Translog function in the expected logarithms of the output and the inputs of the form

$$\mathsf{E}[\ln(Y)] = \bar{\alpha} + \sum_j \widetilde{\beta}_j \mathsf{E}[\ln(X_j)] + \tfrac{1}{2} \sum_j \sum_k \widetilde{\gamma}_{jk} \mathsf{E}[\ln(X_j)]\, \mathsf{E}[\ln(X_k)], \tag{15}$$

where

$$\begin{aligned} \widetilde{\beta}_j &= (1 + a_j)\bar{\beta}_j = (1 + \lambda_{jj}\mu_j\nu_j)\bar{\beta}_j, \\ \widetilde{\gamma}_{jk} &= (1 + c_{jk})\gamma_{jk} = (1 + \rho_{jk}\nu_j\nu_k)\gamma_{jk}. \end{aligned} \tag{16}$$

We can here interpret $\widetilde{\beta}_j$ as macro first-order coefficients and $\widetilde{\gamma}_{jk}$ as macro second-order coefficients and β_j (random) and γ_{jk} (fixed) as the corresponding micro coefficients. This equation exemplifies (4). The stability of the first-order macro coefficients depends on the stability of the a_j's, which depend on the correlation coefficients between the first order coefficients and the log-inputs, λ_{jj}, and the coefficients of variation of the latter, ν_j. The stability of the second-order macro coefficients depends on the stability of the c_{jk}'s, which depend on the correlation coefficients between all log-inputs, ρ_{ij}, and the coefficients of variation of the log-inputs. In particular, $|\widetilde{\gamma}_{jj}| > |\gamma_{jj}|$ since $c_{jj} = \nu_j^2 > 0$, while $\widetilde{\beta}_j - \beta_j$ and $\widetilde{\gamma}_{jk} - \gamma_{jk}$ $(j \neq k)$ may have either sign, depending on the signs of a_j and c_{jk}.

The macro elasticity of output with respect to input j, *input elasticity*, for short, defined as $(\partial \mathsf{E}[\ln(Y)])/(\partial \mathsf{E}[\ln(X_j)])$, and the corresponding aggregated scale elasticity are

$$\widetilde{\epsilon}_j = \frac{\partial \mathsf{E}[\ln(Y)]}{\partial \mathsf{E}[\ln(X_j)]} = \widetilde{\beta}_j + \sum_k \widetilde{\gamma}_{jk} \mathsf{E}[\ln(X_k)]), \tag{17}$$

$$\widetilde{\epsilon} = \sum_j \widetilde{\epsilon}_j = \sum_j \widetilde{\beta}_j + \sum_k \left(\sum_j \widetilde{\gamma}_{jk} \right) \mathsf{E}[\ln(X_k)]). \tag{18}$$

Note that these elasticities are defined subject to changes in the $\mathsf{E}[\ln(X_j)]$'s which leave a_j and c_{jk} unchanged.

4.2 Aggregation in terms of expectations of non-transformed variables

We next express the aggregates in terms of expectations of the non-transformed variables. For this purpose we define

$$\theta_j = \frac{\ln[\mathsf{E}(X_j)] - \mathsf{E}[\ln(X_j)]}{\ln[\mathsf{E}(X_j)]} = \frac{\ln\left[\mathsf{E}\left(e^{x_j - \mathsf{E}(x_j)}\right)\right]}{\ln[\mathsf{E}(e^{x_j})]}, \tag{19}$$

$$\phi = \frac{\ln[\mathsf{E}(Y)] - \mathsf{E}[\ln(Y)]}{\ln[\mathsf{E}(Y)]} = \frac{\ln\left[\mathsf{E}\left(e^{y - \mathsf{E}(y)}\right)\right]}{\ln[\mathsf{E}(e^{y})]}. \tag{20}$$

Since $e^{\mathsf{E}[\ln(X_j)]}$ and $e^{\mathsf{E}[\ln(Y)]}$ can be associated with the geometric means and $\mathsf{E}(X_j)$ and $\mathsf{E}(Y)$ with the arithmetic means of the inputs and output, θ_j and ϕ represent the relative discrepancy between the logs of the arithmetic and the geometric means of the inputs and the output. We will denote θ_i and ϕ as *relative log-of-mean/mean-of-log-differences (RLMML-differences, for short).* The numerator of these expressions, *i.e.*, the logarithm of the ratio between the arithmetic and the geometric mean, is invariant to the choice of measurement scale, but the denominator is not. Consequently, $\theta_j \ln[\mathsf{E}(X_j)]$ and $\phi \ln[\mathsf{E}(Y)]$

are dimensionless numbers, but θ_j and ϕ are not. Although θ_j may be interpreted as a free parameter characterizing the distribution of input j, ϕ is not a free parameter. It is implicitly defined by the aggregation procedure for the inputs and their distribution as well as by the distribution of the random coefficients and the disturbances. Inserting (9) and (11) into (20), we obtain

$$\phi = 1 - \frac{\bar{\alpha} + \sum_j (1+a_j)\bar{\beta}_j \mathsf{E}(x_j) + \frac{1}{2}\sum_j \sum_k (1+c_{jk})\gamma_{jk}\mathsf{E}(x_j)\mathsf{E}(x_k)}{\ln\left[\mathsf{E}\left(e^{\alpha+\sum_j \beta_j x_j + \frac{1}{2}\sum_j \sum_k \gamma_{jk} x_j x_k + u}\right)\right]}, \tag{21}$$

which shows that ϕ is a *'hybrid' parameter* in general.

Since the geometric mean is less than the arithmetic mean whenever these means are defined [cf. Jensen's inequality and Sydsæter, Strøm, and Berck (1999, section 7.1)], we know that ϕ and θ_j are positive provided that $\mathsf{E}(X_j)$ and $\mathsf{E}(Y)$ exceed one. If, in particular, the log-inputs x_j are *normally* distributed, *i.e.*, the inputs X_j are *log-normal*, then $\mathsf{E}(X_j) = \mathsf{E}(e^{x_j}) = e^{[\mathsf{E}(x_j)+\frac{1}{2}\mathrm{var}(x_j)]}$ [cf. *e.g.*, Evans, Hastings, and Peacock (1993, chapter 25)], and so we have $\theta_j = \mathrm{var}(x_j)/[2\mathsf{E}(x_j) + \mathrm{var}(x_j)]$, which is between 0 and 1 if $\mathsf{E}(x_j)$ is positive.

Using (19) and (20), (11) can be written as

$$\begin{aligned}\mathsf{E}[\ln(Y)] = (1-\phi)\ln[\mathsf{E}(Y)] = \bar{\alpha} &+ \sum_j (1+a_j)(1-\theta_j)\bar{\beta}_j \ln[\mathsf{E}(X_j)] \\ &+ \tfrac{1}{2}\sum_j \sum_k (1+c_{jk})(1-\theta_j)(1-\theta_k)\gamma_{jk}\ln[\mathsf{E}(X_j)]\ln[\mathsf{E}(X_k)].\end{aligned} \tag{22}$$

There are two ways of interpreting (22) as an aggregate Translog production function. First, we can, by dividing by $(1-\phi)$, rewrite it in terms of *the logarithms of expectations* as

$$\ln[\mathsf{E}(Y)] = \alpha^* + \sum_j \beta_j^* \ln[\mathsf{E}(X_j)] + \tfrac{1}{2}\sum_j \sum_k \gamma_{jk}^* \ln[\mathsf{E}(X_j)]\ln[\mathsf{E}(X_k)], \tag{23}$$

where

$$\begin{aligned}\alpha^* &= \frac{\bar{\alpha}}{1-\phi}, \\ \beta_j^* &= (1+a_j)\frac{1-\theta_j}{1-\phi}\bar{\beta}_j, \\ \gamma_{jk}^* &= (1+c_{jk})\frac{(1-\theta_j)(1-\theta_k)}{1-\phi}\gamma_{jk}.\end{aligned} \tag{24}$$

The equation we get when taking the antilogarithm in (23), exemplifies (6). If we consider the macro relation as a relationship between the logarithms of arithmetic means, we can interpret α^* as the macro intercept term, β_j^* as macro first-order coefficients and γ_{jk}^* as macro second-order coefficients. The macro intercept term then depends on the RLMML-difference of the output, and since $\phi > 0$, we have $\alpha^* > \bar{\alpha}$. The first-order macro coefficients depend on (i) the correlation between the first-order micro coefficients and the log-inputs and (ii) the RLMML-differences of the inputs and output. The second-order macro coefficients depend on (i) the correlation between all the log-inputs and (ii) the RLMML-differences of the inputs and output. The differences $\beta_j^* - \bar{\beta}_j$ and $\gamma_{jk}^* - \gamma_{jk}$ may be of either sign. The stability of α^*, β_j^*, and γ_{jk}^* depends on the stability not only of the distributional parameters a_j, c_{jk}, θ_j, but also of the hybrid parameter ϕ.

Second, we can interpret (22) as a Translog function *in the expected logarithm of output and the logarithms of expectations of the inputs*, *i.e.* as

$$\mathsf{E}[\ln(Y)] = \bar{\alpha} + \sum_j \beta_j^{\Delta} \ln[\mathsf{E}(X_j)] + \tfrac{1}{2} \sum_j \sum_k \gamma_{jk}^{\Delta} \ln[\mathsf{E}(X_j)] \ln[\mathsf{E}(X_k)], \tag{25}$$

where

$$\begin{aligned} \beta_j^{\Delta} &= (1 + a_j)(1 - \theta_j)\bar{\beta}_j, \\ \gamma_{jk}^{\Delta} &= (1 + c_{jk})(1 - \theta_j)(1 - \theta_k)\gamma_{jk}. \end{aligned} \tag{26}$$

This way of interpreting the aggregate Translog function has the advantage that the macro coefficient (26), unlike (24), do not depend on the hybrid parameter ϕ. In view of (21) they are therefore potentially more stable than β_j^* and γ_{jk}^*. In constructing (25), we *aggregate the inputs and the outputs differently*, which may be unusual, but is perfectly possible. We can interpret $\bar{\alpha}$ both as the micro and the macro intercept term, and interpret β_j^{Δ} as macro first-order coefficients and γ_{jk}^{Δ} as macro second-order coefficients. The first-order macro coefficients depend on (i) the correlation between the first-order micro coefficients and the log-inputs and (ii) the RLMML-differences of the inputs. The second-order macro coefficients depend on (i) the correlation between all the log-inputs and (ii) the RLMML-differences of the inputs. The differences $\beta_j^{\Delta} - \bar{\beta}_j$ and $\gamma_{jk}^{\Delta} - \gamma_{jk}$ may be of either sign.

We can summarize the difference between (15), (23), and (25) as follows. The first can be interpreted as a macro Translog function expressed as a relationship between $\mathsf{E}[\ln(Y)]$ and the $\mathsf{E}[\ln(X_j)]$'s, *i.e.*, between the logs of geometric means. The second can be interpreted as a macro Translog function expressed as a relationship between $\ln[\mathsf{E}(Y)]$ and the $\ln[\mathsf{E}(X_j)]$'s, *i.e.*, between the logs of arithmetic means, all parameters being functions of the hybrid parameter ϕ. This third can be interpreted as an aggregate production function in which the inputs and the output are aggregated differently.

