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PROPERTIES OF DEMAND FUNCTIONS FOR LINEAR CONSUMPTION AGGREGATES*

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ABSTRACT

The starting point is the demand functions for homogeneous goods, with properties derived from standard static consumer theory. A linear consumption aggregate of a commodity group is defined as a weighted sum of the physical quantities of the homogeneous goods in the group. By using different types of weights we obtain for the same commodity group, different consumption aggregates with different demand elasticities relevant for different applications. For example, a linear consumption aggregate of alcoholic beverages can be measured in pure alcohol (for health analysis), in litres (for transportation analysis), in alcohol taxes (for fiscal analysis), or in expenditure at (different sets of) constant prices (for macro economic analysis). We derive properties of the demand functions for a general linear consumption aggregate, and relationships between the demand functions for different aggregates of the same commodity groups and across commodity groups. Results are presented in eight theorems, with comments on possible econometric interpretations. A non-Giffen anti law of demand is derived. A possible interpretation in the case of bread consumption implies that the direct Slutsky elasticity for bread measured in weight (kilograms) is positive, and the direct Cournot elasticity even more so, while the demand elasticities for the Hicksian aggregate of bread have normal signs.

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CONTENTS

1. INTRODUCTION

Our starting point is a system of demand functions for homogeneous goods with well known properties derived from traditional static consumer theory. We partition the goods into groups and define a linear consumption aggregate of a commodity group as a weighted sum of the quantities' consumed of the homogeneous goods within the group, where the weights are some non-negative scalars. By using different types of weights, we get different consumption aggregates for the same commodity group. For an arbitrary linear consumption aggregate we derive Marshallian and Hicksian demand functions and their properties are explored, under different types of conditions. We emphasize relationships between demand functions for different consumption aggregates of the same commodity group, as well as relationship across commodity groups.

The most common procedure in empirical consumption analysis is to use a set of fixed consumer prices, observed in some base period (situation), as weights when constructing a consumption aggregate of a group of goods. This is for example used in the national accounts of most nations, where the standard method is to use Laspeyres volume indexes to measure consumption, cf United Nations (1968). In theoretical consumption analysis it is common to refer to Hicks aggregation theorem, cf Hicks (1939), assuming that the relative prices are constant within commodity groups, and using these fixed relative prices as weights when defining the consumption aggregates. One can then prove that the demand for these (Hicksian) consumption aggregates as functions of group prices and total expenditure have the same properties as the demand for homogeneous goods as functions of the prices of the homogeneous goods and total expenditure. However, if the relative prices (in the base period) used to measure the consumption of aggregates, are different from the relative prices (in the prediction period) used to define the group prices in the relevant demand functions, then the Hicks aggregation theorem does not apply. The results in this essay on the properties of demands for linear consumption aggregates in general are, however, directly applicable to the demand for these "non-Hicksian Laspeyres aggregates".

Using fixed consumer prices as weights can be suitable in many circumstances, but in general the types of weights chosen should depend upon the purpose of the analysis for which the demand function will be applied. An organisation of farmers can be more interested in measuring the consumption of food in terms of producer prices instead of consumer prices. For transportation analysis the most appropriate measure of consumption may be in weight (tons). A nutritionist can be interested in a demand function for food measured in terms of energy, fat and/or proteins, rather than in terms of expenditure or weight. A health department can be interested in the demand for alchoholic beverages measured in litres of pure alchohol. An ecologist may want to measure consumption in terms of energy use and pollution output (e.g. SO_2, NO_x, CO_2). A housing department can be interested in the demand for housing measured both in square meters and in expenditure at constant prices. A chain of retail stores interested in predicting their future profits could use their market share times their profit per unit as weights when aggregating the consumption of specific goods into profit from commodity groups. Numerous of other examples could of course be figured out.

It may often be suitable to work simultaneously with several consumption aggregates of the same commodity group and from these derive information on how the composition of the consumption of the homogenous goods changes as prices and income changes. For example, many econometric analyses of family budgets have shown that the Engel elasticities for food groups are higher when consumption is measured in expenditure than when measured in quantity (kg), implying that rich households buy relatively more of expensive food items than poor households, see e.g. Haavelmo (1939) and Prais and Houthakker (1955,ch.8). This point is elaborated in the empirical illustration in section 6 below.

A linear consumption aggregate of a commodity group will in general be a function of the prices of all the homogeneous goods in the choice set. A simple way to reduce the number of dimensions of the price space is to consider the case where all relative prices within each group remain constant, leaving only one price parameter per commodity group. In applied economics, this is often a relevant problem formulation, e.g. when analyzing the effects of a reduction of the rate of value added tax for a group of food products. This idea is exploited extensively in the present paper. Using the constant relative prices as weights when aggregating the consumption of the goods within a commodity group, the Hicks aggregation theorem is obtained, saying that the demand functions for the aggregated commodities have exactly the same properties as the demand function for the homogeneous goods. However, this is not so for linear consumption aggregates in general. We show that the Slutsky equation and the homogeneity conditions hold good in all cases, but that adding up, symmetry and negativity does not hold in general. Assuming further that the direct utility function is weakly separable between commodity groups we obtain a generalized symmetry condition and strong relationships between demand functions for different linear consumption aggregates of the same commodity group. Results are presented in eight theorems, with comments on some possible econometric interpretations. A non-Giffen anti law of demand is derived (Theorem 6). A possible interpretation in the case of bread consumption implies that the direct Slutsky elasticity for bread measured in weight (kilograms) is positive, and the direct Cournot elasticity even more so, while the demand elasticities for the Hicksian aggregate of bread have normal signs.

Wold (1952,p.109,113,144) is the only reference I have found which defines and analyzes the demand function for a linear consumption aggregate in general. The analysis is not taken far however. He only shows that Engel- and Cournot elasticities for a linear consumption aggregate can be expressed as weighted sums of the corresponding elasticities for the homogeneous goods within the group. Cramer (1971,p.158) has an interesting discussion of the Engel elasticities for expenditure, quantity and unit prices for different food groups, in connection with analysis of family budgets. Aasness (1979) carries this type of analysis further, measuring consumption of food groups also in terms of energy and fat, and analyzing the effects of demographic, regional and seasonal variables as well as total expenditure. The basic theoretical results in the present paper was developed and used when I was confronted with an applied problem as discussed in Aasness (1984). Deaton (1987) uses similar ideas as in the present paper, but his focus is on unit prices within an iteresting econometric application.

2. PROPERTIES OF DEMAND FUNCTIONS FOR HOMOGENEOUS GOODS

In this section we briefly formulate a system of demand functions for homogeneous goods with a set of standard properties. On this basis we will in the following sections define various demand functions for consumption aggregates of commodity groups, and derive properties of these functions.

Let us consider a consumer with a utility function

$$(1) \qquad u = \upsilon(q_1, q_2, \ldots, q_n),$$

and a linear budget constraint

$$(2) \qquad \sum_{i=1}^{n} p_i q_i = y,$$

where q_i is the quantity and p_i the price of commodity (homogeneous good) i, and y is total expenditure (income for short). We assume that the quantities consumed must be non-negative, and that the vector of prices and total expenditure belongs to some subspace of the non-negative orthant of the n+1 dimensional Euklidian space, called the price-income space.

The assumption that a unique solution exists to the problem of maximizing utility subject to the budget constraint, gives the Marshallian demand functions

$$(3a) \qquad q_i = g_i(y, p_1, p_2, \ldots, p_n), \qquad i = 1, \ldots, n.$$

The existence of a unique solution to the dual problem of minimizing total expenditure for a given utility level (indifference curve), gives the Hicksian demand functions

$$(3b) \qquad q_i = h_i(u, p_1, p_2, \ldots, p_n), \qquad i = 1, \ldots, n.$$

In the following we will state (postulate) properties of these demand functions. Some or all of these properties can be derived from different versions of utility theory, cf e.g. Barten and Bohm (1982) or Deaton and Muellbauer (1980), which we will only briefly comment on. Note that assumptions on preferences may be stated as properties directly on the preference relations, or on the direct utility function, or indirectly on the demand functions themselves. We may also start out with the indirect utility function, the cost function, the profit function etc. There exist many duality theorems showing the equivalence of different sets of assumptions. However, not all assumptions are simple (or even possible) to formulate in the different dimensions.

We will first assume that the demand functions are differentiable,

(4) g_i and h_i are continuous differentiable, $i = 1,\ldots,n$.

This is a very convenient assumption, and may be derived by assuming that the direct utility function is sufficiently smooth.

We will also assume that the demands are strictly positive for all goods,

(5) $g_i(y,p_1,\ldots,p_n) > 0$, $h_i(u,p_1,\ldots,p_n) > 0$, $i = 1,\ldots,n$.

Thus we neglect possiblities of corner solutions. We may derive (5) by assuming that the direct utility function is sufficiently steep along the q-axes, or by restricting the price-income space to the subspace where (5) holds true for the utility function. In some applications (5) may be a very strict assumption. Many of our results can be derived without it. However, (4) and (5) make it possible to define demand elasticities, and to express our further assumptions and results through relations between the demand elasticities. This we find very convenient, because many of our theoretical results are easily presented and intuitively grasped when using elasticities, and because demand elasticities are so widely used in empirical and applied economics. (For a mathmatical definition of elasticites and standard rules for operating with them see e.g. Sydsæter (1981,section 3.14.).)

The Slutsky equations in terms of elasticities are

(6) $e_{ij} = \varepsilon_{ij} - E_i w_j$, $i,j = 1,\ldots,n$,

where e_{ij} is a Cournot elasticity, i.e. the elasticity of g_i with respect to p_j, ε_{ij} is a Slutsky elasticity, i.e. the elasticity of h_i with respect to p_j, E_i is a Engel elasticity, i.e. the elasticity of g_i with respect to y, and $w_j = p_j q_j / y$, i.e. the budget share of commodity j.

From the assumption of a unique solution to the optimum problems, it follows that the Marshallian demand functions are homogeneous of degree zero in total expenditure and prices, and that the Hicksian demand functions are homogeneous of degree zero in prices. This homogeneity property implies in terms of Slutsky elasticites that

(7) $\sum_{j=1}^{n} \varepsilon_{ij} = 0$, $i = 1,\ldots,n$.

From (6) and (7) we derive the homogeneity property in terms of Cournot and Engel elasticities, i.e. $E_i + \Sigma_j e_{ij} = 0$.

From the existence of the demand functions (3) and the budget constraint (2), it follows that the demand functions satisfy the adding-up property $\Sigma_i p_i q_i (y, p_1, \ldots, p_n) = y$. In terms of Engel elasticities this implies

$$(8) \qquad \sum_{i=1}^{n} E_i w_i = 1.$$

One may be note that the adding-up property in terms Cournot elasticities, i.e. $\Sigma_i w_i e_{ij} = -w_j$, follows from (6), (7), (8) and (9).

The symmetry property in terms of Slutsky elasticities is

$$(9) \qquad \varepsilon_{ij} w_i = \varepsilon_{ji} w_j, \qquad i,j = 1,\ldots,n,$$

which follows from (4) and Young's theorem.

The standard negativity property states that the matrix of Slutsky derivatives, $\partial h_i / \partial p_j = \varepsilon_{ij} q_i / p_j$, is negative semidefinite, that is, the quadratic form

$$(10) \qquad \sum_{i=1}^{n} \sum_{j=1}^{n} \xi_i \xi_j \varepsilon_{ij} q_i / p_j \leq 0,$$

for any n vector ξ. This implies that the direct Slutsky elasticities are nonpositive,

$$(11) \qquad \varepsilon_{ii} \leq 0, \qquad i = 1,\ldots,n.$$

We shall interpret the goods in the utility function (1) as homogeneous commodities with a single price where the quantities are measured in physical units. This implies that in most economies the number of goods must be very large indeed. In empirical work we are forced to aggregate over commodities. This is the subject for the next section.

