John K. Dagsvik and Astrid L. Mathiassen

Agricultural Production with Uncertain Water Supply

Abstract:
The purpose of this paper is to develop a framework for analysis of multioutput agricultural production when the supply of water is uncertain. Specifically, we assume that the farmer operates as if the decision process takes place in two stages. In stage one the farmer decides how much land to allocate to each crop. However, in this stage he is uncertain about the supply of water during the growth period before harvest. In the second stage when the uncertainty is revealed he adjusts the quantities of (ex-post) input factors (given the allocation in the first stage). The production technology is assumed to be of the Leontief type. We also extend the model to the case with several seasons where one crop is cultivated throughout all season while the remaining crops are seasonal-specific. The empirical model is extended to allow for a particular version of bounded rationality in which the farmer is allowed to make optimization error. This implies that the estimation procedure is considerably simplified.

Address: John K. Dagsvik, Statistics Norway, Research Department.
E-mail: john.dagsvik@ssb.no

1. Introduction

In this paper we develop a modeling framework for analyzing multicrop agricultural production in the presence of uncertainty with respect to supply of water with land as a fixed factor. Specifically, we consider a situation where the decisions of the production process of the farmer can be viewed as a two stage process as follows: In stage one he allocates the appropriate land areas-and other (ex-ante) input factors to the respective crops he plans to cultivate, within the total farmland available to him. This allocation must be carried out before it is revealed how much water is available to him, since he needs to plant and distribute seeds in the beginning of the respective seasons. In the second stage the uncertainty about water supply is revealed and the farmer adjusts the quantities of the ex-post input factors including the allocation of water to the respective cropland areas. Typical ex-post input factors (in addition to water) are pesticides, labor for treshing, etc. The farmer is risk neutral and the production technology is of the Leontief type. Our motivation for this assumption is the fact that in agricultural production there are typically limited possibilities for substitution among key (crop specific) production factors. In particular, the amount of water needed for a given land area and a given crop is approximately fixed.

In the literature on economic analysis of agricultural production a number of studies have considered the issue of modeling multicrop production. Often a production function based on aggregated farm inputs (aggregated across crops) is applied. This is motivated by the fact that data on crop-specific inputs are often not available, see for example Chambers and Just (1989). Just, Silberman and Hockman (1983), and Just et al. (1990) discuss whether separability or nonjointness is a better approach for attaining tractability for multicrop production function estimation. Shumway, Pope and Nash (1984), and Chambers and Just (1989)) discuss the implication for the modeling approach when jointness is a consequence of a fixed but allocable input factor such as land. Moore, Gollehon and Carey (1994a) discuss alternative models of input allocations of multicrop systems, and Moore, Gollehon and Carey (1994b) have conducted an empirical analysis on multicrop irrigated production under the assumption of nonjointness and land constraints with particular focus on the role of water pricing. Moore and Dinar (1995), discuss the case when water is quantity rationed, either due to subsidized pricing or institutional regulations on water supply and they treat water as a fixed rather than a variable input factor. Ballivian and Sickles (1994), analyze the relationship between risk-avoidance behavior and economic jointness in a multioutput agricultural technology. They treat production uncertainty as unobserved stochastic error that is common to all farms in specific regions.

The difference between our approach and the ones referred to above is as follows: We formulate an explicit model in which the farmer's uncertainty with respect to water supply, rationing of

land and the possibility of corner solutions are accounted for. None of the papers referred to above allow for all these aspects in one and the same model. Furthermore, we demonstrate how an empirical version of the model can be estimated from appropriate micro data.

An additional aspect that separates this paper from the mainstream approach to this type of modeling challenges is that we propose a particular form of *bounded rationality*. This means that we allow for optimization error. The motivation for this is two-fold. First, it is the acknowledgement that the farmer may have difficulty with evaluating expected profit conditional on the allocation of land areas to each crop. Furthermore, he may, due to limited computational capacities, not be able to (or care to) even consider evaluating expected utility conditional at all feasible combinations of cropland allocations. Second, this approach is very useful for practical empirical analysis because it yields a convenient representation for the probability distribution of the *chosen* allocation of cropland. By adapting a result of Dagsvik (1994) we demonstrate that one can obtain convenient expressions for the joint probability density for the *chosen* allocation of cropland areas. This choice probability density reflects the inherent randomness in the decision making process. This randomness, due to optimization error, implies that the farmer may make different ex ante allocation decisions under identical conditions.

The paper is organized as follows: In Section 2 we specify the setting and production technology and in Section 3 we derive the conditional expected profit function given the respective areas that are allocated to each crop for the case with two feasible crops and we derive some implications from the model. In Section 4 we extend this analysis to the general case with several feasible crops. In the appendix we consider briefly a particular case with more than one season. In tropical environments a year normally consists of three seasons; monsoon, winter and summer. We take into account that some crops (such as sugarcane) needs more that one season to grow, while most crops ripen within one season. In Section 5 we discuss the application of our approach to *bounded rationality* with particular focus on empirical modeling and estimation. This yields a convenient representation for the probability distribution of the *chosen* allocation of cropland. As a result, the estimation method based on the bounded rationality approach represents an enormous simplification of the estimation procedure.

2. The modeling setting

The model we develop below applies to the dry season, where water is assumed to be distributed through an irrigation system. In this farming context the farmer's decision process takes place in two stages. In stage one he decides how much land to allocate to each crop. In this stage he is uncertain about how much water he will receive during the season. In the second stage he receives the water and

allocation of water takes place. We assume that the farmer's production technology is Leontief. Since there is no scope for substitution between land and water, it follows that the profit maximizing farmer distributes water to the different crops in a hierarchical manner. First, he allocates the appropriate amount of water (if available) to the crop with the highest marginal returns (crop 1). If some water is left it is distributed in an appropriate amount to the crop with the second highest marginal returns (crop 2). If there still is some water left it is distributed to the crop with the third highest marginal returns (crop 3), and so on, until there is no water left or his entire field is irrigated. In the first stage, when the agent allocates the cropland areas, we assume that he takes into account the second stage regime of water allocation. It is also assumed that some of the input factors are chosen in the second stage when the uncertainty about water supply is revealed. The uncertainty the farmer faces is due to several factors. One important source of uncertainty stems from the fact that the weather is stochastic. In many regions it typically rains heavily during a relatively short period of time and water is therefore collected in regional reservoirs or in private wells. These reservoirs are exposed to evaporation and percolation losses. Further sources of uncertainty about the water supply are related to possible corrupt behavior among reservoir administrators, farmers that may illegally tap water, and irregularity in the supply of water dependent on the location of the farm relative to the reservoirs. As a result, the farmer does not know exactly how much water is available to him during the growth period until harvest. The farmer is supposed to have information about the degree of uncertainty of the water supply in the second stage, and is capable of representing this information by a probability distribution function. The production technology parameters may possibly also be uncertain to the farmer, but we shall assume here that the farmer ignores this type of uncertainty.

We assume that there is, conditional on farm production equipment, no production of scope effect. Let x_{rj} denote quantity of input factor r associated with output of type j, $r = -s, -s+1, ..., q$. We shall let x_{0j} represent area of land and x_{1j} represents quantity of water that affects output of type j (sugarcane, maize, rice, etc.). Let x_{rj}, with $r \leq 1$ represent inputs of factors associated with the first stage allocation, such as seeds, etc. Thus, the variables $x_{rj}, r \in \{-s, -s+1, ..., 1\}$ represent inputs of factors, associated with the first stage allocation, while $r = 2, ..., q$, represent second stage input factors such as water, labor, etc.

The Leontief production technology implies that output y_j of type j is given by

(2.1)
$$y_j = \min_{-s \leq r \leq q} \left(b_{rj} x_{rj} \right), \quad b_{rj} > 0$$

where b_{rj} is the marginal productivity of input factor r for production of crop j.

