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Optimal Sequencing of Hydroelectric and Thermal Power Generation under Energy Price Uncertainty and Demand Fluctuations

by

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ABSTRACT

The choice between hydro and thermal power in electricity supply is analyzed under stochastic demand and stochastic fuel cost. The first formulation gives rise to an optimal stopping problem, where the investment in hydro power is not undertaken until the price of natural gas reaches a reservation price. Since investment in hydro power is irreversible, the thermal source should be utilized until the the reservation price is reached, and this reservation price is higher under uncertainty. The second formulation gives rise to a stochastic control problem. In this solution there is an expansion of the hydro power system over time, with investments taking place whenever the random component of demand reaches a boundary. We suggest applications to current problems in the Norwegian energy sector.

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1 Introduction

Technology switching is an important issue in energy planning, and examples like the transition from fossil to synthetic fuel, the prospect of solar energy and the choice between hydro and thermal power in the electricity supply all raise the question of optimal timing of the switching decision. Uncertainties on both demand and supply sides of the energy markets render simple investment criteria invalid. The effects of uncertainty are even more important when the irreversible nature of many energy investments is taken into account. Furthermore,the timing of technology switching is especially important in the management of exhaustible natural resources as the prospect of a new technology may ensure the transition to renewable resources and hence alleviate resource constraints.

In recent literature the question of technology switching — either on the energy sector level or in the investment decision of a firm — is often dealt with in deterministic or static models and the proper extensions to the case of uncertainty and irreversibility are not readily available. In a recent contribution Amit (1986) analyzes the switching from primary to secondary recovery in petroleum extraction in a deterministic framework. Abel (1983) examines the choice of capital-energy-intensity for a firm in a stochastic putty-clay model. In a similar model, Albrecht and Hart (1983) also incorporate the irreversibility aspect as a putty-clay structure; neither of these models address the timing considerations.In the classical text by Massé (1962) the effect of uncertainty on the optimal choice between hydro and thermal power in electricity generation is examined, however, from the viewpoint of parallel development rather than the phasing in of an irreversible investment. The importance of irreversibility is clearly recognized in numerous contributions in environmental economics. Arrow and Fisher (1974) conclude that uncertainty should imply underinvestment when the investments are irreversible; see also recent work by Fisher and Hanemann (1982).

The optimal timing of technology switching is a somewhat neglected problem in recent literature. Technology switching is often modelled as simple technological uncertainty, where the alternative technology is not available at the outset but the innovation will occur at any future time with a given probability. Dasgupta and Heal (1974) and Dasgupta and Stiglitz (1981) consider an exogenous innovation probability whereas Kamien and Schwartz (1978) recognize that the innovation probability depends on research and development devoted to this end. In a similar model Hochman and Zilberman (1985) analyze the transition from an exhaustible resource to a renewable resource in electricity generation. In a more general context Johansen (1978) shows how the exogenous probability of a technological change will increase the social discount rate; hence, the opportunity cost of switching to the alternative technology is higher.

A more realistic but more complicated problem is that the new technology is available but only at a substantial capital cost and once the investment is undertaken, it is irreversible. In contrast, the existing technology has a higher degree of flexibility but with an possibly large uncertainty in output or production cost. This is the problem we will examine in this paper. It is related to the question of investment timing and the option value arising from the option to postpone investment, which has received widespread attention lately, see McDonald and Siegel (1986), Majd and Pindyck (1985) and Venezia and Brenner (1979). In the context of natural resource extraction, Brennan and Schwartz (1985) analyze the investment timing problem and the option value arising from the option to postpone investment. Nevertheless, these works do not explicitly deal with the switching time.

We address the problem in the framework of optimal stopping. This powerful tool of analysis, see Øksendal (1985) and Shiryayev (1978), has not yet found widespread applications in economics,however, important contributions include McDonald and Siegel (1986), Roberts and Weitzman (1981) and a recent work on switching times in resource extraction by Olsen and Stensland (1987).

The methodological approach is to represent the uncertainty as Brownian motions in stochastic processes which in recent years have received widespread attention in economic applications as a powerful tool of analysis. The stochastic assumptions are also of a very stylized character allowing explicit solutions. More realistic solutions may require computer intensive numerical solutions. Although the mathematical requirements for solving these stochastic control problems are quite demanding, the character of the solutions are almost surprisingly simple and open to direct interpretations that could easily be incorporated within traditional planning procedures as rules of thumb.

In this paper we analyze the optimal switching between thermal power and hydro power in the future expansion of electricity supply. Norway has abundant energy supplies consisting of hydro power, crude oil and natural gas and is one of the few countries of the world in which the electricity supply is more than 99 percent based on hydro power. This reflects that hydro power until recently has been the cheapest source for covering a steadily increasing demand for electricity. The recent price fall of crude oil has also brought down the price of natural gas in Western Europe, and actualized the issue of whether further expansion of the electricity supply system in Norway should be based on thermal power generation or on still unexploited hydro sources.