3. DEMAND FUNCTIONS FOR LINEAR CONSUMPTION AGGREGATES

Let us rewrite the utility function (1) through partitioning the n single commodities in N vectors,

$$(12) \qquad u = \upsilon(\underline{q}_1, \underline{q}_2, \ldots, \underline{q}_I, \ldots, \underline{q}_N),$$

where $\underline{q}_1=(q_1,\ldots,q_{n_1})$ is the n_1 vector of the consumption of the first n_1 homogeneous commodities, $\underline{q}_2=(q_{n_1+1},\ldots,q_{n_1+n_2})$ is the n_2 vector of the consumption of the next n_2 commodities etc. Furthermore, we will let SI denote the set of subscripts of the homogeneous goods in group I, thus $S1=\{1,\ldots,n_1\}$, $S2=\{n_1+1,\ldots,n_1+n_2\}$, and

$$(13) \qquad SI = \{\sum_{J=1}^{I-1} n_J + 1, \sum_{J=1}^{I-1} n_J + 2, \ldots, \sum_{J=1}^{I} n_J\}, \qquad I = 2,3,\ldots,N.$$

In our interpretation the quantity consumed of a single commodity, q_i, is measured in physical terms, e.g. in kilograms. We shall allow for different measures of consumption proportional to the reference measure q_i, i.e.

$$(14) \qquad z_i = \theta_i q_i, \qquad \theta_i \geq 0, \qquad i = 1,\ldots,n,$$

where θ_i is a non-negative factor of proportionality for commodity i. We shall let z symbolize an arbitrary consumption measure of the type (14). The consumption concept z can be another type of physical measure, e.g. energy measured in joule, and θ_i will thus be the amount of energy per unit (joule per kilogram) for commodity i. The consumption z can also be measured in economic units, e.g. in expenditure at constant prices, and θ_i will then be a price (e.g. "1980 dollars" per kilogram) for commodity i.

From (3), and (14) we can immediately derive Marshallian and Hicksian demand functions for a homogeneous commodity i, using an arbitrary measure of consumption z_i,

$$(15a) \qquad z_i = \theta_i g_i(y,p_1,\ldots,p_n) \equiv g_i^z(y,p_1,\ldots,p_n), \qquad i = 1,\ldots,n,$$

$$(15b) \qquad z_i = \theta_i h_i(u,p_1,\ldots,p_n) \equiv h_i^z(u,p_1,\ldots,p_n), \qquad i = 1,\ldots,n.$$

It follows immediately from (15) that the demand elasticities for the homogeneous commmodities are the same regardless of the kind of measures of consumption that are used (as long as θ_i is strictly positive). However, this is not so when we consider consumption aggregates of groups of commodities, as we shall see below. We start out our aggregation analysis by introducing the following definition.

Definition 1:

A linear consumption aggregate of a commodity group is a weighted sum of the quantities consumed of the homogeneous commodities within the group. The weights are some non-negative scalars independent of the consumption of the commodities.

Comments on Definition 1:

(i) In our symbols, an arbitrary consumption aggregate z_I of commodity group I can be written

(16) $z_I = \Sigma_{SI} z_i = \Sigma_{SI} \theta_i q_i, \quad \theta_i \geq 0, \quad i \in SI, \qquad I = 1,\ldots,N,$

i.e. a weighted sum of the physical quantities (q_i) of the homogeneous goods in group I, where the weights (θ_i) determine the specific consumption aggregate, and SI is the set of subscripts of the homogeneous goods in group I.

(ii) A common approach in economics is to use a set of constant prices as weights,

(17) $x_I^o = \Sigma_{SI} x_i^o = \Sigma_{SI} p_i^o q_i, \qquad I = 1,\ldots,N,$

i.e. the consumption of the commodity groups are measured in terms of expenditure at a set of constant prices, $\underline{p}^o = (p_1^o,\ldots,p_n^o)$. We could, of course, also measure consumption at another set of prices, say $\underline{p}^1 = (p_1^1,\ldots,p_n^1)$, i.e.

(18) $x_I^1 = \Sigma_{SI} x_i^1 = \Sigma_{SI} p_i^1 q_i, \qquad I = 1,\ldots,N.$

This is the way in which consumption is measured in national accounting, cf for example United Nations (1968), using Laspeyres volume indexes, with more or less frequent changes in base years.

(iii) Another simple example is obtained by setting all the weights equal to one,

(19) $q_I = \Sigma_{SI} q_i,$

i.e. we measure the consumption of the commodity group by the unweighted sum of the physical quantities of the homogeneous goods in the group. For example one may measure the consumption of Bread in terms of the weight (kilograms), summed over the different types of bread, or the consumption of Milk in litres, summed over the different types of milk. In surveys of household expenditure one often measures consumption of different food groups both in terms of expenditure (at constant prices) and in terms of physical quantities. Both type of consumption measures have often been used for Engel curve analysis, see e.g. Wold (1952) and Prais and Houthakker (1955), with substantially different results for the Engel elasticities of the two different consumption measures of the same commodity group.

(iv) It does not seem very meaningful, however, to add kilograms of bread and litres of milk. From a nutritionist point of view it is meaningful to add them in terms of content of energy, fat, proteins etc. Aasness (1979) estimated Engel functions for such types of aggregates, defining commodity groups and weights in close cooperation with experts on dietry and nutrition.

(v) It is often meaningful and interesting to add commodities in terms of energy, and there can be different energy concepts of interest. For example, a nutritionist would be interested in the energy supply to the human body when eating the food, with weights obtained from nutritional theory. While an energy economist or an ecologist could be interested in the energy use in producing the food, where the weights might be obtained from a detailed study of the agricultural production process including an input-output analysis with the rest of the economy.

(vi) It is well known that measures of average consumption on different commodities for the same population of consumers can vary substantially between different data sources. In particular this occurs when comparing consumption data from National accounts with corresponding data from household expenditure data, cf for example Adler and Wolfson (1988). An interpretation of this observation is that one or both data sources are influenced by systematic measurement errors, as opposed to random measurement errors with zero expectation. If one assumes that the (expected value of) the observed consumption is proportional to the true consumption for each homogeneous good (or each commodity group in a detailed grouping), then we can formulate this hypothesis as in (14) and (16) with q_i being the true consumption of commodity i, z_i being the (expected value) of the observed consumption of commodity i from one data source, and $1-\theta_i$ being the rate of systematic measurement error from this data source for commodity i. By assuming that the rates of systematic measurement errors $(1-\theta_i)$ for the homogeneous goods are constant over time and/or across different (subpopulations) of consumers, one can derive a large amount of testable consequences including those given by the theorems in this essay interpreted in terms of the present example. (Stochastic formulation of the theory is beyond the scope of this essay.)

(vii) Other examples of linear consumption aggregates are given in the third paragraph of the introduction to this paper. It should be clear by now, that it is possible to construct numerous examples of different types of linear consumption aggregates which can be of considerable interest for some theoretical, empirical and/or practical issue.

(viii) One may, of course, also construct nonlinear consumption aggregates, for example based on a (sub)utility function for the commodity group, cf (44), but that is not the theme of this essay.

From (3) and (16) we can immediately derive the following type of Marshallian and Hicksian demand functions for an arbitrary linear consumption aggregate z_I,

(20a) $z_I = \Sigma_{SI}\theta_i g_i(y,p_1,\ldots,p_n) = g_I^z(y,p_1,\ldots,p_n)$, $I = 1,\ldots,N$,

(20b) $z_I = \Sigma_{SI}\theta_i h_i(u,p_1,\ldots,p_n) = h_I^z(u,p_1,\ldots,p_n)$, $I = 1,\ldots,N$.

A linear consumption aggregate of a commodity group is thus a function of total expenditure and the prices of all the homogeneous goods.

The number of homogeneous consumption goods in an economy may be a very large number. In order to reduce the number of price variables in our demand functions, we will consider the restrictions in the price space given by the following assumption.

Assumption 1:

There is proportional price variation within each commodity group, i.e

(21) $p_i = P_I\ p_i^o$, $\forall\ i \in SI$, $I = 1,\ldots,N$.

where $p^o = (p_1^o,\ldots,p_n^o)$ is a reference price vector and P_I, $I=1,..,N$ are positive real variables called group prices.

Comments on Assumption 1:

(i) This implies that the relative prices within each group are constant. Changes in the prices of the homogeneous commodities in group I is one-dimensional and go through the group price P_I. We thus restricts the price variation to movements in a N-dimensional hyperplane in the n-dimensional price space. Partial elasticities (or derivatives) with respect to group prices will correspond to directional elasticities (or derivatives) with respect to the prices of the homogeneous goods (cf for example Sydsæter (1981)). Note that in many practical applications one is interested in analyzing consequences of such price changes, e.g. analyzing the effects of changing the rate of value added tax on all food products.

(ii) Assumption 1 may be considered as a <u>definition</u>, providing us with concepts and results which also can be a valuable point of reference when analyzing effects of price variation not satisfying this definition.

(iii) Assumption 1 may also be interpreted as an <u>assumption about the real world</u>, interpreting the theory within some kind of econometric model. Using time series data this implies constant relative prices within commodity groups over time, using cross section data it implies constant relative prices within commodity groups across regions, and using panel data it implies both.

(iv) It may also be possible to <u>relax</u> such a strict interpretation by reformulating (21) with stochastic terms, perhaps interpreting the P_I's as latent variables. One may also introduce some (approximation) model substituting our strict group prices with some kind of price indexes. Such extensions are, however, beyond the scope of this essay.

From (3), (16) and (21) we obtain

(22a) $\quad z_I = \Sigma_{S\,I}\,\theta_i\, g_i\,(y,P_1 p_1^o,\dots,P_N p_n^o) \equiv G_I^z(y,P_1,\dots,P_N), \qquad I = 1,\dots,N,$

(22b) $\quad z_I = \Sigma_{S\,I}\,\theta_i\, h_i\,(u,P_1 p_1^o,\dots,P_N p_n^o) \equiv H_I^z(u,P_1,\dots,P_N), \qquad I = 1,\dots,N.$

In (22) we have defined demand functions for consumption aggregates when all the prices within each commodity group change proportionally. It can also be interesting to analyze how the demand for each homogeneous good within a group responds to a proportional price change within each group, i.e. when (21) holds. From (3) and (21) we obtain

(23a) $\quad q_i = g_i\,(y,P_1 p_1^o,\dots,P_N p_n^o) \equiv G_i\,(y,P_1,\dots,P_N), \qquad i = 1,\dots,n,$

(23b) $\quad q_i = h_i\,(u,P_1 p_1^o,\dots,P_N p_n^o) \equiv H_i\,(u,P_1,\dots,P_N), \qquad i = 1,\dots,n.$

Thus the demand for each homogeneous good depends, in this case, only on total expenditure and the price variable for each commodity group.

We may also note that we can define corresponding demand functions for homogeneous goods when using an arbitrary consumption measure z. Combining (14) and (23) we obtain

(24a) $\quad z_i = \theta_i\, G_i\,(y,P_1,\dots,P_N) \equiv G_i^z(y,P_1,\dots,P_N), \qquad i = 1,\dots,n,$

(24b) $\quad z_i = \theta_i\, H_i\,(u,P_1,\dots,P_N) \equiv H_i^z(u,P_1,\dots,P_N), \qquad i = 1,\dots,n.$

4. PROPERTIES OF DEMAND FUNCTIONS FOR LINEAR CONSUMPTION AGGREGATES

In this section we will derive properties of the functions defined in the previous section. In particular we will analyze the demand functions (22) of linear consumption aggregates as functions of total expenditure (or utility) and of the prices $(P_1 \ldots, P_N)$ of the commodity groups, these group prices being defined by the assumption (21) of constant relative prices within each group. As a first step in this analysis we have deducted properties of the demand functions (23) of homogeneous goods with group prices $(P_1 \ldots, P_N)$ as arguments. These demand functions are also of independent interest, and results are presented in the following theorem. More results on these type of demand functions are given in Theorem 4 in the next section.