We mainly consider the case where agricultural production takes place in only one season. In the appendix we outline how one could allow for several seasons where the cultivation of some crops may extend over several seasons.

3. The case with two feasible crops

In this section we shall derive the conditional expected profit function given the respective inputs of cropland, and we shall subsequently discuss some implications of this function.

For simplicity we start the analysis by considering the case with only two possible crops. Recall that the farmer's decision process takes place in two stages. In the first stage the area of cropland is allocated by maximizing expected profit with respect to cropland areas, as will now be explained. Let $\tilde{\pi}(\mathbf{x})$ denote the conditional profit function given the respective quantities of input, i.e.

$$(3.1) \qquad \tilde{\pi}(\mathbf{x}) = \sum_{j=1}^{2} \left[p_j \min_r \left(b_{rj} x_{rj} \right) - \sum_{r=-s}^{q} w_{rj} x_{rj} \right]$$

where $\mathbf{x} = \left(x_{-s1}, x_{-s+1,1}, ..., x_{01}, x_{11}, x_{21}, ..., x_{02}, x_{12}, ..., x_{q2} \right)$, p_j is the output price of crop j, and w_{rj} is the price of input r of crop j. Prices and technology parameters are assumed known with perfect certainty to the farmer.

The constraints are given by

$$(3.2) \qquad x_{01} + x_{02} \leq a$$

and

$$(3.3) \qquad x_{11} + x_{12} \leq V$$

where a is the area of the land available for crop cultivation and V is the total amount of water. V is a random variable with c.d.f. $F(v)$, $v \geq 0$. The distribution F is the subjective distribution specific to the farmer. Thus, this setup means that we assume that the farmer is capable of representing his uncertainty about water supply by a probability distribution.

Assuming risk neutrality, the agent maximizes the conditional expected profit (given the allocation of water to the crops) with respect to land allocation. Due to the Leontief technology (2.1) the demands satisfy the equations

$$(3.4) \qquad x_{rj} = \frac{b_{0j} x_{0j}}{b_{rj}}, \quad r = -s, ..., -1, \text{ and } x_{rj} = \frac{b_{1j} x_{1j}}{b_{rj}}, \quad r = 2, .., q.$$

Hence, allocation of land determines input of ex-ante input factors. Allocation of water determines use of ex-post input factors. Assume now that the farmer pays for water according to how much he plants. This is realistic if the farmer gets water from a surface system, because there are usually no volumetric measures installed[1]. Thus, expected profit conditional on (x_{01}, x_{02}) equals

(3.5)
$$\pi^*(x_{01}, x_{02}) = \sum_{j=1}^{2}\left(p_j E \min(b_{0j}x_{0j}, b_{1j}x_{1j}) - \sum_{r>1}^{q} w_{rj} E x_{rj} - \sum_{r=-s}^{1} w_{rj} x_{rj} \right)$$
$$= \sum_{j=1}^{2}\left(p_j b_{1j} E \min(c_j x_{0j}, x_{1j}) - b_{1j} E x_{1j} \sum_{r>1} \frac{w_{rj}}{b_{rj}} - \sum_{r=-s}^{1} x_{rj} w_{rj} \right)$$

where $c_j = b_{0j}/b_{1j}$. The interpretation of c_j is the required amount of water per unit land (cubic meter per hectare). If cropland area no. one is the principal cropland we have that

(3.6)
$$x_{11} = \min(z_1, V) \quad \text{and} \quad x_{12} = \min(z_2, (V - z_1)_+),$$

where X_+ means $\max(0, X)$ and $z_j = c_j x_{0j}$, $j = 1, 2$. We shall call z_1 and z_2 the respective water equivalents.

We can now prove the following result:

Proposition 1

Let M_j be defined by

(3.7)
$$M_j = p_j b_{1j} - b_{1j} \sum_{r>1}^{q} \frac{w_{rj}}{b_{rj}},$$

and assume that $M_1 > M_2$.. The conditional profit function $\pi(z_1, z_2)$ given the input of water equivalents, (z_1, z_2), equals

(3.8)
$$\pi(z_1, z_2) \equiv \pi^*\left(\frac{z_1}{c_1}, \frac{z_2}{c_2}\right) = M_1 E \min(z_1, V) + M_2 E \min(z_2, (V - z_1)_+) - h_1 z_1 - h_2 z_2,$$

where

---

[1] If in contrast, irrigation depends on groundwater from own well it is more realistic that the farmer pays for the water he actually uses. In this case the cost of water is the cost of operating the pump of the well. The analysis of this case is completely similar to the case above and will not be discussed here.

$$h_j = b_{1j} \sum_{r \leq l} \frac{w_{rj}}{b_{rj}}.$$

The conditional profit function, $\pi(z_1, z_2)$, can be expressed as

(3.10)
$$\pi(z_1, z_2) = (M_1 - M_2) G(z_1) + M_2 G(z_1 + z_2) - h_1 z_1 - h_2 z_2$$

where

(3.11)
$$G(z) \equiv E \min(z, V) = \int_0^z (1 - F(v)) dv.$$

Since in the second stage supply of water will be the limiting factor it is convenient to express values in terms of returns per unit of water. Clearly, M_j is the operating ex-post marginal return per unit of water used, h_j is ex ante investment costs, also measured per unit of water. Thus, the first bracket in (3.8) yields the expected profit of crop one, viewed from the beginning of stage one and the second bracket yields the same for crop two. Eq. (3.8) follows directly from (3.5) and (3.6). Recall that it follows from the model, and it is also intuitive, that it is most profitable to first irrigate the crop with the highest marginal profit per unit water.

Note that when the amount of water $V \in (v, v + dv)$, $v < z_1$, then $E \min(V, z_1) = v$. The probability of this is $f(v) dv$. When $V > z_1$, $E \min(V, z_1) = z_1$. The probability that this will happen is $1 - F(z_1)$. Summing up we obtain that

$$E \min(V, z_1) = \int_0^{z_1} v f(v) dv + z_1 (1 - F(z_1)) = \int_0^{z_1} (1 - F(v)) dv = G(z_1)$$

where G is defined in (3.11). The last equation follows from integrating by parts. Similarly we obtain that

$$E \min(z_2, (V - z_1)_+) = \int_{z_1}^{z_1 + z_2} (v - z_1) f(v) dv + z_2 (1 - F(z_1 + z_2)) = \int_{z_1}^{z_1 + z_2} (1 - F(v)) dv = G(z_1 + z_2) - G(z_1).$$

The farmer's planning problem is to allocate land to the two crops in stage one so that expected profit is maximized when total available land is given. That is, the problem is to maximize $\pi(z_1, z_2)$ given by (3.10), subject to $z_1 \geq 0$, $z_2 \geq 0$, and

$$(3.12) \qquad \frac{z_1}{c_1} + \frac{z_2}{c_2} \le a \ .$$

As seen from (3.10), the function G plays a crucial role here. G(z) is the truncated mean supply of water below the truncation threshold z. G(z) can be interpreted as the expected "capacity utilization" given the cropland area z.

By applying definition 12.6 in Berck and Sydsæter (1993) it follows readily that $\pi\left(z_1, z_2\right)$ is strictly concave when $M_1 > M_2$. It follows that the first order conditions that correspond to the maximization problem above are given by

$$(3.13) \qquad \frac{\partial \pi\left(z_1, z_2\right)}{\partial z_1} = \left(M_1 - M_2\right) \overline{F}\left(z_1^*\right) + M_2 \overline{F}\left(z_1^* + z_2^*\right) - h_1 = \frac{\lambda}{c_1} - \mu_1$$

and

$$(3.14) \qquad \frac{\partial \pi\left(z_1, z_2\right)}{\partial z_2} = M_2 \overline{F}\left(z_1^* + z_2^*\right) - h_2 = \frac{\lambda}{c_2} - \mu_2$$

where $\overline{F}(z) \equiv 1 - F(z)$ and z_1^* and z_2^* denote the respective values at which optimum is achieved and λ and μ_1, μ_2 are Lagrange multipliers. The multiplier λ is positive when all available cropland is used and zero otherwise, while μ_1 and μ_2 are the Lagrange multipliers associated with non-negative water usage. That is, μ_1 is positive when $z_1^* = 0$, and μ_2 is positive when $z_2^* = 0$. Otherwise, μ_1 and μ_2 are equal to zero. The interpretation of $\overline{F}\left(z_1^*\right)$ and $\overline{F}\left(z_2^*\right)$ is as the probability of receiving enough water to cultivate z_1^* and z_2^*, respectively.