The macro input elasticity of input j, defined as $(\partial \ln[\mathsf{E}(Y)])/(\partial \ln[\mathsf{E}(X_k)])$, and the corresponding scale elasticity can then be written as

$$\epsilon_j^* = \frac{\partial \ln[\mathsf{E}(Y)]}{\partial \ln[\mathsf{E}(X_j)]} = \beta_j^* + \sum_k \gamma_{jk}^* \ln[\mathsf{E}(X_k)], \tag{27}$$

$$(28) \qquad \epsilon^* = \sum_j \epsilon_j^* = \sum_j \beta_j^* + \sum_k \left(\sum_j \gamma_{jk}^* \right) \ln[\mathsf{E}(X_k)].$$

Note that these elasticities are defined subject to changes in $\ln[\mathsf{E}(X_j)]$ which leave a_j, c_{jk}, θ_j, and ϕ unchanged. The macro input elasticity function of input j corresponding to (25), in which the output and the inputs are aggregated differently, and defined as $(\partial\mathsf{E}[\ln(Y)])/(\partial \ln[\mathsf{E}(X_j)])$, and the corresponding scale elasticity can be written as

$$(29) \qquad \epsilon_j^\Delta = \frac{\partial \mathsf{E}[\ln(Y)]}{\partial \ln[\mathsf{E}(X_j)]} = \beta_j^\Delta + \sum_k \gamma_{jk}^\Delta \ln[\mathsf{E}(X_k)],$$

$$(30) \qquad \epsilon^\Delta = \sum_j \epsilon_j^\Delta = \sum_j \beta_j^\Delta + \sum_k \left(\sum_j \gamma_{jk}^\Delta \right) \ln[\mathsf{E}(X_k)].$$

These elasticities are defined subject to changes in $\ln[\mathsf{E}(X_j)]$ which leave a_j, c_{jk}, and θ_j unchanged. We then have given three different definitions of the macro input and scale elasticity functions of the aggregate Translog function. Obviously, we have

$$(31) \qquad \frac{\epsilon^\Delta}{\epsilon^*} = \frac{\epsilon_j^\Delta}{\epsilon_j^*} = \frac{\mathsf{E}[\ln(Y)]}{\ln[\mathsf{E}(Y)]} = 1 - \phi.$$

4.3 Aggregation by analogy and aggregation biases

We finally consider two ways of performing simplified, approximate aggregation. They are intended to mimic the aggregation by analogy often used by macro-economists.

First, assume that we perform aggregation in terms of *geometric means*, and instead of using (15) – (18) aggregate the expected log-output and the input and scale elasticities by using the expected micro coefficients in the following way

$$(32) \qquad \mathsf{E}[\widehat{\ln(Y)}] = \bar{\alpha} + \sum_j \bar{\beta}_j \mathsf{E}[\ln(X_j)] + \tfrac{1}{2} \sum_j \sum_k \gamma_{jk} \mathsf{E}[\ln(X_j)] \mathsf{E}[\ln(X_k)],$$

$$(33) \qquad \widehat{\epsilon}_j = \bar{\beta}_j + \sum_k \gamma_{jk} \mathsf{E}[\ln(X_k)],$$

$$(34) \qquad \widehat{\epsilon} = \sum_j \widehat{\epsilon}_j = \sum_j \bar{\beta}_j + \sum_k \left(\sum_j \gamma_{jk} \right) \mathsf{E}[\ln(X_k)].$$

Eq. (32) exemplifies (7). Second, assume that we perform aggregation in terms of *arithmetic means*, and instead of using (23), (27), and (28), or (25), (29), and (30) aggregate the expected log-output and the input and scale elasticities by means of the expected micro coefficients in the following way

$$(35) \qquad \ln[\widehat{\mathsf{E}(Y)}] = \bar{\alpha} + \sum_j \bar{\beta}_j \ln[\mathsf{E}(X_j)] + \tfrac{1}{2} \sum_j \sum_k \gamma_{jk} \ln[\mathsf{E}(X_j)] \ln[\mathsf{E}(X_k)],$$

(36) $$\widehat{\widehat{\epsilon}}_j = \bar{\beta}_j + \sum_k \gamma_{jk} \ln[\mathsf{E}(X_k)],$$

(37) $$\widehat{\widehat{\epsilon}} = \sum_j \widehat{\widehat{\epsilon}}_j = \sum_j \bar{\beta}_j + \sum_k \left(\sum_j \gamma_{jk} \right) \ln[\mathsf{E}(X_k)].$$

Eq. (35) exemplifies (8).

Consider the aggregation biases in three cases.

Case 1: When we perform *aggregation in terms of geometric means* and use (32) – (34) instead of the exact formulae (15) – (18), the aggregation biases will be, respectively,

(38) $e_1 = \mathsf{E}[\ln(Y)] - \mathsf{E}[\widehat{\ln(Y)}] = \sum_j a_j \bar{\beta}_j \mathsf{E}[\ln(X_j)] + \frac{1}{2} \sum_j \sum_k c_{jk} \gamma_{jk} \mathsf{E}[\ln(X_j)]\, \mathsf{E}[\ln(X_k)],$

(39) $f_{1j} = \widetilde{\epsilon}_j - \widehat{\epsilon}_j = a_j \bar{\beta}_j + \sum_k c_{jk} \gamma_{jk} \mathsf{E}[\ln(X_k)],$

(40) $f_1 = \widetilde{\epsilon} - \widehat{\epsilon} = \sum_j f_{1j}.$

Case 2: When we aggregate by means of (35) – (37) instead of using the exact formulae (25), (29), and (30), mixing geometric and arithmetic means, the aggregation biases will be, respectively,

(41) $$e_2 = \mathsf{E}[\ln(Y)] - \ln[\widehat{\mathsf{E}(Y)}] = \textstyle\sum_j (a_j - \theta_j + \mu_{2j}) \bar{\beta}_j \ln[\mathsf{E}(X_j)] + \frac{1}{2} \sum_j \sum_k [c_{jk} - \theta_j - \theta_k + \xi_{2jk}] \gamma_{jk} \ln[\mathsf{E}(X_j)] \ln[\mathsf{E}(X_k)],$$

(42) $$f_{2j} = \epsilon_j^{\Delta} - \widehat{\widehat{\epsilon}}_j = (a_j - \theta_j + \mu_{2j}) \bar{\beta}_j + \textstyle\sum_k [c_{jk} - \theta_j - \theta_k + \xi_{2jk}] \gamma_{jk} \ln[\mathsf{E}(X_k)],$$

(43) $$f_2 = \epsilon^{\Delta} - \widehat{\widehat{\epsilon}} = \textstyle\sum_j f_{2j},$$

where μ_{2j} and ξ_{2jk} are second order, interaction terms given by

$$\mu_{2j} = (1 + a_j)(1 - \theta_j) - (1 + a_j - \theta_j) = -a_j \theta_j,$$
$$\xi_{2jk} = (1 + c_{jk})(1 - \theta_j)(1 - \theta_k) - (1 + c_{jk} - \theta_j - \theta_k) = (1 + c_{jk}) \theta_j \theta_k - c_{jk}(\theta_j + \theta_k).$$

Case 3: When we perform *aggregation in terms of arithmetic means* and use (35) – (37) instead of the exact formulae (23), (27), and (28), the aggregation biases will be, respectively,

(44) $$e_3 = \ln[\mathsf{E}(Y)] - \ln[\widehat{\mathsf{E}(Y)}] = \textstyle\sum_j (a_j + \phi - \theta_j + \mu_{3j}) \bar{\beta}_j \ln[\mathsf{E}(X_j)] + \frac{1}{2} \sum_j \sum_k [c_{jk} + \phi - \theta_j - \theta_k + \xi_{3jk}] \gamma_{jk} \ln[\mathsf{E}(X_j)] \ln[\mathsf{E}(X_k)],$$

(45) $$f_{3j} = \epsilon_j^* - \widehat{\widehat{\epsilon}}_j = (a_j + \phi - \theta_j + \mu_{3j}) \bar{\beta}_j + \textstyle\sum_k [c_{jk} + \phi - \theta_j - \theta_k + \xi_{3jk}] \gamma_{jk} \ln[\mathsf{E}(X_k)],$$

(46) $$f_3 = \epsilon^* - \widehat{\widehat{\epsilon}} = \textstyle\sum_j f_{3j},$$

where μ_{3j} and ξ_{3jk} are second order, interaction terms given by

$$\mu_{3j} = \frac{(1+a_j)(1-\theta_j)}{1-\phi} - (1+a_j+\phi-\theta_j) = \frac{(\phi+a_j)(\phi-\theta_j)}{1-\phi},$$
$$\xi_{3jk} = \frac{(1+c_{jk})(1-\theta_j)(1-\theta_k)}{1-\phi} - (1+c_{jk}+\phi-\theta_j-\theta_k) = \frac{(1+c_{jk})\theta_j\theta_k + (\phi+c_{jk})(\phi-\theta_j-\theta_k)}{1-\theta}.$$

Why is a comparison of these cases interesting? When we proceed from (e_1, f_{1j}, f_1), via (e_2, f_{2j}, f_2) to (e_3, f_{3j}, f_3), a successively increasing number of terms contributing to the aggregation biases are involved: (e_1, f_{1j}, f_1) only include the effect of the correlation between the first order coefficients and the log-inputs as well as the correlation between the log-inputs, (e_2, f_{2j}, f_2) also include the effect of the RLMML-differences of the inputs, and (e_3, f_{3j}, f_3) in addition include the effect of the RLMML-difference of the output via the hybrid parameter ϕ. This algebraic decomposition, of course, says nothing about the numerical size of the respective biases. An empirical investigation is therefore required.

5 Empirical results

Tables A1 and A2 in Appendix B contain estimates of the parameters of the Translog model for the three manufacturing industries, taken from Biørn, Lindquist and Skjerpen (2002). The estimates in Table A1 are utilized when calculating the exact macro elasticities and the aggregation biases. Table A2 shows estimates of the second order moments of the random coefficients and the disturbances. Tables A3 – A5 contain year-specific estimates of the expected log-inputs and of the distributional parameters c_{jk} $(j, k = K, L, E, M)$ [cf. (12)], whereas Table A6 gives estimates of the RLMML-differences of the output and the inputs [cf. (19) and (20)]. In Tables A3 – A6 we utilize empirical analogs in estimating unknown population parameters. On the one hand, the expectation of the log-inputs and their second order moments in year t are represented by the corresponding empirical means and empirical covariances of the cross-section for that year. On the other hand, in estimating the RLMML-differences we represent the expectations of the non-transformed output and input variables, by the corresponding empirical means. Zero correlation between the β coefficients and the logarithms of the inputs is assumed, *i.e.*, all $a_j = 0$. In Tables A7, A9, and A11 we report annual estimates of the different exact macro elasticities (as well as the expectation of the log of output or the log of the expectation of output) for Cases 1 – 3 for Pulp and paper, Chemicals and Basic metals, respectively. The corresponding estimates of the aggregation biases are given in Tables A8, A10 and A12. Tables 3 and 4 contain, for Cases 1 and 3, summary statistics based on the annual values in Tables A7 – A12.

5.1 Aggregation in terms of geometric means (Case 1)

The six first columns of Table 3 contain summary statistics of the annual estimates of the expected log-output and of the scale and input elasticities in Case 1. The six first columns of Table 4 contain corresponding summary statistics for the annual aggregation biases. In each panel of Table 3, the first row gives the mean of the estimates of the expected log-output and the mean of the elasticities, the second row gives the coefficient of variation, while the third and fourth rows contain the minimum and maximum values. Information about whether there is a significant linear trend in the annual estimates is given in the last row. Some of the rows of Table 4 have an interpretation which differs somewhat from those in Table 3. Note in particular that a rescaled version of the coefficient of variation of the aggregation biases is given in the second row and that the regressand in the trend analysis in the last row is the absolute value of the aggregation bias.

The mean macro scale elasticity is between 1.00 and 1.25 in all the industries. The variation of the scale and input elasticities is smallest in Pulp and paper and substantially larger in Basic metals, in particular. For Chemicals, the largest variation is found in the capital elasticity, whereas the labour elasticity varies most for Basic metals. In some cases these two elasticities have the wrong sign.

