Discussion Papers

Statistics Norway
Research department

No. 709

October 2012

Statistics Norway

John K. Dagsvik and Zhiyang Jia

Labor supply as a discrete choice among latent jobs

Statistisk sentralbyrå

Discussion Papers No. 709, October 2012
Statistics Norway, Research Department

John K. Dagsvik and Zhiyang Jia

Labor supply as a discrete choice among latent jobs

Abstract:
This paper discusses a modeling framework in which workers are assumed to choose their preferred job from latent worker-specific choice sets. This point of departure yields a framework that formalizes the widely used ad hoc approaches (fixed cost of working and dummies at peak hours) in the literature of discrete labor supply models. We discuss the conditions under which the preferences and job opportunity restrictions can be separated using conventional data on hours and wages only. Subsequently, we show that the framework is consistent with stochastic choice sets and a relaxation of the IIA assumption. An empirical model version for married/cohabiting couples is estimated using Norwegian micro data. Based on the empirical model, we discuss further important empirical issues, such as functional form, prediction performance and simulation of counterfactual policy reforms.

Keywords: Labor supply, non-pecuniary job attributes, latent choice sets, random utility models.

JEL classification: J22, C51

Acknowledgements: We thank Jørgen Aasness, Thor Olav Thoresen and Terje Skjerpen for valuable comments.

Address: John K. Dagsvik, Statistics Norway, Research Department. E-mail: john.dagsvik@ssb.no
Zhiyang Jia, Statistics Norway, Research Department. E-mail: zhiyang.jia@ssb.no

Discussion Papers comprise research papers intended for international journals or books. A preprint of a Discussion Paper may be longer and more elaborate than a standard journal article, as it may include intermediate calculations and background material etc.

Abstracts with downloadable Discussion Papers
in PDF are available on the Internet:
http://www.ssb.no
http://ideas.repec.org/s/ssb/dispap.html

For printed Discussion Papers contact:
Statistics Norway
Telephone: +47 62 88 55 00
E-mail: Salg-abonnement@ssb.no

ISSN 0809-733X
Print: Statistics Norway

Sammendrag

Tradisjonelle analyser av arbeidstilbud er basert på antakelsen om at arbeiderne har mulighet til å velge arbeidstid fritt såfremt konsumutgiftene ikke overstiger disponibel inntekt. Denne artikkelen drøfter et rammeverk for modellering av arbeidstilbud der en fundamental egenskap er at arbeidstilbudstilpasningen medfører valg av type jobb. Mer presist er arbeiderne antatt å ha preferanser over jobber (uobserverbare) innen individ-spesifikke valgsett av jobber, hvorfra den mest attraktive jobben velges. Denne artikkelen generaliserer tidligere arbeidstilbudsmodeller basert på diskret valghandlingsteori i flere retninger, med spesielt fokus på identifikasjon og aggregeringsegenskaper. Vi har videre estimert en empirisk modellversjon basert på norske mikro data. Vi har brukt denne modellen til å beregne lønnselastisiteter og til å simulere effekten av eksogene endringer i valgmengdene av tilgjengelige jobber.

1. Introduction

In the traditional approach (the standard approach), individual labor supply is viewed as a choice among feasible leisure and disposable income combinations. This approach has been criticized for ignoring important behavioral aspects: namely, that individuals in the labor market typically have preferences regarding job types and may face restrictions on their choices regarding job opportunities and hours of work. Recently, the discrete choice approach to labor supply modeling has gained widespread popularity, mainly because it is much more practical than the conventional continuous approach based on marginal calculus: see Creedy and Kalb (2005). For example, with the discrete choice approach, it is easy to deal with non-linear and non-convex economic budget constraints, which is not the case with the continuous approach, where kinked and non-convex budget sets represent major difficulties (Bloemen and Kapteyn, 2008). However, from a theoretical perspective, the conventional discrete choice approach represents no essential departure from the standard approach. This is because the only new assumptions postulated are that the set of feasible hours of work is finite and that the random components of the utility function have particular distributional properties.

The main purpose of this paper is to provide an extension of the conventional discrete choice model to accommodate for agents' preferences being dependent on non-pecuniary job attributes, as well as possible restrictions on hours of work opportunities. In the conventional discrete choice labor supply model, it is common practice in empirical applications to introduce dummy variables in the model specification to account for observed concentration of hours of work, although these approaches are ad hoc. In contrast, our approach offers a theoretical rationale and an alternative interpretation for this practice.

The theoretical point of departure is the assumption that a worker's labor supply follows from his or her choice of job. More precisely, labor supply is viewed as resulting from a choice among latent job "packages", each of which is characterized by an offered wage rate, offered hours of work, and non-pecuniary (qualitative) attributes describing the nature of the job-specific tasks to be performed. This starting point can thus be viewed as a version of Lancaster's characteristic approach – see Lancaster (1966, 1971) – where agents not only have preferences over consumption and leisure, but also over job characteristics. The characteristic approach is intuitively appealing because it shifts the focus to qualitative aspects of the labor market that everyday life experiences tell us are important. Examples of such latent job attributes of major importance are job-specific tasks to be performed, location, quality of the social and physical environment, etc.[1] More recently, Farzin (2009) has

---

[1] A further example is provided by the so-called Job Characteristics Model, developed by organizational psychologists Hackman and Oldham (1976). Their model takes a normative characteristics approach to the analysis of job enrichment. They specify five core job dimensions, which include skill variety (the range of tasks performed), task identity (the ability to complete the whole job from start to finish), and task significance (the impact of the job on others).

discussed the effects of including non-pecuniary variables explicitly in the standard labor supply model and argues that ignoring such aspects of jobs can give biased estimates and thus result in misleading policy suggestions. Further related approaches are given by Sattinger (1993, 1995) and van Ophem, Hartog and Vijverberg (1993).

In a modeling context where "job" is allowed to be a decision variable, it is necessary to specify the choice set of available jobs in addition to the budget constraint. The individual-specific sets of feasible jobs are endogenous in the sense that they are determined by market equilibrium conditions and/or by negotiations between unions and employers. However, to the individual agent the set of job opportunities may be viewed as given. The difficulty with applying a modeling approach that addresses the issue of job choice is that one seldom has sufficiently detailed information about non-pecuniary job attributes and choice sets. However, in this context we are not primarily interested in modeling the job choice per se, but only in the wage and hours of work combination that follows from the job choice. That said, it would certainly still be of interest to have detailed information about jobs and choice sets, because such information could prove vital for testing key modeling assumptions. As will be clear below, our particular point of departure allows us to address neglected aspects in traditional labor supply analysis: namely, that workers face important restrictions on their job choice in the labor market. The resulting labor supply model can, in our framework, be represented as a type of multinomial logit model, where the representative utility terms are weighted by a measure (opportunity measure) of job availability. It follows that this framework is flexible and practical to apply, and provides a better analogy to crucial features of the "true" choice setting.

There have already been several interesting attempts in the literature of labor supply to deal with restrictions on hours and job opportunities. They include Ilmakunnas and Pudney (1990), van Soest, Woittiez and Kapteyn (1990), Tummers and Woittiez (1991), Dickens and Lundberg (1993), Stewart and Swaffield (1997), Bloemen (2000, 2008), and Ham and Reilly (2002). All these contributions are versions of discrete choice models, extended to account for particular representations of choice restrictions. In a way, our model is consistent with that of Dickens and Lundberg (1993). However, our model is simpler to interpret and easier to implement in practice.

The analysis in this paper develops further the approach taken by Dagsvik (1994) and Dagsvik and Strøm (2006). The paper contains a number of new contributions. First, we discuss in detail the identification problem, which differs from standard identification results of discrete choice models because the present model contains representations of both preferences and choice constraints. We show that suitable separability conditions are required for non-parametric identification of the model based on typical labor supply data available.

Second, we extend the basic framework in order to make the approach more realistic in the context of empirical applications. We show how one can introduce more general taste distributions that do not require the Independence from Irrelevant Alternatives (IIA) assumption. In addition, we show how one can accommodate unobserved heterogeneity in the choice restrictions: that is, when allowing for stochastic choice sets of job offers. In contrast to the multidimensional Poisson process representation approach applied by Dagsvik and Strøm (2006), our approach is based on a particular random coefficient approach and thus is more familiar to most researchers.

Finally, we illustrate the practical application of the framework by estimating an empirical model version for married/cohabitating couples. Since functional form assumptions on preferences are important in empirical model specifications, we compare two specifications of the deterministic part of the utility function: namely, the quadratic and the generalized Box-Cox function (generalized to allow for interaction terms). We find that both specifications provide good fit to the data, with the generalized Box-Cox slightly outperforming the quadratic specification. However, the estimates of the quadratic specification imply that utility is decreasing in leisure for high levels of leisure. Furthermore, we conduct out-of-sample prediction exercises which show that the model performs rather well. We also show how the model can be applied to simulate the effect of particular changes in choice restrictions.

The paper is organized as follows. In Section 2 we discuss the basic structure of the modeling framework. The section also includes discussion on how the model can be identified. In Section 3 we discuss two extensions: a relaxation of IIA and an introduction of unobserved heterogeneity of choice sets of latent jobs. In Section 4 we discuss aspects on simulation of labor supply effects and in Section 5 we report results from the empirical application. Section 5 also contains out-of-sample prediction exercises and simulations of the effect of particular changes in choice restrictions. Section 6 concludes.

2. The basic framework

In this section we first present the basic structure of our modeling approach, which will serve as a point of departure for the extensions and application discussed below. In contrast to the traditional approach, in which the agent is restricted to having preferences over combinations of total consumption and hours of work, we allow the agent to have preferences over total consumption, hours of work, and non-pecuniary job attributes, such as the nature of the job-specific tasks to be performed and location of the workplace, etc. Thus this approach is similar to the characteristic approach taken by Lancaster (1966, 1971), with job attributes playing a role that is entirely analogous to his characteristics.

Let $U(C,h,z)$ be the (ordinal) utility function of the household, where C denotes household consumption (disposable income) and h is hours of work. The positive indices, z = 1, 2,…, refer to market opportunities (jobs) and $z = 0$ refers to the non-market alternative. For a market opportunity (job) z, associated hours of work and wage are assumed fixed and equal to $(H(z), W(z))$. In this paper, we shall assume that the hours of work and wage take only discrete values in a given set.[2] Let D be the set of possible positive hours of work and G be the set of possible values of wage. The utility function is assumed to have the structure

$$(2.1) \qquad U(C,h,z) = v(C,h)\varepsilon(z),$$

for $z = 0,1,2,...,$ where $v(\cdot)$ is a positive deterministic function and $\{\varepsilon(z)\}$ are positive random taste shifters that are i.i.d. with c.d.f. $\exp(-1/x)$, for positive x.[3] The random taste shifters are assumed to account for unobservable individual characteristics and non-pecuniary job-specific attributes that affect utility, and hence will vary across both households and job opportunities.

For given hours and wage, h and w, the economic budget constraint is represented by

$$(2.2) \qquad C = f(hw, I),$$

where I is non-labor income, C is (real) disposable income, and $f(\cdot)$ is the function that transforms gross income into after-tax household income. The function $f(\cdot)$ can in principle capture all details of the tax and benefit system.

For simplicity, we use the notation

$$(2.3) \qquad \psi(h,w,I) \equiv v\left(f(hw,I),h\right).$$

The term $\psi(h,w,I)$ is the representative utility of jobs with hours of work h, a given wage w, and non-labor income I.

In addition to (2.2), there are restrictions on the set of available market opportunities faced by a specific worker. This is because there are job types for which the worker is not qualified and there may be variations in the set of job opportunities for which he or she is qualified. In addition, due to competition in the labor market, the most preferred type of job for which a worker is qualified may not

[2] A version where hours of work and wage rates are realizations in a continuous set is discussed in Dagsvik and Strøm (2006).

[3] In the terminology of Resnick (1987) this c.d.f. is called type I (standard) extreme value distribution, or Frechet distribution. Other authors call this c.d.f. type III extreme value distribution. Note that the error terms being distributed according to type I extreme value distribution exp(-1/x), for positive x, in a multiplicative formulation of the utility function are equivalent to the error terms being distributed according to the c.d.f. $\exp(-\exp(-x))$, for any real x, in an additive utility formulation. This follows immediately by taking the logarithm of the multiplicative utility function in (2.1).

necessarily be available to him or her. Let $B(h,w)$ denote the agent's set of available jobs with hours of work and wage (h,w): that is, this set contains those jobs z for which $H(z)=h$ and $W(z)=w$. Let $m(h,w)$ be the number of jobs in $B(h,w)$. For the non-market alternative, $m(0,0)$ is normalized to one. In the following we shall call $m(h,w)$ the *opportunity measure*. The choice sets $\{B(h,w)\}$ are unobserved to the researcher. In this section we assume that the choice sets $\{B(h,w)\}$ are equal for observationally identical workers. In Section 3 this assumption will be relaxed to allow for unobserved heterogeneity in the choice sets.