There are three special cases of interest which we shall analyze next.

Case (i): $z_1^* = 0, z_2^* > 0$.

For this case to be true, we see from (3.13) and (3.14) that we must have that $\mu_1 \ge 0$ and $\mu_2 = 0$, so that

$$(3.15) \qquad c_1 \left. \frac{\partial \pi\left(z_1, z_2\right)}{\partial z_1} \right|_{z_1 = 0} \le c_2 \left. \frac{\partial \pi\left(z_1, z_2\right)}{\partial z_2} \right|_{z_1 = 0} \ .$$

Thus, viewed from the planning stage (stage one), no land should be allocated to crop one if the marginal expected profit of allocating land to crop one evaluated at zero land for crop one, is less than

or equal to the marginal expected profit of allocating land to crop two, evaluated at zero land for crop one. From (3.14) we see that when $z_1^* = 0$,

$$M_2 \overline{F}(z_2^*) - h_2 = \frac{\lambda}{c_2} \geq 0$$

where $\lambda > 0$ corresponds to the case where all the land is cultivated, i.e., $z_2^* = a\,c_2$. If $M_2 \overline{F}(a\,c_2) < h_2$, then it is not optimal to use all the land and z_2^* is in this case determined by

$$z_2^* = \overline{F}^{-1}\left(\frac{h_2}{M_2}\right).$$

In general, we can express z_2^* as

$$z_2^* = \min\left(a\,c_2, \overline{F}^{-1}\left(\frac{h_2}{M_2}\right)\right).$$

Case (ii): $z_1^* > 0, z_2^* = 0$.

For this case to be true we must have that $\mu_1 = 0$ and $\mu_2 \geq 0$ in (3.13) and (3.14), and we thus get

(3.16)
$$c_1 \left.\frac{\partial \pi(z_1, z_2)}{\partial z_1}\right|_{z_2=0} \geq c_2 \left.\frac{\partial \pi(z_1, z_2)}{\partial z_2}\right|_{z_2=0}.$$

If marginal expected profit of allocating land to crop two, evaluated at zero land for crop two, is less than or equal to the marginal expected profit of allocating land to crop one, evaluated at zero land for crop two, then $z_2^* = 0$.

Similarly to case (i) the land constraint (3.12) is not binding when $M_1 \overline{F}(a\,c_1) - h_1 < 0$, so that z_1^* can be expressed as

$$z_1^* = \min\left(a\,c_1, \overline{F}^{-1}\left(\frac{h_1}{M_1}\right)\right).$$

Case (iii): $z_1^* > 0, z_2^* > 0$.

This case will occur if the inequalities in both (3.15) and (3.16) are reversed, in which case z_1^* and z_2^* are determined by

$$(3.17) \qquad c_1 \frac{\partial \pi (z_1, z_2)}{\partial z_1} = c_2 \frac{\partial \pi (z_1, z_2)}{\partial z_2}.$$

Eq. (3.17) states that optimal allocation of land, as viewed from stage one, follows from equating marginal expected profit. If sufficient water is received in stage two then all land will be used and allocation must fulfill the land constraint (3.12).

However, if

$$(3.18) \qquad \frac{h_1 - h_2}{M_1 - M_2} < 1 \ \text{ and } \ \frac{h_2}{M_2} < \frac{h_1 - h_2}{M_1 - M_2}$$

then not all land will be used and

$$z_1^* = \overline{F}^{-1}\left(\frac{h_1 - h_2}{M_1 - M_2} \right), \ z_1^* + z_2^* = \overline{F}^{-1}\left(\frac{h_2}{M_2} \right)$$

provided these allocations yield

$$\frac{z_1^*}{c_1} + \frac{z_2^*}{c_2} < a.$$

Note that the conditions in (3.18) are equivalent to

$$M_1 - M_2 > h_1 - h_2 \ \text{ and } \ \frac{h_1}{M_1} > \frac{h_2}{M_2}.$$

The next proposition summarizes the discussion above.

Proposition 2

Assume that $n = 2$, $M_1 > M_2$, and $M_j \geq h_j$, $j = 1,2$. Assume also that the interval $[0, max(ac_1, ac_2)]$ is included in the support of F.

(i) *If*

$$(3.19) \qquad c_2 \left(M_2 \overline{F}(ac_2) - h_2 \right)_+ \geq c_1 \left(M_1 - h_1 - \left(M_2 - h_2 - \left(M_2 \overline{F}(ac_2) - h_2 \right)_+ \right) \right)$$

then $z_1^* = 0$ and $z_2^* > 0$. In particular, if (3.19) holds with $M_2\bar{F}(ac_2) \geq h_2$ then $z_2^* = ac_2$. Otherwise,

if (3.19) holds with $M_2\bar{F}(ac_2) < h_2$ then $z_2^* = \bar{F}^{-1}\left(\dfrac{h_2}{M_2}\right) < ac_2$.

(ii)   If

(3.20)
$$c_1\left(M_1\bar{F}(ac_1) - h_1\right)_+ \geq c_2\left(M_2\bar{F}(ac_1) - h_2\right)$$

then $z_1^* > 0$ and $z_2^* = 0$. In particular, if (3.20) holds with $M_1\bar{F}(ac_1) \geq h_1$, then $z_1^* = ac_1$. Otherwise,

if (3.20) holds with $M_1\bar{F}(ac_1) < h_1$, then $z_1^* = \bar{F}^{-1}\left(\dfrac{h_1}{M_1}\right) < ac_1$.

(iii)   If

(3.21) $c_1\left(M_1\bar{F}(ac_1) - h_1\right) \leq c_2\left(M_2\bar{F}(ac_2) - h_2\right)_+ < c_1\left(M_1 - h_1 - \left(M_2 - h_2 - \left(M_2\bar{F}(ac_2) - h_2\right)_+\right)\right)$

then $z_1^* > 0$ and $z_2^* > 0$.

If

(3.22)
$$\frac{h_1}{M_1} > \frac{h_2}{M_2},\ M_1 - M_2 > h_1 - h_2 > 0$$

and

(3.23)
$$\bar{F}^{-1}\left(\frac{h_1 - h_2}{M_1 - M_2}\right)\left(\frac{1}{c_1} - \frac{1}{c_2}\right) + \bar{F}^{-1}\left(\frac{h_2}{M_2}\right)\frac{1}{c_2} < a$$

then not all the land is used and

$$z_1^* = \bar{F}^{-1}\left(\frac{h_1 - h_2}{M_1 - M_2}\right) > 0 \ \ and \ \ z_2^* = \bar{F}^{-1}\left(\frac{h_2}{M_2}\right) - z_1^* > 0.$$

When interpreting the results in Proposition 2 it is important to recall that since crop 1 has highest ex-post marginal profit per unit water, the farmer will always supply water to this crop first. If for example, he considers to reduce production of crop two in order to start up production of crop one, he must take into account in the planning that crop 1 gets the necessary amount of water first (in the second stage). Since the crops have decreasing marginal expected returns, the farmer will never

produce crop 1 if the marginal expected profit with respect to crop 1, evaluated at zero land use for crop 1 and total land use for crop 2, is less that the marginal expected profit with respect to crop 2, evaluated at zero land use for crop one and total land use for crop two. This is expressed in (3.19). Note that due to uncertainty about water supply the marginal expected profit with respect to crop one, expressed on the right hand side of (3.19), equals the corresponding marginal profit minus a correction term that represents marginal expected "loss" due to the fact that crop two does not receive the "first drop of water". Similarly, in case (ii) the farmer will never produce crop two if the marginal expected profit with respect to crop two, evaluated at zero land use for crop two and total land use for crop one is less than the marginal expected profit with respect to crop one, evaluated at zero land use for crop two and total land use for crop one. In the in-between cases the farmer will produce both crops. Case (iii) summarizes the situation when it is optimal to produce both crops. Unfortunately, a closed form expression for z_1^* and z_2^* in case (iii) can in general not be found.

