Discussion Paper

Central Bureau of Statistics, P.B. 8131 Dep, 0033 Oslo 1, Norway

No. 34 May 1988

A NOTE ON MYOPIC DECISION RULES IN THE NEOCLASSICAL THEORY OF PRODUCER BEHAVIOUR

BY

ERLING HOLMØY AND ØYSTEIN OLSEN

Abstract

The paper presents a theoretical framework for analysing investment behaviour in a perfect neoclassical environment. The model distinguishes explicitly between capital units of different age. In coherence with the neoclassical assumptions, these vintages can be straightforwardly aggregated. Furthermore, perfect second-hand markets for used capital exist. When the capital market is in equilibrium, the producer will be indifferent between investing in different vintages. Given this, it is shown that myopic decisions are consistent with rational, optimizing bahaviour, and that a simple additive user cost formula is valid independently of the form of capital deterioration.

1. Introduction

It has been common knowledge among economists - at least since the work of Arrow (1964) and the formalization by Jorgenson (1965) - that when analysing investment decisions and imposing neoclassical assumptions, myopic behaviour is consistent with rational, i.e. optimizing behaviour. The conclusion rests essentially on the assumption made about perfect second-hand markets for used capital. Then investment decisions are reversible and the optimal capital stock may be adjusted continuously through time responding to changes in exogenous parameters, prices and rate of interest.

The neoclassical model is usually presented with a very special assumption concerning the structure of physical retirement of capital; capital units disappear radioactively according to an exponentially declining survival function. However, several authors (Arrow (1964), Hall (1968), Johansen and Sørsveen (1967) and Biørn (1984)) have generalized the neoclassical production model and introduced a rather general survival function. Clearly, this complicates the formal analysis and the derivation of equilibrium conditions describing optimal investment behaviour. But the fundamental feature of the neoclassical model that investment decisions can be made solely from information of present prices and capital gains still prevails.

This conclusion may seem rather obvious since simple intuition tells us that it is the presence of perfect second-hand markets - not a specific (exponential) survival function - that justifies myopic behaviour. The purpose of the present paper is to analyse this problem and to show that myopic behaviour is rational even when capital disappears according to the general survival function utilized e.g. by Biørn (1983). Our model distinguishes explicitely between capital units of different age (vintages), and stresses the role of well-established markets for used assets. In coherence with the standard neoclassical framework it is assumed that different vintages of capital can be straightforwardly aggregated (Fisher (1965) and Diewert (1983) survey the conditions for consistent capital aggregation). The interpretation underlying this assumption may be that a new unit of capital "produces" a certain amount of capital services. By aging the output of capital services deteriorates according to the assumed survival function.

The theoretical framework presented in the next section builds extensively on several works by E. Biørn (Biørn (1983), (1984)). However, when analysing investment behaviour Biørn (1984) does not include an explicit treatment of second-hand markets, which we believe explains why he does not arrive at the conclusion that myopic behaviour is rational under general assumptions about the physical retirement of capital. The following analysis may thus be seen as a modification of Biørn's results at this point. The analysis is carried out in continuous time. A discrete time version of the production model is presented in Holmøy and Olsen (1986).

A more general model than the present was utilized by Hall (1968), comprising the vintage dimension and focusing particularly on how measures of capital and rents are related to technical change. Hall actually derives, in a slightly different manner, the myopic equilibrium condition and the expression for the user cost of capital that we will arrive at below.

2. The model

In the formal model describing investment behaviour the following concepts are central:

$K(t)$ = (Gross) capital stock at time t. K is measured in such a way that one capital unit produces one unit of capital services per unit of time.

$J(t,\theta)$ = The number of capital units of age θ invested at time t.

$B^*(s,\theta)$ = Survival function for vintage θ. This indicates the proportion of an investment made s years ago in capital of age θ, which still exists as productive capital. The survival function is normalized by setting $B^*(0,\theta) = 1$ for all θ. Analogously to Biørn (1983) $B^*(s,\theta)$ is assumed to be monotonically decreasing in s, and $\lim_{s\to\infty} B^*(s,\theta) = 0$. We also introduce the simplifying notation $B^*(s,0) = B(s)$. In the formal analysis below we will need to modify the vintage specific survival function, so that the productive capacity depends only on total age, $s+\theta$. As a consequence, the survival function becomes one-dimensional.

$q(t,\theta)$ = Price of a capital unit of age θ at time t.