Let $\varphi(h,w)$ denote the probability that the agent chooses a particular job with offered hours h, wage w, given non-labor income I, and individual characteristics. Let D be the maximal set of possible positive hours of work and G the maximal set of positive wages. By applying standard results in discrete choice theory (McFadden, 1973), it follows that

(2.4)
$$\varphi(h,w\mid I)=\sum_{z\in B(h,w)}P\left(\psi(h,w,I)\varepsilon(z)=\max_{x\in D\cup\{0\},y\in G,\,k\in B(x,y)}\left(\psi(x,y,I)\varepsilon(k)\right)\right)$$
$$=\sum_{z\in B(h,w)}\frac{\psi(h,w,I)}{\sum_{y\in G,}\sum_{x\in D,}\sum_{z\in B(x,y)}\psi(x,y,I)+\psi(0,0,I)}=\frac{\psi(h,w,I)m(h,w)}{\psi(0,0,I)+\sum_{y\in G}\sum_{,x\in D}\psi(x,y,I)m(x,y)},$$

for $h,w>0,$ and

(2.5)
$$\varphi(0\mid I)=\frac{\psi(0,0,I)}{\psi(0,0,I)+\sum_{y\in G,}\sum_{x\in D}\psi(x,y,I)m(x,y)},$$

for $h=0.$ The resulting expression is a choice model that is analogous to a multinomial logit model with representative utility terms $\{\psi(h,w,I)\}$, weighted by the frequencies of available jobs, $\{m(h,w)\}$. Note that it is a consequence of our distributional assumptions of the stochastic error terms in the utility function that the respective numbers of available latent jobs, $\{m(h,w)\}$, represent a set of *sufficient* statistics for the corresponding choice sets. Thus the modeling framework above allows the researcher to account for restrictions on latent job opportunities through the opportunity measure.

As mentioned earlier, there are several contributions in the literature that address the issue of restrictions on hours and job opportunities. The one that is closest to ours is Dickens and Lundberg (1993).[4] They propose a model where the choice environment is assumed to consist of a random set of hours of work alternatives. Compared to the modeling framework presented above, Dickens and Lundberg (1993) assume that only hours of work and consumption matter; thus there are no non-pecuniary attributes of the job offers that affect preferences. In addition, the resulting choice

[4] Bloemen (2000) and Tummers and Woittiez (1991) have developed extensions of the approach taken by Dickens and Lundberg (1993).

probabilities are rather complicated and cannot be expressed in closed form as in the framework considered above. On the other hand, their approach has the advantage that the choice sets of offered jobs are random, thus allowing for unobserved heterogeneity across individuals. However, it is possible to extend the framework considered above to allow for random choice sets of job opportunities (Section 3.2).

The conventional discrete choice approach proposed by van Soest (1995) can be seen as a special case of the model above, obtained by letting $m(h,w)=1.$ Evidently, the conventional model with standard assumptions about preferences cannot explain the high concentration of hours of work at full-time, and possibly part-time, a feature which is typical in many countries. Many authors, including van Soest (1995), make use of an ad hoc adjustment by introducing alternative specific dummies in the specification of the representative utilities. When the opportunity measure $m(h,w)$ depends on hours of work only – that is, $m(h,w)\equiv m(h)$ – the alternative specific dummies can be viewed as a reformulation of our framework. In other words, our framework offers a theoretical formalization of the add hoc approaches in the literature.

2.1. Identification

We now turn to a discussion of identification of the model. In our model, the observed wage and hours of work are a result of both preference (utility function) and job choice constraints (opportunity measure). Since the opportunity measure $m(h,w)$ is not directly observable, we need to estimate it simultaneously with the systematic part of the utility function $v(C,h).$ It is thus important to discuss under what conditions we can identify these two factors separately.[5] It follows immediately from (2.4) and (2.5) that

(2.6) $$\frac{v(f(hw,I),h)m(h,w)}{v(f(0,I),0)}=\frac{\varphi(h,w\,|\,I)}{\varphi(0\,|\,I)}.$$

Note that the sample counterpart of $\varphi(h,w\,|\,I)/\varphi(0\,|\,I)$ is the relative share between the number of workers with hours of work and wage pair (*h,w*) and workers who do not work, given the same non-labor income *I*, which is observable. Thus one can identify $v(f(hw,I),h)m(h,w)/v(f(0,I),0)$ non-parametrically.

[5] Recently, several authors have suggested using subjective information such as desired hours of work to estimate labor supply models. See, for example, Bloemen (2008) and references given there. In principle, if one can identify the preferences using desired hours of work first, one can then identify the job offer distributions based on actual observed hours of work.

However, (2.6) does not allow us, without further assumptions, to identify the preference $\psi(h,w) = v(C,h)$ and the choice constraints represented by $m(h,w)$ separately. In the following, we shall first show that although $v(C,h)$ depends on non-labor income I through the budget constraints $C = f(hw, I)$, whereas $m(h,w)$ does not, one still cannot, without further assumptions, recover $v(C,h)$. In fact, one cannot even identify $\partial v(C,h)/\partial C$. However, if we assume that the opportunity measure is multiplicatively separable – that is, $m(h,w) = m_1(h)m_2(w)$ – one can identify $v(C, h)$ non-parametrically up to a multiplicative factor $\delta(h)$, say, that depends on hours of work only: see Appendix A.

Note that for the purposes of simulating solely the effect of counterfactual changes in the tax function it is, however, not necessary to separate $\delta(h)$ from $m_1(h)$. The reason for this is that the tax function enters the model only through $v(C,h)$, and since $v(C,h)$ is identified one can simulate the effects of tax reforms conditional on fixed distribution of offered hours even if $\delta(h)$ and $m_1(h)$ are not identified separately.

One way to obtain full identification is to make parametric functional form assumptions about both $v(C,h)$ and $m(h,w)$. To this end Dagsvik and Strøm (2006), Dagsvik and Røine Hoff (2011), and Dagsvik (2012) have embarked upon a theoretical approach to justify the choice of functional form based on particular invariance principles. These invariance assumptions imply that the systematic term $v(C,h)$ is of a generalized Box-Cox functional form that allows for particular interaction terms in order to accommodate non-separability features: see Dagsvik and Strøm (2006) and Dagsvik and Røine Hoff (2011) for details. Specifically, they show that the resulting non-separable functional form is given by

(2.7) $$\log v(C,h) = \frac{\gamma(C^\alpha - 1)}{\alpha} + \frac{\delta((1-h/M)^\beta - 1)}{\beta} + \frac{\mu(C^\alpha - 1)((1-h/M)^\beta - 1)}{\alpha\beta},$$

where $\alpha, \beta, \gamma, \delta$ and μ are parameters, and M is a predetermined constant representing time which can be allocated to working. If $\alpha, \beta < 1, \gamma, \delta > 0,$ and with suitable restriction imposed on μ, then utility is increasing and concave in consumption and leisure. Another popular candidate is the polynomial function, often quadratic, which has the advantage of being flexible and easy to estimate: see, for example, van Soest, Das and Gong (2002) and Blundell and Shepard (2012).

When the tax function is piecewise linear in both wage income and non-labor income, as many tax systems are, it can shown that if one assumes that (2.7) holds one can identify $v(C, h)$ up to a multiplicative factor $\delta(h)$ even if the opportunity measure is not separable (Appendix A). However, this is not true when we assume polynomial functional form for the deterministic part of the utility function.

In empirical applications (Dagsvik and Strøm, 2006; Dagsvik et al. 2011) it is typically assumed that the above separability condition on opportunity measure is satisfied and $m_1(h)$ is uniform apart from peaks at part-time and full-time hours of work. However, given our identification result above, we can also specify non-separable opportunity measures that allow offered wages to be correlated with offered hours of work,[6] when we assume that (2.7) holds.

One way of rationalizing the multiplicative separability assumption of the opportunity measure is to assume that the marginal product to a firm of hiring a worker is constant. For example, this property follows if production functions exhibit constant returns to scale. The marginal product may be determined by the worker's education and in addition by firm-specific experience. A typical feature of labor markets in Scandinavia is that wage is determined in part by negotiation between labor organizations and in part by the marginal product of the workers, and in these negotiations the workload usually plays no role (apart from overtime recompense). If in addition we assume that the skills required by the respective firms are not correlated with the firm-specific hours of work, then the opportunity measure will clearly be separable: that is, $m(h,w) = m_1(h)m_2(w)$. In the special case where the marginal product depends only on person-specific human capital, wages will depend solely on worker characteristics, but not on firm characteristics. In this case, all firms will offer the same wage to a given worker, which means that $m(h,w) = m_1(h)$ if the agent faces wage w and zero otherwise.

3. Further development of the basic framework

In Section 2 we presented the basic framework of the model. In this section we relax some of the crucial assumptions made above, such as the IIA assumption and the restrictive assumption on heterogeneity in choice opportunities.

Consistent with our empirical application below, we shall henceforth discuss only the special case in which the wage does not vary across jobs for a given agent and the wage is observed (or depends only on observed individual characteristics). As regards the formal treatment, this simplification represents no essential loss of generality, as it is easily seen how the arguments modify in the general case. Thus, from now on and similar to the traditional neoclassical model of labor supply, the agent is assumed to face an individual-specific wage. In this case the model given in (2.4) and (2.5) simplifies to

(3.1) $$\varphi(h \mid w, I) = \frac{\psi(h,w,I)m_1(h)}{\psi(0,0,I) + \sum_{x \in D} \psi(x,w,I)m_1(x)},$$

---

[6] For example, Barzel (1973) argued that wage offers may systematically vary hours of work, typically an inverted U-shaped relationship. See also Wolf (2002) for a selection of arguments.

for positive h, and a similar expression for h = 0, obtained by replacing the numerator in (3.1) by $\psi(0,0,I)$.

3.1. Relaxation of the IIA assumption

In the basic model presented above, we assume that the random components of the utility function are i.i.d. type I extreme value distributed across all jobs. As is well known from the theory of discrete choice, these distributional assumptions are consistent with the Independence from Irrelevant Alternatives (IIA) assumption: see Luce (1959). The basic underlying intuition of the IIA assumption is that the agent's ranking of job opportunities from a subset, say B, within the choice set of feasible jobs with given job-specific hours of work and wage rate does not change if the choice set of feasible jobs is altered. As is also well known, IIA will be violated if the random terms in the utility function are correlated across alternatives. Whereas IIA can be realistic in many empirical applications, one cannot rule out a priori that it may be restrictive in our context. However, the choice model above is in fact consistent with a setting where IIA is *not* satisfied, which we shall now demonstrate.

Assume that the utility function $\psi(h,w,I)\varepsilon(z)$ is replaced by $\psi(h,w,I)\varepsilon(h,z)$, where the joint c.d.f. of the error terms $\{\varepsilon(h,z)\}$ that correspond to jobs within the set of available jobs with hours h, $B(h)$, say, is given by

$$(3.2)\qquad P\left(\bigcap_{z\in B(h)}\left(\varepsilon(h,z)\le x(h,z)\right)\right)=\exp\left(-\left(\sum_{z\in B(h)}x(h,z)^{-1/\alpha}\right)^{\alpha}\right),$$

for $x(h,z)>0$, where the error terms $\varepsilon(z)$ and $\varepsilon(z')$ are independent for $z\in B(h)$ and $z'\in B(h')$, provided that h differs from h. The error term associated with the "not working" alternative is independent of all other error terms. The parameter α belongs to the interval (0,1] and it can be interpreted as

$$(3.3)\qquad Corr(\log\varepsilon(h,z),\log\varepsilon(h,z'))=1-\alpha^2,$$

when $z,z'\in B(h)$. The structure in (3.2) means that within any $B(h)$ the joint distribution of the error terms is symmetric. This type of correlation structure can, for example, be a result of common unobserved factors affecting preferences across jobs with given hours of work. From (3.3) we realize that within each choice set $B(h)$ any correlation between the taste shifters is possible. Let $U(h)=\max_{z\in B(h)}U(h,z)=\psi(h,w)\max_{z\in B(h)}\varepsilon(h,z)$. We realize that $U(h)$ is the utility of the preferred job with hours of work h. The assumption in (3.2) implies that

(3.4) $$P\big(U(h)\le x\big)=P\left(\bigcap_{z\in B(h)}\left(\varepsilon(z)\le \frac{x}{\psi(h,w)}\right)\right)=\exp\left(-\frac{\psi(h,w)m_1(h)^{\alpha}}{x}\right),$$

for $x > 0$. The expression in (3.4) means that we can write

(3.5) $$U(h)=\psi(h,w,I)\tilde{m}_1(h)\tilde{\varepsilon}(h),$$

where $\tilde{m}_1(h)=m_1(h)^{\alpha}$ and where the error terms $\{\tilde{\varepsilon}(h)\}$ are independent and type I extreme value distributed. From (3.4) and well-known results in the theory of discrete choice, it now follows immediately that the choice model that corresponds to agents maximizing the utility function in (3.5) is given by (3.1) with $m_1(h)$ replaced by $\tilde{m}_1(h).$ Thus the structure of the opportunity measure is invariant under the extension represented by (3.2). It is possible to extend the model further by allowing the similarity parameter α to depend on *h*. However, under this extension the separability structure of the opportunity distribution evidently breaks down.