Theorem 1:

Assuming proportional price variation within each commodity group (Assumption 1), there exist differentiable demand functions (23) for homogeneous goods with group prices as arguments. These demand functions satisfy the following properties, for any point the price-income space.

(i) The following Slutsky equations hold,

$$e_{iJ} = \varepsilon_{iJ} - E_i w_J, \qquad i = 1,\ldots,n, \quad J = 1,\ldots,N, \tag{25a}$$

where e_{iJ} is the Cournot elasticity of good i w.r.t. the price P_J of commodity group J (i.e. the elasticity of the function G_i w.r.t. P_J), ε_{iJ} is the Slutsky elasticity of good i w.r.t. the price P_J of commodity group J (i.e. the elasticity of H_i w.r.t. P_J), E_i is the Engel elasticity of good i, (i.e. the elasticity of G_i (or g_i) w.r.t. y), and w_J is the budget share of commodity group J (i.e. $w_J = \Sigma_{S_J} w_j$).

(ii) The demand functions are homogeneous of degree zero in group prices and total expenditure. This homogeneity property implies in terms of Slutsky elasticities that

$$\sum_{J=1}^{N} \varepsilon_{iJ} = 0, \qquad i = 1,\ldots,n, \tag{25b}$$

and in terms of Cournot and Engel elasticities that $\Sigma_J e_{iJ} + E_i = 0$, $i = 1,\ldots,n$.

(iii) The adding-up property $\Sigma_{i=1}^{n} p_i G_i(y, P_1, \ldots, P_N) = y$ holds, which implies in terms of Engel elasticites, $\Sigma_i E_i w_i = 1$, and in terms of Cournot elasticities, $\Sigma_i e_{iJ} = -w_J$.

(iv) The Cournot (Slutsky) elasticity of a homogeneous good w.r.t. the price of an arbitrary commodity group is equal to the sum of the Cournot (Slutsky) elasticities of the homogeneous good w.r.t. the prices of the goods within the commodity group,

(26a) $e_{iJ} = \Sigma_{SJ} e_{ij}$, $i = 1,\ldots,n$, $j \in SJ$, $J = 1,\ldots,N$,

(26b) $\varepsilon_{iJ} = \Sigma_{SJ} \varepsilon_{ij}$, $i = 1,\ldots,n$, $j \in SJ$, $J = 1,\ldots,N$.

Proof: see the appendix.

Comment on Theorem 1:

All the demand elasticities for homogeneous goods are the same irrespective of which consumption measure z_i is used, as long as θ_i is strictly positive. From (15) it follows (using a standard rule for elasticities, cf Sydsæter (1981,3.14.2.(i))):

(27a) $E_i^z = E_i$, $e_{ij}^z = e_{ij}$, $\varepsilon_{ij}^z = \varepsilon_{ij}$, $\theta_i > 0$, $i,j = 1,\ldots,n$,

where E_i^z is the elasticity of g_i^z w.r.t. y, e_{ij}^z is the elasticity of g_i^z w.r.t. p_j, and ε_{ij}^z is the elasticity of h_i^z w.r.t. p_j. Thus all the properties of the demand functions (3) given in section 2, in terms of elasticites, also holds for the demand functions (15). Correspondingly, from (24) it follows that

(27b) $E_i^z = E_i$, $e_{iJ}^z = e_{iJ}$, $\varepsilon_{iJ}^z = \varepsilon_{iJ}$, $\theta_i > 0$, $i=1,\ldots,n$, $J=1,\ldots,N$,

where E_i^z is the elasticity of G_i^z w.r.t. y, e_{ij}^z is the elasticity of G_i^z w.r.t. p_j, and ε_{ij}^z is the elasticity of h_i^z w.r.t. p_j. Thus all the properties of the demand functions (23) given in Theorem 1, in terms of elasticites, also holds for the demand functions (24).

We are now prepared for analyzing linear consumption aggregates as functions of group prices. A special case of linear consumption aggregates is to use the constant relative prices within groups as weights (θ_i), i.e. let (17) and (21) hold simultaneously. We shall call these aggregates Hicksian aggregates, and denote them by x_I, I=1,...,N, in the following. (Thus we drop the superscript ^{o} on the x when combining (17) and (21)). The properties of the demand functions G_I^x, H_I^x, I=1,...,N, of Hicksian aggregates, are well known in the litterature by the often cited Hicks aggregation theorem, cf Hicks (1939). All the properties of the demand functions for the homogeneous goods we stated in section 2 hold for the demand functions G_I^x and H_I^x of the Hicksian aggregates. However, this is not so for linear consumption aggregates in general, and the properties do not seem to be well known, at least I have not found any systematic treatment in the literature. The results of this section are presented in the following theorem. The theorem is somewhat long and detailed, some points are simple corollaries of other points, but it is convenient for later reference to present it all in one place. More results on these type of demand functions are given in Theorem 5, 6 and 7 in the next section.

Theorem 2:

Assuming proportional price variation within each commodity group (Assumption 1), there exist differentiable demand functions (22) for any _linear consumption aggregate_ *satisfying Definition 1, with* _group prices_ *as arguments. These demand functions satisfy the following properties, for any point the price-income space.*

(i) The following _Slutsky equations_ *hold,*

$$(28a) \qquad e^z_{IJ} = \varepsilon^z_{IJ} - E^z_I w_J, \qquad\qquad I,J = 1,\dots,N,$$

where e^z_{IJ} is the Cournot elasticity of consumption aggregate z_I w.r.t. the price P_J of commodity group J (i.e. elasticity of the function G^z_I w.r.t. P_J), ε^z_{IJ} is the Slutsky elasticity of consumption aggregate z_I w.r.t. the price P_J of commodity group J (i.e. the elasticity of H^z_I w.r.t. P_J), E^z_I is the Engel elasticity of consumption aggregate z_I (i.e. the elasticity of G^z_I (or g^z_I) w.r.t. y), and w_J is the budget share of commodity group J.

(ii) The demand functions are homogeneous of degree zero in group prices and total expenditure. This _homogeneity_ *property implies in terms of Slutsky elasticities that*

$$(28b) \qquad \sum_{J=1}^{N} \varepsilon^z_{IJ} = 0, \qquad\qquad I = 1,\dots,N,$$

and in terms of Cournot and Engel elasticities that $\Sigma_J e^z_{IJ}+E^z_I=0$, $I=1,\dots,N$.

(iii) In the special case of Hicksian aggregates (x), the _adding-up_ *property (in terms of Engel elasticities), the* _symmetry_ *property (in terms of Slutsky elasticities) and the* _negativity_ *property (in terms of direct Slutsky elasticites) hold,*

(29a) $\sum_{I=1}^{N} E_I^x w_I = 1,$ $\qquad I = 1,\ldots,N,$

(29b) $\varepsilon_{IJ}^x w_I = \varepsilon_{JI}^x w_J,$ $\qquad I,J = 1,\ldots,N,$

(29c) $\varepsilon_{II}^x \leq 0,$ $\qquad I = 1,\ldots,N.$

These properties *__do not hold__* *for linear consumption aggregates in general.*

(iv) The direct Slutsky elasticity of a linear consumption aggregate is *__non-positive if__* *the Slutsky elasticities of all the homogenous goods within the group with respect to the group price are non-positive,*

(30) $\varepsilon_{II}^z \leq 0 \quad \text{if} \quad [\varepsilon_{iI} \leq 0, \ \forall\, i \in SI],$ $\qquad I = 1,\ldots,N.$

(v) The Engel, Cournot and Slutsky elasticities of linear consumption aggregates are *__weighted averages__* *of corresponding demand elasticities for the homogeneous goods,*

(31a) $E_I^z = \Sigma_{SI} E_i z_i / z_I,$ $\qquad I = 1,\ldots,N,$

(31b) $e_{IJ}^z = \Sigma_{SI} e_{iJ} z_i / z_I,$ $\qquad I,J = 1,\ldots,N,$

(31c) $\varepsilon_{IJ}^z = \Sigma_{SI} \varepsilon_{iJ} z_i / z_I,$ $\qquad I,J = 1,\ldots,N,$

with weights equal to the consumption share of the homogeneous goods in terms of the linear consumption aggregate.

(vi) Corresponding demand elasticities for different linear consumption aggregate of the same commodity group can have quite different size, and may be of opposite sign, but they can only vary within the following *__bounds__* *determined by the demand elasticities of the homogeneous goods in the group,*

(32a) $E_I^z \in [\min_{i \in SI} E_i, \max_{i \in SI} E_i],$ $\qquad I = 1,\ldots,N,$

(32b) $e_{IJ}^z \in [\min_{i \in SI} e_{iJ}, \max_{i \in SI} e_{iJ}],$ $\qquad I,J = 1,\ldots,N,$

(32c) $\varepsilon_{IJ}^z \in [\min_{i \in SI} \varepsilon_{iJ}, \max_{i \in SI} \varepsilon_{iJ}],$ $\qquad I,J = 1,\ldots,N.$

(vii) The *__difference__* *between corresponding demand elasticities for an arbitrary linear consumption aggregate and the Hicksian aggregate, for the same commodity group, can be written*

(33a) $E_I^z - E_I^x = \Sigma_{SI} E_i (z_i/z_I - x_i/x_I)$, $I = 1,\ldots,N$,

(33b) $e_{IJ}^z - e_{IJ}^x = \Sigma_{SI} e_{iJ} (z_i/z_I - x_i/x_I)$, $I,J = 1,\ldots,N$,

(33c) $\varepsilon_{IJ}^z - \varepsilon_{IJ}^x = \Sigma_{SI} \varepsilon_{iJ} (z_i/z_I - x_i/x_I)$, $I,J = 1,\ldots,N$,

that is a type of covariance between the demand elasticities and difference in consumption weights of the homogeneous goods within the group.

(viii) The demand elasticities for a system of linear consumption aggregates will be equal to the corresponding demand elasticities of the system of Hicksian aggregates,

(34) $E_I^z = E_I^x, \quad e_{IJ}^z = e_{IJ}^x, \quad \varepsilon_{IJ}^z = \varepsilon_{IJ}^x$, $I,J = 1,\ldots,N$,

if the following conditions hold

(35a) $\Sigma_{SI} E_i (z_i/z_I - x_i/x_I) = 0$, $I = 1,\ldots,N$,

(35b) $\Sigma_{SI} \varepsilon_{iJ} (z_i/z_I - x_i/x_I) = 0$, $I,J = 1,\ldots,N$,

which means that both the Engel elasticities and the Slutsky elasticities (in demand functions (23)) for the homogeneous goods are uncorrelated with the difference in consumption weights in terms of the linear aggregate (z) and the Hicksian aggregate (x), for each commodity group. For all linear consumption aggregates which satisfy condition (35), the adding-up, symmetry and negativity properties (29) hold.

Proof: see the appendix.

Comments on Theorem 2:

(i) The consumption shares z_i/z_I must be non-negative and no larger than 1,

(36) $$\frac{z_i}{z_I} = \frac{\theta_i q_i}{\Sigma_{SI} \theta_i q_i} \in [0,1], \qquad i \in SI, \quad I = 1,\ldots,N.$$

They must also add to 1, neglecting the case with $\theta_i = 0$ $\forall i \in SI$. These shares may vary considerably with the choice of consumption measure, i.e. the choice of scalars θ_i. For any good we may, in principle, choose $\theta_i = 0$ so that $z_i/z_I = 0$, or choose $\theta_i > 0$ and $\theta_k = 0$ $\forall k \neq i$, $i,k \in SI$, so that $z_i/z_I = 1$. Thus the bounds in (v) cannot be made tighter for linear consumption aggregates in general.

