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LABOR SUPPLY, INCOME DISTRIBUTION AND EXCESS BURDEN OF PERSONAL INCOME TAXATION IN NORWAY*)

BY

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*) A similar tax analysis based on an alternative model was presented at the conference of the European Labor Markets, Florence 18th-20th May 1989. The present paper is an extension of this paper and contains new results.

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ABSTRACT

This paper presents the results of an empirical analysis of labor supply for married couples in Norway based on a new modelling approach. This new framework is particularly convenient for dealing with complicated budget constraints such as the Norwegian case, and it can also account for restrictions on hours of work.

The purpose of the present paper is to apply the estimated labor supply model to investigate the effect from different tax reforms. In particular, we study the effects on labor supply and income distribution when the 1979 tax rules are replaced by proportional taxes on gross earnings and lump-sum taxes. The proportional tax rate is derived under the constraint that the personal income tax revenue should remain unchanged and equal to the revenue in 1979.

The paper also reports an estimate of the cost of taxation from changing the tax system. The estimate is based on the equivalent variation measure.

1. INTRODUCTION

Measuring the cost and redistributive effects of taxation is of major concern for evaluating tax reform policies. This paper presents a particular methology for estimating the excess burden and the redistributive effects of taxation as well as an application of this methodology to the tax system in Norway, 1979. The methodology we propose is based on a labor supply model developed and estimated by Dagsvik and Strøm (1990).

A key assumption in the model is that the individuals choose from a set of infinite and countable alternatives called matches. Each match is characterized by a wage rate, hours of work and non-pecuniary attributes. Thus, our model may be viewed as an extension of a discrete model with finite choice sets.

Welfare measurement with discrete choice models has been studied by Small and Rosen (1981), Hanemann (1982) and Hau (1985). As opposed to Hau (1985) we assume that the individuals have perfect knowledge of their own set of opportunities and tastes. A random formulation arises because the model contains components in the utility structure and in the opportunity set which are not observed by the econometrician.

The estimated model is used in simulations to demonstrate the impact on labor supply and income distribution of replacing the actual tax rules by a proportional tax on gross wage earnings, given a constant total tax revenue. As a money measure of the utility changes we use the equivalent variation. Alternatively, we could have used the compensating variation. For arguments in favor of measuring the cost of taxation by equivalent variation we refer to Kay (1980) and Browning (1987).

Since our model is stochastic it is not obvious how an equivalent variation measure should be defined. Hanemann (1982) suggests three alternative approaches. The first alternative is to derive the distribution of the equivalent variation and then to use the mean of its distribution as basis for measuring excess burden. The second alternative is to use the median of the distribution. The third alternative is obtained by equating the expected utility under the actual and the alternative tax regime. This third alternative is used by Small and Rosen (1981). While the two first alternatives imply that the compensating measure is invariant with respect to a monotone transformation of the utility function, the third one is invariant only in the case of no income effects. Our approach is based on the first alternative. Excess burden is defined as the mean in the distribution of the equivalent variation divided by the mean in the distribution of the tax revenue. The excess burden is calculated within a partial equilibrium framework in the sense that the wage distribution is kept unchanged when taxes are changed.

Sections 2 and 3 present the econometric model. Only a brief description is given and the readers are referred to Dagsvik and Strøm (1990) for further details. Section 4 deals with the estimation results. Section 5 gives the results of various policy simulations including the estimates of the excess burden of taxation.

2. THE MODEL

In this model the individual (couple) is assumed to choose from a set of hours-wage packages called matches. A match is defined as a particular combination of skills offered (by the individual) and qualifications required to perform specific tasks. We assume that the individual has perfect knowledge about his opportunities, but due to unobserved heterogeneity across individuals the set of feasible matches is viewed as random by the econometrician. Let z=1,2,... be an enumeration of the matches. Match z includes fixed hours of work, H(z), wage rate, W(z), qualifications demanded, $T_2(z)$, and skills offered, $T_3(z)$. For non-market matches, $H(z) = W(z) = T_2(z) = T_3(z) = 0$.

The individual's economic budget constraint, conditional on match z, is given by

$$(2.1) \quad C = C(z) = f(H(z)W(z)+I)$$

where C is consumption, I is nonlabor income and $f(\cdot)$ is the function that transforms gross income to income after tax. The form of the function f depends on the tax system and of the rules of social security payments, etc. It may be non-differentiable, non-concave and even discontinuous at some points. Tax rules are described and discussed in Appendix 2. Let

$$(2.2) \quad T_1(z) = \theta(T_2(z),T_3(z))$$

where $\theta(\cdot)$ is a "distance" function in the sense that it attains low value to matches where the difference between skills offered and demanded is large.

The individuals utility function is assumed to have the form

$$(2.3) \quad U(C,h,z) = v(C,h,T_1(z)) + \varepsilon(z)$$

where $v(h,C,t)$ is a deterministic function that is quasi-concave in (h,C), decreasing in h and increasing in C for fixed t. $\varepsilon(z)$ is a random variable that is supposed to account for unobserved heterogeneity in tastes. Moreover, the utility function is supposed to depend on how well the individual is fit for the match measured through $T_1(z)$.