As we have seen above, the structure in (3.2) does not represent any departure from the IIA assumption as regards the final observable properties of the model. However, this invariance depends crucially on the assumed correlation patterns over the error terms. We shall next point out another extension of the model where the error terms have a nested structure that differs from (3.4). Suppose that the joint c.d.f. of the error terms in the utility functions is given by

(3.6) $$P\left(\bigcap_{z\in B(h)}\big(\varepsilon(h,z)\le x(h,z)\big)\right)=\exp\left(-x(0)^{-1}-\left(\sum_{h\in D,}\sum_{z\in B(h)}x(h,z)^{-1/\rho}\right)^{\rho}\right),$$

where ρ similarly to α is a correlation parameter, $\rho\in(0,1]$. The structure in (3.6) means that all the error terms associated with the working alternatives (jobs) are symmetrically correlated, whereas the error term associated with the "not working" alternative is independent of all other error terms. The motivation for this structure is that there may be an unobservable attribute common to all working alternatives which is associated with social aspects and the feeling of meaningfulness achieved by carrying out the specific tasks required in the respective jobs. Note that in this version, preferences over jobs with different hours of work as well as jobs with the same hours of work are allowed to be correlated, whereas the structure in (3.2) allows only jobs with the same hours of work to be correlated. By standard derivation – see McFadden (1984) – it follows that

(3.7) $$\varphi(h\mid w,I)=\frac{\left(\sum_{x\in D}\psi(x,w,I)^{1/\rho}m_1(x)\right)^{\rho-1}\psi(h,w,I)^{1/\rho}m_1(h)}{\psi(0,0,I)+\left(\sum_{x\in D}\psi(x,w,I)^{1/\rho}m_1(x)\right)^{\rho}},$$

and similarly for the probability of not working. We note that when $\rho = 1,$ the formula in (3.7) reduces to the expression in (3.1).

3.2. Unobserved heterogeneity in the choice sets

As we have seen above, our framework allows for more general representations of preferences depending on non-pecuniary attributes and is considerably more practical than previous approaches. However, in the way it has been presented so far it has a serious drawback: namely, that choice sets are identical for observationally identical agents. Dagsvik (1994) and Dagsvik and Strøm (2006) have demonstrated that the choice probabilities in (3.1) (or in (2.4) and (2.5)) are in fact consistent with random choice sets. Their approach to accommodating random choice sets is based on a multidimensional inhomogeneous Poisson process. We shall now present an alternative approach which is a great deal simpler and does not require as much knowledge of probability theory.

Recall that in our framework the respective numbers of available latent jobs, $\{m_1(h)\}$, represent a set of *sufficient* statistics for the corresponding choice sets. So random choice sets across identical individuals can be accounted for by allowing $\{m_1(h)\}$ to be stochastic. Assume now that $m_1(h)$ has the representation $m_1(h) = \bar{m}_1(h)\omega(h),$ where $\omega(h),\ h \in D,$ are positive random variables and $\bar{m}_1(h),\ h \in D,$ are corresponding positive deterministic terms. A challenging issue is how to characterize the distribution of $\{\omega(h)\}.$ Our approach to this end is to postulate plausible properties we believe this distribution should possess and subsequently derive the implications. The properties we postulate are the following: (i) $\omega(h) > 0;$ (ii) for any hours of work, h_1 and h_2, and non-negative constants, b_1 and b_2, $b_1\omega(h_1) + b_2\omega(h_2)$ has the same distribution as $\tau\omega(h_1),$ where τ is a positive constant that may depend on h_1, w_1, h_2, w_2, b_1, and b_2; (iii) the random variables $\omega(h_1)$ and $\omega(h_2)$ are i.i.d. The motivation for (i) is obvious: unless this condition is satisfied, for some hours of work, the conditional choice probabilities would be zero or negative. Conditions (ii) and (iii) mean that for any positive $h_1, h_2, ..., h_r,$ the distribution of the conditional aggregate choice probabilities

$$\sum_{k=1}^{r} \varphi\left(h_k \mid w, I, \{\omega(x), x \in D\}\right)$$

(which are random variables because they depend on $\{\omega(h), h \in D\}$), across unobservable choice sets, belongs to the same family of distributions as the conditional choice probabilities, $\varphi(h_k \mid w, I, \{\omega(x), x \in D\})$. In other words, requirement (ii) implies that the distribution of the conditional choice probabilities is *invariant* under an aggregation of alternatives (a combination of hours of work and wage rates). The motivation for property (ii) is that since the aggregation level within the total set of available hours and wages is somewhat arbitrary, it seems intuitive that the distributional properties of the model should not depend critically on the partition of the set of feasible hours and wages into aggregate alternatives. The independence assumption (iii) is equivalent to asserting that the correlation between the sizes of any two choice sets is independent of which choice sets are compared. It can, however, be demonstrated that one can easily allow for an asymmetric dependence structure of the joint distribution of $\{\omega(h)\}$. Specifically, this can be done by applying multivariate stable distributions. However, to discuss this extension will be left for another occasion.

It can be demonstrated that the postulated assumptions imply that the distribution of $\omega(h)$ is strictly stable, $S_\alpha(1,1,0)$, with $\alpha \in (0,1)$, and totally skewed to the right.[7] Recall that the class of stable distribution is a generalization of the normal distribution and follows from an extended version of the Central Limit Theorem: see, for example, Embrechts, Klüppelberg and Mikosch (1997). From (2.4) and these distributional assumptions, it is shown in Appendix B that the probability of choosing a job with hours and wage (*h*) is given by

$$\text{(3.8)}\qquad \varphi(h \mid w, I) = E\left(\frac{\psi(h,w,I)m_1(h)}{\psi(0,0,I)+\theta\sum_{x\in D}\psi(x,w,I)m_1(x)}\right) = \frac{\psi(h,w,I)^\alpha \tilde{m}_1(h)}{\psi(0,0,I)^\alpha + \sum_{x\in D}\psi(x,w,I)^\alpha \tilde{m}_1(x)},$$

for $h > 0$, with a similar formula for $h = 0$, where $\tilde{m}_1(h) = \bar{m}_1(h)^\alpha$. Thus we realize that the assumptions of this section are equivalent to (3.2) since they yield the same structure on the choice probabilities as the one following from (3.2). From (3.8) we realize, provided that $\log\psi(h,w,I)$ is linear in some of the parameters, that the parameter α is absorbed in these parameters. Thus we have obtained the neat result that the basic model structure is invariant under unobserved heterogeneity in choice sets, provided that particular distributional assumptions are fulfilled. By allowing the random variables $\{\omega(h)\}$ to be multivariate stable, one can allow for correlated choice sets across alternative hours of work.[8]

4. Simulation of labor supply effects

In the extended versions of the model discussed in Section 3, the opportunity measure typically takes the form $m_1(h)^\alpha$, where α is a positive parameter that is less than or equal to one. In these cases, the part which corresponds to the choice constraints, $m_1(h)^\alpha$, no longer represents the number of feasible

[7] The notation $S_\alpha(\sigma,\beta,\mu)$ refers to the Stable distribution with index of stability $\beta \in (0,2]$, scale parameter $\sigma > 0$, skewness parameter $\beta \in [-1,1]$, and location parameter μ.

[8] Under particular assumptions about the dependence structure, one can show that the resulting model is of a multinomial nested logit case.

jobs with hours *h*. However, $m_1(h)^\alpha$ can still be interpreted as an opportunity measure since it is a strictly increasing mapping of the number of feasible jobs with hours *h*.

It is useful to break down the opportunity measure in the following manner: namely,

(4.1) $$m_1(h)^\alpha = \theta g_1(h),$$

where

(4.2) $$g_1(h) = \frac{m_1(h)^\alpha}{\sum_{x \in D} m_1(x)^\alpha}, \quad \text{and} \quad \theta = \sum_{x \in D} m_1(x)^\alpha,$$

where $\alpha \in (0,1].$ We shall call $g_1(h)$ the opportunity distribution of hours. In the basic job choice model, where $\alpha = 1,$ $g_1(h)$ is the proportion of jobs with offered hours equal to *h* available to the worker. The term θ can be interpreted as a measure of the relative size of the market opportunity set with respect to the not working opportunities. It could also be extended to account for the fixed cost of working and the disutility of working. However, in this case we will not be able to identify the fixed cost of working from the "true" working opportunity measure given the typically available data on labor supply. In practice, θ may depend on variables that represent the effects of schooling and experience and characterize local labor market conditions, plus possible variables that represent the fixed cost of working.

Empirical evidence suggests that there is a wide variation in the wage opportunities across agents, and it seems hardly possible to account for this distribution by observed individual characteristics, such as length of schooling and potential experience. According to Mortensen (2003), observable worker characteristics that account for productivity differences typically explain no more than 30 percent of the variation in compensation across workers. The unexplained differences can be due to both unobserved individual abilities and unobserved job-specific effects. To account for unobserved individual-specific abilities in the wage offer distribution, we introduce a random effect in this distribution. Let η be a positive random effect such that $w = \bar{w}\eta$, where $\bar{w}$ is a deterministic term that depends on observed covariates such as experience and length of schooling. Then the unconditional marginal choice probability of hours of work is of course equal to

(4.3) $$\tilde{\varphi}(h \mid \bar{w}, I) = E_\eta \varphi(h \mid \bar{w}\eta, I),$$

where E_η denotes the expectation operator with respect to $\eta.$ One important implication of the random effect specification above is that the resulting choice probabilities will no longer satisfy the IIA assumption. This is the approach used in the empirical part of this paper, as well as in Dagsvik and Jia (2006), Dagsvik and Strøm (2006), Kornstad and Thoresen (2007), and Dagsvik et al. (2011). Another approach is to ignore the unobserved individual effect and attribute the unobserved wage differential solely to job-specific variations. For example, Aaberge, Dagsvik and Strøm (1995) applied such a strategy. Although our conjecture is that inter-individual variation in wages is more important than intra-individual variation in wages, it is pretty much an open question which wage variation is the most important. Evidently, the best "solution" would be to accommodate both types of wage variation. One possibility that we intend to pursue in the future is to apply panel data to identify and separate variations in offered wages across jobs from inter-individual heterogeneity offered wages.[9]

Once an empirical version of the model has been estimated, one can simulate the effects on labor supply of different types of reform, such as the effects of changes in wage rates and the tax system. In the context of counterfactual simulation of pure labor supply effects, it makes sense to keep job opportunities fixed under changes of wages. Since the choice sets are represented by the opportunity distribution, $\{g_1(h)\}$ and θ are kept fixed in this type of simulation.

A particular feature of our model is that it can be used to study the effects on labor supply of changes in the choice sets. If, for example, one assumes that $g_1(h)$ is uniform for all h, it is equivalent to assuming that there are no restrictions on hours of work. In this case the model, conditional on working, expresses the distribution of desired hours of work.[10]

5. An empirical application

5.1. Empirical results for a job choice model for married couples

In this section we report the results from an empirical application based on micro data from the Norwegian Labor Survey 1997. In this application we analyze joint labor supply behavior of married couples. Although the model specified above is only for single-individual households, it can easily be extended to married couples: see Appendix C.

Since there are very few married men who are not working, we have excluded households where the husband is not working and assume that husbands can choose only positive working hours. We specify eight hours of work alternatives, which correspond to annual hours of work at 0, 260, 780, 1,040, 1,560, 1,950, 2,340, and 2,600.

---

[9] This is closely related to the study of wage dispersion. See, for example, Mortenson (2003) for a detailed discussion. In particular, we refer to Abowd, Kramarz and Margolis (1999) and Abowd and Kramarz (2001) for a discussion on the breakdown of the wage dispersion into worker and employer components.

[10] In the presence of Stated Preference data on desired hours of work, one could test our model specification by comparing the predictions with the Stated Preference data. Unfortunately, this kind of data is not yet available to us.

The wages are assumed to be individual-specific and equal across jobs for a given individual, but we allow for unobserved heterogeneity across individuals. To deal with the problem that wages are unobserved for those who do not work, we need to estimate wage equations as well. This, in principle, can be done simultaneously with the estimation of the labor supply model. However, wages may be correlated with the taste-shifters in the utility function, which makes it difficult to implement this strategy in practice. Instead, we follow a three-stage estimation procedure proposed by Dagsvik and Strøm (2004). This means that we first estimate a reduced form participation probability and then estimate the wage equations using the results from the first stage to control for selectivity bias. In the third stage we estimate the labor supply model by the maximum likelihood method after inserting the estimated wage equations into the model. This procedure has the added advantage of reducing the measurement error caused by a negative correlation between hours of work and wages.