This is a question of marginal cost comparison which is basically very simple, but becomes more intricate when the uncertainty aspects of the problem are taken into consideration. The importance of uncertainty is exacerbated by the difference in cost structure between the hydro power plant and the gas fired plant. In our presentation we stylize this difference by representing the cost of hydro power solely as everlasting capital, while only the fuel cost of thermal power is taken into consideration.

The uncertainty of importance in deciding on the composition of the energy supply system comes in our presentation from three sources. The uncertainty of future demand stems from cyclical factors, temperature variations and the residual uncertainty in estimated elasticities of future demand. The uncertainty of the opportunity cost of natural gas stems from the export market which is the alternative outlet for the domestic supplies of natural gas. The export price is by the current type of contract related to the crude oil price. The third source of uncertainty is the variations in annual output from a hydro power system due to variations in rainfall. We concentrate mostly on the first two sources.

We first pose the problem of whether to choose thermal power or hydro power to cover a given expansion of demand when the price of thermal energy is uncertain. The irreversibility of hydro power investment implies that this can be viewed as an option value problem. Once the irreversible investment is undertaken it is not worthwhile to switch back to thermal power even when the opportunity price is below the electricity price. In more general terms the issue is to develop criteria for the optimal sequencing of hydro and thermal power generation in the further expansion of the supply system when all uncertainty factors are taken into consideration.

2 Uncertainty vs. irreversibility: option value in the choice between thermal and hydro power

We assume that an additional unit of demand of electricity should be covered by an increase of the supply capacity, either from hydro power at capital cost K or from gas fired plants with no capital costs, but with stochastic fuel costs Q_s per unit of energy generated. We pose this as a cost minimization problem:

$$C(t,q) = \min_{\tau} E^{t,q}\{\int_t^{\tau} Q_s e^{-r_Q s} ds + K e^{-r_F \tau}\} \tag{1}$$

where r_F is the risk free discount rate, and r_Q is the discount rate on a asset with the same uncertainty as Q_s which is a stochastic process given by

$$dQ_s = \mu Q_s ds + \sigma Q_s dB_{1s}. \tag{2}$$

In (2) the first term expresses the exponential drift of Q_s while the second term represents the uncertainty of Q_s as a Brownian motion (Wiener process) B_{1s}. (2) implies that

$$E\{Q_s \mid Q_t) = Q_t \cdot e^{\mu(s-t)} \text{ and } var(\ln(\frac{Q_s}{Q_t})) = \sigma^2(s-t)$$

(1) can be restated noting that

$$\begin{aligned} E^{t,q}\{\textstyle\int_t^{\tau} Q_s e^{-r_Q s} ds\} &= E^{t,q}\{\textstyle\int_t^{\infty} Q_s e^{-r_Q s} ds\} - E^{t,q}\{\textstyle\int_{\tau}^{\infty} Q_s e^{-r_Q s} ds\} \\ &= \textstyle\int_t^{\infty} E^{t,q}(Q_s) e^{-r_Q s} ds - E^{t,q}\{\textstyle\int_{\tau}^{\infty} E^{\tau,Q_\tau}(Q_s) e^{-r_Q s} ds\} \\ &= \frac{1}{r_Q-\mu}[q e^{-r_Q t} - E^{t,q}\{Q_\tau e^{-r_Q \tau}\}] \end{aligned} \tag{3}$$

i.e. buying gas in the period t to τ is equivalent to buying a right to eternal gas delivery at t and selling this right at τ. Buying the right at $t = 0$ gives us

$$C(0,q) = \frac{1}{r_Q - \mu}[q - \max_{\tau} E^{0,q}\{(Q_\tau - K(r_Q - \mu))e^{-r\tau}\}] \tag{4}$$

where r is the discount rate on the option, as in McDonald and Siegel (1986). The solution to (4) follows immediately from the following optimal stopping theorem:

Theorem 1 *Let*

$$V(t,q;\kappa,r) = \max_{\tau} E^{t,q}\{(Q_\tau - \kappa)e^{-r\tau}\} \quad (5)$$

where:

$$dQ_s = \mu Q_s ds + \sigma Q_s dB_s. \quad (6)$$

Then

$$V(t,q;\kappa,r) = \frac{1}{\gamma-1}(q^*)^{-\gamma}\kappa q^\gamma e^{-rt} \quad (7)$$

where the reservation price $q^* = \frac{\gamma}{\gamma-1}\kappa$, *with*

$$\gamma = \frac{1}{\sigma^2}[-(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}] > 1 \quad (8)$$