As mentioned above the collection of matches feasible to the individual is random and consequently the set of feasible attributes and taste-shifters $\{H(z),W(z),T(z),\varepsilon(z)\}$ where $T(z) = (T_1(z),T_2(z))$ is random. Specifically, we assume that $\{H(z),W(z),T(z),\varepsilon(z)\}$ are the points of a Poisson process on $[0,\bar{h}]x[0,\bar{w}]x[0,1]^2xR$ with intensity measure

$$(2.4a) \quad \lambda(h,w,t_1,t_2)dhdwdt_1dt_2 \cdot e^{-\varepsilon}d\varepsilon$$

for market matches and

$$(2.4b) \quad \lambda(0,0,0,0)e^{-\varepsilon}d\varepsilon$$

when $h=w=t_1=t_2=0$. Eq. (2.4) means that the probability that a match for which

$$(H(z)\in(h,h+dh),W(z)\in(w,w+dw),T(z)\in(t,t+dt),\varepsilon(z)\in(\varepsilon,\varepsilon+d\varepsilon))$$

is feasible, is equal to

$$\lambda(h,w,t_1,t_2)dhdwdt_1dt_2 \cdot e^{-\varepsilon}d\varepsilon + o(dhdwdt_1dt_2d\varepsilon).$$

We assume that

$$(2.5a) \quad \lambda(h,w,t_1,t_2) = \mu g_1 g_2(h)g_3(w|t_2)g_4(t_1)g_5(t_2)$$

and

(2.5b) $\lambda(0,0,0,0) = \mu(1-g_1)$

where $\mu>0$ is a constant $g_1\in[0,1]$, $g_2(h)$, $g_3(w|t_2)$, $g_4(t_1)$ and $g_5(t_2)$ are probability densities. As demonstrated in Dagsvik and Strøm (1990), $g_2(h)$ can be interpreted as the density of feasible hours (relative to the individual) offered by the firms. The densities g_j, $j>2$, can be interpreted similarly. The interpretation of g_1 is as the fraction of feasible matches that are market matches. The particular decomposition (2.5a) means that offered hours and wages are independent. Moreover, offered hours are independent of $\{T(z)\}$ and wages are independent of $\{T_1(z)\}$. These assumptions are justified as follows: Offered hours of work are often determined by the nature of the tasks to be performed and by institutional regulations independent of wages and individual and firm-specific characteristics. However, as demonstrated below the assumption of independence between <u>offered</u> hours and wages does not exclude the possibility of dependence between <u>realized</u> hours and wages. The assumption that $g_3(w|t_2)$ does not depend on t_1 may be more difficult to defend since one may claim that wages may depend on how well the individual is fit for the job. However, if we let $g_3(w|t_2)$ also depend on t_1 we run into serious identification problems. Anyhow, we believe that the main wage determinants are the individual qualifications represented by $\{T_2(z)\}$.

Let us now consider the realized hours and wage distribution in the market. Let $\varphi(h,w)$ be the probability density of the realized hours of work and wages, i.e., the hours-wage combination that corresponds to the match that yields the highest utility. According to Dagsvik (1988) the Poisson process assumption and (2.4) imply that

$$\text{(2.6a)}\quad \varphi(h,w,t_1,t_2)=\frac{\iint e^{\psi(h,w,t_1)}\lambda(h,w,t_1,t_2)dt_1dt_2}{\iiiint e^{\psi(x,y,t_1)}\lambda(x,y,t_1,t_2)dxdydt_1dt_2+e^{\psi(0,0,0)}\lambda(0,0,0,0)}$$

for $h>0$, $w>0$, $t_1>0$, $t_2>0$ and

$$\text{(2.6b)}\quad \varphi(0,0,0,0)=\frac{e^{\psi(0,0,0)}\lambda(0,0,0,0)}{\iiiint e^{\psi(x,y,t_1)}\lambda(x,y,t_1,t_2)dxdydt_1dt_2+e^{\psi(0,0,0)}\lambda(0,0,0,0)}$$

where

(2.7) $\psi(h,w,t_1) = v(f(hw+I),h,t_1).$

The model above is so far a disequilibrium model in which the opportunity densities must be interpreted as exogenously given. Dagsvik and Strøm op.cit. introduced a concept called quasi-equilibrium (QE). By QE we mean that the hours, wages and $\{T(z)\}$ adjust so that the probability density of the realized hours and wages depends solely on the preference term $\psi(\cdot)$ which is fulfilled when offered hours and wages are uniformly distributed. This means that in a "large" sample the realized market distribution of hours and wages coincides with the distribution of preferred hours and wages. However, in a small sample this may not necessarily be true. The relevant interpretation is that due to market imperfections the adjustments cannot take place so rapidly so as to ensure perfect equilibrium.