(ii) Combining (26) in Theorem 1 and (31) in Theorem 2 we obtain

(37a) $e^{z}_{IJ} = \Sigma_{SI} z_i/z_I \ \Sigma_{SJ} e_{ij}$, $\quad i \in SI, \quad j \in SJ, \quad I,J = 1,\ldots,N,$

(37b) $\varepsilon^{z}_{IJ} = \Sigma_{SI} z_i/z_I \ \Sigma_{SJ} \varepsilon_{ij}$, $\quad i \in SI, \quad j \in SJ, \quad I,J = 1,\ldots,N,$

showing how the price elasticities of (22) are aggregates of the price elasticities of (3). Thus the formulas in (26) and (31) perform this aggregation in two steps. These relationsships can be a starting point for interesting theoretical and empirical studies. If group I consists of goods which are close substitutes, then $|\varepsilon_{ii}|$ may be much larger in magnitude than $|\varepsilon_{iI}|$ where opposite effects can cancel each other out in the sum. Correspondingly, $|\varepsilon_{II}|$ can be considerably smaller than the (weighted) average value of $|\varepsilon_{ii}|$ for $i \in SI$. The intervals in (32c) may be much tighter than corresponding intervals of ε_{ij}. Furthermore, $|\varepsilon_{iI}|$ and $|\varepsilon_{II}|$ may typically vary less across the price-income space than $|\varepsilon_{ii}|$.

(iii) The corresponding condition to (35b) for Cournot elasticities, $\Sigma_{SI} e_{iJ}(z_i/z_I - x_i/x_I) = 0$, follows from (35b) and the Slutsky equations (28a).

(iv) The expressions in (33) and (35) can be written as proper covariances, using the fact that $\Sigma_{SI}(z_i/z_I - x_i/x_I) = 1 - 1 = 0$. Thus e.g. (33a) can be rewritten as

$$E^{z}_{I} - E^{x}_{I} = \frac{1}{n_I} \Sigma_{SI}(E_i - \overline{E}_I)(z_i/\overline{z}_I - x_i/\overline{x}_I),$$

where $\overline{E}_I = \Sigma_{SI} E_i/n_I$, $\overline{z}_I = z_I/n_I$ and $\overline{x}_I = x_I/n_I$. Thus $E^{z}_{I} - E^{x}_{I}$ is equal to the covariance between E_i and $(z_i/\overline{z}_I - x_i/\overline{x}_I)$ over homogeneous goods in group I.

(v) It may be a plausible, and testable, hypothesis that (35) holds approximately for a many types of linear aggregates. One example could be Laspeyres volume indexes for consumption groups in national accounts, where changes in relative prices from the base year to the current year might well be approximately uncorrelated with the demand elasticities for the homogeneous goods. But there are also important examples of linear consumption aggregates for which the conditions (35) seems to be systematically violated, cf the comments to Theorem 6 and 7.

(vi) There is a simple special case where (35) holds exactly, namely when

(38) $\theta_i = \theta_I p^{o}_i$, $\quad \forall i \in SI, \quad I = 1,\ldots,N,$

i.e the weights are proportional to the constant relative prices within each group, as for the Hicksian aggregates, but where the weights may be given an independent interpretation, say a purely physical one like energy content. From (38), (21) and (17) it follows that $z_i/z_I = x_i/x_I$, thus all the differences in weights in (35) are zero and thus also the covariances.

It can be of considerably interest to study how linear consumption aggregates depends directly on the prices of the homogeneous goods, not assuming (21). Results are presented in Theorem 3. More results on these type of demand functions follows in Theorem 8 in the next section.

Theorem 3:

For any linear consumption aggregate, satisfying Definition 1, there exist differentiable demand functions (20) with prices of the homogeneous goods as arguments. These demand functions satisfy the following properties, for any point the price-income space.

(i) The following Slutsky equations hold,

$$(39a) \qquad e^z_{Ij} = \varepsilon^z_{Ij} - E^z_I w_j, \qquad I = 1,\dots,N, \quad j = 1,\dots,n,$$

where e^z_{Ij} *is the Cournot elasticity of consumption aggregate* z_I *w.r.t. the price* p_j *of good* j *(i.e. the elasticity of the function* g^z_I *w.r.t.* p_j*),* ε^z_{Ij} *is the Slutsky elasticity of consumption aggregate* z_I *w.r.t. the price* p_j *of good* j *(i.e. the elasticity of* h^z_I *w.r.t.* p_j*),* E^z_I *is the Engel elasticity of consumption aggregate* z_I *(i.e. the elasticity of* g^z_I *(or* G^z_I*) w.r.t.* y*), and* w_j *is the budget share of good* j*.*

(ii) The demand functions are homogeneous of degree zero in total expenditure and the prices of the homogeneous goods. This homogeneity property implies in terms of Slutsky elasticities,

$$(39b) \qquad \sum_{j=1}^{n} \varepsilon^z_{Ij} = 0, \qquad I = 1,\dots,N.$$

and in terms of Cournot and Engel elasticities: $\Sigma_j e^z_{Ij} + E^z_I = 0$, $I = 1,\dots,N$.

(iii) The Engel, Cournot and Slutsky elasticities of linear consumption aggregates are weighted averages of corresponding demand elasticities for the homogeneous goods,

$$(40a) \qquad E^z_I = \Sigma_{SI} E_i z_i / z_I, \qquad i \in SI, \quad I = 1,\dots,N,$$

$$(40b) \qquad e_{Ij} = \Sigma_{SI} e_{ij} z_i / z_I, \qquad i \in SI, \quad I = 1,\dots,N, \quad j = 1,\dots,n,$$

$$(40c) \qquad \varepsilon^z_{Ij} = \Sigma_{SI} \varepsilon_{ij} z_i / z_I, \qquad i \in SI, \quad I = 1,\dots,N, \quad j = 1,\dots,n,$$

with weights equal to the consumption share of the homogeneous goods in terms of the linear consumption aggregate.

Proof: see the appendix.

<u>Comment on Theorem 3</u>:

The relationsships (32a) and (33a) in Theorem 2 are also valid for the demand for linear consumption aggregates as functions of the prices of the homogeneous goods treated in Theorem 3. The relationsships (32b-c) and (33b-c) hold analogously, just substitute the index J of a commodity group with the index j of a homogeneous good.

5. FURTHER RESULTS ASSUMING WEAK SEPARABILITY

We start out by introducing the concept of group expenditure functions, which turns out to be a useful tool when analyzing the implications of the separability assumptions which will be introduced below. The expenditure on commodity group I (y_I) is defined as

$$(41) \qquad y_I = \Sigma_{SI} p_i q_i , \qquad i \in SI, \quad I = 1,\ldots,N.$$

Group expenditure functions are defined by

$$(42a) \qquad y_I = \Sigma_{SI} p_i g_i (y,p_1,\ldots,p_n) = g_I^y(y,p_1,\ldots,p_n), \qquad I = 1,\ldots,N,$$

$$(42b) \qquad y_I = \Sigma_{SI} p_i h_i (u,p_1,\ldots,p_n) = h_I^y(u,p_1,\ldots,p_n), \qquad I = 1,\ldots,N,$$

analogous to the demand functions (20) for linear consumption aggregates. Under Assumption 1 of proportional price variation within each group, the group expenditures are functions of group prices,

$$(43a) \qquad y_I = \Sigma_{SI} p_i g_i (y,P_1 p_1^o,\ldots,P_N p_n^o) = G_I^y(y,P_1,\ldots,P_N), \qquad I = 1,\ldots,N,$$

$$(43b) \qquad y_I = \Sigma_{SI} p_i h_i (u,P_1 p_1^o,\ldots,P_N p_n^o) = H_I^y(u,P_1,\ldots,P_N), \qquad I = 1,\ldots,N,$$

analogous to the demand functions (22) for linear consumption aggregates. We will also use the terms Engel, Cournot and Slutsky elasticities, and the symbols for these terms, analogous to those introduced for the demand functions. For example, the Slutsky elasticity for expenditure on group I with respect to the price of good j, ε_{Ij}^y, is defined as the elasticity of the function h_I^y w.r.t. p_j.

These group expenditure functions are of interest in themselves, but the focus in this essay is to use them as a tool for analyzing the properties of the demand functions introduced in section 3, when assuming weak separability in the utility function (1) generating these demand functions.

Asumption 2:

The utility function is weakly separable in N groups, i.e. there exist functions $f,\upsilon_1,\ldots,\upsilon_N$, such that

$$(44) \qquad u = f(\upsilon_1(\underline{q}_1),\upsilon_2(\underline{q}_2),\ldots,\upsilon_N(\underline{q}_N)).$$

The grouping and notation are the same as introduced in (12). It can be noted that the assumption of a weakly separable utility function is equivalent with an assumption of weakly separable preferences, cf Katzner

(1970,Theorem 2.3-3) or Barten and Böhm (1982,Theorem 6.1).

The implications of weak separabilty on the properties of the demand functions (3) of homogeneous goods are well known. We summarize the main results in the following lemma.

<u>Lemma 1</u>:

If the utility function is weakly separable (Assumption 2), the demand for homogeneous goods as functions of the prices of these goods (3) satisfy the following properties, in addition to those given i section 2.

(i) The demand for good i is a function of the group expenditure (y_I) *on the commodity group to which it belongs and of the prices* ($\underline{p}_I$) *within this group ,*

$$(45a) \qquad q_i = \overset{*}{g}_i(y_I, \underline{p}_I), \qquad\qquad i \in SI, \quad I = 1,\ldots,N.$$

Total expenditure (or utility) and the prices of the goods in the other groups enter only through the group expenditure functions (42). The <u>*conditional demand functions*</u> g_i^* *have "standard" properties of demand functions for homogeneous goods (assuming the subutility functions* υ_i *have "standard" properties), in particular they are homogeneous of degree zero in* y_I *and* $\underline{p}_I$.

(ii) The <u>*cross-price Slutsky*</u> *elasticity of a homogeneous good w.r.t. the price of a good belonging to another group, is equal to the product of four terms:*

$$(45b) \qquad \varepsilon_{ij} = \mu_{IJ} E_i E_j w_j, \qquad\qquad i \in SI, \quad j \in SJ, \quad I \neq J, \quad I,J = 1,\ldots,N,$$

i.e. the Engel elasticities of the two goods, the budget share of the good which price increases, and a parameter (μ_{IJ}) *which are common for all goods belonging to the two groups.*

<u>Comments on Lemma 1</u>:

(i) These types of results are well known, see e.g. Pollak (1971) and Deaton and Muellbauer (1980,section 5.2). However, the restrictions of type (45b) are usually presented in terms of derivatives. I cannot remember to have seen these elasticity relations presented elsewhere, so a simple (and direct) <u>proof</u> of (45b) is included in the appendix.

(ii) We will call the μ_{IJ}'s for <u>substitution parameters</u>, due to their intimate connection with cross-price Slutsky elasticities. These parameters will in general vary across the price-income space, as demand elasticities do. The size of these parameters and how they vary across the price-income space and between different (groups of) consumers, involves interesting theoretical and empirical issues, but are not the subject of this essay. We may note, however, that if a system of demand functions based on Assumption 2 is estimated, then we can of course derive estimates of the μ_{IJ} parameters, and how they vary across the price-income space, just as is often done for demand elasticities. Furthermore, as will be clear from the following theorems, the μ_{IJ} parameters can be identified from the demand functions for linear consumption aggregates of the commodity groups.

