Discussion Papers

Statistics Norway
Research department

No. 701
August 2012

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Faustmann and the Climate

Statistics Norway

Statistisk sentralbyrå

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Abstract:
This paper presents an adjusted Faustmann Rule for optimal harvest of a forest in the presence of a social cost of carbon emissions. A contribution of the paper is to do this within theoretical and numerical frameworks that take account of the dynamics and interactions of the forest's multiple carbon pools within an infinite time horizon model. With our less restrictive assumptions we find that a social cost of carbon has a significantly stronger effect on the optimal harvest age than found in earlier studies. Considered is also how increased use of harvest residues for energy purposes and storage of carbon in building materials and furniture should influence the length of the rotation period. The theoretical results are quantified within a numerical framework.

Keywords: climate, forestry, biofuels, Faustmann, carbon.

JEL classification: Q23, Q54, Q42.

Acknowledgements: The authors gratefully acknowledge comments from Cathrine Hagem as well as participants at the BEER Conference held in Bergen in May 2012 and the SURED Conference held in Ascona June 2012. While carrying out this research the authours have been associated with CREE -Oslo Centre for Research on Environmentally friendly Energy. CREE is supported by the Research Council of Norway.

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ISSN 0809-733X
Print: Statistics Norway

Sammendrag

Denne studien presenterer en justert Faustmann-regel for optimal hogst når man står overfor en samfunnsøkonomisk kostnad knyttet til utslipp av CO_2. Et av studiens bidrag er å gjøre dette innenfor et teoretisk og numerisk rammeverk som tar hensyn til dynamikken og interaksjonen mellom de ulike deler av skogens karbonlager samtidig som man anvender et uendelig tidsperspektiv. Med vår generelle tilnærming finner vi at en samfunnsøkonomisk kostnad av CO2 har en signifikant sterkere virkning på optimalt hogsttidspunkt enn det man har funnet i tidligere studier. Det studeres også hvordan optimalt hogsttidspunkt påvirkes av at man øker uttaket av hogstavfall for energiformål samtidig som man også tar hensyn til at en andel av tømmervirket brukes til varig lagring i møbler og bygninger.

1 Introduction

Inspired by the contributions by Fargione et al. (2008) and Searchinger et al. (2008) an extensive scientific debate has emerged with regard to whether the increasing use of biofuels is to the benefit of the climate, see for example Gibbs et al. (2010), Gurgel et al. (2007), Holtsmark (2012), Lapola et al. (2010), and Melillo et al. (2009). This debate makes topical the question of optimal forest management when there is a social cost of carbon. Earlier studies have concluded that a social cost of carbon emissions should lead to longer rotation periods, or, if the cost of carbon exceeds a certain level, the forests should not be harvested at all, see Asante et al. (2011), Asante and Armstrong (2012), Kötke and Dieter (2010), Karpainen et al. (2004), Price and Willis (2011), van Kooten et al. (1995).

The contribution of the present paper is to analyze this question theoretically with less restrictive assumptions than earlier studies and support the theoretical analysis with numerical simulations. We will show that our less restrictive assumptions turn out to be important for the conclusions.

Our starting point is Faustmann (1849), who has been attributed a formula for determination of the length of the rotation period when a forest owner's goal is to maximize the discounted yield, see also Clark (2010), Samuelson (1976) and Scorgie and Kennedy (1996). We develop an adjusted Faustmann Rule when there is a social cost of carbon emissions, while taking into account the dynamics and interactions of the forest's multiple carbon pools.

A contribution closely related to ours is van Kooten et al. (1995), who also provided a formula for determination of the length of the rotation period when there is a social cost of carbon emissions. However, van Kooten et al. (1995) included the carbon stock of stems only, which meant that important carbon pools as roots, stumps, tops and branches, harvest residues and naturally dead organic matter was not accounted for in their study. Both our theoretical and numerical analyses show that taking account of the interactions and dynamics of the forests' multiple carbon pools are crucial and means significantly longer rotation periods than found by van Kooten et al. (1995). Moreover, inclusion of the multiple carbon pools means that the threshold value of the social cost of carbon above which the forest should not be harvested at all is significantly lower

Asante et al. (2011) and Asante and Armstrong (2012) are two other closely related contributions. They underlined the importance of taking into account the forests' multiple carbon pools, as emphasized in our analysis as well. However, in order to keep the mathematics simple Asante et al. (2011) and Asante and Armstrong (2012) considered a single rotation period only. This explains some of their conclusions, for example that accounting for dead organic matter has the effect of reducing the rotation age (Asante and Armstrong, 2012, p 145). When we apply a multi-rotation, infinite time horizon model we come to the opposite conclusion.

More generally; to our knowledge no one has undertaken a full analysis of optimal forest management in the presence of a social cost of carbon that includes all the following five realistic features, which are all included in our model:

1. Only about half of the carbon in the forests' living biomass is contained in the tree trunks. Tops, branches, roots and stumps constitute the remaining half of the carbon stored in living biomass.

2. Harvest residues will gradually decompose and release carbon to the atmosphere. Moreover, natural deadwood constitutes an important part of the carbon stock of a forest. The dynamics of these carbon pools are included in the analysis.

3. We allow an exogenous fraction of tops, branches, roots and stumps to be harvested and used for energy purposes, and study the consequences of changing this fraction.

4. Tree trunks that are harvested may either be used in a way that immediately releases carbon to the atmosphere (e.g. for energy purposes) or as materials for buildings and furniture. The size of the fraction of the harvest used for such purposes and the lifetime of this carbon stock could be varied. We study different assumptions with regard to these parameters.

5. We apply an infinite time perspective, not only with a single harvest perspective.

Before we embark on the analysis, we should also mention Hartman (1976), who provided an adjusted rule for optimal rotation length. However, he considered a case where a forest provides valuable services when standing in addition to the values provided by timber harvesting and did not focus on a social cost of carbon.

The next section presents our theoretical model and our main theoretical results. The subsequent section presents numerical examples. The final section concludes. Appendix A contains proofs of our main results, while Appendix B provides a background discussion of whether the social cost of carbon is rising over time.

2 A model for calculation of optimal rotation length

2.1 The social cost of carbon is constant over time

We consider a forest where the stock of stems, measured in units of its carbon content, develops according to the function $R(t)$, where t is the time since last harvest, and $R(0) = 0$. It is assumed that the trunks constitute a share $\alpha \in (0, 1)$ of the total stock of living biomass $B(t)$. Other relevant stocks of carbon are harvest residues left in the forest and natural deadwood. In addition the model includes the stock of carbon stored in wood based building materials and furniture with their origin in the considered forest. Below the dynamics of all these stocks of carbon are modeled.

The forest owner is assumed to harvest a share $\sigma \in [0, 1]$ of the residues in addition to the trunks $R(t)$. Hence, in total a share $\alpha + \sigma(1 - \alpha) \in [\alpha, 1]$ of the total living biomass $B(T)$ is harvested, where T is the length of the rotation period.