In reality there are, however, more severe imperfections. Examples are institutional restrictions imposed by unions and government on hours and wages. Involuntarily unemployment is another example. These and other imperfections prevent the equilibrating process of the QE type described above to take place. Hence, a model of labor supply should allow for a possible deviation between the unconditional distributions of realized and preferred hours and wages. In our model we do this by postulating a partial QE. By this we understand that wages adjust so as to give QE within groups of matches. A group is identified by a specific level of $(H(z),T(z))$. We thus assume that the conditional distribution of realized wages, given hours and attributes $(T(z))$, depends solely on preferences. This implies that the wage rate must be a function of individuals qualifications. It can then be shown that (2.6) implies

(2.8) $W(z) = \tilde{w}(T_2(z))$

where $\tilde{w}(\cdot)$ is a function that satisfies

(2.9) $g_3(\tilde{w}(t_2)|t_2) = 1/\bar{w}.$

Thus if (2.8) holds the density of offered wage, conditional on qualifications (as measured by $T_2(z)$), is uniform.

From (2.8) and (2.9) it follows that the unconditional wage distribution across qualification groups takes the form

$$(2.10)\quad \tilde{g}(w) = g_5(\tilde{t}_2(w))\left|\frac{d\tilde{t}_2(w)}{dw}\right|$$

where $\tilde{t}_2(\cdot)$ is the inverse mapping of $\tilde{w}(\cdot)$. By inserting (2.10) into (2.6) we get the partial QE density of realized hours and wages

$$(2.11a)\quad \tilde{\varphi}(h,w) = \frac{g_1 \exp(\tilde{\psi}(h,w))g_2(h)\tilde{g}(w)}{g_1\int\int \exp(\tilde{\psi}(x,y))g_2(x)\tilde{g}(y)dxdy+\kappa(1-g_1)\exp(\tilde{\psi}(0,0))}$$

for h>0, w>0 and

$$(2.11b)\quad \tilde{\varphi}(0,0) = \frac{(1-g_1)\kappa\exp(\tilde{\psi}(0,0))}{g_1\int\int \exp(\tilde{\psi}(x,y))g_2(x)\tilde{g}(y)dxdy+\kappa(1-g_1)\exp(\tilde{\psi}(0,0))}$$

where

$$(2.12)\quad \tilde{\psi}(h,w) = \log(\int \exp(\psi(h,w,t_1))g_4(t_1)dt_1)$$

and

$$(2.13)\quad \kappa = \frac{e^{v(C,0,0)}}{\int e^{v(C,0,t_1)}g_4(t_1)dt_1}$$

The interpretation of κ is as a parameter that accounts for the value of non-market matches relative to the value of the market matches evaluated at h=0. In general κ may depend on C, but as a consequence of separability assumptions made in the empirical specification below it follows that κ is independent of C.

3. EXTENSION OF THE MODEL TO TWO-PERSON HOUSEHOLDS (MARRIED COUPLES)

Let $U(C,h_F,h_M,z)$ denote the household's utility function where h_F and h_M denote the wife's and the husband's hours of work, respectively. C is total consumption of the household and $z = (z_F, z_M)$ indexes the matches of the wife, z_F, and husband, z_M, respectively.

The constraints are given by

$$(3.1) \quad (h_F, h_M) = (H_F(z), H_M(z)),$$

$$(3.2) \quad C(z) = f(H_F(z)W_F(z), H_M(z)W_M(z), I)$$

where $H_F(z)$, $W_F(z)$, $H_M(z)$ and $W_M(z)$ are the match-specific hours of work and wages for the wife and for the husband, respectively, I denotes capital income and $f(\cdot)$ is the function that transforms gross income into consumption. In the calculation of $f(\cdot)$ for alternative values of h_j, j=M,F, the details of the tax structure of 1979 are taken into account.

As above let $T_{1F}(z)$ and $T_{1M}(z)$ represent the "distance" attribute for match z relative to female and male, respectively.

Under assumptions that are straight forward extensions of the assumptions of the preceding section we can write

$$(3.3) \quad U(C(z), H_F(z), H_M(z), z)) = v(C(z), H_F(z), H_M(z), T_{1F}(z), T_{1M}(z)) + \varepsilon(z).$$

The corresponding choice densities under partial QE are straight forward extensions of (2.11), see Dagsvik and Strøm (1990).

4. SUMMARY OF ESTIMATION RESULTS

The estimation of the model is based on a sample of data for married couples where the females are between 27 and 66 years of age and where the main income of the family comes from wage work.