Summary statistics of the aggregation biases in the case of logarithmic aggregation, computed from (38) – (40) (six first columns of Table 4), show that the bias in the scale elasticity (f_1) is more important for Chemicals and Basic metals than for Pulp and paper. For the two former industries, the aggregation bias is positive in all the years, but shows a clear negative trend. Since $a_j = 0$, all contributions come from the correlation pattern of the log-inputs [cf. (38) – (40) and Tables A3 – A5]. The mean aggregation bias in the capital elasticity (f_{1K}) in Chemicals has about the same size as the bias in the scale elasticity. The biases in the labour and energy elasticities (f_{1L} and f_{1E}) are of opposite sign, but fairly equal in absolute value, whereas the bias in the materials elasticity (f_{1M}) seems negligible. In Pulp and paper, the absolute value of the aggregation bias in the scale elasticity never exceeds 0.01, while among the input elasticities, the largest bias is found for the energy elasticity [mainly within the interval (0.02, 0.03)]. The results for the aggregation bias in the logarithm of expected output (e_1) resemble those for the scale elasticity. The bias is largest for Chemicals (although decreasing over time) and Basic metals. In Pulp and paper the aggregation bias is negative and small in absolute value, below 3 % in all the years.

5.2 Mixed aggregation (Case 2)

The six middle columns of Tables A7, A9, and A11 contain the estimated values of the logarithm of expected output and the scale and input elasticities in Case 2, whereas the six middle columns of Tables A8, A10, and A12 contain the corresponding aggregation bias estimates. Recall that under mixed aggregation, we relate the mean of the log-output to the logarithm of the input aggregates constructed from arithmetic means, and hence remove from the aggregation biases the part which is due to the RLMMML-difference in output [cf. (41) – (43)]. As shown in (31), the scale and input elasticities in Case 2 are obtained by multiplying those in Case 3 (to be commented on below) by $(1-\phi)$, which is year specific and belongs to the interval (0,1) for all years and industries considered (cf. Table A6). This implies that the estimated elasticities in Case 2 are always smaller than in Case 3, which is confirmed from Tables A7, A9, and A11.

For all industries, the estimated aggregation biases in the expected output (e_2) and in the scale elasticity (f_2) are negative in all years. The biases in the input elasticities are also generally negative, the main exception being that the bias in the labour elasticity is positive in Basic metals. From (31), (43), and (46) it follows that $f_2 = f_3 - \phi\epsilon^*$, which implies $f_2 < f_3$ since ϕ and ϵ^* are positive. Thus if f_3 is negative, f_2 will be more strongly negative. If f_3 is positive, f_2 may be of either sign, depending on the size of ϕ and ϵ^*. We find that the sign of the former is positive and the sign of the latter is negative, *i.e.* $\phi\epsilon^* > f_3$, for Pulp and paper and Basic metals. For Chemicals, the same sign conclusion holds in the majority of years.

5.3 Aggregation in terms of arithmetic means (Case 3)

The six last columns of Table 3 contain summary statistics for the estimated log of expected output and the scale and input elasticities when aggregating exactly by means of arithmetic means. The corresponding summary statistics for the aggregation biases are reported in the six last columns of Table 4. Case 3 probably most closely mimics the common way of performing aggregation by using time aggregates or time specific means in establishing production functions. As shown by (44) – (46), the biases in this case are affected not only by the input correlation, as in Case 1, but also by the RLMMML-differences of the inputs and the output (cf. Table A6). Generally, the two latter effects increase the aggregation biases substantially.

Comparing the results for Cases 1 and 3 for *Pulp and paper* (Table 3) we find no large discrepancies between the summary statistics. For instance, the mean estimated scale elasticity is 1.19 in Case 3 and 1.06 in Case 1. We obtain for each year an estimated aggregation bias in the scale elasticity (f_3) in the interval (0.09, 0.17) (Table 4). This indicates that systematic underestimation occurs when we use the estimated mean input parameters in combination with year specific arithmetic means. The main contribution to this bias comes from the bias in the materials elasticity (f_{3M}) and, to a smaller degree, from the biases in the capital and labour elasticities (f_{3K} and f_{3L}). The bias in the energy elasticity (f_{3E}) has the opposite sign, but is rather small. Compared with the case with logarithmic aggregation (Case 1), the bias in output is severe, a sizable underestimation occurs in all the years. To a large extent, this seems to be due to the fact that the exactly aggregated expected output depends on the distribution of the production function disturbance, which is neglected when performing aggregation by means of arithmetic means.

For *Chemicals* we find somewhat larger discrepancies between the mean macro elasticities in Cases 1 and 3 than for Pulp and paper, for instance the mean input elasticities of capital and labour are 0.40 and 0.32 in Case 1 and 0.50 and 0.41 in Case 3. In both cases, the scale elasticity increases over time. Unlike the two other industries, the mean aggregation bias of the scale elasticity are fairly equal in Cases 1 and 3, but its variation is much larger in the latter. Note also that while the bias of the scale elasticity contains a significantly negative trend in Case 1, the trend is positive in Case 3. The aggregation bias in the scale elasticity is negative in some years in Case 3. The bias in the capital elasticity, which is estimated to -0.17 in 1972 and shows a clear positive trend in the subsequent years, contributes substantially to this result. As regards the aggregation bias in output (e_3), overestimation occurs in the first part of the sample period and underestimation in the last part.

Finally, in *Basic metals*, the mean scale elasticity as well as the mean of three of the four input elasticities are somewhat larger in Case 3 than in Case 1. The largest variation is found for the labour elasticity. There is a significantly negative linear trend in the macro scale elasticity in Case 3, as in Case 1. The aggregation bias in the scale elasticity in this industry is substantial. It is largest at the start of the sample period, about 0.5 in 1972, and shows a clear negative trend and is reduced to about 0.1 in 1993. The positive bias in the labour elasticity (f_{3L}) contributes markedly to this result. A clear positive contribution also comes from the aggregation bias in the materials elasticity (f_{3M}). For the energy elasticity (f_{3E}) the bias is clearly negative. The bias in expected output (e_3) indicates a substantial underestimation. However, the aggregation bias decreases clearly over the years. Especially in the first part of the sample period, the aggregation bias in expected output is more dramatic for this industry than in the other two.

6 Conclusion

In this paper, using panel data, we consider aggregation of Translog production functions from the plant to the industry level. Plant specific heterogeneity is represented by both random intercepts and random first order coefficients. We show how exact aggregation in different contexts should be carried out and derive the exact formulae for the scale and input elasticities.

In the empirical part of the paper, the main issue is to estimate aggregation biases in expected output and in the scale and input elasticities. Our exact formulae are compared with formulae typically used at the macro level in analogy to the micro level. Three sources of aggregation bias can be distinguished. Considering aggregation in terms of expectations of logarithms and disregarding correlation between log-inputs and their coefficients, only one source is of relevance, that is the correlation between log-inputs. If we consider aggregation in terms of expectations of non-transformed variables, which most closely resembles the aggregation by analogy often used by practitioners, two more sources should be taken into account. The relative discrepancy between the logs of the arithmetic and the geometric means of the inputs and the output, the RLMML-differences, gives important contributions. An intermediate, mixed case in which we take account of the discrepancy between the means of the inputs only, by aggregating the inputs and the output differently, is also considered.

Our framework is applied to data from plants belonging to three unbalanced panel data sets within Norwegian manufacturing in Pulp and paper, Chemicals, and Basic metals. The Translog function parameters are estimated by Maximum Likelihood from data for the years 1972 – 1993. The different aggregation biases are computed for each year, which, *inter alia*, enables us to detect possible trends in the biases. The means of the estimated macro scale elasticity over this 22 year period varies between 1 and 1.5 across the three industries depending on the aggregation procedure.

There is a substantial difference in the size of the aggregation bias depending on whether aggregation is done in terms of expectations of logarithms or in terms of expectations of non-transformed variables. The genuine disturbance in the Translog function plays a different rôle. In the former case, it has no influence on the estimated aggregation bias, since it has zero expectation, whereas in the latter, it should be taken into account. When using aggregation by analogy in macro-economics, one is in general forced to neglect this bias.

When aggregating in terms of expectations of *log-transformed* variables we find that the absolute value of the bias in the scale elasticity never exceeds 1 % in Pulp and paper. It is somewhat higher in the other two industries, but does not exceed 10 %. In the case with aggregation in terms of expectations of *non-transformed* variables, the aggregation bias of the scale elasticity in Pulp and paper is mainly between 10 and 15 %. For Chemicals, it varies considerably, and for two years the aggregation bias exceeds 30 %. The highest aggregation bias of the scale elasticity is found in Basic metals, where it is as high as 50 % at the beginning of the sample period. Over the years there has been a clear negative trend, so at the end of the sample period the size of the aggregation bias has decreased to almost 10 %. *Systematic* aggregation biases, though, may be partly corrected for by, *e.g.*, calculating the overall values of the correction factors in (16), (24), and (26) from within-sample data. In order to obtain such correction factors, however, access to micro data is necessary, or the data producer has to provide correlation coefficients and ratios between geometric and arithmetic means, etc. Still, substantial year-to-year variation in the input elasticities remains.

Finally, it should be recalled that in the empirical part of the paper, unlike its theoretical part, we only consider models in which the plant specific random coefficients and the log-transformed inputs are assumed to be stochastically independent. In future research it may also be of interest to represent the heterogeneity by fixed plant specific intercepts and slope coefficients. We can then calculate correlations between coefficients and log-transformed inputs. A drawback of this approach is, however, that plants with relatively few observations must be omitted because of the degrees of freedom problem. Simulation studies, including bootstrapping, may also be a fruitful way of carrying this research further.

Table 3. Exact aggregation of production, scale and input elasticities. Summary statistics for the period 1972-1993

	Log-linear aggregation (Case 1)						Linear aggregation (Case 3)					
	E[ln(Y)]	$\tilde{\varepsilon}$	$\tilde{\varepsilon}_K$	$\tilde{\varepsilon}_L$	$\tilde{\varepsilon}_E$	$\tilde{\varepsilon}_M$	ln[E(Y)]	ε^*	ε^*_K	ε^*_L	ε^*_E	ε^*_M
Pulp and paper												
Mean	4.224	1.058	0.263	0.128	0.128	0.539	5.748	1.186	0.304	0.158	0.119	0.605
CV[a]	0.104	0.013	0.016	0.151	0.102	0.024	0.082	0.025	0.023	0.163	0.100	0.029
Min	3.607	1.035	0.255	0.096	0.104	0.519	5.172	1.129	0.288	0.113	0.095	0.579
Max	5.030	1.084	0.270	0.167	0.149	0.570	6.570	1.240	0.316	0.210	0.138	0.640
Trend[b]	Pos.	Neg.	-	Neg.	Pos.	Neg.	Pos.	Neg.	-	Neg.	Pos.	Neg.
Chemicals												
Mean	5.066	1.246	0.404	0.316	0.179	0.346	7.475	1.500	0.498	0.411	0.188	0.404
CV[a]	0.187	0.059	0.745	0.169	0.387	0.314	0.161	0.093	0.769	0.174	0.334	0.307
Min	3.873	1.132	-0.063	0.235	0.070	0.169	6.116	1.314	-0.075	0.302	0.084	0.198
Max	6.830	1.364	0.887	0.434	0.279	0.512	10.097	1.828	1.146	0.552	0.276	0.600
Trend[b]	Pos.	Pos.	Pos.	Neg.	Neg.	Neg.	Pos.	Pos.	Pos.	Neg.	Neg.	Neg.
Basic metals												
Mean	3.466	1.099	0.119	0.163	0.342	0.476	5.598	1.309	0.152	0.242	0.343	0.572
CV[a]	0.095	0.050	0.066	0.842	0.044	0.135	0.026	0.137	0.049	0.875	0.033	0.069
Min	3.010	1.005	0.103	-0.052	0.313	0.375	5.234	1.047	0.134	-0.065	0.325	0.500
Max	4.116	1.175	0.129	0.376	0.362	0.590	5.818	1.583	0.168	0.590	0.368	0.653
Trend[b]	Pos.	Neg.	Pos.	Neg.	Pos.	Pos.	Neg.	Neg.	-	Neg.	-	Pos.