The present value of the commercial profits from the next harvest, is

$$V_P(p,T,\sigma) = e^{-\delta T} p \left(1 + \sigma \frac{1-\alpha}{\alpha}\right) R(T), \tag{1}$$

where $\delta \in (0,1)$ is the discount rate and p is commercial profit per unit of harvest. To keep it as simple as possible, we assume that the commercial profit from harvesting of residues is the same as for trunks per unit volume. A more realistic approach would have been to assume a lower per unit profit from residues than from trunks, and in addition per unit profit from residues is a declining function of the share σ. Our simplification at this point is likely to mean that we overestimate the optimal share of residues that should be harvested as well as to what extent increasing the share of harvested residues reduces the optimal harvest age.

Next, assume that a share $\beta \in [0,1]$ of the trunks harvested is used as building materials and furniture. Hence, at time of harvest a stock of building materials and furniture $M(T)$ is generated, and we have that

$$M(T) = \beta R(T). \tag{2}$$

The remaining harvest, $(1 + \sigma(1-\alpha)/\alpha - \beta) R(t)$, is used for energy purposes and combusted immediately after harvesting.

We assume that there is a social cost of carbon emissions $s > 0$ that is constant over time. The present value social cost of immediate combustion of this share of the harvested biomass is

$$V_F(T,s,\beta,\sigma) = e^{-\delta T} s \left(1 + \sigma \frac{1-\alpha}{\alpha} - \beta\right) R(T). \tag{3}$$

Within each time period a share $\kappa \in (0,1)$ of the stock of building materials and furniture is scrapped and combusted. Hence, at time t the remaining stock of building materials/furniture from the first harvest is equal to $e^{-\kappa t} M(T)$, while emissions at time t due to combustion of this wood are $\kappa e^{-\kappa t} M(T)$.

Correspondingly, the amount of harvest residues left in the forest after a single harvest event is

$$D(T) = (1-\sigma) \frac{1-\alpha}{\alpha} R(T). \tag{4}$$

Within each period a share $\omega \in (0,1)$ of the stock of the residues left in the forest decomposes. Hence, at time t the remaining stock of residues from the first harvest is equal to $e^{-\omega t} D(T)$, while emissions at time t due to decomposition of these residues are $\omega e^{-\omega t} D(T)$. It follows that the present value social cost of these emissions from combustion of building materials and furniture, $V_M(T)$, and from decomposition of residues, $V_D(T)$, are:

$$V_M(T,s,\beta) = e^{-\delta T} s \int_0^\infty e^{-\delta x} \kappa e^{-\kappa x} \beta R(T) dx, \tag{5}$$

$$V_D(T,s,\sigma) = e^{-\delta T} s \int_0^\infty e^{-\delta x} \omega e^{-\omega x} (1-\sigma) \frac{1-\alpha}{\alpha} R(T) dx. \tag{6}$$

These expressions are simplified to:

$$V_M(T,s,\beta) = e^{-\delta T} s \frac{\kappa}{\delta+\kappa} \beta R(T), \tag{7}$$

$$V_D(T,s,\sigma) = e^{-\delta T} s \frac{\omega}{\delta+\omega} (1-\sigma) \frac{1-\alpha}{\alpha} R(T). \tag{8}$$

As the forest grows, it will capture and store carbon. The social present value of carbon capture in living biomass over the first rotation is:

$$V_{CC}(T,s) = \frac{s}{\alpha} \int_0^T e^{-\delta x} \dot{R}(x) dx. \tag{9}$$

Finally, we have to take into consideration that the forest contains a stock of naturally dead biomass, denoted by $N(t)$, and with $N(0) = 0$ (ignoring any remaining natural deadwood that might have been generated in earlier rotation periods). We assume that within each period, the amount of natural deadwood generated is equal to a constant share $\gamma \in (0,1)$ of living biomass. Hence, the accumulation of natural deadwood is:[1]

$$\dot{N}(t) = \frac{\gamma}{\alpha} R(t) \text{ for } t \in (0,T). \tag{10}$$

Solving for $N(t)$ from (10), and again ignoring any natural deadwood that might have been generated in earlier rotation periods, the stock of natural deadwood at time of the first harvest is:

$$N(T) = \frac{\gamma}{\alpha} \int_0^T R(x) dx. \tag{11}$$

At time T, when the forest is harvested, accumulation of a new stock of natural deadwood begins. At the same time the stock of natural deadwood from the first

[1] Note that $B(t)$ should be interpreted as the volume of living biomass at the end of period t after deduction of the share γ, which died in period t.

rotation enters a phase of decomposition (see comment on this below), and we assume that natural deadwood decomposes with the same rate ω as harvest residues.

The net accumulation of natural deadwood gives rise to a positive welfare effect through additional carbon capture in the forest. The present social value of carbon capture due to accumulation of natural deadwood during the first rotation period is:

$$V_{NCC}(T,s) = s\frac{\gamma}{\alpha}\int_0^T e^{-\delta x}R(x)dx. \tag{12}$$

It follows that the discounted social cost of emissions from decomposition of natural deadwood that was accumulated during the first rotation cycle is:

$$V_N(T,s) = e^{-\delta T}s\int_0^\infty e^{-\delta x}\omega e^{-\omega x}N(T)dx,$$

which could be written:

$$V_N(T,s) = e^{-\delta T}s\frac{\omega}{\delta+\omega}\frac{\gamma}{\alpha}\int_0^T R(x)dx. \tag{13}$$

Before we proceed, it should be noticed that we make a simplification in the model of natural deadwood accumulation, by no explicit modeling of the decomposition of natural deadwood before the harvest. Hence, with our formulation in (10) the parameter γ represents the *net* accumulation rate. An alternative formulation would be to explicitly model decomposition also in the phase before harvest such that natural deadwood develops according to $\dot{N}(t) = \gamma B(t) - \omega N(t)$, a formulation that is used in Holtsmark et al. (2012). However, this is not necessarily an improvement compared to our formulation in (10). The assumption that the generation of natural deadwood constitutes a constant fraction of the living biomass in each period is also a simplification. Older forests have higher turnover rates than younger forests. This corresponds to γ being increasing in the stand age t. If one was to include the decomposition of natural deadwood also before harvest has taken place, as in the alternative formulation, this would therefore represent a time profile for the net accumulation of natural deadwood with too high stocks of natural deadwood in young stands and vice versa. Thus, even though our formulation represents a simplification, at least it does not draw the results towards a common bias. Finally, it might be considered unrealistic that our model means positive net accumulation of natural deadwood even in very old forests. However, Luyssaert et al. (2008) found that also in very old forests there is a net accumulation of natural deadwood.