The allocation of land depends on the uncertainty related to distribution function, F, and the relation between ex-post profit and investments costs for the crops. Case (i) of Proposition 2 illustrates that even if ex-post marginal profit per unit water is lowest for crop two, i.e. $M_1 - h_1 > M_2 - h_2$, the farmer can still choose only to produce crop two. This can be the case when crop two is the most "water intensive" crop, i.e., c_2/c_1 is sufficiently high and the land constraint is effective (supply of water is high). This result is consistent with part (ii) of Proposition 3 (perfect certainty), stated below. The planting costs affect the risk of producing the respective crops. For example, in case (i) not all land will be used if h_2 is so high that the probability of receiving enough water is less than h_2/M_2, i.e.,

$$P(V > ac_2) = \overline{F}(ac_2) < \frac{h_2}{M_2}.$$

In case (ii) the same situation occurs if

$$P(V > ac_1) = \overline{F}(ac_1) < \frac{h_1}{M_1}.$$

In case (iii) the situation is a bit more complicated. Consider now

$$\overline{F}^{-1}\left(\frac{h_1 - h_2}{M_1 - M_2}\right) \text{ and } \overline{F}^{-1}\left(\frac{h_2}{M_2}\right)$$

as a function of $h_1 - h_2$ and h_2, respectively. Since these functions are decreasing in $h_1 - h_2$ and h_2, respectively, it will be the case that the inequality in (3.23) will be true for sufficiently high $h_1 - h_2$

and h_2 when $c_1 < c_2$. Thus, there is increasing risk when h_2 increases and h_1 increases faster than h_2. However, when $c_1 > c_2$ the left hand side of (3.23) is decreasing in h_2 and increasing in $h_1 - h_2$. Thus, in this case there is increasing risk when h_2 increases and h_1 does not increase as fast as h_2. In this case crop one is the most profitable if sufficient water is supplied. Thus, even a risk neutral farmer may grow both crops when uncertainty about water supply is high.

The next result concerns the special case in which the farmer is assumed to have perfect certainty. Note that *only* part (ii) of Proposition 3 stated below is a special case of Proposition 2.

Proposition 3

Assume that $n = 2$, $M_1 > M_2$ and $M_j > h_j$, $j = 1,2$. Assume furthermore that the farmer knows the supply of water with perfect certainty.

(i)   If $V < min(ac_1, ac_2)$ and $M_j - h_j < M_k - h_k$, $j \neq k$, $j, k = 1, 2$,

then $z_j^* = 0$ and $z_k^* = V$.

(ii)   If $V \geq max(ac_1, ac_2)$ and $c_j(M_j - h_j) < c_k(M_k - h_k)$, $j \neq k$, $j, k = 1, 2$,

then $z_j^* = 0$ and $z_k^* = ac_k$.

(iii)   If $ac_j < V < ac_k$ and $M_j - h_j < M_k - h_k$, $j \neq k$, $j, k = 1, 2$,

then $z_j^* = 0$ and $z_k^* = V$.

(iv)   If $ac_j < V < ac_k$ and $c_j(M_j - h_j) > c_k(M_k - h_k)$, $j \neq k$, $j, k = 1, 2$,

then $z_j^* = ac_j$ and $z_k^* = 0$.

(v)   If $ac_j < V < ac_k$, $c_j(M_j - h_j) > c_k(M_k - h_k)$ and $c_j < c_k$, $j \neq k$, $j, k = 1, 2$,

then $z_j^* = ac_j$ and $z_k^* = V - ac_j$.

The result in (i) of Proposition 3 states that when the land constraint is not binding the farmer will only produce the crop with the highest marginal profit per unit water. In case (ii) the profit from producing crop j only is $ac_j(M_j - h_j)$. Thus, it will be most profitable to produce solely crop j if crop

j has the highest marginal profit per unit land. Case (iii) is similar to case (i) and case (iv) is similar to case (ii). In case (v) we realize that the conditions imply that the low water intensive crop has the highest marginal profit per unit water. Thus, both crops will be produced when the following conditions are fulfilled: The low water intensive crop has highest marginal profit per unit water, supply of water is higher than the requirement to grow this crop on the entire farm, and marginal profit per unit land from the high water intensive crop is higher than for the low water intensive crop.

4. The general case with more than two feasible crops

With the same notation as above we get in this case that

$$(4.1) \quad \pi(\mathbf{z}) = M_1 E \min(z_1, V) + M_2 E \min\left(z_2, (V - z_1)_+\right) + \ldots + M_n E \min\left(z_n, \left(V - \sum_{j=1}^{n-1} z_j\right)_+\right) - \sum_{j=1}^{n} h_j z_j.$$

where $\mathbf{z} = (z_1, z_2, \ldots, z_n)$, and $z_j = x_{0j} c_j$. Similarly to the analysis above we obtain

$$(4.2) \quad \begin{aligned} \pi(\mathbf{z}) &= (M_1 - M_2) G(z_1) + (M_2 - M_3) G(z_1 + z_2) \\ &\quad + \ldots + (M_{n-1} - M_n) G\left(\sum_{j=1}^{n-1} z_j\right) + M_n G\left(\sum_{j=1}^{n} z_j\right) - \sum_{j=1}^{n} h_j z_j \end{aligned}$$

with budget constraint

$$(4.3) \quad \sum_{j=1}^{n} \frac{z_j}{c_j} \leq a.$$

Now assume that $M_1 > M_2 > \ldots > 0$. The conditional profit function is strictly concave in this case and the first order conditions are given by

$$(4.4) \quad (M_1 - M_2) \overline{F}(z_1^*) + (M_2 - M_3) \overline{F}(z_1^* + z_2^*) + \ldots + M_n \overline{F}\left(\sum_{j=1}^{n} z_j^*\right) = h_1 + \frac{\lambda}{c_1} - \mu_1,$$

$$(4.5) \quad (M_2 - M_3) \overline{F}(z_1^* + z_2^*) + \ldots + M_n \overline{F}\left(\sum_{j=1}^{n} z_j^*\right) = h_2 + \frac{\lambda}{c_2} - \mu_2,$$

$$(4.6) \quad (M_3 - M_4) \overline{F}(z_1^* + z_2^* + z_3^*) + \ldots + M_n \overline{F}\left(\sum_{j=1}^{n} z_j^*\right) = h_3 + \frac{\lambda}{c_3} - \mu_3,$$

15

…., etc., and

$$(4.7) \qquad M_n \overline{F}\left(\sum_{k=1}^{n} z_k^*\right) = h_n + \frac{\lambda}{c_n} - \mu_n$$

where λ is the Lagrange multiplier associated with the constraint (4.3) and μ_j is positive when $z_j^* = 0$, and zero otherwise.

It is possible to derive similar results as in Proposition 2 from the first order conditions (4.4) and (4.7). We shall, however, leave this for another occasion.

5. Allowing for bounded rationality

The modeling approach discussed above is based on the assumption that the farmer is able to evaluate perfectly the expected profit conditional on a given allocation of land to each crop. However, many studies as well as laboratory experiments seem to indicate that people are boundedly rational in the sense that when a decision-maker faces replications of the same choice experiment, he may make different choices in different replications. In our case it seems reasonable that there may be a considerable degree of bounded rationality. The reason for this is that the farmer does not have a precise knowledge of the distribution of the supply of water. Even if he did, he may not be able to solve the type of optimization problem we have discussed above (given F, $\{M_j\}$, $\{h_j\}$ and $\{c_j\}$). As we shall discuss in the next section our particular approach to bounded rationality leads to a practical maximum likelihood method that can conveniently be applied to a sample of micro data.