The optimal stopping rule is

$$\tau = inf\{t > 0 : Q_t > q^*\} \quad (9)$$

or, simply, to wait until Q_t *is equal to the reservation price.*

Theorem 1 is a well known result and a special case of the problem solved by McDonald and Siegel(1986). We apply the theorem using $\kappa = (r_Q - \mu)K$. It follows that we should prefer the thermal source until

$$Q_s = q^* = \frac{\gamma(r_Q - \mu)}{\gamma - 1}K \quad (10)$$

The theorem assumes that the discount rate on the option r is given. Using an adjusted growth rate $\hat{\mu}$ from CAPM, and the risk free discount rate, gives an alternative expression for γ. The adjustment is such that $\mu - \hat{\mu} = r_Q - r_F$.

$$\gamma = \frac{1}{\sigma^2}[-(\hat{\mu} - \frac{1}{2}\sigma^2) + \sqrt{(\hat{\mu} - \frac{1}{2}\sigma^2)^2 + 2r_F\sigma^2}] > 1 \quad (11)$$

It follows that $\frac{\gamma}{\gamma-1} > \frac{r_F}{r_F - \hat{\mu}}$, and since $r_Q - \mu = r_F - \hat{\mu}$ the reservation price q^* exceeds $r_F K$, which can be interpreted as the reservation price in the case of no uncertainty. Under uncertainty the hydro power project has an option value and the investment should be postponed until Q_s reaches q^*. As long as $Q_s < q^*$ there is an expected gain if future gas prices increase above q^* since the option then will be exercised. Define $\tilde{Q}_s$ as:

$$\tilde{Q}_s = \begin{cases} r_F K & \text{if } Q_s > q^* \\ Q_s & \text{otherwise} \end{cases} \quad (12)$$

It follows that $E\tilde{Q}_s \leq EQ_s$ since the option to switch to hydro power represents a truncation of the probability distribution of Q_s.

Hence the expected cost is reduced by holding the option. The cost reduction effect is most readily depicted in the case of $\mu = 0$, where the expected cost equals $r_F K$ throughout, see Figure 1(a). Given Q_0, the probability distribution of Q_t is lognormal.

When Q_s reaches q^*, the option should be exercised. The option value of the hydro power project can be expressed as:

$$W(t,q) = E^{t,q}\{\int_t^{\infty} Q_s e^{-r_Q s} ds\} - \min_{\tau} E^{t,q}\{\int_t^{\tau} Q_s e^{-r_Q s} ds + K e^{-r_F \tau}\} \qquad (13)$$

which is the expected gain from having the option to switch to hydro power at τ as compared to a pure thermal based expansion. This option value is illustrated in Figure 1 (b) for the case $\mu = 0$.

As a numerical illustration let us set $r_F = 0.05$,$r_Q = 0.06$, $\mu = 0.03$ and $\sigma = 0.12$. The reservation price under uncertainty is then $0.06324 \cdot K$ as compared with $0.05 \cdot K$ under certainty.

3 Optimization of energy supply over time under uncertain demand and gas price

The setting in this section is the planning problem of the power authority: when to phase in the remaining reserve of hydro power. We assume that the electricity market is in equilibrium. As before we make very stylized assumptions about the cost structure of both hydro and thermal power. Furthermore, we ignore adjustment costs and assume that hydro power capacity can be increased in arbitrary small amounts.We represent both demand and the gas price as continuous stochastic processes with positive drift and non-negligible variance.

For our purpose any reasonably shaped demand curve would suffice. Explicit solutions are facilitated by choosing a particularly simple structure such as

$$D_t = P_t^{-\epsilon} \Theta_t \qquad (14)$$

where P_t is the price of electricity and Θ_t a stochastic term given as an Itô diffusion.:

$$\frac{d\Theta_t}{\Theta_t} = \alpha dt + \beta dB_{2t} \qquad (15)$$

Θ_t takes care of the income effect and other factors which may influence demand.

The hydro power capacity is given by K_t. More capacity is available at increasing costs. The unit cost of another unit of capacity is given by $C(K_t), C'(K_t) > 0$. Our control variable is additional hydro power investment $\kappa_t > 0$.

$$dK_t = \kappa_t dt \qquad (16)$$

The alternative source is thermal power from natural gas at a constant opportunity cost q per unit of power generated. At the end of the section we will consider the case when the gas price is also given as an Itô diffusion.