We assume that the mean in the wage distribution across individuals for gender k, $k = F, M$ is equal to $\bar{w}_k \eta_k$, where $\{\eta_k\}$ are random terms that account for unobserved differences in wages across workers, and we assume that $\log \eta_k$ are independent and normally distributed, $N(0, \sigma_k)$. Furthermore, we assume that $\log \bar{w}_k$ is a linear function of the length of schooling, experience, and experience squared. Experience is defined as age minus years of schooling minus seven. When the wage equations are inserted into the model and the error terms in these equations are integrated out, we obtain an empirical model of labor supply behavior for married couples which is similar to (4.3). In practice, we compute the expectation by Monte Carlo simulation when estimating the model.

Since wages do not vary across different jobs for a given worker in our maintained model, the corresponding opportunity measure depends only on hours of work. Following (4.1), we rewrite the opportunity measures as $\theta_k g_{1k}(h)$ $k = F, M$.[11] The term θ_F, which is supposed to measure the number of jobs available for the female (and possibly account for the fixed cost of working), is assumed to depend on length of schooling and capture partially the (observed) individual differences in choice constraints.[12] The opportunity distributions of hours, $g_{1k}(h)$, $k = F, M$, are uniform except for peaks at full-time and part-time hours. The full-time peak corresponds to 1,950 hours annually (37.5 hours a week), while the part-time peak corresponds to 1,040 hours annually (20 hours a week). The part-time and full-time peaks in the hours distribution capture institutional restrictions and technological

---

[11] Note that when we allow for stochastic choice sets, as discussed in section 3.2, the opportunity measures are individual-specific. Thus the estimates for θ_k can be interpreted as means of the corresponding individual measure for observable identical individuals, while $g_{1k}(h)$ can be seen as the sample average.

[12] Note that since our sample includes only households where husbands are working, we are not able to identify θ_M.

constraints and hence market imperfections in the economy. Recall that this specification of the opportunity distribution is formally equivalent to introducing suitable dummy variables at full-time and part-time hours of work in the utility specification of the conventional discrete choice specification: see, for example, van Soest (1995).

For the structural term of the utility function, many different versions are proposed in the literature, such as polynomials, translog, and generalized Box-Cox functions. Given our identification results above, it would in principle be possible to apply a non-parametric or semi-parametric approach to estimate the utility function, since we have assumed that the opportunity measures are separable. However, due to difficulties in estimation we have chosen instead a parametric approach and apply the generalized Box-Cox functional form. Since the quadratic specification is easier to estimate (because it is linear in parameters) and is widely used in applications, we have also estimated a model version with quadratic utility specification.

The deterministic part of the utility function is allowed to depend on household disposable income, individual leisure for both husband and wife. Disposable income, *C*, is measured as the sum of the annual household wage income after tax, household capital income after tax, and child allowances. Gender specific individual leisure L_k, $k = F, M$, is defined as $L_k = 1 - h_k/3650$. The detailed empirical specification of the model is presented in Appendix D.

Estimates of the wage equations and the parameters of the structural choice model are given in Tables D.1–D.3 in Appendix D. As shown in Table D.1, the selection bias in the wage equations is negligible. The estimates of the variances of the error terms in the wage equations are large. Thus it seems important to account for the error terms in the wage equations when estimating the structural model.

The estimates for the generalized Box-Cox utility specification are reported in Table D.2. The results imply that $\log v(C, h)$ is strictly increasing and concave in consumption and leisure. The marginal utilities of female and male leisure are increasing functions of age. The number of children has a significant effect on the marginal utility of leisure of the female. In contrast, the marginal utility of leisure for the male does not depend significantly on the number of children. This indicates that the female takes more responsibility for the children within the family than the male, which is not a surprising result. The exponent α_1 associated with consumption is significantly different from zero, which implies that the households care about not only relative consumption levels (beyond subsistence) but also absolute levels. The measure of the number of available jobs for females, θ_F, depends positively on the length of schooling (*S*). Higher educational level increases job opportunities for females. Note also that the full-time and part-time peaks in the opportunity density of hours for males are substantially higher than the corresponding peaks for females. The reason for this is that more women work in other labor market sectors (such as health care) than men and therefore may face different choice restrictions. This could be partially due to the differences in gender-specific human capital investments, which are important in shaping the job choice constraints.[13]

Figure 1. Predicted and observed distributions of hours of work for married males, 1997

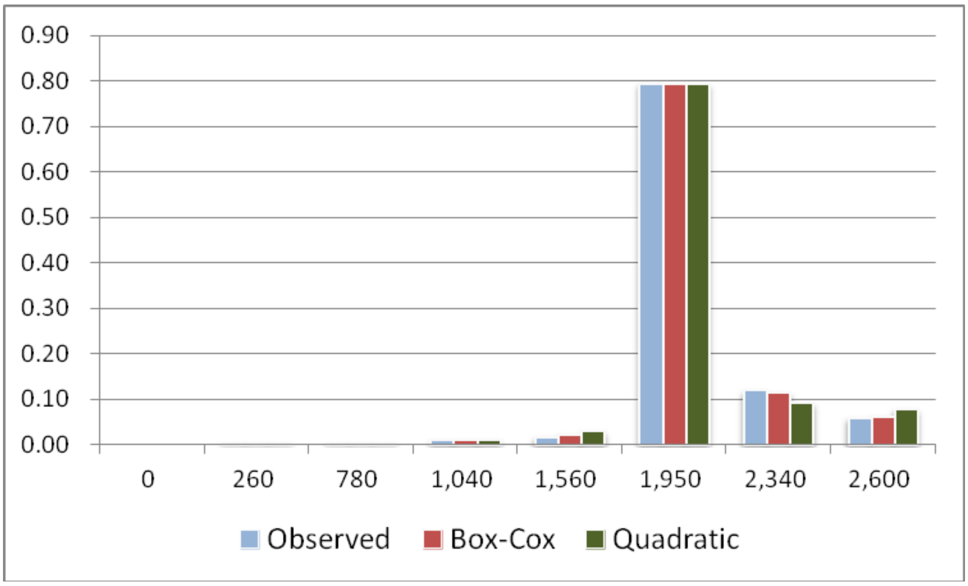
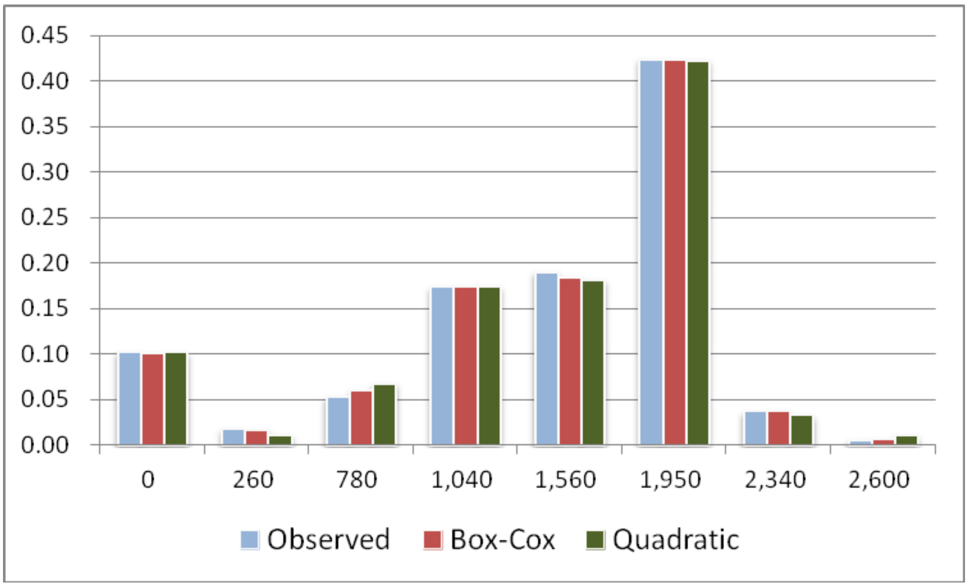


Figure 2. Predicted and observed distributions of hours of work for married females, 1997



[13] In a sector-specific model, similarly to Dagsvik and Strøm (2006) one could obtain explicit sector-specific opportunity measures.

For the quadratic specification of $\log v(C,h),$ the corresponding estimates (Table D.3) imply that the utility function is not monotonic for higher levels of a wife's leisure.

Both specifications fit the data quite well with values of McFadden's ρ^2 about 0.44, with the generalized Box-Cox slightly outperforming the quadratic as measured by the likelihood levels, even though it has two parameters less than the quadratic one. Figures 1 and 2 show the observed and (aggregate) predicted values of participation and hours of work for married couples based on our model. We note that the model predicts these aggregates quite well. Similar results have also been found by other researchers: see Dagsvik and Strøm (2006), Dagsvik et al. (2011), and Mastrogiacomo et al. (2011). However, since the Box-Cox functional form is non-linear in parameters, it is more complicated to estimate than the corresponding quadratic one. Mastrogiacomo et al. (2011) report difficulties with estimating Box-Cox utility specifications in some cases. Blundell and Shepard (2012) found that their Box-Cox utility specification resulted in an unacceptable estimate of one parameter.

5.2. Prediction performance

A test of model performance other than goodness-of-fit is to examine the extent to which the model is able to predict out-of-sample labor supply behavior. Below we report selected prediction results of this type.

During the prediction exercises we keep the parameters of the opportunity measure fixed but allow mean wages to follow the respective observed trends. In the first exercise, we compare model predictions based on data from the same source as our sample used for estimation but nine years later: namely, the Labor Force Survey of 2006. The advantage of using this sample for comparison is that one can construct all variables in the same way as we did for the sample used to estimate the model and apply the same sample-selection rules.

In a second performance assessment we investigate the prediction performance of the model on a completely different dataset: the income tax returns filed in 2003. The problem is that there is no information on labor supply available in this dataset. Instead, we compare the predicted distribution of disposable income with the corresponding empirical income distribution. Thus in this out-of-sample prediction exercise we simulate how predictions of hours and wages affect disposable income. The only sample-selection criterion imposed on this simulation is the requirement that individuals should be wage earners between 26 and 62 years of age.

Two parameters are important when using the model estimated for one year (the base year) to predict labor market behavior and corresponding incomes in another year (the simulation year): namely, the wage growth rate and the inflation rate, both measured from base year to simulation year. We use the observed wage growth rate from the base year (1997) to the year of prediction (2006 or

2003), together with the wage equations estimated in the base year, to generate the wage in the simulation year. Similarly, incomes in the simulation year are adjusted by using the inflation rate.

We also compare predictions from our labor supply models with results from a "naive" prediction procedure, based on the assumption that labor supply behavior remains at the level of 1997. The labor supply model should in principle be superior to such a simple procedure, since it takes into account the effects of changes in tax rules and wages, as well as changes in the demographic composition of the sample. In 2006 a tax reform was introduced that implies a substantial realignment of dividend income and wage income taxation. In addition to a higher effective tax on dividends, there was a substantial reduction in the maximum marginal tax rate, from 55.3 to 47.8 percent, which was expected to increase labor supply. Figures 3 and 4 display the observed and predicted distributions of labor supply for married couples obtained from the first exercise. As we see from these figures, the predictions for married females from both specifications of our model are very similar and are much better than the "naive" method. However, while the quadratic specification performs better than the generalized Box-Cox one for married males, both specifications seem to predict too high an increase in hours of work for married males, which may indicate that our model has overestimated the males' response to wage changes.

Figure 3. Predicted and observed distributions of hours of work for married males, 2006

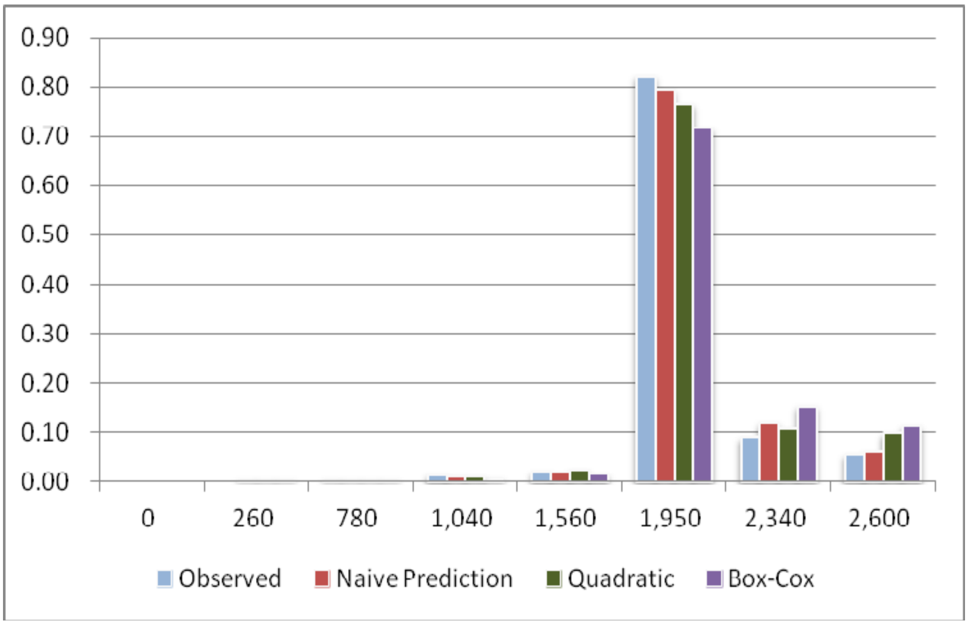


Figure 5 displays the observed and predicted disposable income distribution (adjusted to 1997 prices) obtained in the second exercise. In this case, our model predicts better than the naive method. In particular, the naive method misses the right-shift of the income distribution from 1997 to 2003, which is a result of increased wage level in 2003. In contrast to the naive method, our prediction depends on both the labor supply model (conditional on wages) and the wage distribution for males and females, which leads to a much better fit for the income distribution.