Consider for simplicity the model with 2 possible crops. Let

$$(5.1) \qquad U(z_1, z_2) \equiv \pi(z_1, z_2) + \varepsilon(z_1, z_2)$$

where $\pi(z_1, z_2)$ is given by (3.10) and $\varepsilon(z_1, z_2)$ is a stochastic error term (taste-shifter), and assume now that the farmer maximizes $U(z_1, z_2)$ rather than maximizing $\pi(z_1, z_2)$ (subject to the land constraint (3.12)). As mentioned above, one important contribution to the error term stem from the farmer's inability to assess the correct objective distribution of the water supply. Specifically, let $G_i(z)$ denote the subjective function farmer i applies instead of the objective function, $G(z) = E\min(z, V)$. In general, $G_i(z)$ may differ from $G(z)$. Thus the error term $\varepsilon(z_1, z_2)$ may be interpreted as

$$\varepsilon(z_1, z_2) = (M_1 - M_2)(G_i(z_1) - G(z_1)) + M_2(G_i(z_1 + z_2) - G(z_1 + z_2)) + \eta(z_1, z_2)$$

where the first two terms are due to possible deviation of the subjective functions $\{G_i\}$ (based on the distribution of water supply) from the objective G function and the term $\eta(z_1, z_2)$ is due to optimization error. For farmer i one may have, as a special case, that the two first terms do not vary from one "experiment" to another, that is from one year to another. In this special case the two first terms represent in fact "pure" unobserved heterogeneity. We assume that the farmer is perfectly certain about the values $\{M_j\}$, which he uses to determine the priority rank ordering of the crops with respect to the distribution of water. Let d be the lowest level of land use observed in the data. In addition, we shall also assume that the farmer is boundedly rational in the sense that he does not consider all possible allocation combinations within the set Ω given by

$$(5.2) \qquad \Omega \equiv \left\{ (z_1, z_2) : d \le \frac{z_1}{c_1} + \frac{z_2}{c_2} \le a, \, z_1 \ge 0, \, z_2 \ge 0 \right\},$$

but rather considers only a subset $\hat{\Omega}$ of Ω. The subset $\hat{\Omega}$ is random in the sense that its points are generated by a probability mechanism. This probability mechanism is such that the points in $\hat{\Omega}$ are dispersed randomly. The intuition is that the subset of alternatives the agent actually takes into consideration may vary across identical choice experiments due to psychological processes that are not fully understood—or can be controlled by him. This type of approach has been investigated by Dagsvik (1994). We shall now outline the version of Dagsvik's approach. Let $\left\{ \hat{\mathbf{Z}}_k, k = 1, 2, ... \right\} \equiv \hat{\Omega}$, be an enumeration of the points in $\hat{\Omega}$, where $\hat{\mathbf{Z}}_k = \left(\hat{Z}_{1k}, \hat{Z}_{2k} \right)$. For simplicity write $\hat{\varepsilon}_k \equiv \varepsilon \left(\hat{\mathbf{Z}}_k \right)$. One convenient way of representing random sets consisting of independently scattered points is to apply the formalism of the multidimensional Poisson process. We shall therefore assume that the points $\left\{ \left(\hat{\mathbf{Z}}_k, \hat{\varepsilon}_k \right), k = 1, 2, ... \right\}$ are distributed according to a nonhomogeneous Poisson process on $\Omega \times R$ with intensity $\mu(z)\theta(\varepsilon)$, where $\mu(z)$ and $\theta(z)$ are positive functions defined on Ω and R, respectively. This means that the points $\left\{ \left(\hat{\mathbf{Z}}_k, \hat{\varepsilon}_k \right), k = 1, 2, ... \right\}$ are independently scattered across $\Omega \times R$, and that the "coordinates" $\{\hat{\mathbf{Z}}_k\}$ and $\{\hat{\varepsilon}_k\}$ are independent. The intensity representation given above means that the probability that there is a point $\left(\hat{\mathbf{Z}}_k, \hat{\varepsilon}_k \right)$ of the process for which $\hat{\mathbf{Z}}_k \in (\mathbf{z}, \mathbf{z} + d\mathbf{z})$ and $\hat{\varepsilon}_k \in (\varepsilon, \varepsilon + d\varepsilon)$ equals $\mu(\mathbf{z})\theta(\varepsilon)d\mathbf{z}\,d\varepsilon$. Moreover, the probability that more than one point of the process lies within $(\mathbf{z}, \mathbf{z} + d\mathbf{z}) \times (\varepsilon, \varepsilon + d\varepsilon)$ is negligible. To gain more insight in the properties of the

17

multidimensional (inhomogeneous) Poisson process, let C be a subset of $\Omega \times R$ and $N(C)$ the number of Poisson points located within C. Then the expected number of points within C equals

$$(5.3) \qquad\qquad EN(C) = \int_C \mu(\mathbf{z})\,\theta(\varepsilon)\,d\mathbf{z}\,d\varepsilon\,,$$

and the probability that there are exactly n points within C is Poisson distributed with parameter $EN(C)$, that is, it is given by

$$(5.4) \qquad\qquad P\big(N(C) = n\big) = \exp\big(-EN(C)\big)\frac{\big(EN(C)\big)^n}{n!}\,.$$

Recall that in our context the probability mechanism is meant to account for the heterogeneity in behavior across farmers, and also for one farmer across seemingly identical "choice experiments". For example, if we focus on fluctuations in behavior of one farmer due to bounded rationality, the "empirical" counterpart to the probability density in (5.4) is the fraction of times, in a large number of replications of a choice experiment, the farmers pick n points within C, where each point consists of a combination of a potential allocation $\big(\hat{\mathbf{Z}}_k\big)$ and the associated taste-shifters, $\big(\hat{\varepsilon}_k\big)$. If population heterogeneity has been properly controlled for, heterogeneity across farmers will be indistinguishable from heterogeneity across time for a given farmer.

Assume furthermore that the farmers behavior satisfies the *Independence from Irrelevant Alternatives* assumption (IIA). We can express this formally as follows. Let $A \subset \Omega$. Then

$$(5.5) \qquad P\left(U\big(\hat{\mathbf{Z}}_j\big) = \max_k U\big(\hat{\mathbf{Z}}_k\big) \,\middle|\, \max_{\hat{\mathbf{Z}}_k \in A} U\big(\hat{\mathbf{Z}}_k\big) = \max_k \hat{U}\big(\hat{\mathbf{Z}}_k\big) \right) = P\left(U\big(\hat{\mathbf{Z}}_j\big) = \max_{\hat{\mathbf{Z}}_k \in A} U\big(\hat{\mathbf{Z}}_k\big) \right)$$

for $\hat{\mathbf{Z}}_j \in A$. The condition stated in (5.5) means the following: The left hand side expresses the probability that $\hat{\mathbf{Z}}_j$ is the preferred allocation within $\hat{\Omega}$, given that the preferred allocation lies within the subset $A \cap \hat{\Omega}$. The right hand side of (5.5) states the probability that $\hat{\mathbf{Z}}_j$ is the preferred allocation within the subset A.

Let $\varphi(\mathbf{z})$ be the probability density of the farmer's chosen allocation in $\hat{\Omega}$. That is, $\varphi(\mathbf{z})d\mathbf{z}$ is the probability that the chosen allocation lies within $(\mathbf{z}, \mathbf{z} + d\mathbf{z})$.

Theorem 1

Suppose the choice setting of the farmer can be represented as described above and that IIA holds. Then

$$(5.6) \qquad \varphi(z) = \frac{exp\big(\tau\pi(z)\big)\mu(z)}{\int_\Omega exp\big(\tau\pi(u)\big)\mu(u)\,du}.$$

The proof of Theorem 1 is given in Dagsvik (1994), Section 4.