Summing up, all terms in the net social welfare generated by the first harvest cycle, $V(p,T,s,\beta,\sigma)$, is then:

$$\begin{aligned} V(p,T,s,\beta,\sigma) := {} & V_P(\cdot) + V_{CC}(\cdot) - V_F(\cdot) - V_M(\cdot) \\ & - V_D(\cdot) + V_{NCC}(\cdot) - V_N(\cdot), \end{aligned} \tag{14}$$

where all terms on the right hand side are defined above. Next, define a welfare function including the sum of the discounted welfare of all future rotation cycles:

$$W(p,T,s,\beta,\sigma) := V(\cdot) + e^{-\delta T}V(\cdot) + e^{-\delta 2T}V(\cdot) + ...,$$

which is simplified to:

$$W(p,T,s,\beta,\sigma) = \frac{1}{1-e^{-\delta T}}V(p,T,s,\beta,\sigma). \tag{15}$$

In preparation for our first result, note that if the rotation period T is increased by one time unit, the first harvest takes place one time unit later, the second harvest two time units later, and so forth. A rule of harvesting simply saying that the growth rate of the stock of stems should drop to the level of the discount rate does not account for this. The contribution of the German forester Martin Faustmann (1849) was to take into account the complete added delay of profits from harvesting when the rotation period is prolonged.

When a social cost on carbon emissions is introduced, similar and additional effects come into play. When increasing the rotation period, the amount of carbon stored in the forest at time of harvesting increases, and emissions from immediate combustion, and from combustion of building materials and furniture, in addition to decomposition of harvest residues, are postponed. And these delays apply to future rotations as well. However, the beginning of the process of carbon capture *after* each harvest is also delayed. Furthermore, the process of accumulation of natural deadwood is also affected by increasing the rotation period. In a period of time after harvest there will be net release of CO2 from natural deadwood, as the generation of natural deadwood is small in a young stand. Postponing harvest means an additional period with positive net accumulation of natural deadwood. The trade off between carbon storage now or in the future, as well as between profits now or in the future, determines the optimal length of the rotation period.

Next, define:

$$\Phi(T) := \frac{\int_0^T R'(x)e^{-\delta x}dx}{R(T)}, \tag{16}$$

$$\Theta(T) := \frac{\gamma}{\delta+\omega}\left(1-e^{-\delta T}\right) + \frac{\gamma}{R(T)}\int_0^T \left(\frac{\omega}{\delta+\omega} - e^{-\delta x}\right)R(x)dx, \tag{17}$$

$$\Omega := p\left[\alpha + \sigma(1-\alpha)\right] + s\left[(1-\alpha)(1-\sigma)\left(1-\frac{\omega}{\delta+\omega}\right) + \alpha\beta\left(1-\frac{\kappa}{\delta+\kappa}\right)\right]. \tag{18}$$

Note for later use that $\Phi(t) \leq 1$. Furthermore, $\Omega > 0$ and $\partial\Omega/\partial s \in (0,1)$, which means that the fraction (s/Ω) is monotonically increasing in s. Moreover, $\Theta(T) \geq 0$ and $\partial\Omega/\partial s > 0$, see Appendix for proofs.

We can now state our main theoretical result; an adjusted Faustmann formula taking the social costs of carbon emissions into account:

Proposition 1 *If there exists a T that satisfies:*

$$\frac{R'(T)}{R(T)} = \frac{\delta}{1-e^{-\delta T}}\left(1 - \frac{s}{\Omega}\left(1 - \Phi(T) + \Theta(T)\right)\right), \tag{19}$$

then this T maximizes social welfare $W(p, T, s, \beta, \sigma)$.

If, for all $T > 0$

$$\frac{R'(T)}{R(T)} > \frac{\delta}{1-e^{-\delta T}}\left(1 - \frac{s}{\Omega}\left(1 - \Phi(T) + \Theta(T)\right)\right), \tag{20}$$

then social welfare $W(p, T, s, \beta, \sigma)$ is maximized by never harvesting the forest. All functions and parameters in (19) *and* (20) *are defined above.*

Proof. See Appendix. ■

The right hand side of (20) is monotonically decreasing in s, which means that there is a threshold value, which we label $\bar{s}$, such that if $s > \bar{s}$, then (20) apply for all $T > 0$ and the forest should not be harvested. This also means that if harvesting is not commercially profitable without any subsidies or taxes, harvesting cannot give a social surplus either and the harvesting should not take place.

It follows from Proposition 1 (more precisely from equation (19)) that if $s = 0$, then the rotation period that maximizes social welfare is defined by:

$$\frac{R'(T)}{R(T)} = \frac{\delta}{1-e^{-\delta T}}, \tag{21}$$

which is the classical formula attributed to Faustmann (1849) for maximization of the forest owner's profit. Furthermore, if $s = 0$ and the discount rate δ approaches zero, then

$$\frac{R'(T)}{R(T)} = \frac{1}{T}. \tag{22}$$

This formula gives the maximum sustained yield.

Our next result concerns the effect on the optimal length of the rotation period of an increase in the social cost of carbon, s:

Proposition 2 *If there exists a T that satisfies (19), the length of the rotation period that maximizes social welfare is strictly increasing in the social cost of carbon, s.*

Proof. See Appendix. ■

This result is in accordance with earlier studies. However, the numerical chapter will show that with our combination of a multiple carbon pools with a multiple rotation period approach, the effect of the social cost of carbon s on the length of the rotation period is significantly stronger than found by van Kooten et al. (1995) and by Asante et al. (2011) and Asante and Armstrong (2012). Moreover, the threshold value of s above which the forest should not be harvested is estimated to be significantly lower with our multiple carbon pools approach.

In our next result, we consider how an increase in σ, the share of residues that is harvested, affects the optimal length of the rotation period. In order to focus on the choice of the rotation period, we have treated σ as exogenous. We have also made the assumption that the per unit commercial profit, p, is the same for these residues as for trunks. It would be more realistic to let the marginal profit be decreasing in the share of residues that is harvested, due to increasing marginal costs of harvesting this material. The forest owner would then choose $\sigma \in [0, 1]$ in order to maximize profits. It is therefore of interest to investigate the effect of changes in this parameter.

Proposition 3 *If there exists a T that satisfies (19), an increase in σ, the share of the living biomass that is harvested in addition to trunks, will strictly decrease the optimal length of the rotation period, T, if and only if*

$$\frac{s}{p} < \frac{\delta + \omega}{\delta}. \tag{23}$$

If this inequality does not hold, the optimal length of the rotation period will either be increased or be unaffected by an increase in σ.

Proof. See Appendix. ■

An increase in σ means that more biomass is harvested and used for energy purposes, and less harvest residues are left in the forest. The result is that both commercial profits and emissions immediately after harvest are increased. If the per unit profit is large enough, this decreases the optimal length of the rotation period. However, if the social cost of carbon emissions is large compared to the per unit profit, the optimal length of the rotation period is increased.

2.2 A rising social cost of carbon

Economic theory suggests that $s(t)$ is rising over time, while the present value of $s(t)$ is declining over time (see Appendix B). So far, we have considered the limiting case of $s(t)$ being constant. We now turn to the other limiting case, i.e. the case of $s(t)$ rising at the rate of interest δ, i.e. $s(t) = s_0 e^{\delta t}$.