Figure 4. Predicted and observed distributions of hours of work for married females, 2006

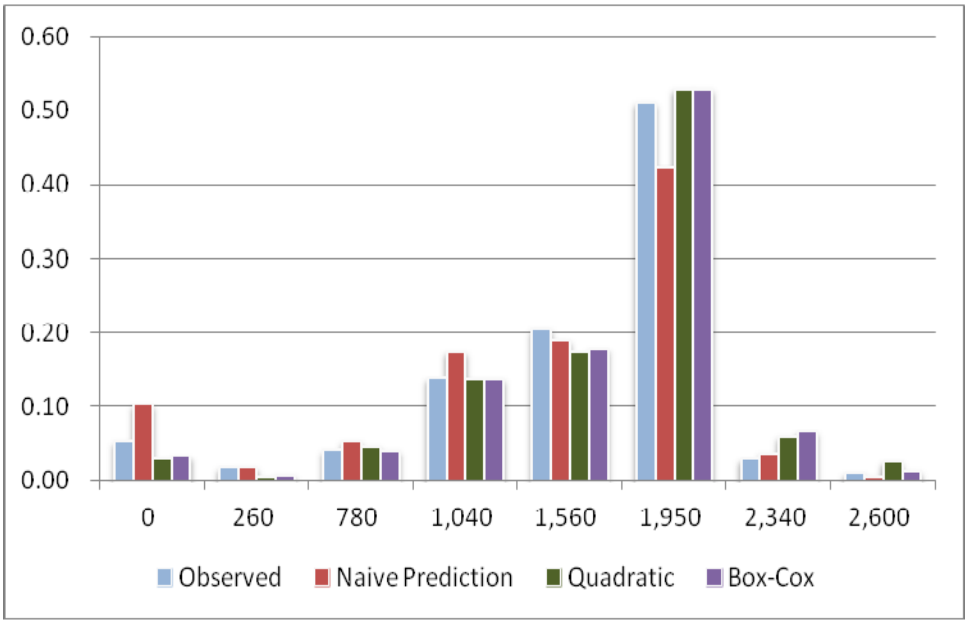
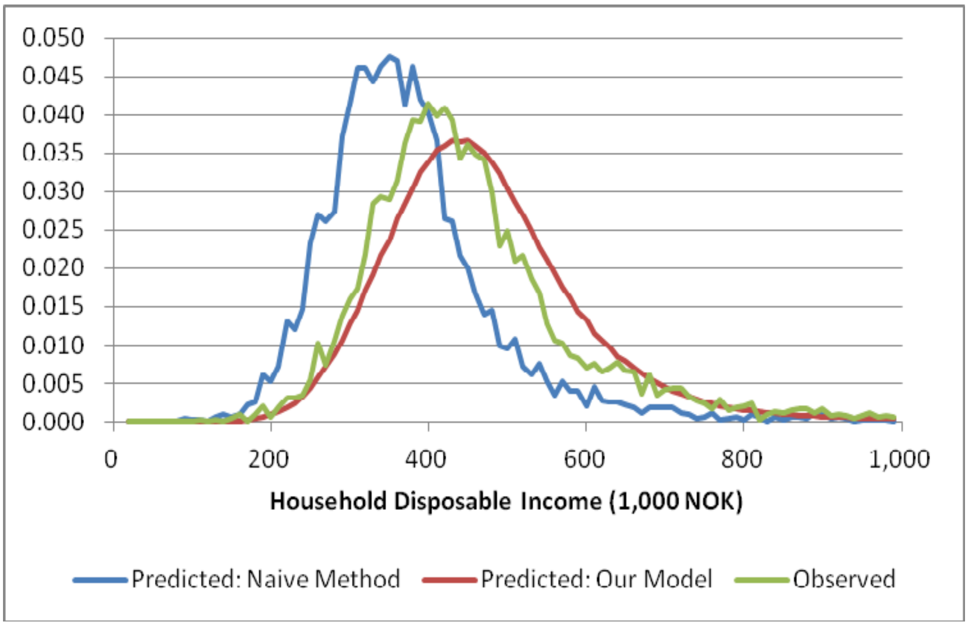


Figure 5. Observed and predicted density of disposable income for married couples, 2003



5.3. Aggregate wage elasticities

In this section we report selected wage elasticities. We have chosen to calculate elasticities that take into account both the systematic terms and the unobservables in the model. This means that we account for how the mean of the *distribution* of labor supply is affected by changes in, say, wage levels. We refer to these elasticities as *aggregate* elasticities because they accommodate unobserved and observed heterogeneity in the population. In Table 1, we report what we term aggregate uncompensated elasticities. They are calculated as follows. For each household, we simulate the change in the choice probabilities of working and the expected hours of work for females and males following a 10 percent increase in wages. We then aggregate over the sample to obtain the corresponding change in the mean probability of working and mean expected hours of work. To obtain elasticities, we multiply these figures by 10 and divide by the respective mean probability of working and the mean expected hours of work.

In general, the tables show that the uncompensated wage elasticities are moderate for married females but small for males and single females. For married females, the own-wage elasticity of the probability of working is equal to 0.33, which means that if the wages of married females were to increase by 5 percent, say, then the aggregate proportion of married females working would increase by 1.5 percent. If both male and female wages were increased, then the corresponding elasticity of the probability of working would be equal to 0.22. This means that the proportion of married females working would increase by 1 percent.

Conditional on working, the wage elasticity of mean hours of work is 0.28 for married females. Note also that the elasticities conditional on income groups decrease slightly by income for females but increase slightly for males. However, the elasticities with respect to change in both wages remain practically constant over income groups. The corresponding unconditional elasticities for the females measure the effect on total mean hours of work of a change in wages. Table 1 shows that the unconditional elasticities for married females range from 0.71 in the lowest decile to 0.52 in the highest decile of disposable income. The figure for the whole population is 0.61. This means that a 5 percent increase in the wage rate of married females increases total mean annual hours of work by 44 hours.

It is important to be aware of the fact that models are non-linear. As also emphasized by van Soest and Das (2001), results will in all likelihood depend on aggregation methods: for instance, whether elasticity estimates are based on average elasticities or elasticities of the average. The elasticities reported in Table 1 are calculated as the average elasticities of the estimation sample and depend crucially on the sample distribution of the exogenous variables, such as age, education, and non-labor income.

Table 1. Uncompensated wage elasticities for married couples

		Female base value	Male base value	Female own-wage elasticity	Female cross-wage elasticity	Male own-wage elasticity	Male cross-wage elasticity	Female elasticity with respect to both wage rates	Male elasticity with respect to both wage rates
Probability of working	Whole sample	0.89		0.33	–0.14			0.22	
	Lowest decile	0.87		0.42	–0.18			0.28	
	2nd to 8th decile	0.90		0.33	–0.14			0.22	
	Highest decile	0.92		0.25	–0.09			0.17	
Mean hours of work conditional on working	Whole sample	1,601	2,015	0.28	–0.09	0.08	–0.02	0.20	0.06
	Lowest decile	1,581	2,002	0.29	–0.09	0.07	–0.02	0.21	0.05
	2nd to 8th decile	1,602	2,015	0.28	–0.09	0.08	–0.02	0.20	0.06
	Highest decile	1,618	2,030	0.27	–0.08	0.09	–0.01	0.19	0.08
Unconditional mean hours of work	Whole sample	1,444		0.61	–0.23			0.42	
	Lowest decile	1,383		0.71	–0.26			0.48	
	2nd to 8th decile	1,445		0.61	–0.22			0.42	
	Highest decile	1,500		0.52	–0.18			0.37	

5.4. Desired hours of work and changes in the opportunity distribution of offered hours of work

As discussed above, an important feature of our job choice model is that it enables us to represent restrictions on hours of work in a convenient way through the opportunity measures. The framework enables us to predict the “desired hours of work”: that is, the labor supply when there are no constraints in the labor market. Figures 6 and 7 show the distribution of desired hours of work for married males and females respectively. For the purposes of comparison, we also show the “actual” hours of work. The predictions of desired hours seem fairly reasonable. The figures show that predictions of desired hours of work vary across utility specifications despite the fact that both specifications yield rather close fits to the data. This is somewhat surprising and shows that choice of functional form may be more crucial than expected. One explanation for this result may be that different functional forms may have differing abilities to capture unobserved population heterogeneity even though the levels of the corresponding likelihood functions are close.

Figure 6. Desired hours of working, married men, 1997

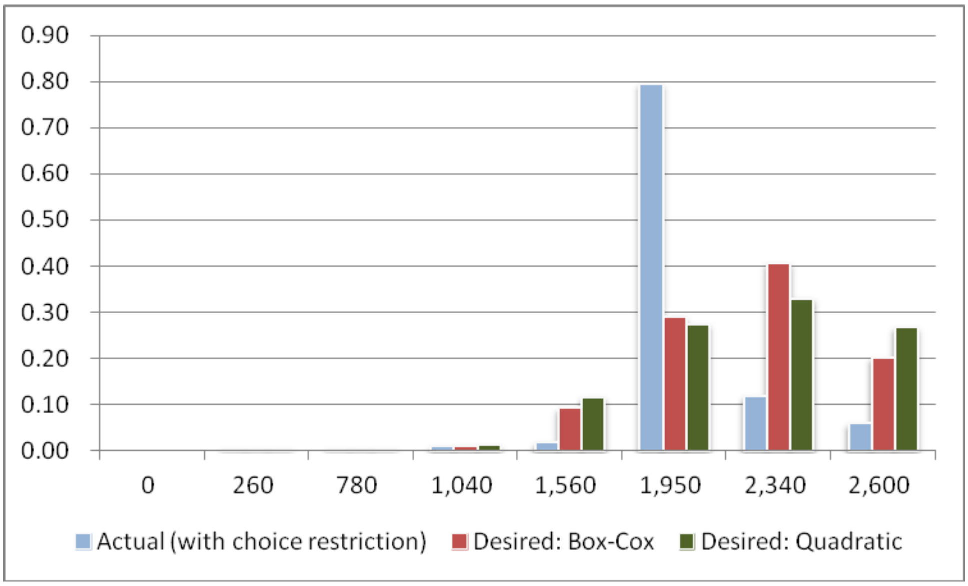
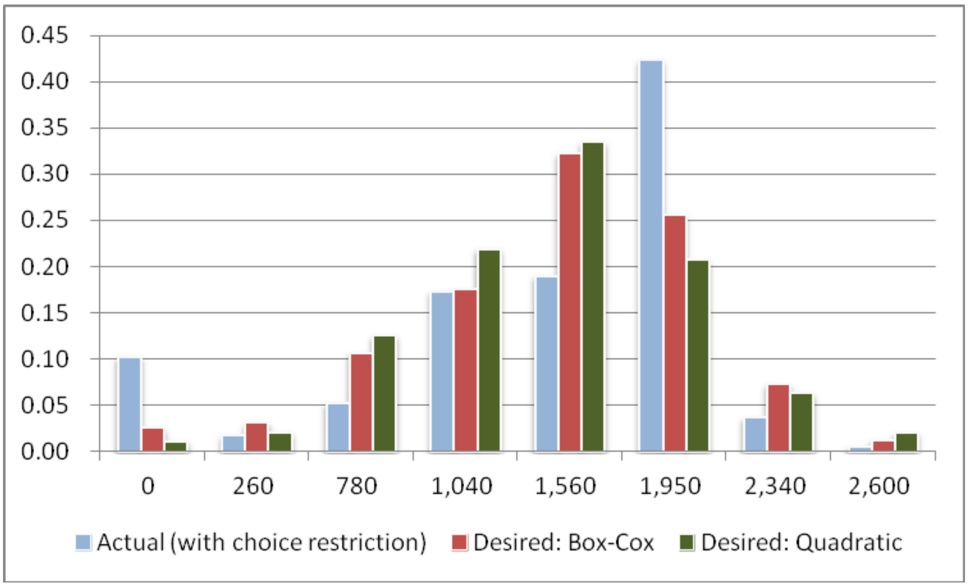
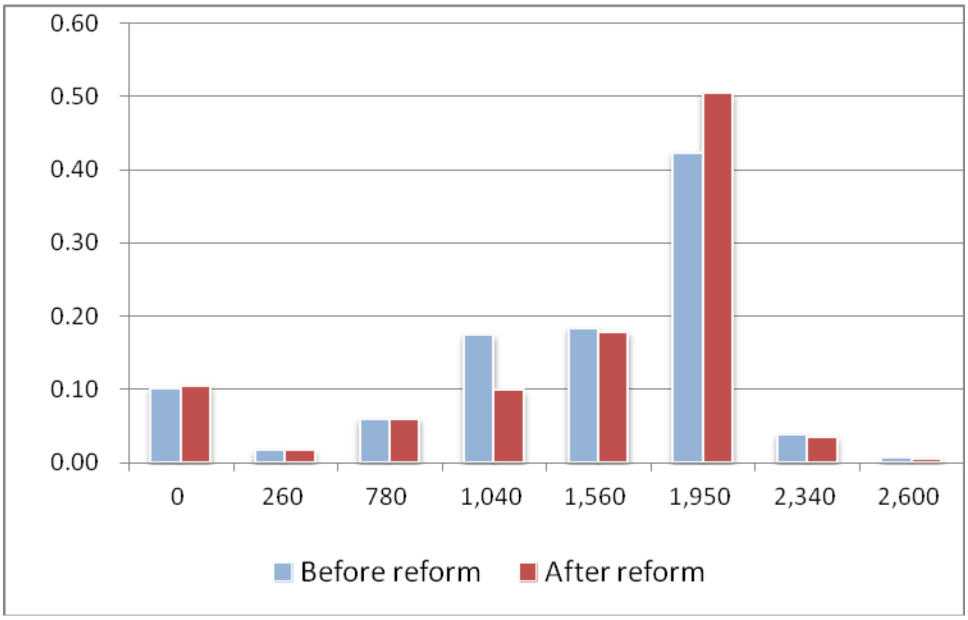