[a] CV is the coefficient of variation.

[b] We estimate regressions of the following type (using the scale elasticity as an example): $\tilde{\xi}_t = a + bt + w_t$, where $\tilde{\xi}_t$ denotes the estimated macro scale elasticity in year t and w_t is an error term. We test H_0 : b=0 against the two-sided alternative H_1: b≠0, using a significance level of 5 percent. If the trend parameter is significant, the estimated sign of b is indicated.

Table 4. Aggregation errors in production, scale and input elasticities. Summary statistics for the period 1972-1993.

	Log-linear aggregation (Case 1)						Linear aggregation (Case 3)					
	e_1	f_1	f_{1K}	f_{1L}	f_{1E}	f_{1M}	e_3	f_3	f_{3K}	f_{3L}	f_{3E}	f_{3M}
Pulp and paper												
Mean	-0.019	0.001	-0.003	-0.011	0.025	-0.011	1.547	0.126	0.038	0.016	-0.019	0.091
CVS[a]	0.001	0.002	0.003	0.015	0.024	0.007	0.038	0.017	0.016	0.059	0.045	0.014
Min	-0.029	-0.003	-0.004	-0.015	0.019	-0.019	1.158	0.092	0.029	-0.001	-0.032	0.076
Max	-0.004	0.008	-0.001	-0.008	0.030	-0.003	2.010	0.164	0.050	0.034	-0.014	0.106
Trend[b]	-	Neg.	-	-	Neg.	Neg.	Neg.	Neg.	-	Neg.	Pos.	-
Chemicals												
Mean	0.245	0.064	0.068	0.014	-0.017	-0.001	0.106	0.069	-0.043	-0.002	0.040	0.073
CVS[a]	0.012	0.015	0.047	0.019	0.032	0.005	0.073	0.073	0.213	0.045	0.044	0.073
Min	0.111	0.025	0.033	0.001	-0.029	-0.003	-0.673	-0.077	-0.166	-0.049	0.026	0.031
Max	0.324	0.089	0.094	0.022	-0.008	0.003	1.377	0.339	0.197	0.028	0.059	0.134
Trend[b]	Neg.	Neg.	Neg.	Neg.	Neg.	Neg.	-	Pos.	-	-	-	Neg.
Basic metals												
Mean	0.102	0.053	0.008	-0.021	0.093	-0.027	1.791	0.294	-0.029	0.284	-0.100	0.139
CVS[a]	0.003	0.005	0.009	0.015	0.037	0.016	0.153	0.102	0.065	0.341	0.048	0.058
Min	0.085	0.043	0.005	-0.028	0.073	-0.038	0.655	0.112	-0.056	0.192	-0.134	0.084
Max	0.138	0.065	0.009	-0.017	0.118	-0.013	3.163	0.507	-0.014	0.438	-0.075	0.188
Trend[b]	Pos.	Neg.	-	-	Neg.	Neg.	Neg.	Neg.	Pos.	Neg.	Pos.	Neg.

[a] CVS is a scaled coefficient of variation. Consider for instance the scale elasticity in Case 1. Let $\tilde{\xi}_t$ denote the mean scale elasticity from Table 3 and let $\bar{f}_1$ denote the mean aggregation bias of the scale elasticity. We then define $CVS = CV * \bar{f}_1 / \tilde{\xi}$.

[b] We estimate regressions of the following type (using the scale elasticity as an example): $|e_{1t}| = c + dt + r_t$, where $|e_{1t}|$ denotes the absolute value of the estimated aggregation bias of the macro scale elasticity in year t and r_t is an error term. We test H_0 : d=0 against the two-sided alternative H_1: d≠0, using a significance level of 5 percent. If the trend parameter is significant, the estimated sign of d is indicated.

Appendix A. Data and empirical variables

We use an unbalanced plant-level panel data set that covers the period 1972 – 1993. The primary data source is the Manufacturing Statistics database of Statistics Norway. Our initial data set includes all large plants, generally defined as plants with five or more employees (ten or more employees from 1992 on), classified under the Standard Industrial Classification (SIC)-codes 341 Manufacture of paper and paper products (Pulp and paper, for short), 351 Manufacture of industrial chemicals (Chemicals, for short) and 37 Manufacture of basic metals (Basic metals, for short). Both plants with contiguous and non-contiguous time series are included.

In the description of the empirical variables below, MS indicates that the data are from the Manufacturing Statistics, and the data are plant specific. NNA indicates that the data are from the Norwegian National Accounts. In this case, the data are identical for plants classified in the same National Account industry. While the plants in our unbalanced panel mainly are collected from 18 different industries at the 5-digit SIC-code level, the plants are classified in 14 different National Account industries. We use price indices from NNA to deflate total material costs, gross investments and fire insurance values. The two latter variables are used to calulate data on capital stocks, applying a variant of the perpetual inventory method.

Y: Output, 100 tonnes (MS)
$K = KB + KM$: Total capital stock (buildings/structures plus
machinery/transport equipment), 100 000 1991-NOK (MS,NNA)
L: Labour input, 100 man-hours (MS)
E: Energy input, 100 000 kWh, electricity plus fuels (excl. motor gasoline) (MS)
$M = CM/QM$: Input of materials (incl. motor gasoline), 100 000 1991-NOK (MS,NNA)
CM: Total material cost (incl. motor gasoline) (MS)
QM: Price of materials (incl. motor gasoline), 1991=1 (NNA)

Output: The plants in the Manufacturing Statistics are in general multi-output plants and report output of a number of products measured in both NOK and primarily tonnes or kg. The classification of products follows The Harmonized Commodity Description and Coding System (HS), and assigns a 7-digit number to each specific commodity. For each plant, an aggregate output measure in tonnes is calculated. Hence, rather than representing output in the three industries by deflated sales, which may be affected by measurement errors [see Klette and Griliches (1996)], our output measures are actual output in physical units, which are in several respects preferable.

Capital stock: The calculations of capital stock data are based on the perpetual inventory method assuming constant depreciation rates. We combine plant data on gross investment with fire insurance values for each of the two categories Buildings and structures and Machinery and transport equipment from the Manufacturing statistics. The data on investment and fire insurance are deflated using industry specific price indices of investment goods from the Norwegian National Accounts (1991=1). The depreciation rate for Buildings and structures is 0.020 in all industries. For Machinery and transport equipment, the depreciation rate is set to 0.040 in Pulp and paper and Basic metals, and 0.068 in Chemicals. For further documentation of the data and the calculations, see Biørn, Lindquist and Skjerpen (2000, Section 4, and 2003).

Other inputs: From the Manufacturing Statistics we get the number of man-hours used, total electricity consumption in kWh, the consumption of a number of fuels in various denominations, and total material costs in NOK for each plant. The different fuels, such as coal, coke, fuelwood, petroleum oils and gases, and aerated waters, are transformed to the common denominator kWh by using estimated average energy content of each fuel [Statistics Norway (1995, p. 124)]. This enables us to calculate aggregate energy use in kWh for each plant. For most plants, this energy aggregate is dominated by electricity. Total material costs is deflated by the price index (1991=1) of material inputs (incl. motor gasoline) from the Norwegian National Accounts. This price is identical for all plants classified in the same National Account industry.

We have removed observations with missing values of output or inputs. This reduced the number of observations by 4 – 8 per cent in the three industries.

Appendix B. Supplementary tables

Table A1. Coefficient estimates for Translog production functions with first order random coefficients						
Coef.	Pulp and paper		Chemicals		Basic metals	
	Estimate	Std. error	Estimate	Std. error	Estimate	Std. error
$\bar{\alpha}^*$	-4.1716	0.7319	1.7184	2.4210	-3.4141	0.7874
κ_1	0.0234	0.0108	-0.0739	0.0361	0.0900	0.0172
$\bar{\beta}_K$	0.5379	0.1984	-1.1664	0.5810	0.3304	0.2201
$\bar{\beta}_L$	0.9479	0.2446	0.5126	0.8271	-0.2718	0.3211
$\bar{\beta}_E$	-0.0255	0.1035	0.1581	0.3527	0.2200	0.1712
$\bar{\beta}_M$	-0.0083	0.1771	0.4553	0.4244	0.9901	0.1712
γ_{KK}	0.0081	0.0421	0.0843	0.1365	-0.0072	0.0452
γ_{LL}	-0.1565	0.0603	-0.1132	0.2235	0.2832	0.0866
γ_{EE}	0.0269	0.0125	-0.0323	0.0436	0.0812	0.0286
γ_{MM}	-0.1298	0.0391	-0.0007	0.0623	0.0880	0.0375
κ_2	0.0013	0.0003	0.0034	0.0013	-0.0024	0.0006
γ_{LK}	-0.0841	0.0399	0.0398	0.1398	-0.0583	0.0495
γ_{EK}	-0.0072	0.0208	-0.0056	0.0625	0.0499	0.0288
γ_{MK}	0.0567	0.0309	0.0344	0.0698	0.0024	0.0353
δ_K	-0.0026	0.0025	0.0397	0.0074	-0.0013	0.0039
γ_{EL}	0.0256	0.0229	0.0770	0.0784	-0.0732	0.0385
γ_{ML}	0.1431	0.0385	0.0034	0.0941	-0.1313	0.0429
δ_L	-0.0058	0.0026	-0.0131	0.0093	-0.0073	0.0045
γ_{ME}	-0.0275	0.0180	-0.0349	0.0484	-0.0267	0.0255
δ_E	0.0028	0.0012	-0.0075	0.0043	-0.0032	0.0021
δ_M	0.0025	0.0020	-0.0165	0.0055	0.0065	0.0024

Table A2. Estimates of second order moments of random coefficients and the variance of the genuine error term			
Parameter	Pulp and paper	Chemicals	Basic metals
$\Omega[1,1]$	5.9590	23.6710	2.7431
$\Omega[2,1]$	-0.4606	-0.2175	-0.0959
$\Omega[3,1]$	-0.7185	-0.8084	-0.6226
$\Omega[4,1]$	0.3611	0.4750	0.2727
$\Omega[5,1]$	0.4157	0.1811	0.1081
$\Omega[2,2]$	0.1182	0.4984	0.1496
$\Omega[3,2]$	-0.0749	-0.2561	-0.5912
$\Omega[4,2]$	-0.4387	-0.0832	-0.6727
$\Omega[5,2]$	-0.5697	-0.3389	0.0952
$\Omega[3,3]$	0.1539	1.2501	0.1422
$\Omega[4,3]$	-0.2442	-0.5478	0.2784
$\Omega[5,3]$	-0.4498	-0.2037	-0.3612
$\Omega[4,4]$	0.0224	0.2660	0.0852
$\Omega[5,4]$	0.1072	-0.3169	-0.6432
$\Omega[5,5]$	0.1045	0.3743	0.1007
σ_{uu}	0.0397	0.2926	0.0968

Table A3. Year-specific estimates of the expectation of log-transformed input variables and of c_{ij}-parameters. Pulp and paper