The empirical counterpart to $\varphi(\mathbf{z})d\mathbf{z}$ is the fraction of farmers in the sample with chosen allocation within the interval $(\mathbf{z}, \mathbf{z} + d\mathbf{z})$. The parameter τ is positive and $1/\tau$ represents the degree of dispersion of the error term $\varepsilon(\mathbf{z})$. In our application it is reasonable to assume that the points $\{\hat{\mathbf{Z}}_k\}$ are evenly distributed in Ω. This means that the distribution $\mu(\mathbf{z})$ is uniform. In this case (5.6) reduces to

$$(5.7) \qquad \varphi(\mathbf{z}) = \frac{exp\big(\tau\pi(\mathbf{z})\big)}{\int_\Omega exp\big(\tau\pi(\mathbf{u})\big)\,d\mathbf{u}}.$$

The analysis carried out above is completely analogous in the case with more than two products.

Above we have treated the alternative allocations within $\hat{\Omega}$ symmetrically. A possible extension would be to allow the error terms to have different correlation patterns in different subsets of Ω as follows:

$$A_1 = \left\{ \mathbf{z} : \big(0 < z_1 \le c_1 a, z_2 = 0\big) \cup \big(z_1 = 0, 0 < z_2 \le a c_2\big) \right\}$$

$$A_2 = \left\{ \mathbf{z} : \frac{z_1}{c_1} + \frac{z_2}{c_2} = a, z_1 > 0, z_2 > 0 \right\}$$

$$A_3 = \left\{ \mathbf{z} : \frac{z_1}{c_1} + \frac{z_2}{c_2} < a, z_1 > 0, z_2 > 0 \right\}$$

A plausible hypothesis may be that the correlation between the error terms for alternatives within A_k is different from the corresponding correlation for alternatives within A_j, $j \ne k$, $j, k \in \{1, 2, 3\}$. This can be

accounted for by applying a nested logit formulation. We shall, however, not discuss this extension further in this paper.

6. Estimation

The theoretical model developed in Section 4 is deterministic in the sense that it is silent about individual unobserved heterogeneity and variations from one year to the next for each farmer. A conventional estimation strategy would for example be to specify how the distribution of water supply vary across farmers and to use the first order conditions to derive the estimation procedure. In general, this will be a rather cumbersome and complicated strategy. In contrast, the estimation procedure based on the random utility model of Section 5 leads to a relative simple estimation procedure.

We shall now discuss how the framework developed in Section 5 can be applied for estimating the unknown parameters. As above we consider the case with only two possible crops, but the general case is completely analogous. The unknown parameters are the parameters associated with the (subjective) distribution of the supply of water. As in the previous section we shall assume that the distribution of V/a is independent of the area a. However, more general regimes may easily be handled. In practice, data on the supply of water is usually not available. If so, one cannot, a priori, obtain estimates of b_{1j} directly from observed quantities. We assume, however, that the farmer knows $\{M_j\}$ and $\{h_j\}$. He uses the information about $\{M_j\}$ to assess the rank ordering of the crops with respect to allocation of water. Note next that we can write

$$(6.1) \qquad \tau\pi(z_1, z_2) = m_1\left(\frac{M_1}{b_{11}}G(z_1) - \frac{h_1}{b_{11}}z_1\right) + m_2\frac{M_2}{b_{12}}\big(G(z_1 + z_2) - G(z_1)\big) - \frac{h_2 z_2}{b_{12}}m_2$$

where $m_1 = \tau b_{11}$ and $m_2 = \tau b_{12}$. Since b_{1j} cancels in M_j/b_{1j} and in h_j/b_{1j} it follows that the right hand side is known to the researcher apart from m_1 and m_2 and the parameters of G (which are the parameters of F). The unknown parameters to be estimated are thus m_1, m_2 and the parameters of F.

In principle, it is now straight forward to apply the density (5.7) in a maximum likelihood estimation procedure. However, the computation of the denominator in (5.7) involves calculations of multiple integrals. Fortunately, this problem is easily dealt with. McFadden (1984) has demonstrated that one can consistently estimate the choice probability densities by replacing the "choice set" Ω by a finite and small subset of Ω which may vary across farmers. The points in this subset can for example be drawn randomly with equal probabilities. In addition, the chosen subset must contain the observed choice (for each farmer). This implies that the multiple integral in (5.7) is replaced by a simple finite sum with a relatively small number of terms.

Ex post, when the crops have been sold, it may be possible to estimate τ because observed profit, $\hat{\pi}$, satisfies

$$(6.2) \qquad E\hat{\pi} = E\left[m_1 \frac{M_1}{b_{11}} \left(\min\left(Z_1^*, V \right) \right) + m_2 \frac{M_2}{b_{12}} \min\left(Z_2^*, \left(V - Z_1^* \right)_+ \right) \right]$$

where E denotes the empirical mean taken across the sample (or subsamples). When m_1 and m_2 have been estimated then one can estimate τ from (6.2). Subsequently, one can estimate b_{11} and b_{12} from the estimates of m_1/τ and m_2/τ.

7. Conclusion

This paper discusses the modeling of multicrop agricultural production when the farmer is constrained with respect to land and is uncertain about his supply of water. We have assumed that the farmer is risk neutral and maximizes expected profit and that the production technology is Leontief with nonjointness production. From the model we have derived a characterization of the farmer's decision rule with respect to allocation of land to the respective crops. Since the crops require different irrigation supply per hectare, the uncertainty about water supply and the investment costs play an important role in the farmer's allocation decision. For example, it is interesting to note that a risk neutral farmer may go from single to multi-crop allocation when uncertainty about water supply increases.

In this paper we have also considered a particular approach to bounded rationality in the sense that the farmer is allowed to make optimization error. This leads to a stochastic decision rule, which we have characterized. The motivation for the bounded rationality approach is that it seems more realistic than the perfect rational setting and it will also simplify empirical analysis and estimation.

Finally, we have discussed how the model can be estimated when micro-data are available. Specifically, we have demonstrated how the bounded rationality approach simplifies the estimation procedure drastically.

The modeling framework presented here seems convenient for analyzing effects of various agricultural policies on distribution of income and agricultural production. In particular it can be used to study and compare effects of various water reforms such as redistribution policies or water pricing.

References

Ballivian, M. A., and R. C. Sickles (1994): Product Diversification and Attitudes Toward Risk in Agricultural Production. *The Journal of Productivity Analysis*, **5**, 271-286.

Berck, P. and K. Sydsæter (1993): *Economists' Mathematical Manual.* Springer-Verlag, Berlin.

Chambers, R.G. and R. E. Just (1989): Estimating Multioutput Technologies. *American Journal of Agricultural Economics*, **71**, 980-995.

Dagsvik, J.K. (1994): Discrete and Continuous Choice, Max-Stable Processes, and Independence from Irrelevant Attributes. *Econometrica*, **62**, 1179-1205.

Just, R. E., D. Zilberman and E. Hochman (1983): Estimation of Multicrop Production Functions. *American Journal of Agricultural Economics*, **65**, 770-780.

Just, R. E., D. Zilberman, E. Hochman and Z. Bar-Shira (1990): Input Allocation in Multicrop Systems. *American Journal of Agricultural Economics*, **72**, 200-209.

McFadden, D. (1984): "Econometric Analysis of Quantitative Response Models". In Z. Griliches and M.D. Intriligator (eds.): *Handbook of Econometrics*, Vol. II, Elsevier Science Publishers BV, New York.

Moore, M.R., N. R. Gollehon and M. B. Carey (1994a): Alternative Models of Input Allocation of Multicrop Systems: Irrigation Water in the Central Plains, United States. *Agricultural Economics*, **11**, 143-158.