Figure 7. Desired hours of working, married women, 1997



Using our framework, we can also simulate the effect on labor supply of changing the opportunity distribution. In Norway, a high proportion of married women work part-time. In our sample, more than 35 percent of the married women work between 20 and 30 hours a week, while only around 40 percent work full-time (37.5 hours a week). Many of those in part-time jobs are employed in the public sector, especially in health care. Whereas Norwegian working environment legislation opens up the possibility of voluntary part-time jobs, an important reason for the high concentration of part-time workers relates to particular institutional regulations in the public health sector. Part-time positions are sometimes the only positions offered by public health care organizations, in particular in more rural areas. Recently, there has been a heated debate between politicians and trade unions about replacing part-time with full-time positions. It has been argued that the current hours of work regulations in the labor market affect women in particular and work against gender equality. Other parties are more concerned about the cost of this type of reform. The prospects of a future shortage of labor supply, particularly in the health sector, have also been an important aspect of discussions. A proper assessment of the potential labor supply effect of a reform where part-time jobs are replaced by full-time ones is therefore of considerable interest.

Figure 8.Change in the aggregated labor supply for married and cohabiting women



We shall now explain how the effect of this reform has been simulated by our model. Recall that in our framework, $g_{1F}(h_F)$ represents the proportion of jobs with hours of work h_F that are available to the wife, whereas the parameter θ_F is a measure of the total number of jobs that are available to the wife. Recall that there are two peaks in the estimated offered hour distribution, $g_{1F}(h_F)$. The proposed reform can be interpreted as a change in the opportunity distribution of hours for the women, obtained by removing the part-time peak and increasing the full-time peak (since the part-time jobs are replaced by full-time ones), while keeping the total number of available jobs unchanged (that is, θ_F is kept unchanged). With the new opportunity distribution, say $g_{1F}^*(h_F)$, which is uniform apart from the full-time peak, one can apply the model to simulate the corresponding realized labor supply distribution. Figure 8 displays the results given the 2004 tax system. As we can see, there is a significant decrease in the share of married/cohabiting women who prefer to work part-time, accompanied by an increase in the share of full-time hours of work of about a similar magnitude. We also observe a slight increase in the share of women who do not work, which indicates that the new opportunity distribution is viewed as more restrictive than the old one. In addition, we found that the corresponding labor supply of the males is changed very little by the introduction of this reform.

However, one needs to be careful when interpreting this result in the context of reforms specific to labor market sectors (such as the health care sector), since the sector-specific preferences and restrictions are not explicitly accounted for in the model. Nevertheless, this simulation exercise clearly illustrates the advantage and potential of our modeling framework.

6. Conclusion

There is a strong tradition in economics of working with highly stylized models. The standard model of labor supply based on consumption/leisure choice is an example of this. In the early days this strategy could perhaps be justified owing to the lack of suitable micro data and the absence of suitable theoretical and empirical methodology. However, this is no longer the case now. First, detailed micro data are available in most countries. Second, as has been demonstrated in this and related papers, the emergence of new econometric methodology (for example, the theory of discrete choice) has drastically simplified the task of quantitative representation of qualitative choice behavior in the presence of different types of choice constraint.

As discussed in the introduction, the traditional models of labor supply, being versions of the theory of consumer demand with two goods, disposable income and leisure, grossly simplify the choice setting. An essential feature of the modeling framework is that it is consistent with the notion of latent job opportunities, which implies that one can easily accommodate restrictions on the set of feasible jobs and peaks in the hours of work distribution (interpreted as owing to restrictions on hours) typically observed in many data sets. As we have explained, our analysis represents a further development of the modeling framework of Dagsvik and Strøm (2006). In contrast to the standard model, where the choice environment is represented by wage and non-labor income, the alternative approach based on the notion of job choice implies that one needs to model a representation of the agent-specific choice sets of available jobs. In this paper we have demonstrated one way of doing this: namely, through what we have defined as the opportunity measure. In the context of policy analysis, this raises new problems, since it is not evident how one should interpret changes in the opportunity measure. In this paper we have focussed on policy simulation of labor supply behavior. This means that we consider simulation of behavior conditional on choice constraints, represented by the opportunity measure and the economic budgetary constraints. In other words, we do not need a full equilibrium representation of the opportunity measure for this purpose.

Finally, we have carried out an empirical application of the framework based on a set of micro data from Norway. Although Dagsvik and Strøm (2006) have conducted an empirical analysis for the single-individual case, our analysis in this paper shows that a corresponding model for married couples can readily be estimated. An additional motivation is to contrast the practical features of this framework with the inherent difficulties of estimating a two-person version of the Hausman model (Hausman and Ruud, 1984): see Bloemen and Kapteyn (2008).

References

Aaberge, R., J.K. Dagsvik and S. Strøm (1995): Labor Supply Responses and Welfare Effects of Tax Reforms. *Scandinavian Journal of Economics*, **97**, 635–659.

Abowd, J. and F. Kramarz (2001): Inter-industry and Firm-size Wage Differentials: New Evidence from Linked Employer-Employee Data. Paper presented at the Econometric Society World Conference, 2000.

Abowd, J., F. Kramarz and D. Margolis (1999): High Wage Workers and High Wage Firms. *Econometrica*, **67**, 251–334.

Barzel, Y. (1973): The Determination of Daily Hours and Wages. *Quarterly Journal of Economics*, **87**, 220–238.

Bloemen, H. (2000): A Model of Labor Supply with Job Offer Restrictions. *Labour Economics*, **7**, 297–312.

Bloemen, H. (2008): Job Search, Hours Restrictions, and Desired Hours of Work. *Journal of Labor Economics*, **26**, 137–179.

Bloemen, H. and A. Kapteyn (2008): The estimation of utility consistent labor supply models by means of simulated scores, *Journal of Applied Econometrics*, **23**, 395–422.

Blundell, R. and A. Shepard (2012): Employment, Hours of Work and the Optimal Taxation of Low-Income Families. *Review of Economic Studies*, **79**, 481–510.

Creedy, J. and G. Kalb (2005): Discrete Hours Labour Supply Modelling: Specification, Estimation and Simulation. *Journal of Economic Surveys,* **19**, 697–738.

Dagsvik, J.K. (1994): Discrete and Continuous Choice, Max-stable Processes and Independence from Irrelevant Attributes. *Econometrica*, **62**, 1179–1205.

Dagsvik, J. K. (2012): Justification of Functional Form Assumptions in Structural Models: a Correction. *Theory and Decision*, DOI 10.1007/s11238-9308-5.

Dagsvik, J.K. and Z. Jia (2006): Labor Supply as a Choice among Latent Job Opportunities: a Practical Empirical Approach, Discussion Papers 481, Research Department of Statistics Norway.

Dagsvik, J. K. and S. Røine Hoff (2011): Justification of Functional Form Assumptions in Structural Models: Applications and Testing of Qualitative Measurement Axioms. *Theory and Decision*, **70**, 215–254.

Dagsvik, J.K. and S. Strøm (2004): Sectoral Labor Supply, Choice Restrictions and Functional Form. Discussion Paper no. 388, Statistics Norway, Oslo.

Dagsvik, J.K. and S. Strøm (2006): Sectoral Labor Supply, Choice Restrictions and Functional Form. *Journal of Applied Econometrics*, **21**, 803–826.

Dagsvik J.K., Z. Jia, K. Orsini and G. van Camp (2011): Subsidies on Low-skilled Workers' Social Security Contributions: the Case of Belgium. *Empirical Economics*, **40**, 779–806.

Dickens, W. and S. Lundberg (1993): Hours Restrictions and Labor Supply. *International Economic Review*, **34**, 169–191.

Embrechts, P., C. Klüppelberg and T. Mikosch (1997): *Modelling Extremal Events.* Springer-Verlag, Berlin.

Farzin, Y.H. (2009): The Effect of Non-pecuniary Motivations on Labor Supply. *Quarterly Review of Economics and Finance,* **49**, 1236–1259.

Hackman, J.R. and G.R. Oldham (1976): Motivation through the Design of Work: Test of a Theory. *Organizational Behavior and Human Performance*, **16**, 250–279.

Ham, J. and K.T. Reilly (2002): Testing Intertemporal Substitution, Implicit Contracts, and Hours Restriction Models of the Labor Market Using Micro Data, *American Economic Review,* **92**, 905–927.

Hausman, J.A. and P. Ruud (1984): Family Labor Supply with Taxes. *American Economic Review*, **74**, 242–253.

Ilmakunnas, S. and S. Pudney (1990): A Model of Female Labour Supply in the Presence of Hours Restrictions. *Journal of Public Economics,* **41**, 183–210.

Kornstad, T. and T.O. Thoresen (2007): A Discrete Choice Model for Labor Supply and Child Care. *Journal of Population Economics*, **20**, 781–803.

Lancaster, K. (1966): A New Approach to Consumer Demand. *Journal of Political Economy*, 74**,** 132–157.

Lancaster, K. (1971): *Consumer Demand: A New Approach*. Columbia University Press, New York.

Luce, R.D. (1959): *Individual Choice Behavior.* Wiley, New York.

Mastrogiacomo, M., N. Bosch, M. Gielen and E. Jongen (2011): Estimating Labour Supply Responses in the Netherlands using Structural Models. CPB Achtergrond Document, CPB Netherlands Bureau for Economic Policy Analysis, The Hague, Netherlands.

McFadden, D. (1973): Conditional Logit Analysis of Qualitative Choice Behavior. In P. Zarmebka (ed.), *Frontiers in Econometrics*, Academic Press, New York.

McFadden, D. (1984): Econometric Analysis of Qualitative Response Models. In Z. Griliches and M. Intilligator (eds.), *Handbook of Econometrics*, Volume III, North-Holland, Amsterdam.

Mortensen, D.T. (2003): *Wage Dispersion: Why Are Similar Workers Paid Differently?*. MIT Press, MA, USA.

Resnick, S.I. (1987): *Extreme Values, Regular Variation, and Point Processes*. Springer-Verlag, Berlin.

Samorodnitsky, G. and M.S. Taqqu (1994): *Stable Non-Gaussian Random Processes.* Chapman & Hall, London.

Sattinger, M. (1993): Assignment Models of the Distribution of Earnings. *Journal of Economic Literature*, **31**, 831–880.

Sattinger, M. (1995): Search and the Efficient Assignment of Workers to Jobs. *International Economic Review,* **36**, 283–302.

Stewart, M.B. and J.K. Swaffield (1997): Constraints on the Desired Hours of Work of British Men. *Economic Journal*, **107**, 520–535.