Year	$E(x_K)$	$E(x_L)$	$E(x_E)$	$E(x_M)$	c_{KK}	c_{LL}	c_{EE}	c_{MM}	c_{LK}	c_{EK}	c_{MK}	c_{EL}	c_{ML}	c_{ME}
1972	6.333	6.944	4.188	5.211	0.084	0.034	0.368	0.116	0.046	0.153	0.086	0.094	0.057	0.177
1973	6.348	6.958	4.165	5.278	0.084	0.033	0.423	0.111	0.046	0.166	0.085	0.098	0.055	0.186
1974	6.256	6.853	4.090	5.107	0.092	0.040	0.441	0.146	0.054	0.183	0.104	0.116	0.070	0.223
1975	6.322	6.800	4.030	4.958	0.087	0.040	0.457	0.155	0.053	0.182	0.105	0.119	0.071	0.234
1976	6.355	6.734	4.077	4.931	0.087	0.041	0.433	0.160	0.054	0.177	0.106	0.115	0.073	0.229
1977	6.510	6.807	4.280	5.037	0.078	0.038	0.376	0.126	0.049	0.155	0.090	0.099	0.063	0.183
1978	6.522	6.716	4.337	5.047	0.076	0.040	0.348	0.127	0.049	0.149	0.089	0.099	0.066	0.184
1979	6.576	6.803	4.407	5.242	0.072	0.038	0.356	0.114	0.047	0.144	0.083	0.095	0.060	0.175
1980	6.715	6.820	4.501	5.399	0.068	0.039	0.320	0.101	0.046	0.132	0.074	0.090	0.055	0.153
1981	6.728	6.776	4.355	5.422	0.069	0.039	0.359	0.103	0.046	0.144	0.077	0.097	0.056	0.167
1982	6.738	6.726	4.245	5.361	0.068	0.039	0.383	0.118	0.045	0.145	0.079	0.101	0.061	0.185
1983	6.964	6.888	4.648	5.658	0.058	0.034	0.285	0.088	0.040	0.114	0.065	0.077	0.047	0.131
1984	6.948	6.914	4.642	5.708	0.058	0.032	0.285	0.084	0.040	0.115	0.064	0.076	0.045	0.132
1985	6.965	6.901	4.720	5.788	0.059	0.033	0.277	0.083	0.041	0.115	0.064	0.077	0.046	0.130
1986	6.987	6.884	4.806	5.730	0.059	0.034	0.269	0.085	0.042	0.114	0.065	0.077	0.048	0.130
1987	7.007	6.861	4.701	5.666	0.058	0.034	0.281	0.093	0.041	0.117	0.068	0.080	0.051	0.140
1988	7.015	6.827	4.600	5.648	0.058	0.034	0.326	0.098	0.040	0.125	0.070	0.083	0.051	0.155
1989	7.054	6.838	4.629	5.747	0.057	0.033	0.317	0.088	0.039	0.123	0.064	0.083	0.048	0.144
1990	6.998	6.779	4.520	5.693	0.059	0.035	0.349	0.091	0.041	0.131	0.067	0.090	0.050	0.151
1991	7.068	6.784	4.702	5.715	0.059	0.035	0.317	0.092	0.041	0.125	0.065	0.084	0.049	0.143
1992	7.328	6.966	5.038	6.001	0.043	0.026	0.213	0.058	0.029	0.087	0.045	0.056	0.033	0.091
1993	7.195	6.901	4.924	6.018	0.051	0.029	0.237	0.059	0.034	0.101	0.050	0.062	0.036	0.098

Table A4. Year-specific estimates of the expectation of log-transformed input variables and of c_{ij}-parameters. Chemicals

Year	$E(x_K)$	$E(x_L)$	$E(x_E)$	$E(x_M)$	c_{KK}	c_{LL}	c_{EE}	c_{MM}	c_{LK}	c_{EK}	c_{MK}	c_{EL}	c_{ML}	c_{ME}
1972	6.757	6.696	5.031	4.714	0.105	0.059	0.290	0.190	0.072	0.153	0.119	0.117	0.094	0.182
1973	6.801	6.716	4.723	4.848	0.092	0.060	0.342	0.180	0.068	0.151	0.107	0.125	0.092	0.173
1974	6.675	6.600	4.567	4.738	0.101	0.064	0.381	0.201	0.073	0.169	0.120	0.138	0.098	0.192
1975	6.654	6.606	4.506	4.732	0.093	0.057	0.365	0.183	0.068	0.160	0.112	0.126	0.091	0.184
1976	6.568	6.559	4.478	4.656	0.101	0.063	0.368	0.226	0.074	0.169	0.132	0.133	0.107	0.222
1977	6.748	6.678	4.844	4.772	0.090	0.058	0.281	0.180	0.067	0.135	0.107	0.108	0.088	0.167
1978	7.025	6.871	5.111	4.912	0.090	0.053	0.258	0.211	0.064	0.135	0.113	0.105	0.091	0.173
1979	7.096	6.886	5.227	5.325	0.084	0.054	0.242	0.178	0.062	0.130	0.103	0.106	0.088	0.161
1980	7.150	6.918	5.224	5.437	0.080	0.051	0.238	0.145	0.059	0.124	0.092	0.101	0.075	0.143
1981	7.207	6.936	5.223	5.475	0.075	0.049	0.242	0.129	0.056	0.122	0.086	0.100	0.067	0.136
1982	7.206	6.940	5.243	5.402	0.074	0.046	0.225	0.140	0.054	0.119	0.089	0.093	0.068	0.140
1983	7.258	6.923	5.392	5.479	0.070	0.046	0.204	0.139	0.052	0.109	0.086	0.088	0.066	0.127
1984	7.343	6.928	5.513	5.744	0.060	0.043	0.193	0.100	0.046	0.087	0.067	0.077	0.055	0.096
1985	7.293	6.906	5.668	5.762	0.063	0.045	0.140	0.103	0.048	0.082	0.069	0.070	0.056	0.086
1986	7.232	6.771	5.602	5.519	0.069	0.047	0.158	0.129	0.053	0.088	0.083	0.073	0.066	0.097
1987	7.629	7.066	5.929	6.062	0.055	0.040	0.141	0.081	0.043	0.076	0.058	0.062	0.046	0.071
1988	7.696	7.085	5.949	6.199	0.050	0.037	0.134	0.069	0.039	0.073	0.050	0.058	0.040	0.063
1989	7.764	7.166	5.930	6.328	0.047	0.036	0.147	0.058	0.037	0.073	0.044	0.059	0.037	0.063
1990	7.844	7.229	6.017	6.335	0.040	0.032	0.124	0.059	0.032	0.060	0.039	0.049	0.034	0.049
1991	7.744	6.979	5.781	6.083	0.040	0.037	0.140	0.070	0.033	0.064	0.044	0.056	0.041	0.064
1992	8.005	7.206	6.085	6.312	0.032	0.025	0.104	0.060	0.023	0.047	0.029	0.038	0.027	0.045
1993	8.016	7.226	6.359	6.327	0.034	0.025	0.070	0.066	0.024	0.036	0.033	0.028	0.029	0.033

Table A5. Year-specific estimates of the expectation of log-transformed input variables and of c_{ij}-parameters. Basic metals

Year	$E(x_K)$	$E(x_L)$	$E(x_E)$	$E(x_M)$	c_{KK}	c_{LL}	c_{EE}	c_{MM}	c_{LK}	c_{EK}	c_{MK}	c_{EL}	c_{ML}	c_{ME}
1972	5.943	7.130	4.382	5.139	0.156	0.063	0.431	0.199	0.090	0.228	0.158	0.151	0.103	0.266
1973	5.922	7.042	4.299	5.172	0.157	0.069	0.461	0.201	0.098	0.243	0.163	0.162	0.108	0.268
1974	6.070	7.168	4.536	5.360	0.144	0.059	0.377	0.172	0.088	0.210	0.146	0.135	0.093	0.226
1975	6.063	7.029	4.210	5.035	0.145	0.066	0.499	0.233	0.092	0.243	0.171	0.165	0.115	0.305
1976	6.194	7.020	4.275	5.079	0.131	0.065	0.463	0.203	0.087	0.224	0.152	0.159	0.105	0.276
1977	6.226	6.914	4.424	4.920	0.132	0.072	0.431	0.229	0.092	0.224	0.162	0.161	0.117	0.284
1978	6.275	6.907	4.544	4.946	0.129	0.061	0.370	0.208	0.084	0.205	0.150	0.138	0.102	0.254
1979	6.415	7.008	4.731	5.170	0.124	0.063	0.364	0.206	0.084	0.200	0.147	0.140	0.102	0.252
1980	6.497	7.041	4.771	5.340	0.119	0.065	0.350	0.176	0.082	0.193	0.133	0.139	0.094	0.227
1981	6.519	7.033	4.776	5.244	0.124	0.063	0.345	0.179	0.084	0.195	0.138	0.136	0.096	0.225
1982	6.548	6.940	4.673	5.154	0.123	0.062	0.355	0.185	0.083	0.196	0.139	0.137	0.096	0.231
1983	6.737	6.943	4.792	5.197	0.104	0.059	0.341	0.191	0.075	0.177	0.127	0.131	0.092	0.224
1984	6.849	7.059	4.999	5.475	0.101	0.058	0.313	0.163	0.074	0.168	0.118	0.126	0.086	0.202
1985	6.962	7.116	5.093	5.636	0.095	0.054	0.275	0.143	0.068	0.149	0.107	0.112	0.078	0.169
1986	7.076	7.189	5.237	5.731	0.090	0.050	0.264	0.138	0.064	0.145	0.102	0.107	0.074	0.164
1987	6.997	7.068	5.115	5.655	0.099	0.056	0.284	0.154	0.071	0.155	0.112	0.117	0.083	0.181
1988	7.099	7.155	5.270	5.814	0.091	0.051	0.262	0.148	0.065	0.143	0.105	0.107	0.078	0.171
1989	7.266	7.131	5.323	5.921	0.077	0.049	0.240	0.136	0.055	0.125	0.088	0.098	0.073	0.154
1990	7.181	7.101	5.288	5.862	0.081	0.050	0.263	0.144	0.061	0.137	0.096	0.103	0.075	0.165
1991	7.217	7.059	5.247	5.786	0.080	0.051	0.274	0.145	0.059	0.136	0.094	0.107	0.078	0.174
1992	7.564	7.361	5.686	6.276	0.060	0.033	0.206	0.092	0.043	0.105	0.065	0.074	0.049	0.115
1993	7.832	7.091	5.133	5.917	0.050	0.037	0.290	0.116	0.029	0.075	0.047	0.094	0.058	0.153

Table A6. Year specific estimates of ϕ and θ_i-parameters

Year	Pulp and paper					Chemicals					Basic metals				
	ϕ	θ_K	θ_L	θ_E	θ_M	ϕ	θ_K	θ_L	θ_E	θ_M	ϕ	θ_K	θ_L	θ_E	θ_M
1972	0.280	0.171	0.092	0.331	0.185	0.341	0.220	0.162	0.395	0.252	0.464	0.267	0.157	0.415	0.285
1973	0.258	0.168	0.092	0.354	0.183	0.341	0.209	0.154	0.348	0.242	0.469	0.269	0.166	0.435	0.283
1974	0.301	0.177	0.102	0.354	0.198	0.391	0.216	0.162	0.376	0.260	0.442	0.254	0.151	0.406	0.262
1975	0.300	0.173	0.102	0.357	0.204	0.386	0.214	0.161	0.370	0.252	0.466	0.255	0.160	0.442	0.287
1976	0.311	0.170	0.105	0.353	0.209	0.385	0.219	0.158	0.384	0.271	0.446	0.243	0.159	0.436	0.279
1977	0.273	0.158	0.097	0.328	0.186	0.365	0.213	0.153	0.366	0.250	0.449	0.241	0.165	0.418	0.295
1978	0.273	0.157	0.100	0.319	0.188	0.335	0.206	0.137	0.308	0.261	0.451	0.237	0.157	0.405	0.283
1979	0.272	0.154	0.095	0.317	0.178	0.337	0.196	0.135	0.297	0.237	0.425	0.228	0.155	0.391	0.275
1980	0.260	0.152	0.095	0.304	0.169	0.312	0.190	0.133	0.283	0.227	0.397	0.224	0.156	0.386	0.260
1981	0.280	0.157	0.100	0.326	0.179	0.305	0.183	0.129	0.291	0.224	0.388	0.227	0.155	0.383	0.265
1982	0.281	0.158	0.103	0.339	0.190	0.307	0.182	0.126	0.279	0.232	0.389	0.227	0.158	0.393	0.264
1983	0.248	0.143	0.093	0.292	0.157	0.310	0.176	0.125	0.273	0.236	0.393	0.209	0.155	0.386	0.266
1984	0.251	0.144	0.091	0.299	0.163	0.300	0.164	0.122	0.257	0.205	0.352	0.204	0.152	0.374	0.250
1985	0.246	0.144	0.092	0.297	0.159	0.295	0.167	0.122	0.232	0.203	0.348	0.196	0.138	0.359	0.233
1986	0.252	0.144	0.095	0.296	0.162	0.347	0.173	0.120	0.232	0.222	0.333	0.190	0.131	0.345	0.223
1987	0.262	0.145	0.095	0.307	0.174	0.240	0.143	0.099	0.200	0.159	0.346	0.199	0.139	0.359	0.237
1988	0.266	0.147	0.096	0.328	0.178	0.273	0.138	0.096	0.199	0.143	0.341	0.191	0.129	0.342	0.231
1989	0.263	0.146	0.096	0.325	0.172	0.285	0.132	0.100	0.216	0.126	0.302	0.175	0.127	0.336	0.225
1990	0.264	0.152	0.101	0.340	0.175	0.298	0.124	0.088	0.192	0.136	0.300	0.185	0.126	0.341	0.229
1991	0.259	0.150	0.099	0.332	0.172	0.363	0.128	0.098	0.206	0.161	0.312	0.183	0.126	0.345	0.237
1992	0.223	0.135	0.082	0.278	0.139	0.344	0.110	0.078	0.173	0.144	0.256	0.155	0.095	0.297	0.182
1993	0.234	0.150	0.090	0.305	0.140	0.276	0.110	0.077	0.135	0.149	0.292	0.127	0.109	0.356	0.217