Completed hydro power investments are sunk cost and maximum benefit requires the full capacity to be used. The planning problem is given by

$$H(t,\theta,k;q) = max_\kappa E^{t,\theta,k}\{\int_t^\infty (P_s D_s - C(K_s)\kappa_s - q(D_s - K_s))e^{-rs}ds\} \tag{17}$$

The price will either be q - when demand is high enough - or the equilibrium price with no use of thermal power, i.e.

$$P_t = \min\{[\frac{\Theta_t}{K_t}]^{\frac{1}{\epsilon}}, q\} \tag{18}$$

In either case we have

$$P_t D_t - q(D_t - K_t) = P_t K_t \tag{19}$$

The Hamilton-Jacobi-Bellman (HJB) equation corresponding to (17) is

$$\sup_\kappa\{(pk - C(k)\kappa)e^{-rt} + \frac{\partial H}{\partial t} + \kappa\frac{\partial H}{\partial k} + \alpha\theta\frac{\partial H}{\partial \theta} + \frac{1}{2}(\beta\theta)^2\frac{\partial^2 H}{\partial \theta^2}\} = 0 \tag{20}$$

We can safely assume that $H_k - C(k)e^{-rt} \leq 0$. If not, K_t would adjust immediately to fulfill this condition. In figure 2 this means that we are outside the area marked as $\mathcal{A}$. In fact the solution of the problem derived below is valid only outside $\mathcal{A}$. Optimal hydro power investment implies that κ_t should be set such that

$$\kappa_t(H_k - C(k)e^{-rt}) = 0$$

This simplifies the HJB equation considerably and it now becomes:

$$pke^{-rt} + \frac{\partial H}{\partial t} + \alpha\theta\frac{\partial H}{\partial \theta} + \frac{1}{2}(\beta\theta)^2\frac{\partial^2 H}{\partial \theta^2} = 0 \tag{21}$$

Time enters the equation only through the discount rate. We try $H(t,\theta,k)\cdot e^{rt} = G(t,\theta,k) = G(\theta,k)$. (18) implies that we have to solve (21) for two separate regions:

$$\begin{array}{lllll} \theta^{\frac{1}{\epsilon}}k^{1-\frac{1}{\epsilon}} & -rG + \alpha\theta\frac{\partial G}{\partial \theta} + \frac{1}{2}(\beta\theta)^2\frac{\partial^2 G}{\partial \theta^2} & = & 0 & \text{for } \theta < q^\epsilon k \quad (a) \\ qk & -rG + \alpha\theta\frac{\partial G}{\partial \theta} + \frac{1}{2}(\beta\theta)^2\frac{\partial^2 G}{\partial \theta^2} & = & 0 & \text{otherwise} \quad (b) \end{array} \tag{22}$$

Under our stylized assumptions these differential equations can easily be solved. The particular solutions are for case (a)

$$G(\theta,k) = \frac{\epsilon}{\epsilon r - \alpha - \varsigma}\theta^{\frac{1}{\epsilon}}k^{1-\frac{1}{\epsilon}},$$

with $\varsigma = \frac{1}{2}\beta^2(\frac{1}{\epsilon} - 1)$, and for case (b)

$$G(\theta,k) = \frac{q}{r}k.$$

The general solutions are

$$\begin{array}{lll} G_a(\theta,k;q) & = & \frac{\epsilon}{\epsilon r-\alpha-\varsigma}\theta^{\frac{1}{\epsilon}}k^{1-\frac{1}{\epsilon}} + c_1(k;q)\theta^{\gamma_1} + c_2(k;q)\theta^{\gamma_2} \\ G_b(\theta,k;q) & = & \frac{q}{r}k + c_3(k;q)\theta^{\gamma_1} + c_4(k;q)\theta^{\gamma_2} \end{array} \tag{23}$$

with $\gamma_2 < 0 < \gamma_1$ the roots of the following equation:

$$\frac{1}{2}\beta^2\gamma^2 + (\alpha - \frac{1}{2}\beta^2)\gamma - r = 0 \tag{24}$$

Noting that in case (a) we must have $G(0,k) = 0$, it follows that $c_2(k;q) = 0$. By considering case (b) for high values of θ, we must have

$$\lim_{\theta\to\infty} G_b(\theta,k) = \frac{q}{r}k \tag{25}$$

outside $\mathcal{A}$ in diagram 1. (As mentioned our solution is not valid within $\mathcal{A}$.) (25) implies that $c_3(k;q) = 0$.