Tummers, M. P. and I. Woittiez (1990): A Simultaneous Wage and Labor Supply Model with Hours Restrictions. *Journal of Human Resources*, **26**, 393–423.

van Ophem, H., J. Hartog and W. Vijverberg (1993): Job Complexities and Wages. *International Economic Review,* **34**, 853–873.

van Soest, A. (1995): Structural Models of Family Labor Supply. A Discrete Choice Approach. *Journal of Human Resources*, **30**, 63–88.

van Soest, A. and M. Das (2001): Family Labor Supply and Proposed Tax Reforms in the Netherlands. *De Economist*, **149**, 191–218.

van Soest, A., M. Das and X. Gong (2002): A Structural Labour Supply Model with Flexible Preferences. *Journal of Econometrics*, **107**, 345–374.

van Soest, A., I. Woittiez and A. Kapteyn (1990): Labour Supply, Income Taxes and Hours Restrictions in the Netherlands. *Journal of Human Resources*, **25**, 517–558.

Wolf, E. (2002): Lower Wage Rates for Fewer Hours? A Simultaneous Wage-hours Model for Germany. *Labour Economics*, **9**, 643–663.

Appendix A

Proof of the identification results

For a general differentiable function $F(x)$ in several variables, let $F_k'(x)$ denote the partial derivative with respect to the k-th component of x. Assume the tax function $f(x,y)$ is monotonic and differentiable everywhere except at a finite number of points. Recall that from (2.7) we have that

$$\text{(A.1)} \qquad \frac{v(f(hw,I),h)m(h,w)}{v(f(0,I),0)} = \frac{\varphi(h,w\mid I)}{\varphi(0\mid I)}.$$

By taking the logarithm transformation of (A.1) and differentiating with respect to I we obtain

$$\text{(A.2)} \qquad \frac{v_1'(f(hw,I),h)f_2'(hw,I)}{v(f(hw,I),h)} - \frac{v_1'(f(0,I),0)f_2'(0,I)}{v(f(0,I),0)} = \frac{\partial \log(\varphi(h,w\mid I)/\varphi(0\mid I))}{\partial I} \overset{def}{=} \xi(h,w,I).$$

Now let $\kappa(C,I)$ be the wage income which is needed to achieve disposable income C at given non-labor income level I, that is, $\kappa(C,I)$ satisfies $C = f(\kappa(C,I),I).$ In a way, this function can been as the inverse of the budget function given non-labor income I. By inserting $wh = \kappa(C,I)$ into (2.7) we get

$$\text{(A.3)} \qquad \frac{v_1'(C,h)f_2'(\kappa(C,I),I)}{v(C,h)} - \frac{v_1'(f(0,I),0)f_2'(0,I)}{v(f(0,I),0)} = \frac{\partial \log(\varphi(h,\kappa(C,I)/h\mid I)/\varphi(0\mid I))}{\partial I}$$

$$= \xi(h,\kappa(C,I)/h,I).$$

Next, by dividing (A.3) by $f_2'(\kappa(C,I),I)$ and rearranging, (A.3) becomes

$$\text{(A.4)} \qquad \frac{v_1'(C,h)}{v(C,h)} = \frac{v_1'(f(0,I),0)f_2'(0,I)}{v(f(0,I),0)f_2'(\kappa(C,I),I)} + \frac{\xi(h,\kappa(C,I)/h,I)}{f_2'(\kappa(C,I),I)}.$$

Note that the left-hand side of (A.4) does not depend on I. We can therefore keep I fixed and equal to I_0 (say). Let $r = v_1'(f(0,I_0),0)f_2'(0,I_0)/v(f(0,I_0),0),$ which we note is an *unknown* constant. We have

$$\text{(A.5)} \qquad \frac{\partial \log v(C,h)}{\partial C} = \frac{r}{f_2'(\kappa(C,I_0),I_0)} + \frac{\xi(h,\kappa(C,I_0)/h,I_0)}{f_2'(\kappa(C,I_0),I_0)}.$$

(i) *The nonparametric case.*

Suppose that no assumption about functional form is made. Let

$$\log \zeta(C) = \int_a^C \frac{dz}{f_2'(\kappa(z,I_0),I_0)}, \text{ and } \log \lambda(C,h) = \int_a^C \frac{\xi(h,\kappa(z,I_0)/h,I_0)dz}{f_2'(\kappa(z,I_0),I_0)},$$

where a is a suitable given real number. Since $f(hw, I)$ is known and $\xi(h,w,I)$ can be identified through (A.3), both $\zeta(C)$ and $\lambda(C,h)$ are *identified.* By integrating (A.5) with respect to C and taking the exponential transformation we get

(A.5) $$v(C,h) = \zeta(C)^r \lambda(C,h)\delta(h),$$

where *r* is an *unknown* constant and $\delta(h)$ is an *unknown* function of *h*, whereas $\zeta(C)$ and $\lambda(C,h)$ are *known* functions. Thus we have proved that the structure of the model implies that one can identify the structural part of the utility function up to the *unknown* power *r* of a power transform of $\zeta(C)$ and a multiplicative term that depends on hours of work.

(ii) *Separable opportunity measure*

To identify the constant *r* one needs further assumptions. One possibility is to assume that the opportunity measure is multiplicatively separable: that is, $m(h,w) = m_1(h)m_2(w)$. From (2.4) and (A.9) it then follows that

$$\frac{\varphi(h,w\,|\,I)}{\varphi(0\,|\,I)} = \frac{v(f(hw,I),h)m_1(h)}{v(f(0,I),0)} = \frac{\zeta(f(hw,I))^r \lambda(f(hw,I)\delta(h)m_1(h)}{\zeta(f(0,I))^r \lambda(f(0,I),0)\delta(0)},$$

which, after taking the logarithm transformation and differentiating with respect to *w*, yields

(A.7) $$\partial \log\left(\frac{\varphi(h,w\,|\,I)\lambda(f(0,I),0)}{\varphi(0\,|\,I)\lambda(f(hw,I),h)}\right)/\partial w = r\partial \log\left(\frac{\zeta(f(hw,I))}{\zeta(f(0,I))}\right)/\partial w.$$

From (A.7) we see that *r* is identified. According to (A.5), $v(C,h)$ is therefore identified up to a multiplicative term $\delta(h)$ that is solely a function of hours of work. Using (2.4), we see immediately that $m_2(w)$ is identified. However, similarly to the parametric case considered above, we cannot without further assumptions separate $\delta(h)$ from $m_1(h)$.

(iii) *The Generalized Box-Cox functional form assumption*

For simplicity we shall in this section assume that the tax function is piecewise linear in both non-labor income and wage income. This assumption represents in practice no essential restriction since most tax systems have this property. In this case, the marginal tax rate with respect to non-labor income $f_2'(x,y)$ is constant within certain intervals.

Since $\kappa(C,I)$ is continuous, one can find an interval (C_-,C_+) (say) such that $f_2'(\kappa(C,I_0),I_0) = r_0$, when $C \in (C_-,C_+)$. In this case, (A.5) can be rewritten as

(A.8) $$\frac{\partial \log v(C,h)}{\partial C} = \frac{r}{r_0} + \frac{\xi(h,\kappa(C,I_0)/h,I_0)}{r_0}$$

where r is a *unknown* constant and r_0 is a *known* constant. Assume now that $\log v(C,h)$ has a generalized Box-Cox structure as in (2.7). It then follows from (2.7) and (A.8) that

(A.9) $$\gamma C^{\alpha-1}(1+\frac{\mu}{\gamma\beta}(M-h)^{\beta}-1)=\frac{r}{r_0}+\frac{\xi(h,\kappa(C,I_0)/h,I_0)}{r_0}$$

Now by differentiating (A.9) with respect to *C* yields

(A.10) $$\gamma(\alpha-1)C^{\alpha-2}(1+\frac{\mu}{\gamma\beta}(M-h)^{\beta}-1)=\frac{\partial\xi(h,\kappa(C,I_0)/h,I_0)/\partial C}{r_0}\overset{def}{=}\zeta(C,h)$$

From (A.9) it follows that further differentiation with respect to *C* implies that

(A.11) $$\frac{\alpha-2}{C}=\frac{\zeta_1'(C,h)}{\zeta(C,h)},$$

which identifies α. Also from (A.9) it follows by differentiation with respect to *h* that

(A.12) $$\frac{\mu}{\gamma}(M-h)^{\beta-1}=\frac{\partial\xi(h,\kappa(C,I)/h,I)/\partial h}{r_0C^{1-\alpha}}.$$

Eq. (A.12) shows that when α has been identified then β and μ/γ are identified. Furthermore, we realize that by (A.10) also γ is identified. Unfortunately, we still cannot identify $\delta(h)$, so that $v(C,h)$ is still identified only up to a function of *h*.

Thus, under the particular Generalized Box-Cox functional form assumption, the separability condition on $m(h,w)$ is no longer required.

(iv) *Quadratic specification*

Consider now the quadratic specification. The cases with higher order polynomials are similar. Thus we assume that

(A.13) $$\log v(C,h)=\beta_0+\alpha_1C+\alpha_2C^2+\beta_1h+\beta_2h^2+\gamma Ch,$$

Then we can rewrite (A.8) as:

(A.14) $$\alpha_1+2\alpha_2C+\gamma h=\frac{r}{r_0}+\frac{\xi(h,\kappa(C,I_0)/h,I_0)}{r_0}.$$

Using (A.14), it is possible to identify α_2 and γ similar to the case of the generalized Box-Cox function. However, we can never identify α_1, since r is *unknown*.

Appendix B.

Properties of the family of stable distributions and Stochastic Choice Sets.

A stable distribution with parameters α, σ, β, and μ is often denoted by $S_\alpha(\sigma,\beta,\mu)$. The parameter α is restricted such that $0<\alpha\le 2$ and is an index that characterizes the heaviness of the tails, whereas σ is a positive scale parameter that is similar to the standard deviation. The parameter β is restricted to the closed interval [−1,1] and it characterizes the skewness of the distribution: $\beta=0$ implies symmetry, whereas $\beta=1$ $(\beta=-1)$ implies that the distribution is maximally skewed to the right (left). The parameter μ is a location parameter that coincides with the expectation when $\alpha>1$, whereas the expectation is not defined when $\alpha\le 1$. When $\alpha=2$, the distribution reduces to the normal distribution, in which case β vanishes. However, when $\alpha<2$, the variance is infinite. It also follows from condition (ii) that $\mu=0$. If we impose (i), it follows in addition that $\alpha<1$ and $\beta=1$: see Samorodnitsky and Taqqu (1994).

Proof of equation (3.8)

For ease of exposition, we simplify notation in proving the result. Consider the choice among M discrete alternatives and assume that alternative j has "utility" of the form $U_j=v_j\bar{m}_j\omega_j\varepsilon_j$, where $\varepsilon_j,\ j=1,2,...,M$ are i.i.d. positive random variables with c.d.f. $\exp(-1/x)$, for $x>0$, $\{v_j\}$ and $\{\bar{m}_j\}$ are positive deterministic terms, and ε_j and $\omega_j, 1,2,...,M$ are independent of $\{\bar{m}_j\}$ and $\{v_j\}$, and distributed according to $S_\alpha(1,1,0)$. Moreover $\{\omega_j\}$ and $\{\varepsilon_j\}$ are independent. Consider the c.d.f. of $\omega_j\varepsilon_j$. Because ω_j and ε_j are independent, it follows from Proposition 1.2.12 in Samorodnitsky and Taqqu (1994, p. 15) that

(B.1) $$P\left(\omega_j\varepsilon_j\le x\right)=EP\left(\varepsilon_j\le\frac{x}{\omega_j}\middle|\omega_j\right)=E\exp\left(-\omega_j x^{-1}\right)=\exp\left(-sx^{-\alpha}\right),$$

where $s=1/\cos(\alpha\pi/2)$. Eq. (A.1) implies that $\omega_j\varepsilon_j$ has the same distribution as $s^{1/\alpha}\varepsilon_j^{1/\alpha}$. Furthermore, this implies that U_j is equivalent to the "utility" function $\tilde{U}_j=v_j^\alpha\bar{m}_j^\alpha k\varepsilon_j$ because $s^{-1}U_j^\alpha$ has the same distribution as $\tilde{U}_j$. According to results that are well known, it follows that

(B.2) $$P\left(U_j=\max_k U_k\left|\{\omega_k\}\right.\right)=\frac{v_j\bar{m}_j\omega_j}{\sum_k v_k\bar{m}_k\omega_k}$$

and that

(B.3) $$P\left(\tilde{U}_j = \max_k \tilde{U}_k\right) = \frac{v_j^\alpha \bar{m}_j^\alpha s}{\sum_k v_k^\alpha \bar{m}_k^\alpha s} = \frac{v_j^\alpha \bar{m}_j^\alpha}{\sum_k v_k^\alpha \bar{m}_k^\alpha}.$$

Consequently, it follows that

(B.4) $$E\left(\frac{v_j \bar{m}_j \omega_j}{\sum_k v_k \bar{m}_k \omega_k}\right) = EP\left(U_j = \max_k U_k \middle| \{\omega_k\}\right)$$
$$= P\left(U_j = \max_k U_k\right) = P\left(\tilde{U}_j = \max_k \tilde{U}_k\right) = \frac{v_j^\alpha \bar{m}_j^\alpha}{\sum_k v_k^\alpha \bar{m}_k^\alpha}.$$

The result in (3.8) follows immediately from (B.4). Hence, the proof is complete. Q.E.D.