The deterministic part of the utility function is assumed to have a Box-Cox form separable in consumption and hours, i.e.,

$$(4.1)\quad \tilde{v}(C,h_F,h_M) = \log\int \exp(v(C,h_F,h_M,t_{1F},t_{1M}))g_4(t_{1F},t_{1M})dt_{1F}dt_{1M}$$

$$= \alpha_2\left(\frac{(10^{-4}C)^{\alpha_1}-1}{\alpha_1}\right) + \left(\frac{L_M{}^{\alpha_3}-1}{\alpha_3}\right)(\alpha_4 + \alpha_5\log A_M$$

$$+ \alpha_6(\log A_M)^2) + \left(\frac{L_F{}^{\alpha_7}-1}{\alpha_7}\right)(\alpha_8 + \alpha_9\log A_F + \alpha_{10}(\log A_F)^2$$

$$+ \alpha_{11}CU6 + \alpha_{12}CO6) + \alpha_{13}L_F L_M$$

where A_F, A_M are the age of the wife and the husband, respectively, CU6 and CO6 are number of children less than 6 and above 6 years, L_k is leisure for gender k = M,F, defined as

$$L_k = 1 - h_k/8760,$$

and α_j, j = 1,2,...13, are unknown parameters. If $\alpha_1<1$, $\alpha_3<1$, $\alpha_7<1$, $\alpha_2>0$,

$$\alpha_4 + \alpha_5\log A_M + \alpha_6(\log A_M)^2 > 0,$$

and

$$\alpha_8 + \alpha_9\log A_F + \alpha_{10}(\log A_F)^2 + \alpha_{11}CU6 + \alpha_{12}CO6 > 0$$

then $\tilde{v}(C,h_F,h_M)$ is increasing in C, decreasing in (h_F,h_M) and strictly concave in (C,h_F,h_M).

The densities of offered hours, $g_{3k}(h_k)$, k=F,M, are assumed uniform except for a peak at full-time hours for males and peaks at full-time and part-time hours for females. The peaks reflect our assumption that observed concentrations of hours around these two working loads are due to restrictions set by firms, unions and government.

Above we assumed that the opportunity distributions for hours were uniform except for full-time and part-time peaks. Unless this or analogous assumptions are made it is not possible to separate some of the structural coefficients in the mean utility function from the parameters of the opportunity densities for hours.

It is of interest to note that since the logarithm of the opportunity density of hours and the utility function enter symmetrically into

(2.11a) it would be possible to interpret the peaks as stemming from preferences in which case the offered hours would be generated by a uniform distribution. In fact, if preferences and the opportunity density of hours are kept fixed we can perform policy simulations with respect to changes in demographic variables, taxes and wage rates based on the estimated model that are consistent with either interpretation.

The wage densities are specified as follows

(4.2) $$\log W_k(z) = \beta_{0k} + \beta_{1k} s_k + \beta_{2k} Exp_k + \beta_{3k} (Exp_k)^2 + \eta_k(z)$$

$k = F,M$, where $(\eta_F(z), \eta_M(z))$ are jointly normally distributed, s_k denote years of schooling, gender k, and Exp_k = experience = $A_k - s_k - 6$. Moreover

(4.3) $$\log\left(\frac{g_{10}\kappa_F}{g_{11}}\right) = \alpha_{14} + \alpha_{15} s_F,$$

(4.4) $$\log\left(\frac{g_{01}\kappa_M}{g_{11}}\right) = \alpha_{16}$$

and

(4.5) $$\log\left(\frac{g_{00}\kappa_{FM}}{g_{11}}\right) = \alpha_{14} + \alpha_{15} s_F + \alpha_{16} + \alpha_{17}.$$

According to the discussion in Dagsvik and Strøm (1990) it is possible to separate κ_F, κ_M and κ_{FM} from the opportunity densities g_{ij}, $i,j=0,1$, by applying data on unemployment. However, since the unemployment rate in 1979 was rather low, close to 1 per cent, we found it of minor importance to separate κ_F, κ_M and κ_{FM} from g_{ij}.

The estimation is based on a procedure suggested by McFadden (1978) which yields results that are close to the full information maximum likelihood method. We are not able to use the exact likelihood function to estimate the model because the evaluation of the integrals in the denominator of the two-person household version of (2.11a) would be too costly and cumbersome. The estimation procedure applied replaces the continuous fourtuple integral in the denominators of the densities by a sum over 30, (alternatively 70), random points, where each term in the sum is adjusted by appropriate weights. In other words, the continuous logit-type model is replaced by a discrete version. McFadden has demonstrated that this method yields consistent and asymptotically normal parameter estimates.

The results of the estimation are reported in Table 1 and 2.

Note that most parameters are rather precisely determined (apart from the cross leisure term) and they have the theoretically expected signs.

Table 1. Estimates of the parameters of the utility function and of the opportunity density

Variables	Coefficients	Estimates	t-values
Consumption	α_1	0.895	19.8
	α_2	1.881	9.2
Male leisure	α_3	-15.531	7.6
	α_4	4.429	1.4
	α_5	-2.299	1.4
	α_{6},	0.306	1.4
Female leisure	α_7	-2.125	4.3
	α_8	222.935	2.7
	α_9	-120.628	2.8
	α_{10}	17.042	2.4
	α_{11}	5.510	7.2
	α_{12}	1.495	4.7
Leisure interaction term	α_{13}	2.179	0.5
Female opportunity density	α_{14}	1.699	2.2
	α_{15}	-0.247	3.3
Male opportunity density	α_{16}	2.644	6.8
Interaction	α_{17}	1.423	3.8
Full-time peak, males	α_{18}	0.499	3.8
Full-time peak, females	α_{19}	0.866	4.1
Part-time peak, females	α_{20}	0.278	2.0