$X(t)$ = Profits of the firm for a given capital stock $K(t)$.

Introducing the (neoclassical) assumption that capital units of different vintages are perfect substitutes, these can be aggregated by simple summation:

$$(1) \qquad K(t) = \int_0^\infty \int_0^\infty B^*(s,\theta)\, J(t-s,\theta)\, d\theta\, ds$$

where

$$(2) \qquad J(t-s,\theta) = B(\theta)\, J(t-s-\theta)$$

$J(t-s-\theta)$ is the initial investment made at time $t-s-\theta$. $J(t-s,\theta)$ measures the remaining units of capital of this investment θ periods later.

We consider an individual firm which faces given prices on all inputs and outputs. As our focus is on the adjustment of the capital stock, we assume that output and all other (variable) inputs are optimally adjusted at each point of time, expressed formally by a restricted profit function

$$(3) \qquad X(t) = F[K(t)]$$

where the F-function has the usual neoclassical properties, i.e. $F' > 0$, $F'' < 0$.

When deciding on investments the producer is supposed to maximize the present value of the net cash flow W. Formally we have the following optimization problem:.

$$(4) \qquad \text{Max } W = \int_0^\infty e^{-rt} \left[X(t) - \int_0^\infty q(t,\theta)\, J(t,\theta)\, d\theta \right] dt$$

subject to (1) and (3) where maximization is with respect to $J(t,\theta)$, $X(t)$ and $K(t)$.

When solving this constrained intertemporal maximization problem we follow Biørn (1979) and write the Lagrangian as

(5) $$U = \int_0^\infty [\, e^{-rt}\{\, F[K(t)] - \int_0^\infty q(t,\theta)\, J(t,\theta)\, d\theta \,\} - \lambda(t)\{\, K(t) - \int_0^\infty \int_0^\infty B^*(s,\theta)\, J(t-s,\theta)\, d\theta\, ds \,\}]\, dt$$

where $\lambda(t)$ is the Lagrange parameter associated with the constraint given by (1). Define

(6) $$K_0(t) = \int_t^\infty \int_0^\infty B^*(s,\theta)\, J(t-s,\theta)\, d\theta\, ds \;,$$

i.e. as the part of K(t) that is predetermined from decisions taken before time t=0, when optimization is performed. The Lagrangian U can then be written

(7) $$U = \int_0^\infty [\, e^{-rt}\{\, F[K(t)] - \int_0^\infty q(t,\theta)\, J(t,\theta)\, d\theta \,\} - \lambda(t)\{\, K(t) - K_0(t) - \int_0^t \int_0^\infty B^*(s,\theta)\, J(t-s,\theta)\, d\theta\, ds \,\}]\, dt$$

By changing limits in the last double integral in (7) the Lagrangian can be rewritten in the following additive form

(8) $$U = \Phi_1[K(t),\lambda(t),t] + \Phi_2[J(t,\theta),\, \alpha(t,\theta)]$$

where

(9) $$\Phi_1[K(t),\lambda(t),t] = \int_0^\infty [e^{-rt}\, F[K(t)] - \lambda(t)\{K(t) - K_0(t)\}]dt$$

(10) $$\Phi_2[J(t,\theta),\alpha(t)\theta] = \int_0^\infty \int_0^\infty J(t,\theta)\, \alpha(t,\theta)\, d\theta dt$$

(11) $$\alpha(t,\theta) = \int_0^\infty \lambda(t+s)\, B^*(s,\theta)ds - e^{-rt}\, q(t,\theta)$$

From (8) it is seen that the optimization problem is separable; Φ_1 is maximized with respect to K(t) and Φ_2 is maximized with respect to $J(t,\theta)$. Intuitively, since capital by assumption is homogenous with respect to productive capacity, the producer first determines the optimal age composition of any given capital stock, and then in the second stage decides on the level of the optimal total stock. As we shall see below, in the former problem indeterminateness prevails.

Since $F[K(t)]$ is concave, Φ_1 is concave in $K(t)$. Consequently, we can apply the necessary first order condition

(12) $$e^{-rt} F'[K(t)] = \lambda(t)$$

to caracterize the relationship between $K(t)$ and $\lambda(t)$ along the optimal path.