In order to insure continuity of $G(\theta,k)$, G_a and G_b must coincide on the boundary $\theta = q^\epsilon k$, that is

$$G_a(q^\epsilon k, k) = G_b(q^\epsilon k, k) \text{ for all } k \tag{26}$$

Furthermore, it seems reasonable to assume also that the shadow price of capital is continuous on the boundary. (The proof will be that the sufficiency conditions of the solution are fulfilled.):

$$\frac{\partial G_a}{\partial k}(q^\epsilon k, k) = \frac{\partial G_b}{\partial k}(q^\epsilon k, k) \text{ for all } k \tag{27}$$

From (26) and (27) follows that

$$\begin{aligned} c_1(k;q) &= \eta_1 k^{1-\gamma_1} q^{1-\epsilon\gamma_1} \\ c_4(k;q) &= \eta_2 k^{1-\gamma_2} q^{1-\epsilon\gamma_2} \end{aligned} \tag{28}$$

with:

$$\eta_1 = -\frac{r - \gamma_2(\alpha + \varsigma)}{r(\epsilon r - \alpha - \varsigma)(\gamma_1 - \gamma_2)} \tag{29}$$

and

$$\eta_2 = -\frac{r - \gamma_1(\alpha + \varsigma)}{r(\epsilon r - \alpha - \varsigma)(\gamma_1 - \gamma_2)} \tag{30}$$

It can easily be shown that this solution implies that G is twice continuously differentiable at the boundary between (a) and (b), $\theta = q^\epsilon k$, which means that $G \in C^2$, a sufficient condition for the validity of the solution, as shown in Øksendal(1985, theorem 10.2).

In figure 2 the diagonal $k = q^{-\epsilon}\theta$ is the boundary between the regions (a) where the electricity price is determined by the equilibrium condition and (b) where the electricity price equals the fuel cost q. The net shadow price $G_k - C(k)$, is positive in $\mathcal{A}$, zero along the boundary given by the concave curve marked K_t^* and negative above this curve. As explained above, investments in hydro power are undertaken when the net shadow price equals zero, i.e. along the curve K_t^*.

The line $K^{\max}$ represents the present value of future fuel costs which is an upper limit for future expansion of the hydro power system. The vertical difference between

$K^{\max}$ and K_t^* in region (b) represents the irreversibility premium; i.e. the reduction in optimal hydro power investment due to demand uncertainty.

In order to illustrate the time dimension of the investment process, figure 3 gives K_t^* and the actual capital stock K_t as time functions. Hydro power investment are undertaken only when $K_t^* \geq K_t$, and K_t is constant on the intervals where $K_t^* < K_t$, i.e. $K_t = \sup\{K_s^* : s \leq t\}$.

The effect of uncertainty on the hydro power investments is most readily interpreted in region (b), where the shadow value of the hydro power capital under certainty equals $\frac{q}{r}k$. The effect of uncertainty enters only through the term η_2, which represents the reduction in the shadow value under uncertainty. With an inelastic demand curve (ϵ small) the stochastic fluctuation induced by θ_t has large effect on the electricity price and hence, on the investment volume. As a consequence, hydro power investment should be smaller the smaller is ϵ. When ϵ is large, the stochastic fluctuations have a negligible effect on the electricity price and the shadow value is close to the opportunity cost under certainty $\frac{q}{r}k$. The impact of the demand elasticity on the uncertainty term η_2 is illustrated in figure 4.

We will now consider also the gas price as stochastic, i.e. as an Itô diffusion:

$$dQ_t = \mu Q_t dt + \sigma Q_t dB_{1t} \tag{31}$$

This requires only a slight revision of the preceding results. The solution will have the same general form. The particular solution in case (a) is unchanged (as q does not enter), while in case (b) it becomes

$$G_b(\theta, k, q) = \frac{q}{r - \mu} k \tag{32}$$

The HJB equations for case (a) and (b) are now

$$\begin{array}{lll} \theta^{\frac{1}{\epsilon}} k^{1-\frac{1}{\epsilon}} - rG + \alpha\theta \frac{\partial G}{\partial \theta} + \frac{1}{2}(\beta\theta)^2 \frac{\partial^2 G}{\partial \theta^2} + \mu q \frac{\partial H}{\partial q} + \frac{1}{2}(\sigma q)^2 \frac{\partial^2 H}{\partial q^2} & = 0 & \text{for } \theta < q^\epsilon k \quad (a) \\ qk - rG + \alpha\theta \frac{\partial G}{\partial \theta} + \frac{1}{2}(\beta\theta)^2 \frac{\partial^2 G}{\partial \theta^2} + \mu q \frac{\partial H}{\partial q} + \frac{1}{2}(\sigma q)^2 \frac{\partial^2 H}{\partial q^2} & = 0 & \text{otherwise} \quad (b) \end{array} \tag{33}$$

As we have found how q appears in the solution when $\mu = \sigma = 0$, we guess that the new general solution is of the same form. Then $\gamma_2 < 0 < \gamma_1$ must be the solutions of

$$-r + \mu + (\alpha - \epsilon\mu - \frac{1}{2}(\beta^2 + \epsilon \cdot \sigma^2))\gamma + \frac{1}{2}(\beta^2 + (\epsilon\sigma)^2)\gamma^2 = 0 \tag{34}$$