Table 2. Wage opportunity density. Simultaneous ML estimation procedure versus OLS*)

	Males		Females	
	OLS	Simul-taneous ML	OLS	Simul-taneous ML
Intercept	3.036 (84.8)	2.711 (39.6)	2.657 (53.5)	2.730 (35.2)
Education	0.036 (13.7)	0.045 (13.3)	0.051 (13.1)	0.047 (9.8)
Experience	0.018 (9.7)	0.022 (6.4)	0.018 (8.8)	0.008 (2.1)
(Experience squared).10^{-2}	-0.036 (8.1)	-0.036 (6.2)	-0.030 (7.5)	-0.012 (1.7)
Standard error		0.101 (36.0)		0.172 (33.6)
R^2	0.23		0.23	

*) t-values in parenthesis.

Figures 1 and 2 give the observed and simulated distributions for hours of work. These figures demonstrate that the model is able to reproduce the observed distributions quite well.

In Table 3 we report what we have called <u>aggregate elasticities</u>. By this we understand the elasticity of the <u>mean</u> (male and female) labor supply with respect to 1 per cent changes in the individual wage rates, respectively. The Cournot elasticity of, for example, female labor supply is obtained by calculating the relative change in the mean female labor supply (over all females in the sample) that results from a 10 per cent wage increase. The Slutsky elasticities are derived in an analogous way except that for each household the utility is kept fixed at the pre-wage-increase level. Note that the "estimates" in Tables 3 and 4 are based on 10 sets of simulations and that the standard deviations inform about the simulation uncertainty. Table 4 provides information about elasticities for male and female members of poor and rich households.

As reported above the deterministic part of the utility function or, more precisely, the mean utility across feasible matches for given w and h, is a concave function in C and h. The random tasteshifter and the latent rationing on hours will, however, counteract this concavity. The aggregate Slutsky elasticity derived from this labor supply model can thus be negative.

Table 3. Aggregate labor supply elasticities*)

Type of elasticity		Male elasticities: Own wage elast.	Male elasticities: Cross elast.	Female elasticities: Own wage elast.	Female elasticities: Cross elast.
Elasticity of the probability of participation	Cournot	0.251 (0.004)	-0.078 (0.004)	0.735 (0.009)	-0.219 (0.008)
	Slutsky	0.222 (0.021)	-0.084 (0.015)	0.738 (0.027)	-0.158 (0.011)
Elasticity of conditional expectation of total supply of hours	Cournot	0.085 (0.002)	-0.039 (0.004)	0.741 (0.007)	-0.204 (0.008)
	Slutsky	0.101 (0.010)	-0.023 (0.014)	0.791 (0.039)	-0.122 (0.013)
Elasticity of unconditional expectation of total supply of hours	Cournot	0.338 (0.005)	-0.116 (0.005)	1.531 (0.011)	-0.418 (0.014)
	Slutsky	0.326 (0.023)	-0.107 (0.017)	1.587 (0.051)	-0.278 (0.018)

*) Standard deviations in parenthesis.

As seen from Table 3 female labor supply is considerably more elastic than male labor supply. The elasticity of unconditional expectation of total supply of hours is 1.5 for females and 0.3 for males. Note that the cross elasticities are substantial and negative. The impact of an overall increase in wage levels will thus be smaller than increases in male and female wages taken separately.

A striking result reported in Table 4 is that the wage elasticities are declining with household income. This is the case for Cournot as well as Slutsky elasticities. The elasticities among the poorest in the population is an order of magnitude higher than among the rich. The high values of the utility constant elasticities, the Slutsky elasticities, among the poorest individuals indicate that there might be a substantial loss in taxing wage earnings of the poorest in the society. The own wage elasticities among the 10 per cent richest are close to zero.

The cost of taxation depends on the labor supply responses to changes in tax rates across individuals as well as on the structure and level of taxes. The Norwegian tax system as of 1979 was rather complex with high marginal tax rates but generous rules of deductions. Thus, the elasticities in Tables 3 and 4 give imcomplete information about the cost of taxation. A separate analysis of the cost of taxation is, however, reported in Section 5.

Table 4. Aggregate labor supply elasticities*) for male and female members of the 10 per cent poorest and the 10 per cent richest households under the 1979-rules