(iii) The substitution parameters (μ_{IJ}) can be restricted by introducing <u>stronger separability</u> assumptions, cf. the last paragraph of this section and the example in section 6.

(iv) Relations (45b) points out that <u>Engel elasticities can contain much information on price elasticities</u>, a feature that will also appear in all the theorems in this section.

Combining the assumption of weak separabililty with our earlier assumptions and concepts we can obtain a lot of new properties of the demand functions introduced in section 3. We present results in form of five theorems, and start out with properties of the demand for homogeneous goods as functions of group prices.

<u>*Theorem 4*</u>:

If the utility function is weakly separable in N groups (Assumption 2) and the relative prices within each group are constant (Assumption 1), then the demand for the <u>homogeneous goods as functions of group prices</u> (23) satisfy the following properties, in addition to those given in Theorem 1.

(i) The demand for good i is a function of the demand for the Hicksian aggregate (x_I) of the commodity group to which it belongs,

$$q_i = \overset{*}{g}_i(x_I, \underline{p}_I^o) = \overset{*}{G}_i(x_I), \qquad i \in SI, \quad I = 1,\dots,N, \tag{46a}$$

which is a <u>conditional Engel function</u>, where the Hicksian aggregate (x_I) is determined by the demand functions (22) with properties given in Theorem 2 and 5.

(ii) The <u>cross-price Slutsky</u> elasticity (ε_{iJ}) of a homogeneous good i with respect to the price (P_J) of a another commodity group is equal to the product of four terms:

$$\varepsilon_{iJ} = \mu_{IJ} E_i E_J^x w_J, \qquad i \in SI, \quad I \neq J, \quad I,J = 1,\dots,N, \tag{46b}$$

i.e. the Engel elasticity of the homogeneous good (E_i)*, the Engel elasticity of the Hicksian aggregate of the commodity group whose price increases* (E^x_J)*, the budget share of the same commodity group* (w_J)*, and the substitution parameter* (μ_{IJ}) *between the two commodity groups.*

(iii) Every Engel, Cournot and Slutsky elasticity for an arbitrary homogeneous good is <u>*proportional*</u> *to the corresponding Engel, Cournot or Slutsky elasticity for the Hicksian aggregate of the commodity group to which the homogeneous good belongs,*

$$\text{(47a)} \quad E_i = \overset{*}{E}_i \, E^x_I \qquad i \in SI, \quad I = 1,\ldots,N,$$

$$\text{(47b)} \quad e_{iJ} = \overset{*}{E}_i \, e^x_{IJ} \qquad i \in SI, \quad I,J = 1,\ldots,N,$$

$$\text{(47c)} \quad \varepsilon_{iJ} = \overset{*}{E}_i \, \varepsilon^x_{IJ} \qquad i \in SI, \quad I,J = 1,\ldots,N,$$

with the same factor of proportionality, which is the conditional Engel elasticity ($\overset{*}{E}_i$) *of the homogeneous good, i.e. the elasticity of* $\overset{*}{G}_i$.

(iv) Any Cournot or Slutsky elasticity (e_{iJ}, ε_{iJ}) *of a homogeneous good has the* <u>*opposite sign*</u> *of the corresponding Cournot or Slutsky elasiticty* ($e^x_{IJ}, \varepsilon^x_{IJ}$) *of the Hicksian aggregate of the commodity group of which the homogeneous good belongs, if and only if the Engel elasticity* (E_i) *of the homogeneous good has the opposite sign of the Engel elasticity* (E^x_I) *of the Hicksian aggregate.*

(v) The demand for a homogenous good as functions of group prices satisfy the <u>*negativity*</u> *property* $\varepsilon_{iI} \leq 0$, $i \in SI$, *if the Engel elasticity* (E_i) *of the homogeneous good has the same sign as the Engel elasticity* (E^x_I) *for the Hicksian aggregate of the commodity group to which the good belongs.*

Proof: see the appendix.

<u>Comments on Theorem 4</u>:

(i) Property (i) implies that if we know (or have estimated) a system of demand functions (22) for Hicksian aggregates, then we only need in addition to know (estimate) <u>one conditional Engel function</u> for a homogeneous good in order <u>to derive the demand function of type (23)</u> for this homogeneous good. (This conditional Engel function must, however, be known (estimated) at the same relative prices within the group as those relative prices defining the Hicksian aggregate of the group.) It is interesting to note in this respect, that the econometric litterature abounds with estimates of systems of demand functions for broad aggregates (which may be interpreted as Hicksian aggregates), but few studies on systems of demand functions with detailed commodity groups exist. On the other hand, Engel functions have often been estimated on more detailed commodity groups, based on cross section data assuming constant relative prices. Furthermore, it is possible to get empirical information on Engel curves on much more detailed commodity groups than is traditionally used, based on inexpensive research techniques and from a large amount of existing surveys. (For example, Aasness (1977) estimated Engel functions for 230 different groups of food, beverages and tobacco.) Theorem 4 can provide a starting point for combining these two different types of empirical information.

(ii) It follows from Theorem 1 and 4 that all Slutsky and Cournot elasticities of the demand functions (23) can be expressed as simple functions of substitution parameters (μ_{IJ}), Engel elasticities (E_i, E_I^x) and budget shares (w_J). The relations for the cross-price Slutsky elasticities are given by (46b), inserting these in the homogeneity property (25b) we obtain the relations for the direct Slutsky elasticities,

$$(48) \qquad \varepsilon_{iI} = -E_i \sum_{\forall J \neq I} \mu_{IJ} E_J^x w_J, \qquad\qquad I = 1,\ldots,N,$$

while the relation for an arbitrary Cournot elasticity can be obtained by inserting the relation for the correponding Slutsky elasticity into the Slutsky equation (25a).

Theorem 5:

If the utility function is weakly separable in N groups (Assumption 2) and the relative prices within each group are constant (Assumption 1), then the demand for linear consumption aggregates as functions of group prices (22) satisfy the following properties, in addition to those given in Theorem 2.

(i) The demand for any linear consumption aggregate (z_I) is a function of the demand for the Hicksian aggregate (x_I) of the same commodity group,

$$(49a) \qquad z_I = \Sigma_{SI} \theta_i \overset{*}{G}_i (x_I) = \overset{*z}{G}_I (x_I), \qquad i \in SI, \quad I = 1,\ldots,N,$$

which is a conditional Engel function for the linear consumption aggregate, where the Hicksian aggregate (x_I) is determined by the demand functions (22) with properties given in Theorem 2 and 5 (below).

(ii) The cross-price Slutsky elasticity (ε_{IJ}^z) of a linear consumption

aggregate (z_I) *with respect to the price* (P_J) *of another commodity group is equal to the product of four terms:*

(49b) $\quad \varepsilon^z_{IJ} = \mu_{IJ} E^z_I E^x_J w_J, \qquad I \neq J, \quad I,J = 1,\ldots,N,$

i.e the Engel elasticity of the linear consumption aggregate (E^z_I), *the Engel elasticity of the Hicksian aggregate of the commodity group whose price increases* (E^x_J), *the budget share of the commodity group whose price increases* (w_J), *and the substitution parameter* (μ_{IJ}) *between the two commodity groups.*

(iii) Every Engel, Cournot and Slutsky elasticity for an arbitrary linear consumption aggregate is ***proportional*** *to the corresponding Engel, Cournot or Slutsky elasticity for the Hicksian aggregate of the same commodity group,*

(50a) $\quad E^z_I = \overset{*}{E}{}^z_I \, E^x_I \qquad I = 1,\ldots,N,$

(50b) $\quad e^z_{IJ} = \overset{*}{E}{}^z_I \, e^x_{IJ} \qquad I,J = 1,\ldots,N,$

(50c) $\quad \varepsilon^z_{IJ} = \overset{*}{E}{}^z_I \, \varepsilon^x_{IJ} \qquad I,J = 1,\ldots,N,$

with the same factor of proportionality, which is the conditional Engel elasticity ($\overset{*}{E}{}^z_I$) *of the linear consumption aggregate, i.e. the elasticity of* $\overset{*}{G}{}^z_I$.

(iv) Any Cournot and Slutsky elasticity ($e^z_{IJ}, \varepsilon^z_{IJ}$) *of a linear consumption aggregate has the* ***opposite sign*** *of the corresponding Cournot or Slutsky elasiticty* ($e^x_{IJ}, \varepsilon^x_{IJ}$) *of the Hicksian aggregate of the same commodity group, if and only if the Engel elasticity* (E^z_I) *of the linear consumption aggregate has the opposite sign of the Engel elasticity* (E^x_I) *of the Hicksian aggregate.*

(v) The demand functions of a linear consumption aggregate satisfy the ***negativity*** *property* ($\varepsilon^z_{II} \leq 0$), *if the Engel elasticity* (E^z_I) *of the linear consumption aggregate has the same sign as the Engel elasticity* (E^x_I) *of the Hicksian aggregate of the same commodity group.*

(vi) The Slutsky elasticities (ε^z_{IJ}) *of linear consumption aggregates satisfy the following* ***generalized symmetry*** *property,*

(51) $\quad \varepsilon^z_{IJ} \, E^x_I \, E^z_J \, w_I = \varepsilon^z_{JI} \, E^z_I \, E^x_J \, w_J, \qquad I \neq J, \quad I,J = 1,\ldots,N,$

which is a generalization of the symmetry condition (34) for Hicksian aggregates, involving the four Engel elasticities E^z_I, E^x_I, E^z_J, *and* E^x_J.

Proof: see the appendix.

Comments on Theorem 5:

(i) One characteristic feature of the results in the above theorem is the strong informational content Engel functions have on full demand functions for linear consumption aggregates. Assume that we know a complete system of demand functions for a set of Hicksian aggregates, and introduce one more linear consumption aggregate (e.g. the protein contents in food consumption or the CO_2 output from transportation consumption), then we only need to know one (conditional) Engel function for this linear consumption aggregate to be able to derive its full demand function of type (22). Such Engel functions may be estimated directly from data on the linear consumption aggregate, or indirectly by first estimating the Engel functions for the homogeneous goods and then aggregating these Engel functions in accordance with (49a).

(ii) The properties presented in Theorem 5 can be transformed to testable statistical hypotheses by means of an econometric model. For example one may test if all the demand elasticities of some linear consumption aggregate is proportional to the corresponding demand elasticities to the Hicksian aggregate of the same commodity group (50). If the hypothesis is rejected a possible interpretation is that Assumption 2 of weakly separable preferences (utility) is incorrect. As far as I know, this represents a new idea for testing separability assumptions. For each commodity group there exist in principle infinitely many different linear consumption aggregates, and the relations shall hold for all of the inifinetly many points in the price-income space. Thus there are plentiful of opportunities for testing specified versions of the consumer theory presented in this essay.

(iii) It follows from Theorem 2 and 5 that all Slutsky and Cournot elasticities of the demand functions (22) can be expressed as simple functions of substitution parameters (μ_{IJ}), Engel elasticities (E_I^z, E_I^x) and budget shares (w_J). The relations for the cross-price Slutsky elasticities are given by (49b), inserting these in the homogeneity property (28b) we obtain the relations for the direct Slutsky elasticities,

$$(52) \qquad \varepsilon_{II}^{z} = -E_I^z \sum_{\forall J \neq I} \mu_{IJ} E_J^x w_J, \qquad I = 1,\dots,N.$$

while the relation for an arbitrary Cournot elasticity can be obtained by inserting the relation for the correponding Slutsky elasticity into the Slutsky equation (28a).