The corresponding values of η_1 and η_2 are

$$\eta_1 = -\frac{r - \mu - \gamma_2(\alpha + \varsigma - \epsilon\mu)}{(r - \mu)(\epsilon r - \alpha - \varsigma)(\gamma_1 - \gamma_2)} \tag{35}$$

$$\eta_2 = -\frac{r - \mu - \gamma_1(\alpha + \varsigma - \epsilon\mu)}{(r - \mu)(\epsilon r - \alpha - \varsigma)(\gamma_1 - \gamma_2)} \tag{36}$$

4 Final remarks

The preceding sections have focused entirely on two aspects of the optimal sequencing of energy supply projects: the uncertainty of demand and of the price of natural gas. The setting of the problem has been highly simplified. The purpose has been to highlight some of the features that tend to be subdued in current government planning. The energy sector in Norway is dominated by government owned companies and the planning of future energy supply has been wholly in the hands of the government. Traditional planning methods tend to underplay considerably the role of uncertainty and be biased towards overinvestment in hydro power capacity. This bias is more costly when other energy sources are becoming more competitive, when future demand is more uncertain, and when new hydro power developments are more expensive. In our paper we focus entirely on the uncertainty problems, ignoring many other important aspects of the overall energy planning.

There are some important aspects of the interaction between hydro power and thermal power that deserve to be mentioned and should be taken into consideration in a more fully developed planning model. Hydro power has its own uncertainty caused by variations in rainfall over the year and between years. The normal seasonal variations are considerable implying that the marginal cost of hydro power varies over the year. This implies that an alternative source of supply with low capital cost may have a role to play in an integrated supply system even when its marginal cost is higher than the annual average marginal cost of hydro power. The variations in marginal cost of hydro power over the year is exacerbated by seasonal demand variations.

The variations in hydro power supply between years is also considerable and is countered in a pure hydro power system by large and expensive reservoirs. This provides a premium for a thermal source that has not been taken into consideration above. These uncertainty aspects are somewhat more complicated to deal with analytically than those studied above.

Furthermore, hydro power was dealt with above as if it could be infinitesimally expanded. In fact, hydro power plants are large units that take long time to approve, plan and build, say 5-10 years. The construction lag does not change, however, the criteria developed above for choosing between a hydro plant and thermal power. The indivisibility of hydro plants changes the problem in a more substantial way.

On the gas side there are also a number of issues that makes the treatment above stand out as highly simplified. The opportunity cost of using gas for electricity generation is not well defined. Some actual gas field would be unconnected to market grid in the foreseeable future, while others produce associated gas that would otherwise be flared (if flaring was allowed). Export prospect for gas to Norway's next door neighbor Sweden is unresolved at the moment. There are thus many other uncertainty issues involved in the optimal sequencing as a practical issue.

References

Abel, A.B. (1983): Energy Price Uncertainty and Optimal Factor Intensity: A Mean-Variance Analysis, *Econometrica*,51, 1839 – 1845

Albrecht, J.W. and A.G. Hart (1983): A Putty-Clay Model of Demand Uncertainty and Investment, *Scandinavian Journal of Economics*, 85(3), 393 – 402

Amit, R. (1986): Petroleum Reservoir Exploitation: Switching from Primary to Secondary Recovery, *Operations Research*, 34, 534 – 549

Arrow, K.J. and A.C. Fisher (1974): Environmental Preservation, Uncertainty and Irreversibility, *Quarterly Journal of Economics*, 88, 312 – 319

Brennan, M.J. and E.S. Schwartz (1985): Evaluating Natural Resource Investments, *Journal of Business*, 58, 135 – 157

Dasgupta, P. and G. Heal (1974): The Optimal Depletion of Exhaustible Resources, *Review of Economic Studies*, 41, *Symposium*, 3 – 28

Dasgupta, P. and J. Stiglitz (1981): Resource Depletion under Technological Uncertainty, *Econometrica*, 49, 85 – 104

Fisher, A.C. and M. Hanemann (1982): Valuing Pollution Control: The Hysteresis Phenomenon in Aquatic Ecosystems, Working paper, University of California, Berkeley

Hochman, E. and D. Zilberman (1985): Optimal Exploitation of Energy Resources: Solar Power and Electricity Generation in below Sea Level Basins, *Journal of Environmental Economics and Management*, 12, 322 – 337

Johansen, L. (1978): *Lectures on Macroeconomic Planning, Volume 2*, North Holland

Kamien, M.I. and N.L. Schwartz: Optimal Exhaustible Resource Depletion with Endogenous Technical Change, *Review of Economic Studies*, 45, 179 – 196

Majd, S. and R.S. Pindyck (1985): Time to Build, Option Value and Investment Decisions, Working Paper, Energy Laboratory, MIT

Massé, P. (1962): *Optimal Investment Decisions: Rules for Action and Criteria for Choice*, Prentice - Hall

McDonald, R and D. Siegel (1986): The Value of Waiting to Invest, *Quarterly Journal of Economics*, 101, 707 – 727

Øksendal, B. (1985): *Stochastic Differential Equations: An Introduction with Applications*, Springer-Verlag.