Total welfare for one rotation period is given by

$$V_P(\cdot) + s_0 \Sigma(\cdot), \tag{24}$$

where $V_P(\cdot)$ is defined by (1) and where

$$\Sigma(\cdot) := -V_F^*(\cdot) - V_M^*(\cdot) - V_D^*(\cdot) + V_{CC}^*(\cdot) + V_{NCC}^*(\cdot) - V_N^*(\cdot), \tag{25}$$

where all $V_i^*(\cdot)$ are defined as $V_i(\cdot)$ in the previous subsection except that we replace s with $e^{\delta t}$. This gives

$$\begin{aligned}
V_F^*(\cdot) &= \left(1 + \sigma\frac{1-\alpha}{\alpha} - \beta\right) R(T) \\
V_M^*(\cdot) &= \beta R(T) \\
V_D^*(\cdot) &= (1-\sigma)\frac{1-\alpha}{\alpha} R(T) \\
V_{CC}^*(\cdot) &= \frac{1}{\alpha} R(T) \\
V_{NCC}^*(\cdot) &= \frac{\gamma}{\alpha}\int_0^T R(x)dx \\
V_N^*(\cdot) &= \frac{\gamma}{\alpha}\int_0^T R(x)dx
\end{aligned}$$

It follows that $\Sigma(\cdot) = 0$. This means that the social welfare for a infinite horizon constant rotation forest is given by

$$W(p, T, \sigma) = \frac{1}{1 - e^{-\delta T}} V_P(p, T, \sigma), \tag{26}$$

and the value of T that maximizes this is simply the standard Faustmann rule given in (21), independent of the size of s_0.

This result is not surprising. Consider again the one period rotation model: We start out with zero carbon tied up in biomass. As time passes, carbon in biomass increases. Once the forest is harvested, all of the carbon is released to the atmosphere (some immediately and some only gradually). As long as the present value of the social cost of carbon is constant, the initial increase of carbon in biomass has exactly the same social value as the later reduction. Hence, the one rotation period social welfare is independent of the level of the social cost of carbon. It immediately follows that the same must be true of the present social value for the infinite horizon constant rotation period case.

3 Numerical illustrations

In order to provide further intuition to the theoretical results in section 2, this section provides numerical simulations of the consequences of implementation of a social cost of carbon for optimal harvest from a forest. We will in this section only consider cases where the social cost of carbon is constant over time.

3.1 Model and parameter values

After a harvest the stock of stems is assumed to develop along the function

$$R(t) = v_1(1 - e^{-v_2 t})^{v_3}.$$

We have followed Asante et al. (2011) in choice of parameter values, which are as follows: $v_1 = 500.4, v_2 = 0.027, v_3 = 4.003$. The chosen numerical representation gives maximum sustained yield at 88 year old stands. Hence, it is representative for a Scandinavian forest where the dominating spruce and pine forests typically are mature after 80 – 110 years. With regard to development of the stock of other living biomass, it is assumed that the trunks constitute 48 percent of total biomass in the forest, i.e. $\alpha = 0.48$ (NCPA, 2010).

With regard to the stock of natural deadwood, it is assumed that $\gamma = 0.001$, see equation (10) for definition. This parameter value gives an accumulation of natural deadwood corresponding to what is found in Asante et al. (2011). The decomposition rate for deadwood, ω, is set to 0.04 (Holtsmark 2012).

With regard to the share β of the harvested stems that are used for building materials and furniture, based on NCPA (2011) it is assumed that $\beta = 0.25$ in the base case. However, simulations are provided where other values of this parameter is applied. We have assumed that building materials and furniture are reasonably durable goods in the sense that only a share κ =0.014 of this stock of wood is scrapped and combusted annually.

The amount of residues harvested is determined by the share σ, which is set to 0.2 in the base case. This implies that 20 percent of other living biomass than the stems are harvested. However, additional simulations are carried out considering higher and lower assumptions with regard to the value of σ. Figure 1 provides a description of how the different components of the forest's carbon stock develop if the rotation length is 150 years.

In the simulations presented in the next subsection it is assumed that the forest owner's net profit is 15 USD/m^3 wood harvested, for short labeled the (net) price of wood. This corresponds to 20.45 USD/tCO_2 or 75 USD/tC, as one cubic meter of wood contains approximately 0.2 tonnes carbon, corresponding to 0.733 tonnes CO_2. Note that only the relative price of the social cost of carbon, s/p, matters.

The discount rate is set to 0.05 in all simulations.

3.2 Simulation results

Figure 2 shows the results of simulations carried out in a case where 20 percent of residues are harvested, i.e. $\sigma = 0.2$. The solid curve shows the case where $\beta = 0$, i.e. the share of the harvested stems that are used for building materials and furniture is zero. The dashed curve shows the case where β =0.25, while the dotted curve shows the case where $\beta = 0.5$.

Figure 1. The development of the components of the stock of carbon in the forest and in building materials/furniture with a rotation length of 150 years

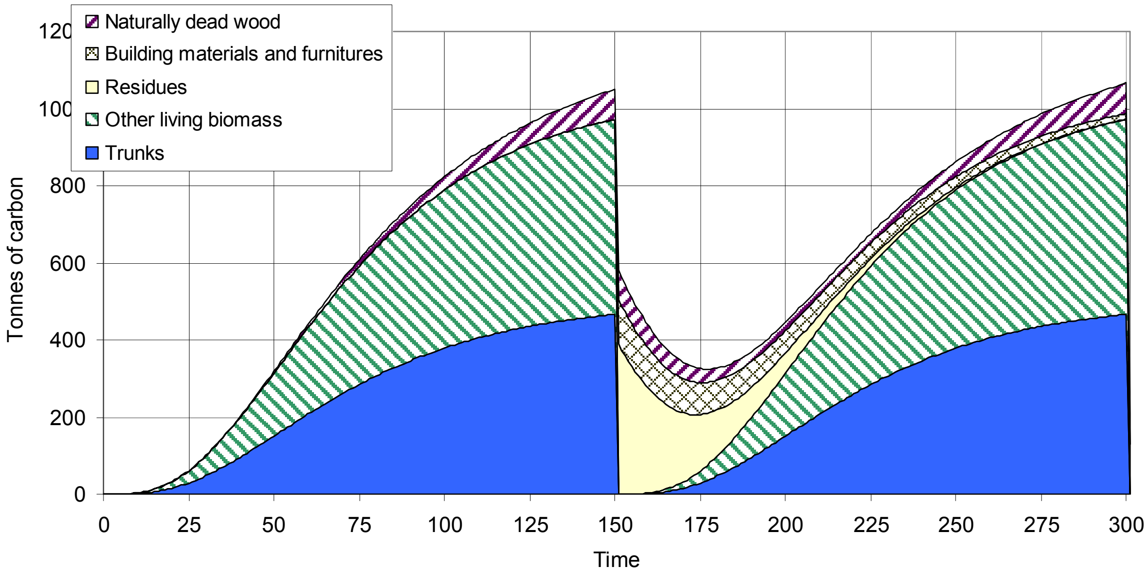


Table 1. Optimal length of the rotation period (*T*) with regard to different values of the social cost of carbon (*s*), as well as different values of the share of residues harvested (σ)[1]

Social cost of carbon		The share of residues harvested (σ)		
s/p	(USD/tCO_2)	0	0.25[2]	0.5[3]
0	0	39	39	39
0.49	10	65	61	57
0.73	15	86	77	71
1.00	20.45	146	110	96
1.22	25	∞	∞	∞

[1] The share of the harvested trunks that are used for durable storage in buildings and furniture (*β*) is set to 0.25 in all simulations presented in this table.
[2] σ=0.25 means that all tops and branches are harvested.
[3] σ=0.5 means that a share of stumps and roots is harvested in addition to tops and branches.