Type of elasticity			Male elasticities		Female elasticities	
			Own wage elasticities	Cross elasticities	Own wage elasticities	Cross elasticities
Elasticity of the probability of participation	Cournot	I	1.89 (.06)	-1.04 (.05)	1.85 (.07)	-1.44 (.07)
		II	0.09 (.003)	-0.08 (.004)	0.66 (.008)	-0.29 (.005)
		III	0.03 (.004)	0.01 (.003)	0.07 (.006)	-0.03 (.011)
	Slutsky	I	2.71 (.35)	0.41 (.09)	2.62 (.28)	0.21 (.21)
		II	0.07 (.01)	-0.12 (.01)	0.73 (.03)	-0.19 (.02)
		III	0.01 (.01)	-0.05 (.02)	0.03 (.03)	-0.11 (.03)
Elasticity of conditional expectation of total supply of hours	Cournot	I	0.29 (.02)	-0.15 (.02)	1.04 (.04)	-1.04 (.07)
		II	0.07 (.002)	-0.09 (.004)	0.78 (.009)	-0.29 (.006)
		III	0.03 (.005)	-0.01 (.005)	0.12 (.013)	-0.06 (.017)
	Slutsky	I	1.11 (.12)	0.47 (.11)	2.39 (.24)	0.42 (.24)
		II	0.09 (.02)	-0.05 (.01)	0.97 (.05)	-0.17 (.01)
		III	0.01 (.01)	-0.02 (.01)	0.05 (.01)	-0.04 (.02)
Elasticity of unconditional expectation of total supply of hours	Cournot	I	2.23 (.06)	-1.18 (.05)	3.09 (.08)	-2.33 (.11)
		II	0.16 (.003)	-0.17 (.004)	1.49 (.012)	-0.57 (.009)
		III	0.06 (.006)	-0.01 (.004)	0.19 (.013)	-0.08 (.017)
	Slutsky	I	4.15 (.49)	0.88 (.15)	5.68 (.58)	0.68 (.45)
		II	0.16 (.02)	-0.17 (.02)	1.77 (.07)	-0.35 (.03)
		III	0.02 (.02)	-0.07 (.02)	0.07 (.03)	-0.15 (.05)

Note that I = 10 per cent poorest household
II = 80 per cent in the middle of the consumption distribution
III = 10 per cent richest household

*) Standard deviations in parenthesis.