It follows already from Theorem 2 that the "law of demand" (i.e.: direct price elasticities are non-positive) does not hold for linear consumption aggregates in general (not even in terms of direct Slutsky elasticities). Theorem 5 give a basis for characterizing an interesting case where the law of demand does not hold. Since the law of demand has such a central place in economics we phrase this result in a separate theorem. To be concrete one may read "commodity group B" as "bread", the "homogeneous goods" within the group as "types of bread", and "linear consumption aggregate z_B" as "bread consumption measured in weight (kilograms)".

Theorem 6: *(A non-Giffen anti law of demand)*

The demand for a linear consumption aggregate z_B of commodity group B increases if all the prices of the homogenous goods within the group increase proportionally (Assumption 1), with direct Slutsky (ε^z_{BB}) and Cournot (e^z_{BB}) elasticities satisfying the inequalities:

(53a) $0 < \varepsilon^z_{BB} < e^z_{BB}$,

if:

(i) the Engel elasticity of the linear consumption aggregate is negative and the Engel elasticity of the Hicksian aggregate is positive:

(53b) $E^z_B < 0 < E^x_B$,

(ii) the direct Slutsky elasticity of the Hicksian aggregate is strictly negative ($\varepsilon^x_{BB}<0$), and

(iii) the utility function (preferences) is weakly separable with respect to commodity group B (Assumption 2).

Proof: Theorem 6 follows from Theorem 5 (iii) and Theorem 2 (i).

Comments on Theorem 6:

(i) For different food groups there are much empirical evidence in favor of the hypothesis that the Engel elasticity for the consumption measured in weight is less than the Engel elasticity for consumption measured in expenditure at constant prices, see e.g. Wold and Jureen (1952), Prais and Houthakker (1955), Cramer (1971), and Aasness (1979). It may well happen for some of these food groups, say Bread, that these elasticities also have different sign so that condition (53b) is fulfilled.

This means that cheap types of bread are inferior ($E_i<0$), expensive types of bread are normal ($E_i>0$) and the consumption shares of the different types are such that $E_I^z < 0 < E_I^x$. The empirical results on inferior goods are more variable, and less reliable. Aasness (1983, section 6) showed that the use of standard methods tend to disguise the existence of inferior goods.

(ii) Almost every textbook in microeconomics mentions the Giffen good, i.e. a good where the direct Cournot elasticity is positive due to a positive income effect (negative Engel elasticity) which dominate a negative Slutsky elasticity. The favorite examples are bread (England, 18th century) and potatoes (Ireland, 19th century). There is also a substantial literature analyzing empirical evidence for the existence of such Giffen goods, cf for example Walker (1987). None seems to have mentioned in this connection (nor in any other relation) the result in Theorem 6. To assume that commodity groups like bread and potatoes are homogeneous goods seems to me to be inappropriate when testing hypotheses on existence of Giffen goods. It would not surprise me, if much of the old consumption data are (partly) based on aggregating homogeneous goods by weight and not by expenditure at constant prices. Thus it might be rewarding to reexamine old empirical evidence on the Giffen good, and to look for new data for testing the existence of Giffen goods and the existence of goods satisfying conditions (53). A starting point could be to assume some kind of model with a "representative consumer" as a maintained hypothesis and to test the following hypotheses: (a) there has never existed any "real" economy with a Giffen good defined by $e^x_{II}>0$, (b) for every "real" economy there exist a food group B where the consumption (z_B) measured in terms of weight or in terms of energy satisfy conditions (53).

Theorem 6 is an example of a type of theorem where we make assumptions on the size of Engel elasticities and derives results on the size of price elasticities. We will present one more simple example of such a type of theorems. For concreteness one may read "commodity group F" as "food" and "linear consumption aggregate z_F" as "consumption of food measured in terms of energy".

<u>Theorem 7</u>: (Relative inelastic linear consumption aggregates)

Every Cournot and Slutsky elasticity of a linear consumption aggregate z_F of commodity group F w.r.t. group prices are smaller in absolute value *than corresponding Cournot and Slutsky elasticities for the Hicksian aggregate of the same commodity group:*

$$(54a)\quad |e^{z}_{FJ}| < |e^{x}_{FJ}|, \qquad |\varepsilon^{z}_{FJ}| < |\varepsilon^{x}_{FJ}|, \qquad J = 1,\ldots,N,$$

if Assumption 1 and 2 holds and in addition:

$$(54b)\quad 0 < E^{z}_{F} < E^{x}_{F} ,$$

i.e. the Engel elasticity for the linear consumption aggregate is less than the Engel elasticity of the Hicksian aggregate for commodity group F.

Proof: Theorem 7 follows from Theorem 5 (iii).

<u>Comment on Theorem 7</u>:

As mentioned in comment (i) to Theorem 6 there exist much empirical evidence supporting the hypothesis $E^z_F<E^x_F$ where F is some food group and z_F is measured in weight or in energy. There are also much empirical evidence in favor of the hypothesis that $0<E^z_F<E^x_F$ for many food groups, and for food as one group when z_F is measured in terms of energy (cf. Aasness (1979)). I will also conjecture that (54b) holds true when (a) F is housing and z_F is consumption of housing measured in square meters of the dwelling area, (b) F is clothing and z_F is consumption of clothing measured in terms of weight of the clothes consumed, (c) F is transport and z_F is transport consumption measured in terms of CO_2 output from the transport activites, and (d) F is wine and z_F is wine consumption measured in terms of pure alcohol.

It may sometimes be of considerable interest to see how consumption of aggregates are influenced by prices of (more) homogeneous good. For example one could analyse how the consumption of transport, measured in expenditure at constant prices, person-kilometers and implied CO_2-output, relates to user costs of different types of cars, gasoline prices, prices of collective transportation etc. Theorem 8 is concerned with such types of demand functions.

<u>*Theorem 8*</u>*:*

If the utility function is weakly separable in N groups (Assumption 2), the demand for <u>linear consumption aggregates as functions of the prices of the homogeneous goods</u> (20), satisfy the following properties, in addition to those given in Theorem 3.

(i) The demand for any linear consumption aggregate (z_I) *of a commodity group, is a function of the expenditure* (y_I) *on the commodity group and the prices* ($\underline{p}_I$) *on the goods within this group,*

(55a) $$z_I = \Sigma_{SI} \theta_i \overset{*}{g}_i (y_I, \underline{p}_I) = \overset{*z}{g}_I (y_I, \underline{p}_I), \qquad i \in SI, \quad I = 1,\dots,N,$$

Total expenditure (or utility) and the prices of the goods in the other groups enter only through the group expenditure functions (42). The *conditional demand functions* $\overset{*z}{g}_I$ *are homogeneous of degree zero in* y_I *and* $\underline{p}_I$.

(ii) The *cross-price Slutsky* *elasticity of a linear consumption aggregate with respect to the price of a homogeneous good in another group is equal to the product of four terms:*

(55b) $$\varepsilon^z_{Ij} = \mu_{IJ} E^z_I E_j w_j, \qquad j \in SJ, \quad I \neq J, \quad I,J = 1,\dots,N,$$

i.e. the Engel elasticity of the linear consumption aggregate (E^z_I), *the Engel elasticity of the homogeneous good which price increases* (E_j), *the budget share of the commodity group which price increases* (w_j) *and the substitution parameter* (μ_{IJ}) *between the two commodity groups.*

(iii) The Engel elasticity and every cross-price Cournot and Slutsky elasticity for an arbitrary linear consumption aggregate (z_I) *is* *proportional* *to the corresponding Engel, Cournot or Slutsky elasticity for the group expenditure* (y_I) *of the same commodity group:*

(56a) $$E^z_I = \overset{*z}{E}_I \, E^y_I \qquad I = 1,\dots,N,$$

(56b) $$e^z_{Ij} = \overset{*z}{E}_I \, e^y_{Ij} \qquad j \in SJ, \quad J \neq I, \quad I,J = 1,\dots,N,$$

(56c) $$\varepsilon^z_{Ij} = \overset{*z}{E}_I \, \varepsilon^y_{Ij} \qquad j \in SJ, \quad J \neq I, \quad I,J = 1,\dots,N,$$

with the same factor of proportionality, which is the conditional Engel elasticity ($\overset{*z}{E}_I$) *of the linear consumption aggregate, i.e. the elasticity of* $\overset{*z}{g}_I$ *w.r.t.* y_I.

(iv) Any cross-price Cournot and Slutsky elasticity ($e^z_{Ij}, \varepsilon^z_{Ij}, j \neg\in I$) *of a linear consumption aggregate has the* *opposite sign* *of the corresponding Cournot or Slutsky elasiticty* ($e^y_{Ij}, \varepsilon^y_{Ij}$) *for the expenditure on the the same commodity group, if and only if the Engel elasticity* (E^z_I) *of the linear consumption aggregates has the opposite sign of the Engel elasticity* (E^y_I) *of the group expenditure.*

(v) The cross-price Slutsky elasticity ($\varepsilon^z_{Ij}, j \neg\in I$) *of a linear consumption aggregate w.r.t. the price* (p_j) *of a homogeneous good, is*

proportional to the cross-price Slutsky elasticity $(\varepsilon^z_{IJ}, J \neq I)$ *of the same linear consumption aggregate w.r.t. the group price* $(P_J, j \in J)$ *of the commodity group of which the homogeneous good belongs,*

$$(57) \qquad \varepsilon^z_{Ij} = \varepsilon^z_{IJ}\,(E_j w_j)/(E_J \tilde{w}^x_J), \qquad j \in SJ, \quad I \neq J, \quad I,J = 1,\ldots,N,$$

where the factor of proportionality $(E_j w_j / E^x_J w_J)$ *is the ratio between the Engel elasticity times the budget share of the homogeneous good whose price increases and the Hicksian Engel elasticity times the budget share of the commodity group to which this homogeneous good belongs. (Assuming that the price vector p belongs to the hyperplane given by Assumption 1, where* ε^z_{IJ} *"points within" this plane (from an arbirtry point in the plane) while* ε^z_{Ij} *"points out of" this plane (from the same point in the plane).)*

Proof: see the appendix.

Comments on Theorem 8:

(i) Note that (56) is analogous to (47) and to (50) except for the following two points. (a) We have not used the Hicksian aggregate as a point of reference, since Hicksian aggregates are restricted to the price space given by (21) which is no longer assumed. (b) (56) does not hold for I=J. If we exploit the homogeneity property, we (only) obtain the following restiction on the sum of the "within group" Slutsky elasticities:

$$(58) \qquad \sum_{j \in SI} \varepsilon^z_{Ij} = -\,E^z_I \sum_{j \neg\in SI} \mu_{IJ} E_j w_j\,,$$

which is proven by inserting (55b) into (39b).

(ii) Note that all our elasticities and other parameters refer to some point in the space of prices and total expenditure (or utility). When combining two formulas of elasticities we (implicitly) assume that they refer to the same point in this space. Note further that ε^z_{IJ} are defined only in the hyperplane determined by (21), they are partial elasticities in the space of group prices $(P_1,\ldots,P_N)$, and directional elasticities in the space of prices $(p,\ldots,p_n)$ of homogeneous goods with directions only within the hyperplane (21). The elasticities ε^z_{Ij}, on the other hand, are also defined outside the hyperplane (21), and since they are defined as partial elasticities in the space of prices of homogeneous goods they point in a direction not included in the hyperplane (21). When proving (57) we combined (55b) and (49b), which means that all the elasticities and budget shares in (57) are calculated at the same (arbitrary) point of prices belonging to the hyperplane (21) and where ε^{z}_{IJ} points along this plane while ε^{z}_{Ij} points out from this plane.