Olsen, T.E. and G. Stensland (1987): *Optimal Sequencing of Resource Pools under Uncertainty*, Working Paper

Roberts, K. and M.L. Weitzman (1981): Funding Criteria for Research, Development and Exploration Projects, *Econometrica*, 49, 1261 – 1288

Shiryayev, A.N. (1978): *Optimal Stopping Rules*, Springer-Verlag

Venezia, I. and M. Brenner (1979): The Optimal Duration of Growth Investment and Search, *Journal of Business*, 52, 393 – 407

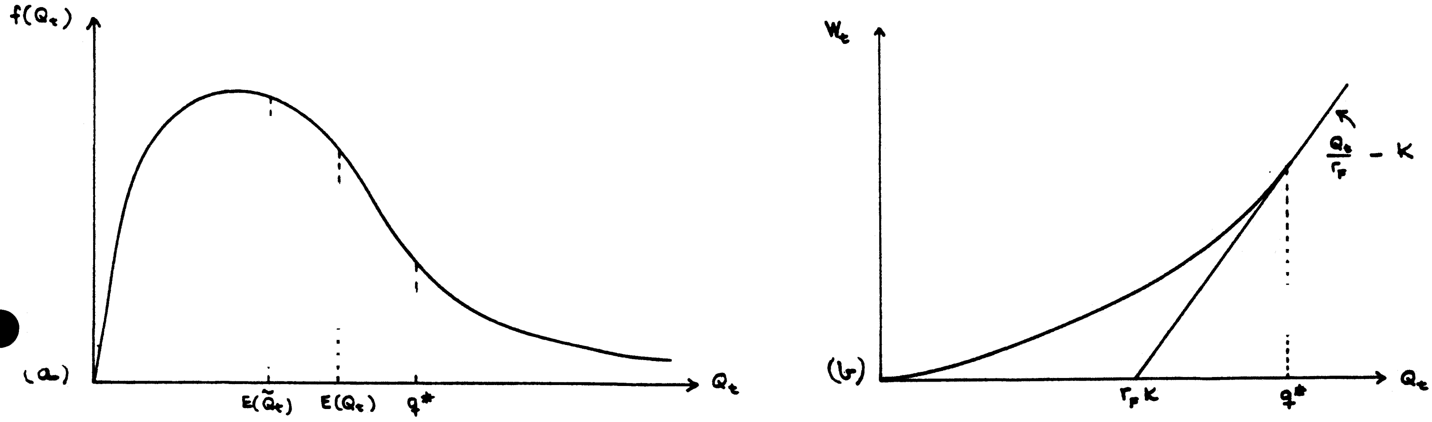

Figure 1:
Expected cost reduction (a) and option value of hydro power (b) in the case of zero drift in the gas price