Figure 2. The optimal length of the rotation period given different shares of the harvest that are used for durable storage in buildings and furniture. The net commercial profit to the forest owner of 15 USD/m^3 wood, which corresponds to 20.45 USD/t CO_2. Hence, *s/p* = 1 if the social cost of carbon is 20.45 USD/t CO_2

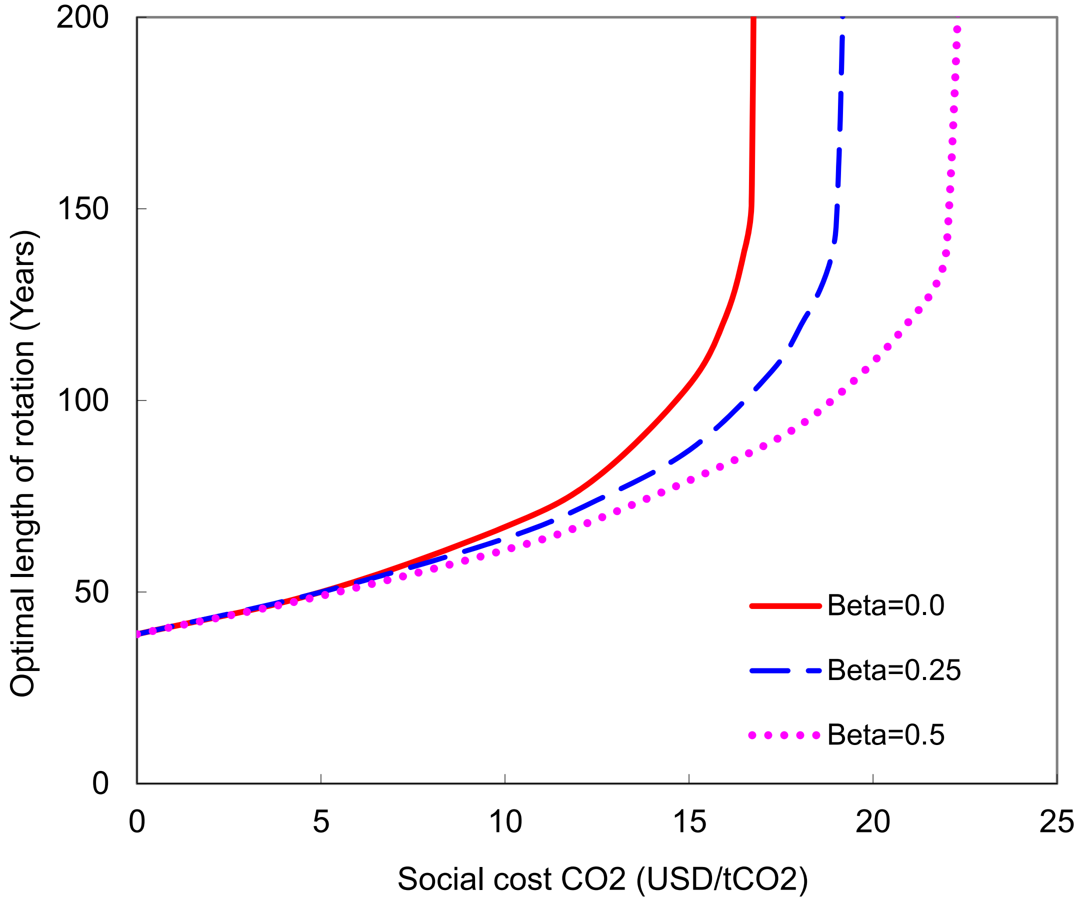


Figure 3. The optimal length of the rotation period in the main multiple carbon pool case (the double lined curve) and cases where one or more carbon pools are not included in the analysis

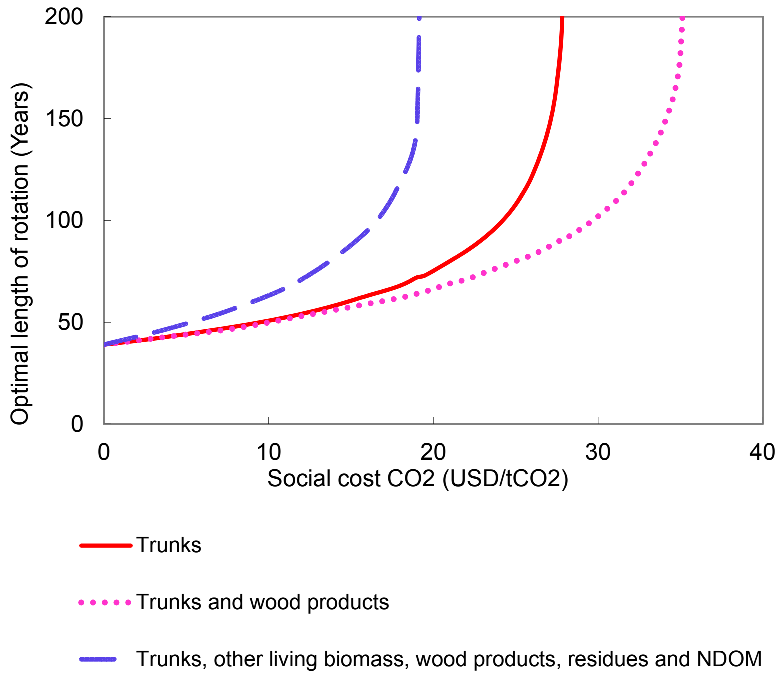
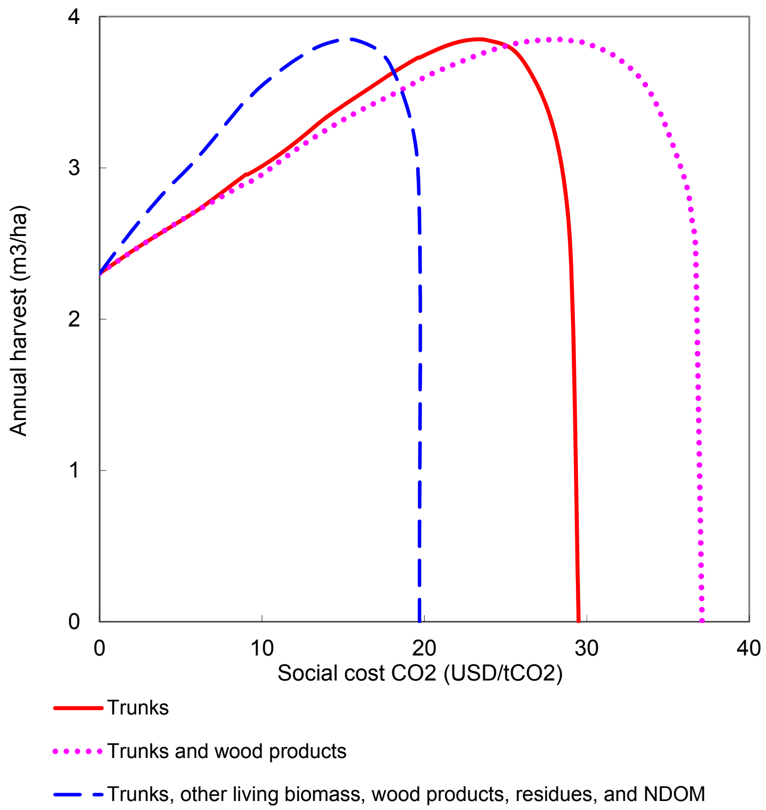