Turning to the other function Φ_2, this is linear in $J(t,\theta)$, which means that if $\alpha(t,\theta) \neq 0$ for θ^*, then it is possible to increase Φ_2 (and U) beyond all limits by increasing $|J(t,\theta^*)|$ infinitely, with sign $[J(t,\theta^*)]$ = sign $[\alpha(t,\theta^*)]$. Note that this does not contradict the constraints given by (1), since a finite capital stock may be maintained by disinvesting in vintages other than θ^*.

Thus, if $\alpha(t,\theta) \neq 0$ for some θ, Φ_2 and U have no maxima, and an optimal investment policy does not exist. In order to indentify an equilibrium solution one must require that the condition

(13) $$\alpha(t,\theta) = 0$$

is satisfied for all θ and t.

(13) is a <u>necessary condition</u> for the existence of an optimal solution. However, when it is fullfilled, $J(t,\theta)$ is <u>indeterminate</u>, and the producer is indifferent with respect to the age structure of the capital stock.

The economic interpretation of $\alpha(t,\theta)$ and the function Φ_2 is quite transparent. Substituting the optimum condition (12) into (11), yields

(14) $$\alpha(t,\theta) = e^{-rt}\{\int_0^\infty e^{-rs} F'[K(t+s)] B^*(s,\theta)ds - q(t,\theta)\}$$

The rhs. of (14) is the present value of the marginal net income generated by a marginal capital unit of age θ, invested at time t. In other words, $\alpha(t,\theta)$ is the present value of the gain from keeping a capital unit of age θ employed in the firm throughout its remaining life time instead of selling it at price $q(t,\theta)$ in the second-hand market.

If the exogenous variables $q(t,\theta)$ and $B(s,\theta)$ are fixed so that with the optimal level of K, $\alpha(t,\theta) \neq 0$ for one or more θ, then the firm can increase its present value of the net cash flow infinitely through arbitrage in capital of different ages. Thus, (13) is an equilibrium condition preventing such outcomes, i.e. it ensures that the firm's cash flow has "productive ativity" as its only source.

By combining (13) and (14) we arrive at the necessary condition for optimal capital adjustment:

$$(15)\qquad \int_0^\infty e^{-rs} F'[\hat{K}(t+s)]\, B^*(s,\theta)ds = q(t,\theta) \quad \text{for all } \theta \text{ and } t.$$

where $\hat{K}$ denotes the optimal capital stock. We assume the existence of $\hat{K}(t)$ and that $\hat{K}(t)$ is uniquely determined by the functional equation (15).

When the path for the optimal capital stock is found, total investments are derived from (1).

From the equilibrium condition (15) it may seem as when deciding on investments the producer needs to make expectations of prices and to assess the impacts of marginal changes in the capital stock for all future periods. This, however, is rather contraintuitive in a world where perfect capital markets exist so that producers are always able to supplement or shrink their capital stocks through market transactions. And, in fact, by restricting the survival function $B^*(s,\theta)$ to be one-dimensional, the equilibrium condition can be transformed to a relation describing a myopic decision rule.

To show this, we first rewrite (15) as

$$(15')\qquad \int_t^\infty e^{-r(u-t)} F'[K(u)]\, B^*(u-t,\theta)\, du = q(t,\theta)$$

Taking the derivative of (15') with respect to t yields

$$(16)\qquad \frac{\partial q(t,\theta)}{\partial t}$$

$$= - F'[K(t)] + \int_0^\infty e^{-rs}F'[K(t+s)][rB^*(s,\theta) + b^*(s,\theta)]ds$$

$$= - F'[K(t)] + rq(t,\theta) + \int_0^\infty e^{-rs}F'[K(t+s)]\, b^*(s,\theta)\, ds$$

where $b^*(s,\theta) = -\frac{\partial B^*(s,\theta)}{\partial s}$ which can be interpreted as the physical retirement function (Hall (1968), Biørn (1983)). The last integral in (16) is the present value of the loss due to physical retirement from investing in a marginal unit of capital vintage θ at time t.