Table A7. Exact aggregation of production, scale and input elasticities. Pulp and paper

Year	Case 1						Case 2						Case 3					
	E[ln(Y)]	$\tilde{\varepsilon}$	$\tilde{\varepsilon}_K$	$\tilde{\varepsilon}_L$	$\tilde{\varepsilon}_E$	$\tilde{\varepsilon}_M$	E[ln(Y)]	ε^{Δ}	ε^{Δ}_K	ε^{Δ}_L	ε^{Δ}_E	ε^{Δ}_M	ln[E(Y)]	ε^{*}	ε^{*}_K	ε^{*}_L	ε^{*}_E	ε^{*}_M
1972	3.817	1.082	0.267	0.167	0.104	0.544	3.817	0.886	0.221	0.151	0.070	0.444	5.300	1.231	0.307	0.210	0.097	0.616
1973	3.861	1.084	0.267	0.167	0.110	0.541	3.861	0.886	0.222	0.151	0.071	0.442	5.206	1.195	0.299	0.204	0.095	0.596
1974	3.703	1.077	0.263	0.159	0.112	0.543	3.703	0.867	0.217	0.143	0.072	0.435	5.299	1.240	0.310	0.204	0.104	0.622
1975	3.622	1.068	0.258	0.134	0.116	0.560	3.622	0.854	0.213	0.120	0.075	0.446	5.176	1.220	0.305	0.172	0.107	0.637
1976	3.607	1.065	0.259	0.132	0.117	0.557	3.607	0.850	0.215	0.118	0.076	0.441	5.238	1.234	0.313	0.172	0.110	0.640
1977	3.758	1.067	0.255	0.120	0.123	0.570	3.758	0.869	0.215	0.108	0.083	0.464	5.172	1.196	0.295	0.149	0.114	0.638
1978	3.762	1.068	0.260	0.130	0.122	0.557	3.762	0.871	0.219	0.117	0.083	0.452	5.174	1.198	0.302	0.161	0.114	0.622
1979	3.918	1.068	0.261	0.135	0.125	0.547	3.918	0.878	0.221	0.123	0.085	0.450	5.380	1.206	0.303	0.168	0.117	0.617
1980	4.055	1.064	0.265	0.137	0.124	0.539	4.055	0.882	0.225	0.124	0.086	0.447	5.478	1.192	0.304	0.167	0.116	0.604
1981	4.072	1.064	0.270	0.137	0.122	0.535	4.072	0.872	0.227	0.124	0.083	0.439	5.658	1.212	0.316	0.172	0.115	0.610
1982	4.044	1.059	0.270	0.131	0.122	0.536	4.044	0.860	0.227	0.117	0.081	0.434	5.621	1.195	0.316	0.163	0.112	0.604
1983	4.348	1.056	0.269	0.130	0.127	0.530	4.348	0.885	0.230	0.118	0.090	0.447	5.779	1.176	0.306	0.157	0.119	0.594
1984	4.404	1.055	0.267	0.130	0.128	0.530	4.404	0.879	0.228	0.118	0.090	0.443	5.877	1.173	0.305	0.158	0.120	0.591
1985	4.484	1.056	0.269	0.138	0.130	0.519	4.484	0.883	0.230	0.125	0.092	0.436	5.949	1.172	0.305	0.166	0.121	0.579
1986	4.505	1.054	0.264	0.126	0.136	0.528	4.505	0.878	0.226	0.114	0.096	0.442	6.022	1.174	0.302	0.153	0.128	0.591
1987	4.498	1.048	0.261	0.113	0.137	0.537	4.498	0.864	0.223	0.103	0.095	0.443	6.098	1.171	0.303	0.139	0.129	0.601
1988	4.509	1.044	0.262	0.108	0.139	0.535	4.509	0.854	0.224	0.097	0.094	0.440	6.145	1.165	0.305	0.133	0.128	0.600
1989	4.621	1.046	0.264	0.111	0.141	0.530	4.621	0.860	0.225	0.101	0.095	0.439	6.269	1.166	0.306	0.137	0.129	0.595
1990	4.598	1.045	0.263	0.108	0.144	0.531	4.598	0.853	0.223	0.097	0.095	0.438	6.248	1.159	0.303	0.132	0.129	0.596
1991	4.688	1.040	0.260	0.102	0.149	0.529	4.688	0.851	0.221	0.092	0.099	0.438	6.326	1.148	0.299	0.124	0.134	0.591
1992	5.025	1.035	0.259	0.096	0.149	0.532	5.025	0.877	0.224	0.088	0.107	0.458	6.469	1.129	0.288	0.113	0.138	0.590
1993	5.030	1.041	0.262	0.108	0.149	0.522	5.030	0.873	0.222	0.099	0.103	0.449	6.570	1.140	0.290	0.129	0.135	0.586

Table A8. Aggregation errors in production, scale and input elasticities. Pulp and paper

Year	Case 1						Case 2						Case 3					
	e_1	f_1	f_{1K}	f_{1L}	f_{1E}	f_{1M}	e_2	f_2	f_{2K}	f_{2L}	f_{2E}	f_{2M}	e_3	f_3	f_{3K}	f_{3L}	f_{3E}	f_{3M}
1972	-0.011	0.004	-0.001	-0.009	0.026	-0.011	-1.320	-0.194	-0.050	-0.026	-0.041	-0.076	1.775	0.150	0.036	0.032	-0.014	0.096
1973	-0.004	0.008	-0.002	-0.009	0.030	-0.012	-1.330	-0.196	-0.049	-0.033	-0.046	-0.069	1.449	0.112	0.029	0.020	-0.022	0.085
1974	-0.017	0.002	-0.002	-0.008	0.029	-0.017	-1.414	-0.210	-0.050	-0.028	-0.046	-0.086	1.944	0.164	0.044	0.034	-0.015	0.101
1975	-0.018	0.002	-0.002	-0.008	0.030	-0.019	-1.415	-0.215	-0.048	-0.029	-0.047	-0.092	1.881	0.151	0.044	0.023	-0.015	0.099
1976	-0.023	-0.001	-0.001	-0.008	0.028	-0.019	-1.437	-0.220	-0.047	-0.032	-0.048	-0.093	2.010	0.164	0.050	0.022	-0.014	0.106
1977	-0.009	0.005	-0.003	-0.011	0.028	-0.009	-1.300	-0.194	-0.042	-0.023	-0.047	-0.082	1.613	0.133	0.039	0.018	-0.016	0.093
1978	-0.015	0.002	-0.003	-0.011	0.025	-0.009	-1.311	-0.195	-0.043	-0.024	-0.047	-0.082	1.580	0.132	0.040	0.020	-0.015	0.088
1979	-0.015	0.003	-0.003	-0.011	0.027	-0.009	-1.284	-0.189	-0.043	-0.027	-0.046	-0.073	1.633	0.139	0.040	0.019	-0.014	0.095
1980	-0.028	-0.001	-0.004	-0.015	0.025	-0.008	-1.270	-0.181	-0.042	-0.021	-0.045	-0.074	1.504	0.128	0.037	0.022	-0.014	0.083
1981	-0.024	0.001	-0.003	-0.013	0.027	-0.010	-1.342	-0.192	-0.045	-0.027	-0.047	-0.073	1.725	0.148	0.044	0.021	-0.015	0.098
1982	-0.024	0.000	-0.002	-0.010	0.027	-0.015	-1.395	-0.203	-0.046	-0.032	-0.049	-0.076	1.646	0.132	0.042	0.014	-0.018	0.094
1983	-0.029	-0.003	-0.003	-0.013	0.023	-0.010	-1.226	-0.171	-0.037	-0.017	-0.046	-0.070	1.421	0.120	0.038	0.021	-0.017	0.077
1984	-0.024	-0.001	-0.003	-0.012	0.022	-0.009	-1.263	-0.179	-0.042	-0.027	-0.048	-0.062	1.428	0.115	0.034	0.012	-0.018	0.086
1985	-0.026	-0.002	-0.003	-0.012	0.022	-0.009	-1.259	-0.176	-0.041	-0.025	-0.049	-0.061	1.375	0.113	0.034	0.016	-0.019	0.082
1986	-0.020	0.000	-0.003	-0.012	0.022	-0.006	-1.273	-0.176	-0.039	-0.023	-0.052	-0.063	1.430	0.120	0.037	0.016	-0.020	0.086
1987	-0.016	0.001	-0.002	-0.009	0.022	-0.009	-1.327	-0.188	-0.042	-0.031	-0.053	-0.062	1.502	0.119	0.037	0.005	-0.020	0.096
1988	-0.019	0.000	-0.002	-0.009	0.024	-0.014	-1.372	-0.198	-0.042	-0.036	-0.058	-0.061	1.495	0.113	0.039	-0.001	-0.024	0.098
1989	-0.015	0.002	-0.002	-0.009	0.025	-0.011	-1.349	-0.191	-0.042	-0.034	-0.058	-0.058	1.481	0.116	0.039	0.002	-0.024	0.099
1990	-0.016	0.003	-0.003	-0.011	0.028	-0.011	-1.387	-0.195	-0.041	-0.030	-0.061	-0.063	1.427	0.111	0.039	0.005	-0.027	0.094
1991	-0.024	-0.001	-0.003	-0.011	0.026	-0.013	-1.393	-0.196	-0.040	-0.030	-0.064	-0.063	1.353	0.101	0.038	0.002	-0.029	0.090
1992	-0.020	-0.001	-0.002	-0.011	0.019	-0.007	-1.174	-0.160	-0.035	-0.014	-0.056	-0.056	1.158	0.092	0.029	0.012	-0.025	0.076
1993	-0.017	0.001	-0.003	-0.014	0.021	-0.003	-1.253	-0.164	-0.035	-0.005	-0.064	-0.060	1.207	0.103	0.033	0.025	-0.032	0.077

Table A9. Exact aggregation of production, scale and input elasticities. Chemicals