Moore, M.R., N. R. Gollehon and M. B. Carey (1994b): Multicrop Production Decisions in Western Irrigated Agriculture: The Role of Water Price. *American Journal of Agricultural Economics*, **76**, 859-874.

Moore, M.R. and A. Dinar (1995): Water and Land as Quantity-Rationed Inputs in California Agriculture: Empirical Tests and Water Policy Implications. *Land Economics*, **71**, 445-461.

Shumway, C.R., R. Pope and E. Nash (1984): Allocatable Fixed Inputs and Jointness in Agricultural Production: Implications for Economic Modeling. *American Journal of Agricultural Economics*, **66**, 72-78.

The presence of seasonal specific crops

So far we have ignored the fact that farm production is heavily influenced by variations in weather conditions. We shall now modify the framework above to account for the fact that some crops, such as sugar, needs two seasons of cultivation before it can be harvested, while other crops can be harvested after one season of cultivation. Typically, sugar needs a whole year of cultivation, but since during the monsoon period irrigation is normally not required[2], we only assume a setting with two seasons.

Here we shall only consider the case where $n = 3$ and that "sugar" is the only crop that needs all seasons to ripen. Assume also that "sugar" is indexed as crop no. 3 and that $\min(M_1, M_2) > M_3$. This is not necessarily the case. Here it is understood that prices and production technology remains constant through the year. We assume furthermore that there are only two seasons, in which crop 1 is grown only in the first season and crop 2 is cultivated only in the second season. Thus

(A.1)
$$\pi(z_1, z_2, z_3) = M_1 E \min(z_1, V_1) - h_1 z_1 - h_2 z_2 - 2h_3 z_3$$
$$+ M_2 E \min(z_2, V_2) + M_3 E \min\left(2z_3, (V_1 + V_2 - z_1 - z_2)_+\right)$$

where

$$z_j = c_j x_{0j}$$

and

(A.2)
$$\frac{z_1}{c_1} + \frac{z_3}{c_3} \leq a$$

must hold in the first season, while

(A.3)
$$\frac{z_2}{c_2} + \frac{z_3}{c_3} \leq a$$

must hold in the second season. Here V_j is the total amount of water available in season j, $j = 1, 2$. We assume here that z_1, z_2 and z_3 are all determined at the beginning of the year. Hence, in this setup, there are no possibilities for the farmer to adjust the allocation of land to the second crop in the second season. From (A.1) we get that

---

[2] In the monsoon period the problem is one with too much water supply.

(A.4)

$$\pi(z_1, z_2, z_3) = M_1 G_1(z_1) + M_2 G_2(z_2) + M_3 G_{12}(2z_3 + z_1 + z_2) - M_3 G_{12}(z_1 + z_2) - h_1 z_1 - h_2 z_2 - 2h_3 z_3$$

where

(A.5)
$$G_j(z) = \int_0^z \overline{F}_j(v) dv,$$

(A.6)
$$G_{12}(z) = \int_0^z \overline{F}_{12}(v) dv = \int_0^z \overline{F}_1(v) F_2(z - v) dv,$$

$F_{12}(v)$ is the c.d.f. of $V_1 + V_2$ and $F_j(v)$ is the c.d.f. of V_j, $j = 1, 2$.

The corresponding first order conditions are given by

(A.7)
$$M_1 \overline{F}_1(z_1^*) + M_3 \left(\overline{F}_{12}(2z_3 + z_1 + z_2) - \overline{F}_{12}(z_1 + z_2) \right) = h_1 + \frac{\lambda_1}{c_1} - \mu_1$$

(A.8)
$$M_2 \overline{F}_2(z_2^*) + M_3 \left(\overline{F}_{12}(2z_3 + z_1 + z_2) - \overline{F}_{12}(z_1 + z_2) \right) = h_2 + \frac{\lambda_2}{c_2} - \mu_2$$

and

(A.9)
$$2M_3 \overline{F}_{12}(2z_3 + z_1 + z_2) = 2h_3 + \frac{\lambda_1 + \lambda_2}{c_3} - \mu_3$$

where λ_1, λ_2, μ_1, μ_2 and μ_3 are the Lagrange multipliers; λ_j is positive when

$$\frac{z_j^*}{c_j} + \frac{z_3^*}{c_3} = a,$$

and zero otherwise, $j = 1, 2$, and μ_k is positive when $z_k^* = 0$, and zero otherwise, $k = 1, 2, 3$.

Recent publications in the series Discussion Papers

279   K. Nyborg and M. Rege (2000): The Evolution of Considerate Smoking Behavior

280   M. Søberg (2000): Imperfect competition, sequential auctions, and emissions trading: An experimental evaluation

281   L. Lindholt (2000): On Natural Resource Rent and the Wealth of a Nation. A Study Based on National Accounts in Norway 1930-95

282   M. Rege (2000): Networking Strategy: Cooperate Today in Order to Meet a Cooperator Tomorrow

283   P. Boug, Å. Cappelen and A.R. Swensen (2000): Expectations in Export Price Formation: Tests using Cointegrated VAR Models

284   E. Fjærli and R. Aaberge (2000): Tax Reforms, Dividend Policy and Trends in Income Inequality: Empirical Evidence based on Norwegian Data

285   L.-C. Zhang (2000): On dispersion preserving estimation of the mean of a binary variable from small areas

286   F.R. Aune, T. Bye and T.A. Johnsen (2000): Gas power generation in Norway: Good or bad for the climate? Revised version

287   A. Benedictow (2000): An Econometric Analysis of Exports of Metals: Product Differentiation and Limited Output Capacity

288   A. Langørgen (2000): Revealed Standards for Distributing Public Home-Care on Clients

289   T. Skjerpen and A.R. Swensen (2000): Testing for long-run homogeneity in the Linear Almost Ideal Demand System. An application on Norwegian quarterly data for non-durables

290   K.A. Brekke, S. Kverndokk and K. Nyborg (2000): An Economic Model of Moral Motivation

291   A. Raknerud and R. Golombek: Exit Dynamics with Rational Expectations

292   E. Biørn, K-G. Lindquist and T. Skjerpen (2000): Heterogeneity in Returns to Scale: A Random Coefficient Analysis with Unbalanced Panel Data

293   K-G. Lindquist and T. Skjerpen (2000): Explaining the change in skill structure of labour demand in Norwegian manufacturing

294   K. R. Wangen and E. Biørn (2001): Individual Hetero-geneity and Price Responses in Tobacco Consumption: A Two-Commodity Analysis of Unbalanced Panel Data

295   A. Raknerud (2001): A State Space Approach for Estimating VAR Models for Panel Data with Latent Dynamic Components

296   J.T. Lind (2001): Tout est au mieux dans ce meilleur des ménages possibles. The Pangloss critique of equivalence scales

297   J.F. Bjørnstad and D.E. Sommervoll (2001): Modeling Binary Panel Data with Nonresponse

298   Taran Fæhn and Erling Holmøy (2001): Trade Liberalisation and Effects on Pollutive Emissions and Waste. A General Equilibrium Assessment for Norway

299   J.K. Dagsvik (2001): Compensated Variation in Random Utility Models

300   K. Nyborg and M. Rege (2001): Does Public Policy Crowd Out Private Contributions to Public Goods?

301   T. Hægeland (2001): Experience and Schooling: Substitutes or Complements

302   T. Hægeland (2001): Changing Returns to Education Across Cohorts. Selection, School System or Skills Obsolescence?

303   R. Bjørnstad: (2001): Learned Helplessness, Discouraged Workers, and Multiple Unemployment Equilibria in a Search Model

304   K. G. Salvanes and S. E. Førre (2001): Job Creation, Heterogeneous Workers and Technical Change: Matched Worker/Plant Data Evidence from Norway

305   E. R. Larsen (2001): Revealing Demand for Nature Experience Using Purchase Data of Equipment and Lodging

306   B. Bye and T. Åvitsland (2001): The welfare effects of housing taxation in a distorted economy: A general equilibrium analysis