Figure 4. The long run supply of wood given different social costs of carbon when different carbon pools are included in the model



The curves in Figure 2 confirm the result of Proposition 2, that increasing the social cost of carbon s should lead to longer rotation periods. This applies also in the case where a reasonable share of the harvested stems in some way or another are converted to a permanent carbon storage, i.e. when $\beta > 0$. In addition Figure 2 illustrates that increasing β,i.e. the share of the harvested stems that are used for building materials and furniture, has a significant effect and draws in the direction of shorter rotation.

Table 1 presents results of a number of model simulations given different levels of the share of the residues that are harvested as well as different levels of the social cost of carbon. In these simulations it is assumed that the share β of the harvested trunks that are used as building materials and furniture is fixed at 0.25, as this is likely to med close to a realistic level (NCPA 2011). Table 1 shows that the optimal length of the rotation period is significantly influenced by the share of the residues that are harvested. It should, however, be noted that we ignore that harvesting of residues is likely to influence the carbon balance of the soil, and might lead to release of carbon to the atmosphere. The carbon stock of the soil constitutes a significant share of the carbon stock of boreal and temperate forests (Kasischke 2000). Hence, this effect might be significant (Nakane and Lee 1995, Palosuo et al. 2001, Nilsen et al. 2008). Moreover, as mentioned in section 2, we assumed that the unit costs related to harvesting of residues are constant to scale and that the commercial profit from harvesting residues is as high as the commercial profit from harvesting stems (per m^3). These simplifications have a common bias and draw in the direction of too high estimates of to what extent increasing the share of residues harvested should reduce the rotation period.

Both Table 1 and Figure 2 illustrate that the social carbon cost has a certain threshold value above which the forest should not be harvested. The higher is the share of the harvest stored in furniture and buildings, the higher is the mentioned threshold value.

It is here appropriate to recall that only the relative price of wood matters. Hence, if we for example are considering a marginal forest in the sense that the commercial profit from harvesting is low, then the threshold value of the social cost of carbon, above which the forest should not be harvested, is lower than found in the presented simulation. And correspondingly, if we consider a forest with high commercial profit from harvesting, the threshold value is higher than found here.

In this paper we have emphasized the importance of taking account of the forests' different carbon pools, not only the trunks. Figure 3 shows the importance of this. The solid curve in Figure 3 shows the estimates of optimal rotation period in the case where all carbon pools other than the trunks are ignored. The dotted curve shows the estimates when only the trunks and the pool of wooden products are included. Finally, the dashed curve shows the result when all carbon pools are taken account of. The figure shows that these choices influence the estimates of the optimal rotation period significantly. The inclusion of the wood product pool means shorter rotation and a higher threshold value above which the forest should not be harvested. Inclusion of harvest pools as other living biomass than the stems, harvest residues and NDOM draws in the direction of significantly longer rotation periods and a significantly lower threshold value above which the forest should not be harvested.

As mentioned in the introduction, our results with regard to the effects of inclusion of dead organic matter in the analysis contrast the main finding in Asante and Armstrong (2012) and Asante et al. (2011). They found that incorporating dead organic matter has the effect of reducing the rotation period. In addition, they found that a high initial stocks of dead organic matter and wood products have the effect of reducing the rotation period. With regard to the latter result, Holtsmark et al. (2012) show that it follows from the consideration of a single rotation period only and the fact that Asante and Armstrong (2012) and Asante et al. (2011) ignore the release of carbon from decomposition of dead organic matter after the time of the first harvest T. With that simplification it is obvious that a large initial stock of dead organic matter draws in the direction of earlier harvest. Holtsmark et al. (2012) show if it had been taken into account that the time profile of the decomposition of the initial carbon pools over the infinite time horizon $t\epsilon(0,\infty)$ is not influenced by the harvest age, the size of the initial carbon pools has no effect on the optimal harvest age. The first mentioned result in Asante and Armstrong (2012) and Asante et al. (2011) with regard to the effects of incorporating multiple carbon pools in the analysis should also be considered in the light of their fail to see the importance of the release of carbon from dead organic matter after time T.

As underlined by van Kooten et al. (1995), longer rotation periods do not necessarily reduce the supply of timber in the long term. Figure 4 illustrates this. When the social cost increases from zero the long term supply of timber is firstly increasing before a maximum is reached. If the social cost of carbon is further increased, the long term supply is reduced and is zero if the social cost of carbon settles above the mentioned threshold value. Figure 4 also illustrates the importance of taking the forests' multiple carbon pools into account.

4 Discussion and conclusion

The increasing use of subsidies in order to encourage the use of biofuels, including wood fuels from forests, calls for a theoretical clarification of how a social cost of carbon should influence forest managment. Searchinger et al. (2009) claim that current regulation regimes might lead to overharvesting of the worlds forests. In order to increase the insight on the issue this paper provides a theoretical model of the relationship between forest management and the interaction and dynamics of the forest's multiple carbon pools. The theoretical analysis leads to an adjusted Faustmann Rule for optimal harvest when there is a social cost of carbon emissions.

Compared to other studies, our contribution is to do this in a considerably less restrictive theoretical framework. We take into account that less than half of the carbon in the forests' biomass is contained in the tree trunks. Tops, branches, roots and stumps constitute approximately half of the carbon stored in living biomass, and to the extent that these components are not harvested together with the trunks, they will gradually decompose and release carbon to the atmosphere. The dynamics of these carbon pools as well as the stock of natural deadwood is included in both the theoretical and numerical analyses. In addition, we allow an exogenous fraction of tops, branches, roots and stumps to be harvested and used for energy purposes. And finally, the dynamics of a stock of carbon stored in building materials and furniture is also taken into account.

The adjusted Faustmann Rule presented confirms earlier results saying that a social cost of carbon emissions should lead to longer rotation periods than what follows from the original rule provided by Faustmann (1849). Moreover, if the social cost of carbon is sufficiently high, the forest should never be harvested. However, with our less restrictive approach, taking both multiple rotation periods and multiple of carbon pools into the analysis, the threshold value of the social cost of carbon above which harvest should not take place is significantly lower than found in studies with a more restrictive approach. The multiple carbon pool approach also means that the effect of a social cost of carbon on the length of the rotation period is significantly stronger than found in previous studies.

We also found that increasing the share of residues harvested and/or the share of stems used for durable storage in buildings and furniture, the effect of a social cost of carbon on the optimal rotation period is smaller.