Figure 1. Observed and simulated hours of work for females

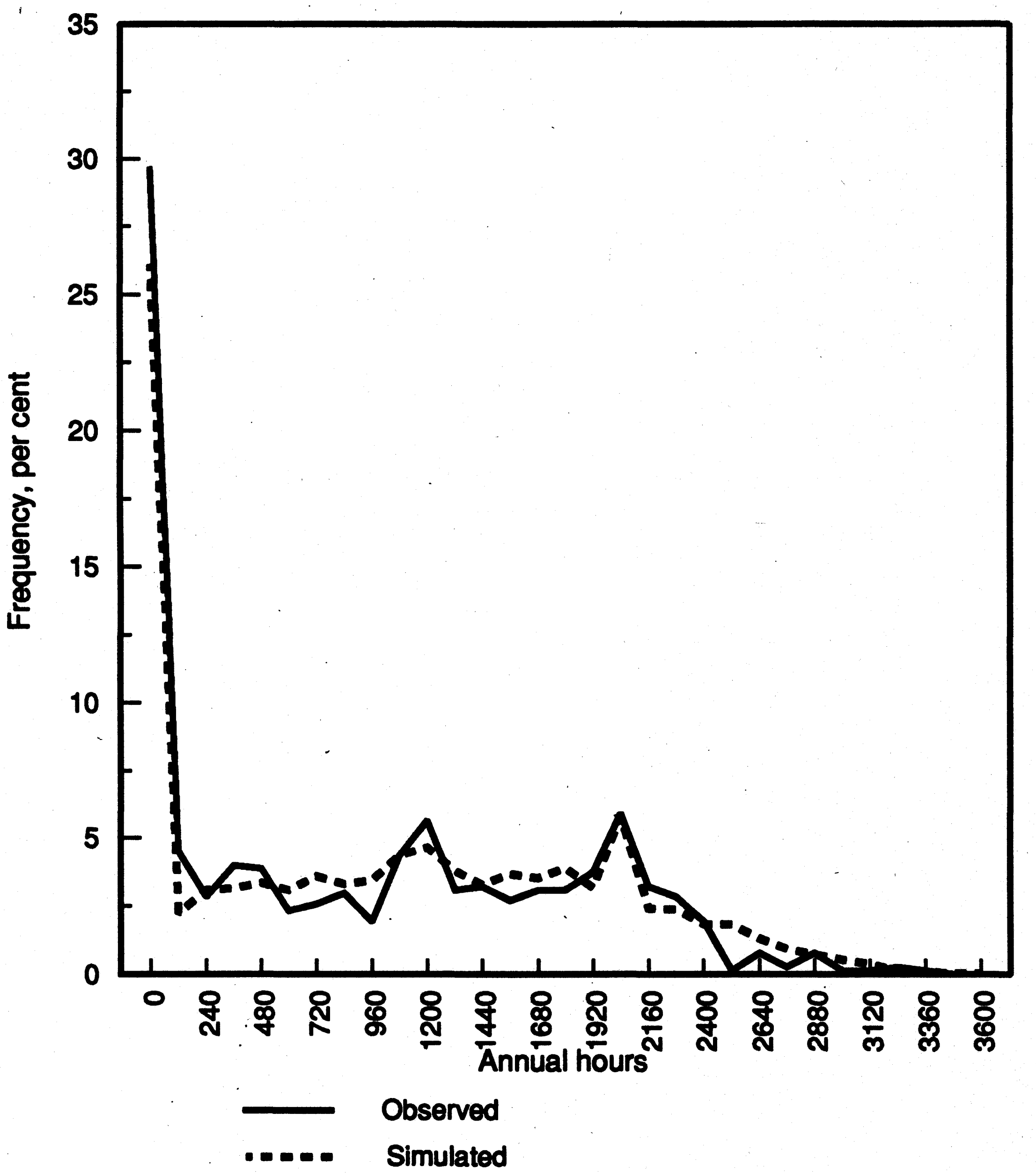
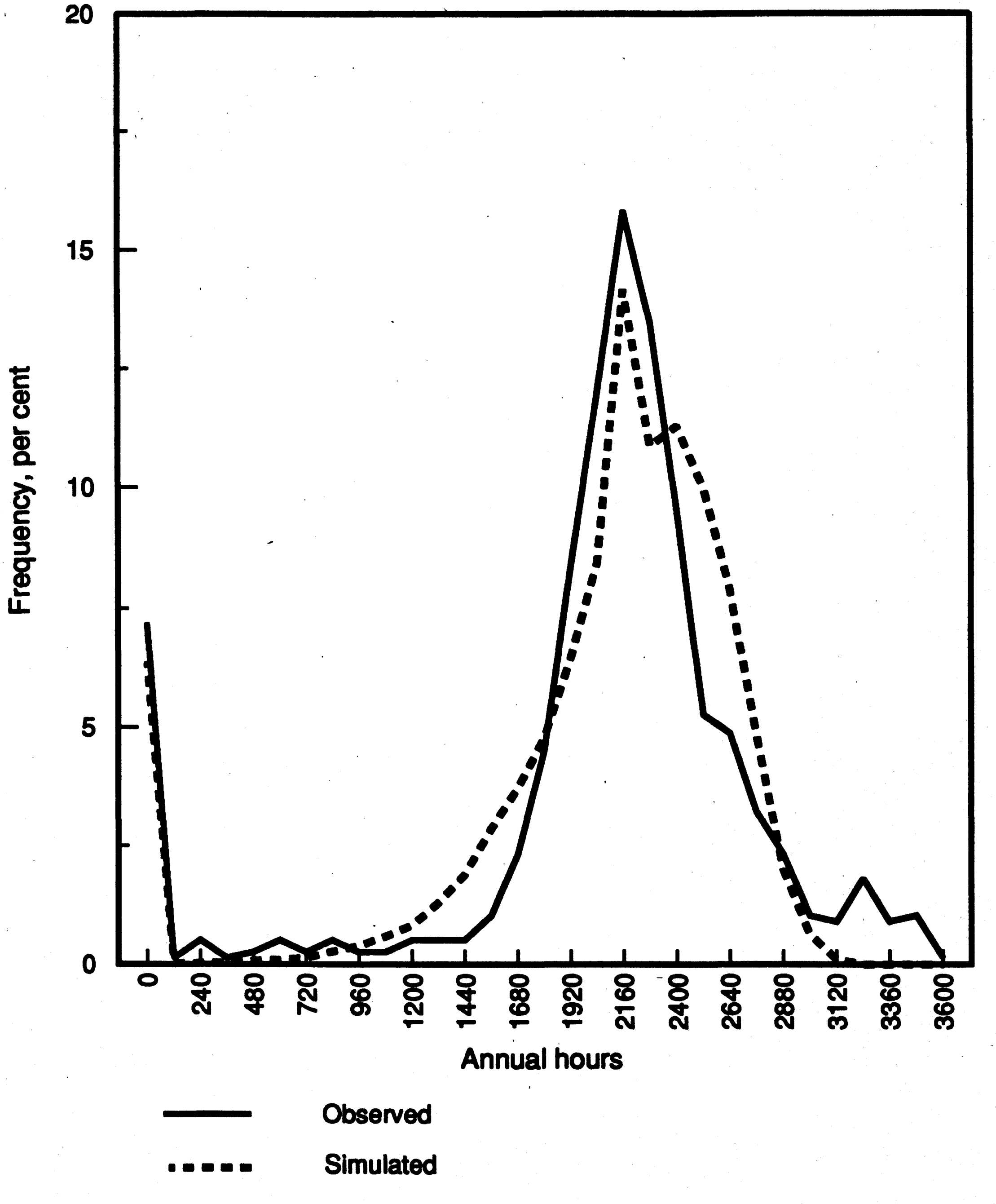
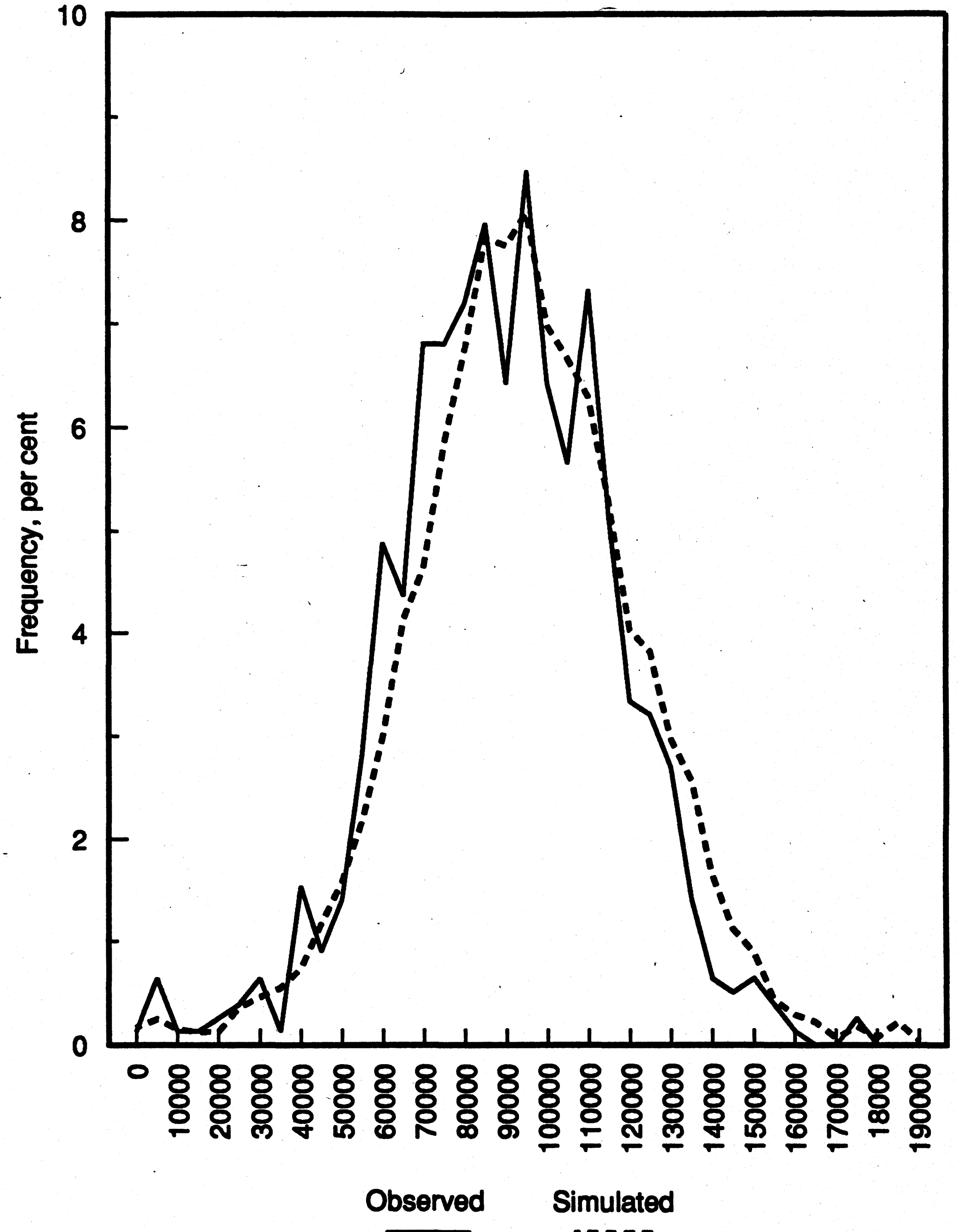