Year	Case 1						Case 2						Case 3					
	E[ln(Y)]	$\tilde{\varepsilon}$	$\tilde{\varepsilon}_K$	$\tilde{\varepsilon}_L$	$\tilde{\varepsilon}_E$	$\tilde{\varepsilon}_M$	E[ln(Y)]	ε^{Δ}	ε^{Δ}_K	ε^{Δ}_L	ε^{Δ}_E	ε^{Δ}_M	ln[E(Y)]	ε^{*}	ε^{*}_K	ε^{*}_L	ε^{*}_E	ε^{*}_M
1972	4.076	1.162	-0.063	0.434	0.279	0.512	4.076	0.866	-0.049	0.364	0.169	0.383	6.182	1.314	-0.075	0.552	0.256	0.581
1973	4.029	1.161	-0.022	0.397	0.278	0.509	4.029	0.885	-0.017	0.335	0.181	0.386	6.116	1.343	-0.027	0.509	0.275	0.586
1974	3.925	1.150	0.006	0.381	0.269	0.494	3.925	0.857	0.005	0.319	0.168	0.365	6.446	1.407	0.007	0.524	0.276	0.600
1975	3.895	1.140	0.038	0.360	0.264	0.478	3.895	0.856	0.030	0.302	0.166	0.358	6.342	1.393	0.048	0.492	0.270	0.582
1976	3.873	1.132	0.074	0.347	0.253	0.458	3.873	0.840	0.058	0.292	0.156	0.334	6.297	1.365	0.094	0.474	0.254	0.544
1977	4.073	1.159	0.126	0.351	0.245	0.437	4.073	0.879	0.099	0.297	0.156	0.327	6.414	1.385	0.156	0.468	0.245	0.516
1978	4.360	1.213	0.203	0.352	0.237	0.421	4.360	0.940	0.161	0.304	0.164	0.311	6.555	1.413	0.242	0.457	0.246	0.468
1979	4.612	1.226	0.259	0.350	0.214	0.402	4.612	0.969	0.208	0.303	0.151	0.307	6.956	1.461	0.314	0.457	0.227	0.463
1980	4.716	1.234	0.304	0.335	0.206	0.388	4.716	0.985	0.246	0.291	0.148	0.300	6.854	1.432	0.358	0.423	0.215	0.436
1981	4.790	1.240	0.346	0.323	0.198	0.373	4.790	0.994	0.283	0.281	0.141	0.290	6.895	1.431	0.407	0.405	0.202	0.417
1982	4.826	1.241	0.383	0.310	0.192	0.356	4.826	0.996	0.313	0.271	0.138	0.273	6.962	1.436	0.452	0.390	0.200	0.395
1983	4.958	1.250	0.425	0.311	0.178	0.337	4.958	1.008	0.350	0.272	0.129	0.257	7.184	1.461	0.507	0.394	0.187	0.373
1984	5.142	1.256	0.471	0.307	0.158	0.320	5.142	1.035	0.394	0.270	0.117	0.254	7.349	1.480	0.563	0.386	0.168	0.364
1985	5.249	1.260	0.508	0.304	0.150	0.298	5.249	1.043	0.423	0.267	0.115	0.238	7.450	1.480	0.600	0.379	0.164	0.337
1986	5.210	1.258	0.535	0.299	0.140	0.283	5.210	1.034	0.443	0.263	0.108	0.220	7.977	1.583	0.678	0.403	0.165	0.338
1987	5.752	1.311	0.624	0.294	0.124	0.268	5.752	1.125	0.535	0.265	0.100	0.225	7.564	1.479	0.703	0.348	0.131	0.296
1988	5.921	1.318	0.670	0.282	0.113	0.253	5.921	1.140	0.578	0.255	0.091	0.216	8.141	1.568	0.795	0.351	0.125	0.298
1989	6.107	1.326	0.720	0.262	0.106	0.238	6.107	1.151	0.625	0.236	0.083	0.208	8.541	1.610	0.874	0.330	0.116	0.291
1990	6.291	1.337	0.763	0.249	0.102	0.223	6.291	1.170	0.668	0.228	0.083	0.193	8.966	1.668	0.952	0.324	0.118	0.274
1991	6.192	1.316	0.777	0.241	0.090	0.209	6.192	1.141	0.677	0.217	0.071	0.175	9.722	1.791	1.063	0.341	0.112	0.275
1992	6.624	1.352	0.844	0.235	0.082	0.191	6.624	1.199	0.752	0.216	0.068	0.164	10.097	1.828	1.146	0.330	0.103	0.249
1993	6.830	1.364	0.887	0.237	0.070	0.169	6.830	1.213	0.790	0.219	0.061	0.144	9.437	1.676	1.092	0.302	0.084	0.198

Table A10. Aggregation errors in production, scale and input elasticities. Chemicals

	Case 1						Case 2						Case 3					
Year	e_1	f_1	f_{1K}	f_{1L}	f_{1E}	f_{1M}	e_2	f_2	f_{2K}	f_{2L}	f_{2E}	f_{2M}	e_3	f_3	f_{3K}	f_{3L}	f_{3E}	f_{3M}
1972	0.324	0.089	0.094	0.021	-0.023	-0.003	-1.927	-0.524	-0.141	-0.237	-0.061	-0.086	-0.673	-0.077	-0.166	-0.049	0.027	0.112
1973	0.296	0.080	0.084	0.020	-0.022	-0.002	-1.735	-0.494	-0.150	-0.174	-0.068	-0.102	-0.469	-0.035	-0.159	0.000	0.026	0.098
1974	0.315	0.087	0.091	0.022	-0.024	-0.002	-1.835	-0.512	-0.158	-0.186	-0.067	-0.100	-0.295	0.039	-0.156	0.019	0.041	0.134
1975	0.286	0.078	0.084	0.020	-0.025	-0.002	-1.803	-0.507	-0.167	-0.176	-0.069	-0.096	-0.284	0.031	-0.148	0.014	0.036	0.129
1976	0.299	0.081	0.092	0.020	-0.029	-0.003	-1.860	-0.518	-0.172	-0.189	-0.060	-0.096	-0.333	0.008	-0.136	-0.007	0.038	0.113
1977	0.276	0.075	0.083	0.016	-0.021	-0.002	-1.853	-0.507	-0.185	-0.195	-0.048	-0.080	-0.316	-0.002	-0.128	-0.024	0.042	0.109
1978	0.309	0.082	0.086	0.020	-0.022	-0.002	-1.803	-0.491	-0.200	-0.159	-0.034	-0.098	-0.297	-0.018	-0.119	-0.006	0.048	0.059
1979	0.301	0.080	0.082	0.019	-0.020	-0.003	-1.741	-0.463	-0.202	-0.152	-0.027	-0.083	-0.081	0.029	-0.096	0.003	0.050	0.073
1980	0.291	0.077	0.078	0.019	-0.018	-0.002	-1.696	-0.447	-0.206	-0.136	-0.027	-0.078	-0.162	-0.001	-0.094	-0.005	0.040	0.058
1981	0.275	0.071	0.073	0.019	-0.019	-0.003	-1.701	-0.442	-0.209	-0.141	-0.022	-0.070	-0.177	-0.005	-0.085	-0.017	0.040	0.058
1982	0.269	0.069	0.073	0.018	-0.020	-0.002	-1.699	-0.439	-0.216	-0.133	-0.018	-0.071	-0.148	0.002	-0.077	-0.014	0.043	0.050
1983	0.260	0.067	0.070	0.017	-0.017	-0.001	-1.705	-0.431	-0.223	-0.134	-0.008	-0.066	-0.074	0.021	-0.066	-0.012	0.050	0.049
1984	0.223	0.057	0.061	0.013	-0.016	-0.001	-1.595	-0.402	-0.217	-0.125	-0.009	-0.052	0.037	0.042	-0.048	-0.009	0.041	0.058
1985	0.244	0.066	0.063	0.011	-0.009	0.001	-1.548	-0.393	-0.225	-0.112	-0.004	-0.052	0.084	0.045	-0.047	0.000	0.045	0.048
1986	0.264	0.072	0.069	0.012	-0.013	0.003	-1.564	-0.401	-0.233	-0.115	0.002	-0.055	0.468	0.148	0.002	0.024	0.059	0.062
1987	0.222	0.057	0.057	0.011	-0.012	0.001	-1.340	-0.339	-0.203	-0.100	-0.001	-0.035	0.026	0.016	-0.034	-0.016	0.030	0.036
1988	0.203	0.051	0.052	0.009	-0.011	0.001	-1.308	-0.329	-0.202	-0.099	0.000	-0.028	0.366	0.099	0.015	-0.003	0.034	0.053
1989	0.183	0.045	0.049	0.010	-0.013	-0.001	-1.320	-0.325	-0.201	-0.102	-0.006	-0.017	0.516	0.134	0.049	-0.008	0.027	0.066
1990	0.165	0.040	0.042	0.008	-0.010	0.001	-1.268	-0.309	-0.200	-0.093	0.004	-0.020	0.747	0.189	0.084	0.003	0.039	0.062
1991	0.149	0.036	0.042	0.007	-0.013	0.000	-1.348	-0.328	-0.215	-0.096	0.006	-0.023	1.261	0.323	0.171	0.028	0.047	0.077
1992	0.111	0.025	0.033	0.005	-0.012	-0.001	-1.215	-0.290	-0.197	-0.085	0.011	-0.019	1.377	0.339	0.197	0.028	0.046	0.066
1993	0.131	0.032	0.036	0.001	-0.008	0.002	-1.185	-0.281	-0.202	-0.068	0.013	-0.024	0.752	0.183	0.100	0.016	0.036	0.031

Table A11. Exact aggregation of production, scale and input elasticities. Basic metals

Year	Case 1						Case 2						Case 3					
	E[ln(Y)]	$\tilde{\varepsilon}$	$\tilde{\varepsilon}_K$	$\tilde{\varepsilon}_L$	$\tilde{\varepsilon}_E$	$\tilde{\varepsilon}_M$	E[ln(Y)]	ε^{Δ}	ε^{Δ}_K	ε^{Δ}_L	ε^{Δ}_E	ε^{Δ}_M	ln[E(Y)]	ε^{*}	ε^{*}_K	ε^{*}_L	ε^{*}_E	ε^{*}_M
1972	3.010	1.175	0.110	0.376	0.315	0.375	3.010	0.849	0.080	0.316	0.184	0.268	5.613	1.583	0.150	0.590	0.344	0.500
1973	3.023	1.169	0.109	0.348	0.317	0.396	3.023	0.832	0.080	0.290	0.179	0.284	5.690	1.567	0.150	0.546	0.337	0.534
1974	3.246	1.152	0.111	0.325	0.313	0.402	3.246	0.841	0.083	0.276	0.186	0.297	5.818	1.508	0.148	0.495	0.333	0.532
1975	3.024	1.166	0.103	0.339	0.321	0.403	3.024	0.828	0.077	0.285	0.179	0.288	5.667	1.552	0.144	0.534	0.335	0.539
1976	3.101	1.155	0.104	0.316	0.321	0.413	3.101	0.824	0.079	0.266	0.181	0.298	5.593	1.487	0.143	0.480	0.327	0.538
1977	3.055	1.157	0.116	0.289	0.339	0.413	3.055	0.818	0.088	0.241	0.197	0.291	5.548	1.485	0.160	0.438	0.358	0.529
1978	3.113	1.149	0.121	0.261	0.339	0.427	3.113	0.821	0.092	0.220	0.202	0.306	5.672	1.496	0.168	0.401	0.368	0.558
1979	3.325	1.135	0.124	0.231	0.345	0.436	3.325	0.817	0.095	0.195	0.210	0.316	5.779	1.419	0.166	0.339	0.365	0.550
1980	3.454	1.122	0.122	0.210	0.339	0.451	3.454	0.813	0.094	0.177	0.208	0.333	5.732	1.350	0.157	0.294	0.346	0.553
1981	3.405	1.119	0.120	0.209	0.342	0.449	3.405	0.810	0.093	0.176	0.211	0.330	5.561	1.323	0.152	0.288	0.344	0.539
1982	3.319	1.117	0.119	0.191	0.343	0.465	3.319	0.803	0.092	0.161	0.208	0.342	5.431	1.314	0.150	0.263	0.341	0.560
1983	3.402	1.114	0.123	0.158	0.355	0.478	3.402	0.799	0.097	0.134	0.218	0.351	5.604	1.317	0.160	0.220	0.359	0.578
1984	3.608	1.090	0.124	0.127	0.353	0.486	3.608	0.792	0.099	0.108	0.221	0.364	5.565	1.221	0.152	0.166	0.341	0.562
1985	3.719	1.074	0.122	0.105	0.347	0.500	3.719	0.795	0.098	0.090	0.222	0.384	5.702	1.218	0.150	0.138	0.341	0.589
1986	3.809	1.065	0.125	0.085	0.353	0.503	3.809	0.797	0.101	0.074	0.231	0.390	5.712	1.194	0.151	0.111	0.347	0.585
1987	3.683	1.059	0.124	0.066	0.349	0.521	3.683	0.777	0.099	0.057	0.224	0.397	5.629	1.187	0.151	0.086	0.342	0.607
1988	3.798	1.046	0.125	0.043	0.349	0.529	3.798	0.775	0.101	0.037	0.230	0.406	5.765	1.176	0.154	0.057	0.349	0.617
1989	3.869	1.034	0.127	0.007	0.351	0.549	3.869	0.770	0.105	0.006	0.233	0.426	5.545	1.103	0.150	0.009	0.334	0.610
1990	3.788	1.038	0.127	0.002	0.354	0.555	3.788	0.766	0.104	0.002	0.233	0.428	5.409	1.094	0.148	0.002	0.333	0.611
1991	3.686	1.032	0.126	-0.006	0.355	0.558	3.686	0.756	0.103	-0.006	0.232	0.426	5.359	1.099	0.150	-0.008	0.338	0.619
1992	4.116	1.014	0.129	-0.041	0.362	0.566	4.116	0.788	0.109	-0.037	0.254	0.463	5.529	1.059	0.146	-0.050	0.341	0.622
1993	3.706	1.005	0.109	-0.052	0.358	0.590	3.706	0.741	0.095	-0.046	0.230	0.462	5.234	1.047	0.134	-0.065	0.325	0.653