Appendix

Proofs

Proof of proposition 1. We want to find the T that maximizes $W(p, T, s, \beta, \sigma)$. In order to simplify, define the functions:

$$W_i(\cdot) := \frac{1}{1 - e^{-\delta T}} V_i(\cdot), \ \ i = P, CC, F, M, D, NCC, N.$$

Next, define $W_H(T) := W_P(\cdot) - W_F(\cdot)$. Then we have that:

$$W_H(\cdot) = \left(p \left(1 + \sigma \frac{1-\alpha}{\alpha} \right) - s \left(1 + \sigma \frac{1-\alpha}{\alpha} - \beta \right) \right) \frac{1}{e^{\delta T} - 1} R(T), \tag{A.1}$$

$$W_M \left(\cdot\right) = s\beta \frac{\kappa}{\delta + \kappa} \frac{1}{e^{\delta T} - 1} R(T), \tag{A.2}$$

$$W_D \left(\cdot\right) = s \frac{\omega}{\delta + \omega} \frac{1}{\alpha} \left(1 - \alpha\right) \left(1 - \sigma\right) \frac{1}{e^{\delta T} - 1} R(T), \tag{A.3}$$

$$W_{CC}(\cdot) = \frac{s}{\alpha} \frac{1}{1 - e^{-\delta T}} \Phi\left(T\right) R(T). \tag{A.4}$$

$$W_{NCC}(\cdot) = s \frac{\gamma}{\alpha} \frac{1}{1 - e^{-\delta T}} \int_0^T e^{-\delta x} R(x) dx, \tag{A.5}$$

$$W_N(\cdot) = \frac{1}{e^{\delta T} - 1} s \frac{\gamma}{\alpha} \frac{\omega}{\delta + \omega} \int_0^T R(x) dx. \tag{A.6}$$

It follows that:

$$\frac{\partial W_H(\cdot)}{\partial T} = \left(p\left(1+\sigma\frac{1-\alpha}{\alpha}\right) - s\left(1+\sigma\frac{1-\alpha}{\alpha} - \beta\right)\right)\frac{1}{e^{\delta T}-1}\left(R'\left(T\right) - \frac{\delta}{1-e^{-\delta T}}R\left(T\right)\right),$$

$$\frac{\partial W_M(\cdot)}{\partial T} = s\beta\frac{\kappa}{\delta+\kappa}\frac{1}{e^{\delta T}-1}\left(R'\left(T\right) - \frac{\delta}{1-e^{-\delta T}}R\left(T\right)\right) \quad \text{(A.7)}$$

$$\frac{\partial W_D(\cdot)}{\partial t} = s\frac{\omega}{\delta+\omega}\frac{1}{\alpha}\left(1-\alpha\right)\left(1-\sigma\right)\frac{1}{e^{\delta T}-1}\left(R'\left(T\right) - \frac{\delta}{1-e^{-\delta T}}R\left(T\right)\right), \quad \text{(A.8)}$$

$$\frac{\partial W_{CC}(\cdot)}{\partial T} = \frac{s}{\alpha}\frac{1}{e^{\delta T}-1}\left(R'(T) - \frac{\delta}{1-e^{-\delta T}}\Phi\left(T\right)R(T)\right), \quad \text{(A.9)}$$

$$\frac{\partial W_{NCC}(\cdot)}{\partial T} = \frac{1}{e^{\delta T}-1}s\frac{\gamma}{\alpha}\left(R(T) - \frac{\delta}{1-e^{-\delta T}}\int_0^T e^{-\delta x}R(x)dx\right), \quad \text{(A.10)}$$

$$\frac{\partial W_N(\cdot)}{\partial T} = s\frac{\omega}{\delta+\omega}\frac{1}{e^{\delta T}-1}\frac{\gamma}{\alpha}\left(R(T) - \frac{\delta}{1-e^{-\delta T}}\int_0^T R(x)dx\right). \quad \text{(A.11)}$$

Next, define:

$$\Delta_1 := R'(T) - \left(1 - \frac{s}{\Omega}\left(1-\Phi(T)+\Theta\left(T\right)\right)\right)\frac{\delta}{1-e^{-\delta T}}R(T). \quad \text{(A.12)}$$

Then we could write the first order condition:

$$\frac{\partial W(p,T,s,\beta,\sigma)}{\partial T} = \frac{\Omega}{\alpha}\frac{1}{e^{\delta T}-1}\Delta_1 = 0, \quad \text{(A.13)}$$

which gives (19). Furthermore, the inequality in (20) is equivalent to

$$\frac{\partial W(p,T,s,\beta,\sigma)}{\partial T} > 0. \quad \text{(A.14)}$$

If this inequality applies for all $T > 0$, then the first order condition (19) does not hold for any $T > 0$, and social welfare is maximized by never harvesting. □

Proof of proposition 2. From $(A.13)$ it follows that the second order condition for the maximization problem can be written as:

$$\frac{\partial^2 W(p,T,s,\beta,\sigma)}{\partial T^2} = \frac{\Omega}{\alpha}\frac{\partial}{\partial T}\left(\frac{1}{e^{\delta T}-1}\right)\cdot\Delta_1 + \frac{\Omega}{\alpha}\frac{1}{e^{\delta T}-1}\cdot\frac{\partial \Delta_1}{\partial T} \leq 0. \quad \text{(A.15)}$$

It follows from the first order condition $(A.13)$ that $\Delta_1 = 0$. Moreover, as the large bracket in Ω is a weighted average of $(1-\alpha)$ and α, it is easily verified that $\Omega > 0$. Hence, the second order condition is reduced to $\partial\Delta_1/\partial T \leq 0$. Define:

$$\Delta_2 := \left(\frac{R''(T)}{R(T)} - \left(\frac{R'(T)}{R(T)}\right)^2 + \frac{\delta}{e^{\delta T}-1}\frac{R'(T)}{R(T)} - \frac{s}{\Omega}\left(\Phi'(T) - \Theta'(T)\right)\frac{\delta}{1-e^{-\delta T}}\right) R(T).$$

It is straight forward to show that

$$\frac{\partial \Delta_1}{\partial T} = \Delta_2. \tag{A.16}$$

Furthermore, when taking the derivative of (19) with respect to s, we find that:

$$\frac{\partial T}{\partial s} = \frac{\delta}{1-e^{-\delta T}}\frac{1}{\Delta_2}\frac{1}{\Omega}\left(\frac{s}{\Omega}\frac{\partial \Omega}{\partial s} - 1\right)\left(1 - \Phi(T) + \Theta(T)\right) > 0. \tag{A.17}$$

To show that $\partial T/\partial s > 0$, we start checking that $(1 - \Phi(T) + \Theta(T)) > 0$. With regard to $\Theta(T)$ some reorganizing gives that:

$$\begin{aligned}\Theta(T) = {} & \frac{1}{\delta+\omega}\left(\left(1-e^{-\delta T}\right) - \frac{\delta}{R(T)}\int_0^T e^{-\delta x}R(x)dx\right) \\ & + \frac{1}{\delta+\omega}\left(\frac{\omega}{R(T)}\int_0^T \left(1-e^{-\delta x}\right)R(x)dx\right).\end{aligned}$$

It is easily seen that the second term here is positive. With regard to the first term, using l'Hospital's rule we find that

$$\lim_{T\to 0}\left(\left(1-e^{-\delta T}\right) - \frac{\delta}{R(T)}\int_0^T e^{-\delta x}R(x)dx\right) = -\lim_{T\to 0}\frac{\delta e^{-\delta T}R(T)}{R'(T)} = 0. \tag{A.18}$$

Hence, as T approaches 0, the first term approaches zero. Moreover, the derivative of the first term with respect to T is:

$$\frac{R'(T)}{(R(T))^2}\int_0^T e^{-\delta x}R(x)dx, \tag{A.19}$$

which is positive. Hence, we have that $\Theta(T)$ is positive for any $T > 0$.As we easily see that $1 - \Phi(T) \geq 0$, we have that $(1 - \Phi(T) + \Theta(T)) > 0$.