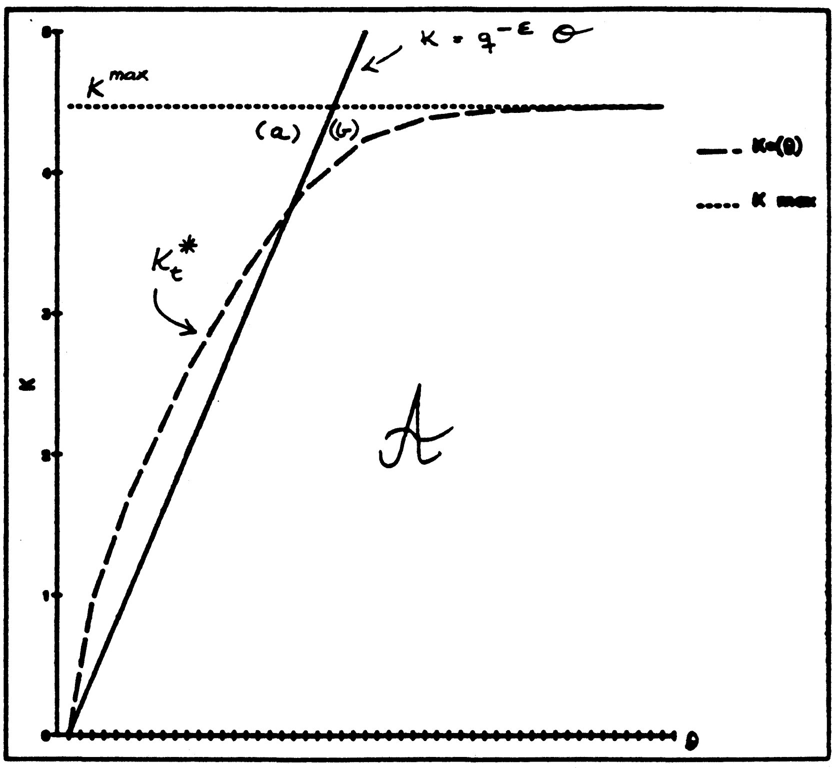

Figure 2: Optimal capital stock K_t^* in hydro power

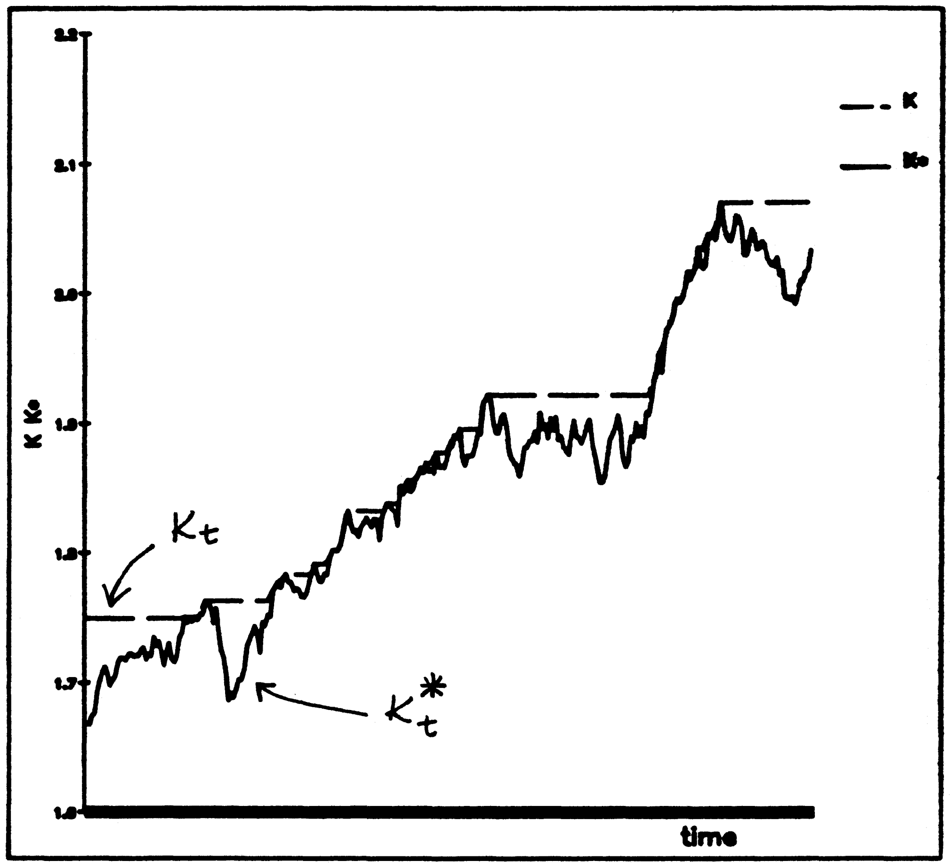

Figure 3: Optimal (K_t^*) and actual (K_t) capital stock in hydro power

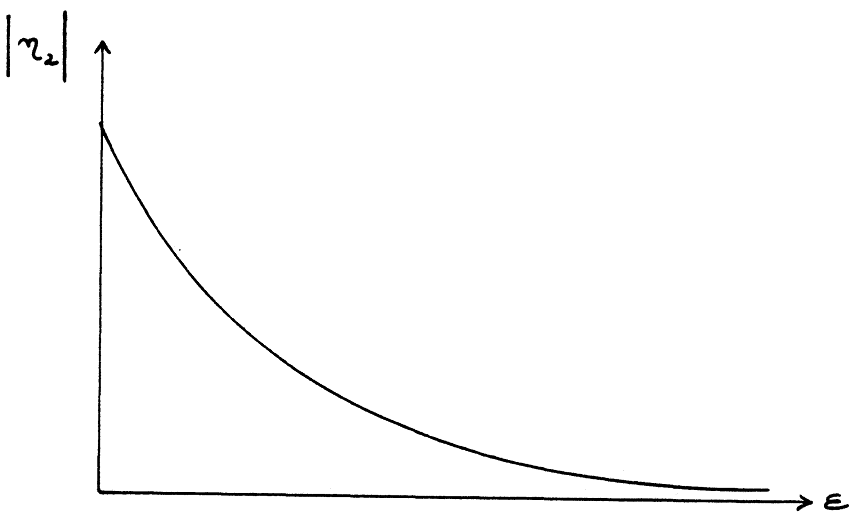

Figure 4: The effect of the demand elasticity on the uncertainty correction η_2

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