Table A12. Aggregation errors in production, scale and input elasticities. Basic metals

Year	Case 1						Case 2						Case 3					
	e_1	f_1	f_{1K}	f_{1L}	f_{1E}	f_{1M}	e_2	f_2	f_{2K}	f_{2L}	f_{2E}	f_{2M}	e_3	f_3	f_{3K}	f_{3L}	f_{3E}	f_{3M}
1972	0.085	0.056	0.008	-0.022	0.105	-0.035	-2.316	-0.227	-0.089	0.165	-0.234	-0.069	3.163	0.507	-0.019	0.438	-0.075	0.162
1973	0.094	0.062	0.007	-0.021	0.112	-0.037	-2.391	-0.240	-0.095	0.162	-0.246	-0.062	3.139	0.494	-0.025	0.418	-0.088	0.188
1974	0.087	0.054	0.007	-0.021	0.099	-0.031	-2.235	-0.223	-0.092	0.166	-0.239	-0.059	2.844	0.444	-0.026	0.385	-0.091	0.176
1975	0.091	0.065	0.009	-0.028	0.118	-0.035	-2.359	-0.240	-0.096	0.171	-0.247	-0.069	2.986	0.483	-0.029	0.420	-0.090	0.182
1976	0.096	0.062	0.008	-0.022	0.111	-0.035	-2.314	-0.240	-0.095	0.166	-0.247	-0.065	2.610	0.423	-0.031	0.380	-0.101	0.175
1977	0.096	0.056	0.008	-0.021	0.106	-0.038	-2.357	-0.245	-0.089	0.161	-0.237	-0.081	2.552	0.422	-0.017	0.358	-0.076	0.157
1978	0.092	0.052	0.009	-0.024	0.097	-0.030	-2.292	-0.240	-0.090	0.168	-0.242	-0.076	2.651	0.435	-0.014	0.349	-0.076	0.176
1979	0.099	0.053	0.009	-0.024	0.097	-0.030	-2.255	-0.230	-0.087	0.166	-0.233	-0.076	2.300	0.373	-0.017	0.310	-0.078	0.157
1980	0.111	0.055	0.009	-0.017	0.095	-0.031	-2.174	-0.222	-0.083	0.156	-0.228	-0.067	1.948	0.314	-0.021	0.273	-0.091	0.153
1981	0.101	0.052	0.008	-0.020	0.095	-0.032	-2.182	-0.224	-0.083	0.159	-0.227	-0.073	1.719	0.289	-0.024	0.270	-0.093	0.136
1982	0.103	0.054	0.008	-0.022	0.098	-0.030	-2.187	-0.231	-0.084	0.156	-0.234	-0.068	1.650	0.280	-0.026	0.258	-0.101	0.149
1983	0.117	0.057	0.009	-0.022	0.095	-0.023	-2.183	-0.226	-0.085	0.164	-0.232	-0.073	1.752	0.291	-0.022	0.250	-0.091	0.154
1984	0.107	0.051	0.008	-0.021	0.090	-0.026	-2.148	-0.220	-0.085	0.164	-0.232	-0.067	1.246	0.210	-0.031	0.222	-0.112	0.131
1985	0.104	0.048	0.006	-0.018	0.082	-0.023	-2.024	-0.206	-0.085	0.176	-0.229	-0.067	1.357	0.218	-0.033	0.224	-0.110	0.137
1986	0.103	0.047	0.008	-0.021	0.082	-0.022	-1.959	-0.197	-0.082	0.176	-0.224	-0.067	1.243	0.201	-0.032	0.213	-0.108	0.128
1987	0.095	0.046	0.007	-0.022	0.084	-0.023	-2.068	-0.208	-0.086	0.187	-0.230	-0.079	1.245	0.203	-0.034	0.217	-0.112	0.131
1988	0.092	0.043	0.007	-0.024	0.080	-0.020	-2.010	-0.195	-0.084	0.198	-0.221	-0.088	1.290	0.207	-0.031	0.217	-0.102	0.123
1989	0.104	0.045	0.008	-0.018	0.073	-0.018	-1.951	-0.188	-0.082	0.200	-0.218	-0.088	0.836	0.145	-0.036	0.203	-0.118	0.097
1990	0.115	0.050	0.008	-0.023	0.083	-0.018	-1.987	-0.188	-0.085	0.212	-0.221	-0.095	0.733	0.140	-0.041	0.213	-0.121	0.088
1991	0.107	0.048	0.009	-0.023	0.084	-0.021	-2.033	-0.193	-0.086	0.221	-0.221	-0.108	0.807	0.150	-0.039	0.219	-0.116	0.086
1992	0.138	0.053	0.009	-0.020	0.076	-0.013	-1.643	-0.152	-0.079	0.205	-0.202	-0.075	0.655	0.119	-0.042	0.192	-0.115	0.084
1993	0.112	0.050	0.005	-0.019	0.077	-0.014	-1.924	-0.193	-0.095	0.230	-0.229	-0.099	0.672	0.112	-0.056	0.211	-0.134	0.092

References

Allen, R.G.D. (1964): *Mathematical Economics.* London: Macmillan.

Biørn, E., Lindquist, K.-G., and Skjerpen, T. (2000): Micro Data on Capital Inputs: Attempts to Reconcile Stock and Flow Information. Discussion Paper No. 268, Statistics Norway.

Biørn, E., Lindquist, K.-G., and Skjerpen, T. (2002): Heterogeneity in Returns to Scale: A Random Coefficient Analysis with Unbalanced Panel Data. Forthcoming in *Journal of Productivity Analysis.*

Biørn, E., Lindquist, K.-G., and Skjerpen, T. (2003): Random Coefficients and Unbalanced Panels: An Application on Data From Norwegian Chemical Plants. Forthcoming in *Annales d'Économie et de Statistique.*

Evans, M., Hastings, N., and Peacock, B. (1993): *Statistical Distributions*, Second Edition. New York: Wiley.

Fortin, N.M. (1991): Fonctions de production et biais d'agrégation. *Annales d'Économie et de Statistique*, 20/21, 41-68.

Gorman, W.M. (1981): Some Engel Curves. In Deaton, A.S. (ed.): Essays in the Theory and Measurement of Consumer Behaviour. Cambridge: Cambridge University Press, pp. 7-29.

Green, H.A.J. (1964): *Aggregation in Economic Analysis: An Introductory Survey.* Princeton: Princeton University Press.

Hildenbrand, W. (1981): Short-Run Production Functions Based on Microdata. *Econometrica*, 49, 1095-1125.

Hildenbrand, W. (1998): How Relevant Are Specifications of Behavioral Relations on the Micro Level for Modelling the Time Path of Population Aggregates? *European Economic Review*, 42, 437-458.

Hildenbrand, W., and Kneip, A. (1999): Demand Aggregation under Structural Stability. *Journal of Mathematical Economics*, 31, 81-111.

Johansen, L. (1972): *Production Functions. An Integration of Micro and Macro, Short Run and Long Run Aspects.* Amsterdam: North-Holland.

Jorgenson, D.W. (1986): Econometric Methods for Modeling Producer Behavior. Chapter 31 in: Griliches, Z. and Intriligator, M.D. (eds.): *Handbook of Econometrics, Vol. III.* Amsterdam: North-Holland.

Jorgenson, D.W., Lau, L.J., and Stoker, T.M. (1982): The Transcendental Logarithmic Model of Aggregate Consumer Behavior. In Basmann, R.L., and Rhodes, C. (eds.): *Advances in Econometrics*, vol. 1. Greenwich, CT: JAI Press, pp. 97-238.

Klein, L.R. (1946a): Macroeconomics and the Theory of Rational Behavior. *Econometrica*, 14, 93-108.

Klein, L.R. (1946b): Remarks on the Theory of Aggregation. *Econometrica*, 14, 303-312.

Klein, L.R. (1974): *A Textbook of Econometrics.* Englewood Cliffs: Prentice-Hall.

Klette, T.J. and Griliches, Z. (1996): The Inconsistency of Common Scale Estimators when Output Prices Are Unobserved and Endogenous. *Journal of Applied Econometrics*, 11, 343-361.

Koebel, B. (1998): Tests of Representative Firm Models: Results for German Manufacturing Industries. *Journal of Productivity Analysis*, 10, 251-270.

Lau, L.J. (1982): A Note on the Fundamental Theorem of Exact Aggregation. *Economics Letters*, 9, 119-126.

Lewbel, A. (1992): Aggregation with Log-Linear Models. *Review of Economic Studies*, 59, 635-642.

Littell, R.C., Milliken, G.A., Stroup, W.W., and Wolfinger, R.D. (1996): *SAS System for Mixed Models*. Cary, NC: SAS Institute.

Muellbauer, J. (1975): Aggregation, Income Distribution and Consumer Demand. *Review of Economic Studies*, 62, 525-543.

Muysken, J. (1987): The Distribution Approach to the Aggregation of Production Functions. *Recherches Économiques de Louvain*, 53, 269-282.

Sato, K. (1975): *Production Functions and Aggregation*. Amsterdam: North-Holland.

Statistics Norway (1995): *Energy Statistics 1995*. Official Statistics of Norway, NOS C 347.

Stoker, T.M. (1984): Completeness, Distribution Restrictions, and the Form of Aggregate Functions. *Econometrica*, 52, 887-907.

Stoker, T.M. (1993): Empirical Approaches to the Problem of Aggregation Over Individuals. *Journal of Economic Literature*, 31, 1827-1874.

Sydsæter, K., Strøm, A., and Berck, P. (1999): *Economists' Mathematical Manual*, Third Edition. Berlin: Springer.

Theil, H. (1971): *Principles of Econometrics*. New York: Wiley.

Wet, G.L. de (1976): The Production Function and Aggregation. A Further Comment. *South African Journal of Economics*, 44, 304-312.

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