It is easily seen that $(s/\Omega)(\partial\Omega/\partial s) - 1 < 0$, as $\alpha + \sigma(1-\alpha) \in (0,1)$ and

$$\frac{\partial \Omega}{\partial s} = (1-\alpha)(1-\sigma)\left(1 - \frac{\omega}{\delta+\omega}\right) + \alpha\beta\left(1 - \frac{\kappa}{\delta+\kappa}\right). \tag{A.20}$$

From the second order condition (A.15) we have that $\Delta_2 \leq 0$. Hence, finally

$$\frac{\partial T}{\partial s} \geq 0.\square$$

Proof of proposition 3. In line with the proof of proposition 2, taking the derivative of (19) with respect to σ and rearranging yields:

$$\frac{\partial T}{\partial \sigma} = \frac{\delta}{1-e^{-\delta T}} \frac{1}{\Delta_2} \frac{s(1-\Phi(T)+\Theta(T))}{\Omega^2} \frac{\partial \Omega}{\partial \sigma}. \tag{A.21}$$

We have that:

$$\frac{\partial \Omega}{\partial \sigma} = (1-\alpha)\left(p - s\left(1-\frac{\omega}{\delta+\omega}\right)\right) \begin{cases} > 0 \text{ if } s/p < \frac{\delta+\omega}{\delta} \\ \leq 0 \text{ if } s/p \geq \frac{\delta+\omega}{\delta} \end{cases}, \tag{A.22}$$

and it follows that

$$\frac{\partial T}{\partial \sigma} \begin{cases} < 0 \text{ if } s < \frac{\delta+\omega}{\delta} \\ \geq 0 \text{ if } s \geq \frac{\delta+\omega}{\delta} \end{cases}, \tag{A.23}$$

which is equivalent to the statement in Proposition 3. $\square$

The social cost of carbon

The social cost of carbon is the present value of all future climate costs caused by one unit of current emissions. In formal notation this is often written as

$$s(t) = \int_t^\infty e^{-(\delta+\rho)(\tau-t)} C'(A(\tau)) d\tau \tag{A.24}$$

where δ is the discount rate, ρ is the depreciation rate for carbon in the atmosphere, $A(\tau)$ is the stock of carbon at date τ (above natural or preindustrial level) and C is a measure of climate costs, assumed at any time to depend on the stock of carbon in the atmosphere at that time.

The size of the appropriate discount rate has been discussed extensively in the literature, and we have nothing to add to this discussion. The formula above is based on the assumption that an amount $\rho A(\tau)$ of the carbon in the atmosphere at date τ is transferred from the atmosphere to other carbon sinks (in particular to the ocean). Although used frequently in economic models, it is well-known that this assumption is a very inaccurate description of the true carbon cycle. In particular, the assumption means that if emissions drop to zero, the amount of carbon in the atmosphere will eventually drop down to its preindustrial level. The assumption also implies that if emissions are constant and equal to $\rho A(\tau)$ from τ onwards, carbon in the atmosphere will remain constant from τ onwards.

It is true that a rapid increase of carbon in the atmosphere will gradually decline over time, as it is transferred to other sinks. However, a significant portion (about 25% according to e.g. Archer, 2005) remains in the atmosphere for ever (or at least for thousands of years). Even if emissions are constant the carbon in the atmosphere will eventually grow; the only possibility for the amount of carbon in the atmosphere to be constant for a long period is to have zero emissions. Moreover, for a given amount of fossil fuels extracted, there is a corresponding long-run increase in the amount of carbon in the atmosphere.

From the discussion above it is clear that $\rho = 0$ in many ways gives a better representation of some important features than $\rho > 0$. Some analyses explicitly take into account the fact that some but not all carbon emissions remain in the atmosphere, see e.g. Farzin and Tahvonen (1996). In our subsequent discussion we simply assume $\rho = 0$, so that (A.24) implies

$$\dot{s}(t) = \delta s(t) - C'(A(t)) \tag{A.25}$$

An immediate conclusion from this is that the present value of $s(t)$ declines over time provided $C' > 0$. To be able characterize the path of $s(t)$ any further we must first discuss the properties of the climate cost function $C(A)$.

The function $C(A)$ is typically assumed increasing and convex−often strictly convex. The background for this is that the global temperature increase above preindustrial average is rising in A, and that climate costs−and probably marginal climate costs−are increasing in the temperature increase. Even if climate costs are an increasing and strictly convex function of the temperature increase, it is not obvious that $C''(A) > 0$. The reason for this is that there is a complex and non-linear relationship between A and temperature increase. In particular, radiative forcing, which is the prime cause of the temperature increase, is a logarithmic function of A. If climate costs were approximately proportional to temperature increase, this suggests $C''(A) < 0$. Although it hence is not obvious that $C''(A) \geq 0$, we shall stick to this assumption as it is frequently used elsewhere in the literature, and will hold if marginal climate costs rise sufficiently with increased temperature.

For he limiting case of $C'' = 0$ it follows from (A.24) that $s(t)$is constant (equal to C'/δ for $\rho = 0$). For the more general case of $C''(A) > 0$, it follows from (A.24) that $s(t)$ must be rising as long as emissions are positive and hence $A(t)$ is increasing (for $\rho = 0$). However, the growth rate of $s(t)$ will be below δ as long as $C' > 0$.

It is sometimes assumed that there is a climate goal of a maximum permitted temperature increase, and that one is not concerned about the temperature increase as long as this limit is not violated. This corresponds to a maximal limit on A, and $C(A) = 0$ below this limit. For this case $C'(A) = 0$ as long as A is below its maximal limit, implying that $\dot{s}(t) = \delta s(t)$ as long as A is below its maximal limit. While the case of a constant present value of the social cost of carbon is of some interest as a limiting case, this case is not particularly relevant in practice: Even if one has a goal of a maximal permitted temperature increase, one would usually also have some concern of temperature increases below this level. If so, $C' > 0$ and

$\dot{s}(t) = \delta s(t)$ also when A is below its maximal limit.

To conclude: The reasoning above suggests that $s(t)$ is rising over time, while the present value of $s(t)$ is declining over time. Our analysis considers the two limiting cases of $s(t)$ constant and $\dot{s}(t) = \delta s(t)$.

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ISSN 0809-733X

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