307   R. Aaberge, U. Colombino and J.E. Roemer (2001): Equality of Opportunity versus Equality of Outcome in Analysing Optimal Income Taxation: Empirical Evidence based on Italian Data

308   T. Kornstad (2001): Are Predicted Lifetime Consumption Profiles Robust with respect to Model Specifications?

309   H. Hungnes (2001): Estimating and Restricting Growth Rates and Cointegration Means. With Applications to Consumption and Money Demand

310   M. Rege and K. Telle (2001): An Experimental Investigation of Social Norms

311   L.C. Zhang (2001): A method of weighting adjustment for survey data subject to nonignorable nonresponse

312   K. R. Wangen and E. Biørn (2001): Prevalence and substitution effects in tobacco consumption. A discrete choice analysis of panel data

313   G.H. Bjertnær (2001): Optimal Combinations of Income Tax and Subsidies for Education

314   K. E. Rosendahl (2002): Cost-effective environmental policy: Implications of induced technological change

315   T. Kornstad and T.O. Thoresen (2002): A Discrete Choice Model for Labor Supply and Child Care

316   A. Bruvoll and K. Nyborg (2002): On the value of households' recycling efforts

317   E. Biørn and T. Skjerpen (2002): Aggregation and Aggregation Biases in Production Functions: A Panel Data Analysis of Translog Models

318   Ø. Døhl (2002): Energy Flexibility and Technological Progress with Multioutput Production. Application on Norwegian Pulp and Paper Industries

319   R. Aaberge (2002): Characterization and Measurement of Duration Dependence in Hazard Rate Models

320   T. J. Klette and A. Raknerud (2002): How and why do Firms differ?

321   J. Aasness and E. Røed Larsen (2002): Distributional and Environmental Effects of Taxes on Transportation

322   E. Røed Larsen (2002): The Political Economy of Global Warming: From Data to Decisions

323   E. Røed Larsen (2002): Searching for Basic Consumption Patterns: Is the Engel Elasticity of Housing Unity?

324   E. Røed Larsen (2002): Estimating Latent Total Consumption in a Household.

325 E. Røed Larsen (2002): Consumption Inequality in Norway in the 80s and 90s.

326 H.C. Bjørnland and H. Hungnes (2002): Fundamental determinants of the long run real exchange rate:The case of Norway.

327 M. Søberg (2002): A laboratory stress-test of bid, double and offer auctions.

328 M. Søberg (2002): Voting rules and endogenous trading institutions: An experimental study.

329 M. Søberg (2002): The Duhem-Quine thesis and experimental economics: A reinterpretation.

330 A. Raknerud (2002): Identification, Estimation and Testing in Panel Data Models with Attrition: The Role of the Missing at Random Assumption

331 M.W. Arneberg, J.K. Dagsvik and Z. Jia (2002): Labor Market Modeling Recognizing Latent Job Attributes and Opportunity Constraints. An Empirical Analysis of Labor Market Behavior of Eritrean Women

332 M. Greaker (2002): Eco-labels, Production Related Externalities and Trade

333 J. T. Lind (2002): Small continuous surveys and the Kalman filter

334 B. Halvorsen and T. Willumsen (2002): Willingness to Pay for Dental Fear Treatment. Is Supplying Fear Treatment Social Beneficial?

335 T. O. Thoresen (2002): Reduced Tax Progressivity in Norway in the Nineties. The Effect from Tax Changes

336 M. Søberg (2002): Price formation in monopolistic markets with endogenous diffusion of trading information: An experimental approach

337 A. Bruvoll og B.M. Larsen (2002): Greenhouse gas emissions in Norway. Do carbon taxes work?

338 B. Halvorsen and R. Nesbakken (2002): A conflict of interests in electricity taxation? A micro econometric analysis of household behaviour

339 R. Aaberge and A. Langørgen (2003): Measuring the Benefits from Public Services: The Effects of Local Government Spending on the Distribution of Income in Norway

340 H. C. Bjørnland and H. Hungnes (2003): The importance of interest rates for forecasting the exchange rate

341 A. Bruvoll, T.Fæhn and Birger Strøm (2003): Quantifying Central Hypotheses on Environmental Kuznets Curves for a Rich Economy: A Computable General Equilibrium Study

342 E. Biørn, T. Skjerpen and K.R. Wangen (2003): Parametric Aggregation of Random Coefficient Cobb-Douglas Production Functions: Evidence from Manufacturing Industries

343 B. Bye, B. Strøm and T. Åvitsland (2003): Welfare effects of VAT reforms: A general equilibrium analysis

344 J.K. Dagsvik and S. Strøm (2003): Analyzing Labor Supply Behavior with Latent Job Opportunity Sets and Institutional Choice Constraints

345 A. Raknerud, T. Skjerpen and A. Rygh Swensen (2003): A linear demand system within a Seemingly Unrelated Time Series Equation framework

346 B.M. Larsen and R.Nesbakken (2003): How to quantify household electricity end-use consumption

347 B. Halvorsen, B. M. Larsen and R. Nesbakken (2003): Possibility for hedging from price increases in residential energy demand

348 S. Johansen and A. R. Swensen (2003): More on Testing Exact Rational Expectations in Cointegrated Vector Autoregressive Models: Restricted Drift Terms

349 B. Holtsmark (2003): The Kyoto Protocol without USA and Australia - with the Russian Federation as a strategic permit seller

350 J. Larsson (2003): Testing the Multiproduct Hypothesis on Norwegian Aluminium Industry Plants

351 T. Bye (2003): On the Price and Volume Effects from Green Certificates in the Energy Market

352 E. Holmøy (2003): Aggregate Industry Behaviour in a Monopolistic Competition Model with Heterogeneous Firms

353 A. O. Ervik, E.Holmøy and T. Hægeland (2003): A Theory-Based Measure of the Output of the Education Sector

354 E. Halvorsen (2003): A Cohort Analysis of Household Saving in Norway

355 I. Aslaksen and T. Synnestvedt (2003): Corporate environmental protection under uncertainty

356 S. Glomsrød and W. Taoyuan (2003): Coal cleaning: A viable strategy for reduced carbon emissions and improved environment in China?

357 A. Bruvoll T. Bye, J. Larsson og K. Telle (2003): Technological changes in the pulp and paper industry and the role of uniform versus selective environmental policy.

358 J.K. Dagsvik, S. Strøm and Z. Jia (2003): A Stochastic Model for the Utility of Income.

359 M. Rege and K. Telle (2003): Indirect Social Sanctions from Monetarily Unaffected Strangers in a Public Good Game.

360 R. Aaberge (2003): Mean-Spread-Preserving Transformation.

361 E. Halvorsen (2003): Financial Deregulation and Household Saving. The Norwegian Experience Revisited

362 E. Røed Larsen (2003): Are Rich Countries Immune to the Resource Curse? Evidence from Norway's Management of Its Oil Riches

363 E. Røed Larsen and Dag Einar Sommervoll (2003): Rising Inequality of Housing? Evidence from Segmented Housing Price Indices

364 R. Bjørnstad and T. Skjerpen (2003): Technology, Trade and Inequality

365 A. Raknerud, D. Rønningen and T. Skjerpen (2003): A method for improved capital measurement by combining accounts and firm investment data

366 B.J. Holtsmark and K.H. Alfsen (2004): PPP-correction of the IPCC emission scenarios - does it matter?

367 R. Aaberge, U. Colombino, E. Holmøy, B. Strøm and T. Wennemo (2004): Population ageing and fiscal sustainability: An integrated micro-macro analysis of required tax changes

368 E. Røed Larsen (2004): Does the CPI Mirror Costs.of.Living? Engel's Law Suggests Not in Norway

369 T. Skjerpen (2004): The dynamic factor model revisited: the identification problem remains

370 J.K. Dagsvik and A.L. Mathiassen (2004): Agricultural Production with Uncertain Water Supply