At the end of this section we shall note that it is possible to reduce the number of substitution parameters (μ_{IJ}) by introducing further separability assumptions. Let us consider the case where the utility function can be written:

(59a) $$u = f^{*}(f_A(\upsilon_1(\underline{q}_1),\ldots,\upsilon_K(\underline{q}_K)),\upsilon_{K+1}(\underline{q}_{K+1}),\ldots,\upsilon_N(\underline{q}_N)).$$

We see that this is a special case of (41), assuming further that a set A of the commodity groups is separable from all the other commodity groups. By the same argumentation this again implies (45) with the further restrictions that

(59b) $$\mu_{IJ} = \mu_{AJ}, \qquad I \in \{1,2,\ldots,K\}, \quad J \in \{K+1,\ldots,N\}.$$

We may say that (59) is an example of hierarchical separability, and there are of course lots of possibilities for introducing more complex hierarchies, obtaining further restrictions on the substitution parameters (μ_{IJ}).

6. AN EMPIRICAL ILLUSTRATION

In order to indicate the potential power and relevance of the previous theoretical results for econometric work, we shall present a simple numerical example of a system of demand elasticities with two types of consumption concepts. We have not specified a full econometric model and estimated all the parameters simultaneously from a single data set. But we have pooled different types of empirical information and arrived at a consistent set of demand elasticities, exploiting our theoretical results. (This example was earlier presented in Aasness (1984), and it was in this connection I developed the basic ideas in this essay.)

Consider the following utility function,

$$u = f^{*}[f_A(f_B[\upsilon_1(\underset{\sim}{q}_1),\upsilon_2(\underset{\sim}{q}_2)],\upsilon_3(\underset{\sim}{q}_3)),\upsilon_4(\underset{\sim}{q}_4)]. \tag{60}$$

This type of hierachical separability implies the following restrictions on the Slutsky elasticities for commodity aggregates of the four groups:

$$(61)\quad \begin{cases} \varepsilon^{z}_{4J} = \mu_{4A}\; E_4\; E^{z}_{J}\; w^{x}_{J}, & J = 1,2,3, \\ \varepsilon^{z}_{3J} = \mu_{3B}\; E_3\; E^{z}_{J}\; w^{x}_{J}, & J = 1,2, \\ \varepsilon^{z}_{21} = \mu_{21}\; E_3\; E^{z}_{1}\; w^{x}_{1}. & \end{cases}$$

Thus we have only three substitution parameters (μ_{4A}, μ_{3B}, μ_{21}) in this model. When the budget shares, Engel elasticities and substitution parameters are given, all the price elasticities can be computed from (41), (31c) and (30c).

In our application we shall consider the consumer as a representative consumer for Norway, simulating how per capita demand changes with per capita total expenditure and proportional price changes within the four commodity groups. These groups are Fish, Meat, Other foods and Other goods. For all four groups the consumption is measured as expenditure at constant prices, and Fish and Meat are also measured in kilograms (of the eatable parts of the different products). Numerical values of the demand elasticities and budget shares are presented in table 1. The budget shares are taken from the 1980-1982 survey of consumer expenditure in Norway, cf CBS (1984). The Engel elasticities are based upon regression analysis of Norwegian surveys of family budgets from the period 1973-76, cf Aasness (1977, 1979). The substitution parameters are given the following values: μ_{21}=40, μ_{3B}=8, μ_{4A}=1/2. The ranking reflects the idea that Fish and Meat are close substitutes, and Other foods is a closer substitute to Fish and Meat than to Other goods. The magnitude of the substitution parameters are chosen so that the models generates direct price elasticities which are in reasonable correspondence with numerous empirical studies in Norway and other countries, relying on the authors personal judgement when pooling the information that was available.

We shall give a few comments to the demand elasticities in table 1. The Engel elasticity for Fish is twice as big when measuring consumption in expenditure at constant prices than when measuring it in quantity (kilograms). This reflects the fact that cheap fish products have low Engel elasticities and expensive products have high Engel elasticities. For example Aasness (1977) estimated the Engel elasticity to be 0.03 for Frozen saithe (cheap in Norway) and 1.16 for Fresh salmon and trout (expensive).

TABLE 1
A SYSTEM OF SLUTSKY, COURNOT, AND ENGEL ELASTICITIES WITH TWO LINEAR CONSUMPTION AGGREGATES FOR TWO OF THE COMMODITY GROUPS

		Slutsky elasticities				Cournot elasticities					
		Change in price of				Change in price of					
Linear consumption aggregate	Commodity group	Fish	Meat	Other foods	Other goods	Fish	Meat	Other foods	Other goods	Engel elasticities	Budget shares
Expenditure	Fish	-0,61	0,36	0,11	0,14	-0,61	0,34	0,07	-0,10	0.30	0,015
at constant	Meat	0,09	-0,50	0,18	0,23	0,08	-0,53	0,12	-0,17	0,50	0,060
prices	Other foods	0,01	0,09	-0,26	0,16	0,01	0,06	-0,30	-0,12	0,35	0,130
	Other goods	0,00	0,02	0,03	-0,05	-0,01	-0,05	-0,13	-0,97	1,16	0,795
Quantity	Fish	-0,30	0,18	0,05	0,07	-0,31	0,17	0,04	-0,05	0,15	
(kg)	Meat	0,07	-0,40	0,15	0,18	0,07	-0,43	0,09	-0,13	0,40	

With a proportional increase in the prices of all fish products the consumption of Fish decreases twice as much when measured in expenditure at constant prices than when measured in quantity. Thus the consumption of expensive fish products decreases relatively more. The consumption of inferior fish products will even increase, when the prices of all fish products increase proportionally, which can be easily shown in our model.

When the prices of all meat products increase proportionally the consumption of Fish increases, and most so if measured in expenditure at constant prices. Thus the consumption of expensive fish products increases relatively more than cheap fish products when meat prices increases.

7. CONCLUSIONS

1. In this essay, we have introduced the concept of a linear consumption aggregate of a commodity group, defined as a weighted sum of the quantities consumed of the (homogeneous) goods within the group, where the weights are some non-negative scalars independent of the consumption of the goods. Hicksian aggregates in classical consumer theory, and Laspeyres volume indexes of consumption categories in systems of national accounts are well known special cases of linear consumption aggregates. We have also given many examples of linear consumption aggregates with physical interpretations, e.g. energy content in the consumed quantities. It is indeed possible to give numerous examples of different types of linear consumption aggregates which can be of considerable interest for some theoretical, empirical and/or practical issue. It thus seeems obvious that developing general results in consumer theory for linear consumption aggregates could be fruitful. This essay shows that many interesting theorems, not found elsewhere in the literature, can be easily derived, and I believe more will be developed in the future.

2. We have introduced several different types of demand functions, including demand for linear consumption aggregates as functions of prices of the homogeneous goods, demand for linear consumption aggregates as functions of group prices, and demand for homogeneous goods as functions of group prices. These group prices are defined through assuming constant relative prices within each group. Although the latter may not be an appropriate assumption as a description of the price fluctuations of a real economy, it may be a useful analytic device, and it may also be a suitable approximation for some empirical analysis. Properties of the demand functions are presented in eight theorems. All the demand functions satisfy Slutsky equations and homogeneity properties as in the standard theory, but adding-up, negativity and symmetry are not satisfied in general. Some or all of these latter properties hold, however, when further restrictions are imposed, of which several examples are given. One type of such (testable) restrictions involves certain covariances of demand elasticities and consumption shares for homogeneous goods (cf. theorem 2 and 3).

3. Combining an assumption of weak separability of the direct utility function for a set of commodity groups, with constant relative prices within each groups, we obtain strong results. For example, if we have a complete system of demand functions for Hicksian aggregates of the

commodities, the only additional information needed is one Engel function for each linear consumption aggregate we introduce, in order to derive the full demand functions for these linear consumption aggregates. Any Slutsky or Cournot elasticity for a linear consumption aggregate will be proportional to the corresponding Slutsky or Cournot elasticity for the Hicksian aggregate, the factor of proportionality being the ratio between the Engel elasticities of the linear consumption aggregate and of the Hicksian aggregate.

4. Linear consumption aggregates can have upward sloping demand curves even if the Giffen case assumptions are excluded. The demand for a linear consumption aggregate (e.g. consumption of bread measured in weight (kilograms)) will increase if the prices of all the homogeneous goods within the group (e.g. types of bread) increase proportionally, provided that (i) the utility function is weakly separable w.r.t. the commodity group, (ii) the Engel elasticity for the linear consumption aggregate is negative and the Engel elasticity for the corresponding Hicksian aggregate is positive, and (iii) the Slutsky elasticity for the Hicksian aggregate is strictly negative. In this case the positive direct Cournot elasticity of the linear consumption aggregate is equal to the sum of a positive direct Slutsky elasticity (substitution effect) and a positive income effect. This might be a starting point for a reexamination of old empirical evidence for positive sloping demand curves, and for looking for new data to test such hypotheses.

5. The theory presented provides a framework and/or a starting point for formulating many types of econometric models to be estimated and hypotheses to be tested. In the previous sections some comments are given indicating possible directions of such analysis, but detailed and systematic discussion of these issues and empirical analysis is left for future research. This essay has provided a framework for combining, in a unified analysis, consumption measures of different types. For example one may supplement Laspeyres aggregates for all commodity groups with physical aggregates for some groups, in order to test more hypotheses, make estimation more efficient, and/or increase the applicability of the results. Sometimes Lapeyres aggregates are not available for some groups, while physical aggregates are. These two pieces of information can be combined in a consistent system of demand functions using the theoretical results above. Often different data sources, e.g. national accounts and household expenditure surveys, give quite different estimates for e.g.

average expenditure on the same commodity group. This may be due to systematic measurement errors in one or both (or all) sources. By assuming that the systematic measurement errors are proportional to the true consumption of the homogeneous goods (or detailed commodity groups) the theorems in this essay can be used to reconcile seemingly conflicting results, and to formulate new testable hypotheses in an important but almost neglected research area in consumer econometrics. Furthermore, we may estimate some part of a model from one data source, say a system of demand functions for 10 Hicksian aggregates from national accounts data, and another part of the model from another data source, say Engel functions for 500 "homogeneous" goods (which can be aggregated to the 10 groups) from one cross section of family budgets. Applying theorems 4 and 5, these parts can be combined into a consistent model of the demand for 500 goods as functions of 10 group prices, and where the demand function for any linear consumption aggregate can be computed by adding assumptions about the weights (θ_i) for the relevant "homogeneous" goods. It could also be noted that the theory provides new ways of testing separability assumptions.

6. Some decision makers (e.g. a ministry of finance), or their economic advisers, are often interested in some promptly delivered rough estimates of price elasticities based on available empirical studies and their own judgement. This essay provides the formulas necessary to calculate the relevant Slutsky and Cournot elasticities from Engel elasticities, budget shares and substitution parameters (μ_{IJ}) for separable groups. Estimates of budget shares and Engel elasticities are often available, substitution parameters can be restricted by assumptions of e.g. hierarchical weak separability, and calibrated by available empirical evidence on price elasticities (using e.g. a work sheet program with our formulas implemented). A simple case study of such an approach is presented, providing a numerical example of a system of price elasticites for different linear consumption aggregates, which are interpreted. This example should make it clear that demand functions for two (or more) linear consumption aggregates of the same commodity groups can provide information on the changes in the consumption shares of the homogeneous goods within the commodity group when group prices or total expenditure change.

APPENDIX: PROOF OF THEOREMS

First we note that the existence and differentiability of all the demand functions introduced in section 3, follows from our assumptions and basic calculus, since these functions can be considered as composite functions of the "elementary" functions given by (3), (14) and (21) which are all continuous differentiable.

Proof of Theorem 1

(iv) Equations (26) can be derived directly from (23) by using the "chain rule" for elasticities, cf Sydsæter (1981,p 136).