Next, we also take the derivative of (15) with respect to θ and get

$$(17)\qquad \frac{\partial q(t,\theta)}{\partial\theta} = \int_0^\infty e^{-rs} F'[K(t+s)] \frac{\partial B^*(s,\theta)}{\partial\theta} ds$$

In order to proceed we make the reasonable assumption that the survival profile depends on <u>total</u> time of use only and is independent of <u>where</u> the capital actually is employed. By this simplification it is possible to transform the two-dimensional survival function into an one-dimensional, so that we can write

$$(18)\qquad B^*(s,\theta)\, B(\theta) = B(s+\theta)$$

As expressed by this relation it is the <u>total</u> age $s+\theta$ of an asset that determines its productive capacity, and this is independent of when it is bought and invested in a specific firm. From (13) we have

$$(19)\qquad \frac{\partial B^*(s,\theta)}{\partial\theta} = \frac{1}{[B(\theta)]^2} [B(s+\theta)\, b(\theta) - B(\theta)\, b(s+\theta)].$$

Using the relationships

$$b(s+\theta) = -\frac{\partial B(s+\theta)}{\partial s} = -\frac{\partial B(s+\theta)}{\partial\theta} = B(\theta)\, b^*(s,\theta)\ ,$$

(19) can be inserted in (17) to yield

$$(20)\qquad -\frac{\partial q(t,\theta)}{\partial\theta} + \frac{b(\theta)}{B(\theta)} q(t,\theta) = \int_0^\infty e^{-rs}\, F'[K(t+s)]\, b^*(s,\theta)\, ds.$$

By substituting (20) into (16) we finally get, for all t and θ:

$$(21)\qquad F'[K(t)] = c(t) = q(t,\theta)\, [\, r + \delta(\theta) - (\, q_\theta(t,\theta) + q_t(t,\theta)\,)\,]$$

where $\delta(\theta) = \frac{b(\theta)}{B(\theta)}$, $q_\theta(t,\theta) = \frac{\frac{\partial q(t,\theta)}{\partial\theta}}{q(t,\theta)}$ and $q_t(t,\theta) = \frac{\frac{\partial q(t,\theta)}{\partial t}}{q(t,\theta)}$.

$\delta(\theta)$ is the rate of retirement per capital unit of vintage θ. q_θ and q_t are growth rates of $q(t,\theta)$ due to increasing age and change of time respectively.

From (21) we see that we have arrived at an equilibrium condition and a general expression for the user cost of capital that is strikingly similar to the simple text-book formula in the case of an exponentially declining survival function. The user cost is the sum of an interest component, the rate of retirement and the two partial "growth rates" for the capital price. q_t is the potential capital gain from increasing prices that is abolished by a marginal investment, while q_θ reflects the marginal loss of value of a capital unit due to aging. (21) thus expresses a myopic decision rule: The producer adjusts the level of the capital stock so that the value of the marginal productivity of capital equals the user cost. It is unnecessary for the decision maker to assess more than the marginal instantaneous changes in $q(t,\theta)$. Nor is information about the whole survival function necessary; only the current retirement rate matters.

Since (21) holds for all θ, the user costs for all vintages are identical. This is another way of expressing that in equilibrium nothing can be gained by trading in different vintages. Consequently, the firm in our model may calculate the relevant user cost by regarding an arbitrary vintage, e.g. new capital units. On the other hand, the possibility of disequilibrium in the capital market obviously gives high incentives to gather information about the complete age dimension of $q(t,\theta)$ and $B(s,\theta)$.

With exponential retirement, i.e. $B^*(s,\theta) = B(s) = e^{-\delta t}$ independently of θ, $\delta(\theta) = \delta$, i.e. constant. Furthermore, it is easily verified by inspection of (15) that the present value of income generated by a marginal capital unit is independent of age, θ. Thus capital units of different age constitute homogenous goods, and the "law of indifference" guaranties that in this case the price per capital unit is independent of age. It is then easily seen that the right hand side of (21) coincides with the well known text book user cost formula, given as

(22) $$c(t) = q(t)\ [r + \delta - q_t]$$

3. Concluding remarks

In Biørn (1983), a specific hypothesis regarding the structure of prices of capital units is introduced, namely that the price $q(t,\theta)$ is proportional to the remaining discounted flow of capital services. This assumption is used by Biørn to eliminate the vintage dimension in the user cost expression. In the present paper we derive a condition for the producer being indifferent between vintages which is equivalent to Biørn's hypothesis for the structure of capital prices. Our model clearly reveals why this "indifference condition" is essential to the neoclassical model of producer behaviour when capital units of different age are assumed to be perfect substitutes. When the capital market is in equilibrium in this respect, the essential conclusion reached in this paper is that myopic behaviour is consistent with rational, optimizing behaviour, and that a simple additive user cost formula remains valid independently of the form of the survival function.

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