Appendix C.

Data Description

The data are obtained by merging the Labor force survey 1997 with two different register data sets that contain additional information about incomes, family composition, children and education. The concepts of the Labor force survey are in accordance with official statistics from Statistics Norway and recommendations given by ILO. Here we only note that persons are asked about their attachment to the labor market during a particular week.

Information about actual and formal working time in main as well as second job and background variables such as demographic characteristics and occupation is also included in the Labor force survey. Conditional on labor market participation, respondents are also asked whether they consider themselves as self-employed or employee, and based on this information we have excluded self-employed persons in the estimation. Working time is measured as formal hours of work in both main as well as second job. If this information is missing and the respondent is participating in the labor market, information about actual working time is used.

Information on education is obtained from the National Education database, a register database that can be linked to the Labor force survey using the system with personal identification numbers. Work experience is defined as age minus years of education minus preschool age.

Table C. 1. Summary statistics for married couples in the sample, Norway 1997

	Female not working		Female working	
	Mean	St. dev.	Mean	St. dev.
Male Age	44.06	9.41	45.07	8.44
Male Education	12.34	2.62	12.63	2.78
Male Experience	24.72	9.92	25.43	9.08
Male non-labor income	10796	15543	6320	12029
Male wage rate	169.11	65.47	153.80	52.977
Male weekly hours of work	39.16	5.96	38.43	5.38
Female Age	41.68	09.70	42.76	8.39
Female Education	10.97	2.14	12.18	2.64
Female Experience	23.71	10.66	23.58	9.32
Female non-labor income	27689	21701	17648	16558
Female wage rate			120.17	37.86
Female weekly hours of work			30.45	8.93
Number of Children 0–7	0.84	0.98	0.44	0.75
Number of children 8–18	0.81	0.93	0.78	0.93
Number of Households	256		2255	

Whereas the Labor force survey yields detailed information about employment and hours of work, it does not provide information about annual labor incomes that can be used in the calculations of (average) gross wage rates, and non-labor income. To obtain this information we apply the Tax Return Register (includes more detailed information about employee income, self-employment income, taxable pensions etc.) These data can be linked to the Labor force survey using personal identification numbers. Nominal hourly wage rates are measured as labor incomes (for main as well as second job) divided by (formal) total annual hours of work (for main and second job). The sample includes persons with age between 26–62 years. The motivation for this is that for women under 26 years of age education is an important activity and for those more than 62 years of age early retirement is rather frequent. The number of children includes all children with age less than 19. A person is defined as working if he works at least one hour per week. Households where one of the adults has income from self-employment higher than NOK 80 000 are excluded. So also are households where one of the adults has hours of work higher than 80, or wage rate less than NOK 50 or higher than NOK 400. In Table C.1 we report the summary statistics for the sample used in estimating the labor supply model.

Appendix D.

Model for Married Couples with estimation results

The modeling framework for two-person households is completely similar to the case for single individual households. Let $U\left(C, h_F, h_M, z\right)$ denote the utility function of a household, where h_F and h_M are hours of work for female and male and $z=\left(z_F, z_M\right)$ indexes the combination of jobs for the female and male in the household, respectively. Similarly to the single individual households, assume that Let $U\left(C, h_F, h_M, z\right)=v\left(C, h_F, h_M\right)\varepsilon(z)$, with interpretation that is completely analogous to the case above. The budget constraint in this case can be written as

(D.1) $$C=f\left(h_F W_F, h_M W_M, I\right)$$

where W_F and W_M are the respective wage rates for female and male and $f(\cdot)$ is the function that transforms gross income to disposable income for the household. Let $\varphi\left(h_F, h_M \mid W_F, W_M, I\right)$ be the joint density of hours of work for female and male in the household, given wage rates and non-labor income. The empirical counterpart of this density is the fraction of couples where the husband works h_F hours and the wife works h_M hours, within the subpopulation of couples with wage rates and non-labor income equal to $\left(W_F, W_M, I\right)$. We assume furthermore that the offered hours H_F and H_M are independent.

(D.2) $$\psi\left(h_F, h_M; W_F, W_M, I\right)=v\left(f\left(h_F W_F, h_M W_M, I\right), h_F, h_M\right).$$

Then, under assumptions that are completely similar to the ones for single individual households we get that the conditionally density of $\left(h_F, h_M\right)$, given that $h_M>0$, equals

(D.3) $$\varphi\left(h_F, h_M \mid W_F, W_M, I\right)=\frac{\psi\left(h_F, h_M; W_F, W_M, I\right) m_F\left(h_F\right) m_M\left(h_M\right)}{M},$$

for $h_F>0, h_M>0$, and

(D.4) $$\varphi\left(0, h_M \mid W_F, W_M, I\right)=\frac{\psi\left(0, h_M; W_F, W_M, I\right) m_M\left(h_M\right)}{M}$$

for $h_F=0$ and $h_M=0$, where

(D.5) $$M\left(W_F, W_M, I\right)=\sum_{y>0} \psi\left(0, y; W_F, W_M, I\right) m_M(y)+\sum_{x, y>0,} \psi\left(x, y \mid W_F, W_M, I\right) m_F(x) m_M(y).$$

Model specification:

The opportunity measure for women, θ_F is assumed to depend on the wages solely through the amount of schooling. Specifically, we assume that

(D.6) $$\log\theta_F = f_{F1} + f_{F2}S\,,$$

where *S* is the length of education. Furthermore, we specify $v(\cdot)$ to be of the form

(D.7) $$\log v(C,h_F,h_M) = \alpha_2\left(\frac{\left[10^{-4}(C-C_0)\right]^{\alpha_1}-1}{\alpha_1}\right)+\left(\frac{(L_F)^{\alpha_3}-1}{\alpha_3}\right)\beta_F + \beta_M\left(\frac{(L_M)^{\alpha_4}-1}{\alpha_4}\right)+\alpha_{15}\left(\frac{(L_M)^{\alpha_4}-1}{\alpha_4}\right)\left(\frac{(L_F)^{\alpha_3}-1}{\alpha_3}\right)$$

where $\beta_F = \alpha_5 + \alpha_6\log A_F + \alpha_7(\log A_F)^2 + \alpha_8 CU6 + \alpha_9 CO6$

and $\beta_M = \alpha_{10} + \alpha_{11}\log A_M + \alpha_{12}(\log A_M)^2 + \alpha_{13}CU6 + \alpha_{14}CO6$

C_0 are subsistence level and $A_k,\ k = F, M,$ is age for gender *k* divided by 10, *CU*6 and *CO*6 are the number of children below or equal to and above the age of six respectively, *C* is given by the budget constraint similarly to (2.2), $L_k,\ k = F, M,$ is leisure for gender *k*, with $L_k = 1 - h_k/3650$, and $\alpha_j,\ j = 1, 2, ..., 15$, are unknown parameters. Note that we have subtracted from total annual hours a "subsistence" level, L_0 = 5,110 hours, which allows for sleep and rest. This corresponds to about 14 hours per day reserved for sleep and rest. We have chosen C_0 to be approximately NOK 40,000 $\sqrt{N}$, where *N* is the number of persons in the household. Disposable income, *C*, is measured as the sum of the annual household wage incomes after tax, household capital income after tax, and child allowances.

The Quadratic Specification

The opportunity measure is specified the same as in the Box-Cox case. Only thing different is the deterministic part of utility function $v(\cdot)$ is specified as the following:

(D.8) $$\log v(C,h_F,h_M) = \alpha_1 C + \beta_M L_M + \beta_F L_F + \alpha_2 C^2 + \beta_{FM} L_F L_M + \beta_{M^2} L_M^2 + \beta_{F^2} L_F^2 + \alpha_{MC} L_M C + \alpha_{FC} L_F C.$$

where variables and parameters defined as in (D.7).

Table D.1. Estimates of wage equations, females and males, 1997

Variables	Males		Females		Females (selection corrected)	
	Estimate	*t*-value	Estimate	*t*-value	Estimate	*t*-value
Constant	4.08	135.1	4.10	132	4.11	109
Experience in years/10	0.22	12.2	0.143	8.6	0.141	7.8
(Experience in years/10)^2	–0.03	–10.1	–0.022	–6.6	–0.022	–6.1
Education in years	0.044	26.9	0.0388	23.1	0.0386	19.7
Married	0.05	6.02	–0.022	–2.67	–0.21	–2.37
Log(P)					0.013	0.3
Variance of error term	0.3029		0.2755		0.2755	
No. observations	5,448		5,074		5,074	
R^2	0.15		0.10		0.10	

Table D.2. Estimates of the parameters of the utility function. Married couples, 1997. Quadratic Utility Function.

	Parameter	Married couples	
		Estimate	Standard error
Preferences:			
Consumption (10^{-4})			
Linear	α_1	0.3834	0.512
Quadratic	α_2	–0.0003	0.003
Female leisure: Linear			
Constant	α_5	53.8310	14.670
Log(age/10)	α_6	–45.3454	11.875
Log(age/10) squared	α_7	18.3212	4.258
No. children below or equal 6 years	α_8	2.5597	0.328
No. children above 6 years	α_9	1.3089	0.232
Female leisure: Quadratic	$\hat{a}_{F2}$	–23.6216	2.603
Male leisure: Linear			
Constant	α_{10}	39.3266	23.357
Log(age/10)	α_{11}	–63.4562	23.488
Log(age/10) squared	α_{12}	23.5614	8.278
No. children below or equal 6 years	α_{13}	–0.2039	0.578
No. children above 6 years	α_{14}	–0.5795	0.411
Male leisure: Quadratic	α_{15}	–13.8534	5.275
Leisure interaction	α_{16}	23.1948	7.126
Consumption Male Leisure interaction	α_{17}	0.5304	0.274
Consumption Female Leisure interaction	α_{18}	0.2181	0.175
The parameters θ_F; $\log\theta_F = f_{F1} + f_{F2}S$			
Constant	f_{F1}	–4.5023	0.495
Education	f_{F2}	1.3450	0.395
Opportunity density of offered hours			
Male full-time peak		2.4090	0.096
Female full-time peak		1.3464	0.060
Male part-time peak		1.2592	0.291
Female part-time peak		0.3789	0.071
Number of Observations		2,511	
Log likelihood		–5,728.2	
McFadden's ρ^2		0.44	

Table D.3. Estimates of the parameters of the utility function. Married couples, 1997. Generalized Box-Cox specification

	Parameter	Married couples	
		Estimate	Standard error
Preferences:			
Consumption			
Exponent	α_1	0.6643	0.054
Scale 10^{-4}	α_2	1.8411	0.352
Subsistence	C_0	$40{,}000\sqrt{N}$	
Female leisure			
Exponent	α_3	–0.8334	0.182
Constant	α_5	11.8387	1.888
Log(age/10)	α_6	–12.5285	1.945
Log(age/10) squared	α_7	5.2456	0.733
No. children below or equal 6 years	α_8	0.9682	0.168
No. children above 6 years	α_9	0.5075	0.094
Male leisure			
Exponent	α_4	–1.8043	0.430
Constant	α_{10}	3.8929	1.112
Log(age/10)	α_{11}	–4.3054	1.142
Log(age/10) squared	α_{12}	1.6682	0.444
No. children below or equal 6 years	α_{13}	0.0547	0.051
No. children above 6 years	α_{14}	0.0083	0.029
Leisure interaction	α_{15}	0.2047	0.147
Leisure subsistence	L_0	5,110	
The parameters θ_F; $\log\theta_F = f_{F1} + f_{F2}S$			
Constant	f_{F1}	–3.5041	0.435
Education	f_{F2}	1.2389	0.366
Opportunity density of offered hours			
Male full-time peak		2.3769	0.086
Female full-time peak		1.4380	0.296
Male part-time peak		1.0960	0.063
Female part-time peak		0.5622	0.067
Number of Observations		2,511	
Log likelihood		–5,706.5	
McFadden's ρ^2		0.44	

B *Returadresse*:
Statistisk sentralbyrå
NO-2225 Kongsvinger

Statistics Norway

Oslo:
PO Box 8131 Dept
NO-0033 Oslo
Telephone: + 47 21 09 00 00
Telefax: + 47 21 09 00 40

Kongsvinger:
NO-2225 Kongsvinger
Telephone: + 47 62 88 50 00
Telefax: + 47 62 88 50 30

E-mail: ssb@ssb.no
Internet: www.ssb.no

ISSN 0809-733X

Statistisk sentralbyrå
Statistics Norway