(i) The Slutsky equations (25a) is proven by combining (26a), (6) and (26b) in this order: $e_{iJ} = \Sigma_{SJ} e_{ij} = \Sigma_{SJ}\varepsilon_{ij} - \Sigma_{SJ} E_i w_j = \varepsilon_{iJ} - E_i w_J$.

(ii) The homogeneity property in terms of Slutsky elasticties (31a) is proven easily by inserting (26b) in the left hand side of (25b) and using (7): $\Sigma_{J=1}^{N}\varepsilon_{iJ} = \Sigma_{J=1}^{N}\Sigma_{j\in SJ}\varepsilon_{ij} = \Sigma_{j=1}^{n}\varepsilon_{ij} = 0$. Homogeneity in terms of Cournot Engel elasticities follows from homogeneity in terms the Slutsky elasticities (25b) and the Slutsky equations (25a). We can also prove the homogeneity property directly in terms of the function G_i, using (21) and (23) and the fact that g_i is homogeneous of degree zero: $G_i(ky,kP_1,\dots,kP_N) = g_i(ky,kP_1p_1^o,\dots,kP_Np_n^o) = g_i(y,P_1p_1^o,\dots,P_Np_1^o) = G_i(y,P_1,\dots P_N)$, where k is some positive scalar. The homogeneity of H_i can be proven correspondingly. The elasticities formulas can alternatively be derived from these relations by taking the elasticity w.r.t. the scalar k and using the chain rule for elasticities.

(iii) The adding-up property follows by inserting (23) in (2), and using standard rules for elasticities (cf Sydsæter (1981,section 3.14)).

Proof of Theorem 2

(v) Equations (31) follows directly from (22) and standard rules for elasticities (cf Sydsæter (1981,section 3.14)).

(i) The Slutsky equations (30c) are proven by combining (31b), (25a), (31c) and (31a): $e_{IJ}^z = \Sigma_{SI} e_{iJ} z_i/z_I = \Sigma_{SI}\varepsilon_{iJ} z_i/z_I - \Sigma_{SI} w_J E_i z_i/z_I = \varepsilon_{IJ}^z - E_I^z w_J$, $i\in SI$, $I,J=1,\dots,N$.

(ii) The homogeneity property in terms of Slutsky elasticities (28b) is proven by substituting (31c) into the left hand side of (28b), rearranging and using (25b): $\Sigma_{J=1}^{N}\varepsilon_{IJ}^z = \Sigma_{J=1}^{N}\Sigma_{i\in SI}\varepsilon_{iJ} z_i/z_I = \Sigma_{i\in SI} z_i/z_I\ \Sigma_{J=1}^{N}\varepsilon_{iJ} = 0$.

Other versions of homogeneity can be proved analogously to the proof of Theorem 1 (ii).

(iii) For Hicksian aggregates the following property hold,

$$(\$) \qquad \frac{x_i}{x_I} = \frac{w_i}{w_I}, \qquad \forall i \in SI, \quad I = 1,\dots,N,$$

i.e. the consumption share of commodity i in commodity group I is equal to the budget share of commodity i in group I. This follows from substituting (17) in the left hand side of ($), multiplying by P_I, using (21) and the definitions of budget shares:

$$\frac{x_i}{x_I} = \frac{p_i^o q_i}{\Sigma_{SI} p_i^o q_i} = \frac{P_I p_i^o q_i}{\Sigma_{SI} P_I p_i^o q_i} = \frac{p_i q_i}{\Sigma_{SI} p_i q_i} = \frac{w_i}{w_I}.$$

This is the main property we shall exploit below which is specific to Hicksian aggregates.

The adding-up property (29a) for Hicksian aggregates is proven by substituting (31a), for the special case of Hicksian aggregates, in the left hand side of (29a), then using ($) and at last (8): $\Sigma_{I=1}^{N} E_I^x w_I = \Sigma_{I=1}^{N} \Sigma_{i \in SI} E_i w_I x_i / x_I = \Sigma_{i=1}^{n} E_i w_i = 1$. The adding-up condition (29a) will not hold for linear consumption aggregates in general, counter examples are easily found. For example, if $E_n \neq E_N^x$ we may choose a z^o with weights $\theta_n = 1$, $\theta_i = 0$ $i \in SN\text{-}\{n\}$, $\theta_i = p_i^o$ $i \in SI$, $I = 1,\dots,N\text{-}1$. Thus, according to (31a), $E_N^{zo} = E_n \neq E_N^x$ and $E_I^{zo} = E_I^x$, $I = 1,\dots,N\text{-}1$. From this and (29a) it follows that $\Sigma_{I=1}^{N} E_I^{zo} w_I = 1 + E_N^x - E_n \neq 1$.

The symmetry property (29b) for Hicksian aggregates is proven by exploiting in the following order (31c), (26b), ($), (9), ($), (26b) and (31c): $\varepsilon_{IJ} w_I = w_I \Sigma_{SI} \varepsilon_{iJ} x_i / x_I = w_I \Sigma_{SI} x_i / x_I \Sigma_{SJ} \varepsilon_{ij} = \Sigma_{SI} \Sigma_{SJ} w_i \varepsilon_{ij}$ $= \Sigma_{SI} \Sigma_{SJ} w_j \varepsilon_{ji} = w_J \Sigma_{SJ} x_j / x_J \Sigma_{SI} \varepsilon_{ji} = w_J \Sigma_{SJ} \varepsilon_{jI} x_j / x_J^x = \varepsilon_{JI} w_J$, $i \in SI$, $j \in SJ$, $I, J = 1,\dots,N$. The symmetry (29b) will not hold for linear consumption aggregates in general, counter examples are easily found. For example, using the z^o aggregate above and assuming $\varepsilon_{nJ} \neq \varepsilon_{NJ}^x$, we obtain using (31c) and (29b): $\varepsilon_{NJ}^{zo} w_N = \varepsilon_{nJ} w_N \neq \varepsilon_{NJ}^x w_N = \varepsilon_{JN}^x w_J = \varepsilon_{JN}^{zo} w_J$, $J = 1,\dots,N\text{-}1$.

The negativity property (29c) for Hicksian aggregates is proven by combining (31c) and (26b), then using (21) and rearranging terms,

$$\varepsilon_{II}^x = \Sigma_{SI} \Sigma_{SI} \frac{x_i}{x_I} \varepsilon_{ij} = \frac{P_I P_J}{x_I} \Sigma_{SI} \Sigma_{SI} p_i^o p_j^o \varepsilon_{ij} q_i / p_j,$$

and observing that the last expression is proportional to the left hand side of (10), setting $\xi_i = p_i^o$ $\forall$ $i \in SI$ and $\xi_i = 0$ $\forall$ i not belonging to SI.

(iv) The conditional negativity property (30) for linear consumption aggregates follows from (31c). If the conditions in the brackets is not true, i.e. there exist a good $k \in SI$ such that $\varepsilon_{kI}>0$, then we may choose an aggregate z^1 such that $\theta_k>0$, $\theta_i=0$ $\forall$ $i \in SI-\{k\}$, so that, using (31c), $\varepsilon_{II} = \varepsilon_{kI} > 0$. Thus the negativity property (29c) will not hold for linear consumption aggregates in general.

(vi) The relationsships (32) follows from (31) and comment (i) on Th.2.

(vii) Equations (33) follows from subtracting from (31) the corresponding equations in the special case of Hicksian aggregates.

(viii) Relations (34-35) follows from (33) and the Slutsky equations (28a), cf comment (iii) on Theorem 2.

Proof of Theorem 3

The proof is analogous to the proof of the corresponding results in Theorem 2, substitute the subscript J with j.

Proof of Lemma 1 (ii)

In order to make the proof as simple as possible we shall assume that all Engel elasticities are different from zero. Define μ_{IJ}^{ij} by

(*) $\quad \mu_{IJ}^{ij} \equiv \varepsilon_{ij}/E_i E_j w_j, \qquad i \in SI, \quad j \in SJ, \quad I \neq J, \quad I,J = 1,\ldots,N.$

We shall prove that μ_{IJ}^{ij} is independent of i and j. From Lemma 1 (i), and the chain rule for elasticities, it follows that $\varepsilon_{ij} = \varepsilon_{Ij}^{y}\overset{*}{E}_i = \varepsilon_{Ij}^{y}E_i/E_I^{y}$. Inserting this in (*) we obtain $\mu_{IJ}^{ij} = \varepsilon_{Ij}^{y}/E_I^{y}E_j w_j \equiv \mu_{IJ}^{j}$ which is independent of i. Correspondingly, we have that $\varepsilon_{ji} = \varepsilon_{Ji}^{y}\overset{*}{E}_j = \varepsilon_{Ji}^{y}E_j/E_J^{y}$, and inserting this in (*) after using the symmetry condition (9) we obtain: $\mu_{IJ}^{ij} = \varepsilon_{ji}/E_i E_j w_i = \varepsilon_{Ji}^{y}/E_J^{y}E_i w_i \equiv \mu_{IJ}^{i}$ which is independent of j. Thus: $\mu_{IJ}^{ij} = \mu_{IJ}^{j} = \mu_{IJ}^{i} \equiv \mu_{IJ}$ which is indpendent of i and j, as was to be proven.

Proof of Theorem 4:

(i) From (45a) and the homogeneity property it follows that $q_i = g_i^*(y_I/P_I, \underline{p}_I/P_I)$, this and (17) and (21) implies that $q_i = g_i^*(x_I, p_I^o)$.

(ii) Equations (46b) follows from applying in the following order (26b), (45b), ($) and (31a):

$$\varepsilon_{iJ} = \Sigma_{SJ}\varepsilon_{ij} = \Sigma_{SJ}\mu_{IJ}E_i E_j w_j = \mu_{IJ}E_i w_J \Sigma_{SJ}E_j x_j/x_J = \mu_{IJ}E_i E_J^x w_J.$$

(iii) Equations (47) follows from (46a) and (43) and the "chain rule" for elasticities, cf Sydsæter (1981,p 136). ((47) can alternatively be derived from (46b), the homogeneity property (25b) and the Slutsky equations (25a).)

(iv) follows from (iii).

(v) follows from (47c) and the negativity property (29c) for Hicksian aggregates.

Proof of Theorem 5:

(i) follows from (16) and Theorem 4(i).

(ii) Equations (49b) follows from applying in the following order (31c), (46b), and (31a):

$$\varepsilon^z_{IJ} = \Sigma_{SI}\varepsilon_{iJ}z_i/z_I = \Sigma_{SI}\mu_{IJ}E_i E^x_J w_J z_i/z_I = \mu_{IJ}E^x_J w_J \Sigma_{SI}E_i z_i/z_I = \mu_{IJ}E^z_I E^x_J w_J .$$

(iii) Equations (50) follows from (49a) and (43) and the "chain rule" for elasticities, cf Sydsæter (1981,p 136). ((50) can alternatively be derived from (49b), the homogeneity property (28b) and the Slutsky equations (28a).)

(iv) follows from (iii).

(v) follows from (50c) and the negativity property (29c) for Hicksian aggregates.

(vi) The generalized symmetry property (51) is immediately obtained by combining (49b) for ε^z_{IJ} and ε^z_{JI}.

Proof of Theorem 8:

(i) follows from (16) and Lemma 1 (i).

(ii) The proof is analogous to the proof of Theorem 5 (ii), just substitute the index J of a commodity group with the index j of a homogeneous good.

(iii) Equations (56) follows from (55a) and (42) and the "chain rule" for elasticities, cf Sydsæter (1981,p 136).

(iv) follows from (iii).

(v) Equation (57) is obtained by combining (49b) and (55b). An interpretation which makes such a combination meaningful is given in comment (ii) on Theorem 8.

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