Figure 2. Observed and simulated hours of work for males



Figur 3. Observed and simulated consumption for married couples



5. POLICY SIMULATIONS

This section describes the nature and the purpose of the simulation experiments. It should be noted that our model is a labor supply model which treats wages as exogenous variables. Hence we are only able to perform conditional simulation experiments, given the wage distribution or given specified changes in wage levels.

For expository reasons consider the one-person household model. Let $V(h,w)$ denote the conditional indirect utility defined by

$$(5.1)\qquad V(h,w) = \max_{z}(v(C(z),H(z),T_1(z)) + \varepsilon(z)).$$

given $H(z) = h, W(z) = w$.

It can easily be demonstrated that

$$V(H(z),W(z)) = \tilde{\psi}(H(z),W(z)) + \tilde{\varepsilon}(z)$$

where $\tilde{\psi}$ is defined by (2.12) and where $\{H(z),W(z),\tilde{\varepsilon}(z)\}$ are the points of a Poisson process with intensity measure

$$\mu g_2(h)\tilde{g}(w)dhdw \cdot e^{-\varepsilon}d\varepsilon \ .$$

Since we have estimated $g_2(h),\tilde{g}_3(w)$, $\kappa(1-g_1)/g_1$ and $\tilde{\psi}(h,w)$ we are able to perform policy simulations (changes in tax rates and in the wage distribution) provided it makes sense to keep the opportunity density $g_2(h)$, κ, and the fraction of feasible market matches, g_1, unchanged. The density of offered hours, $g_2(h)$, is assumed to be determined by institutional constraints and firm-specific hours of regulations. These constraints are not likely to change as a consequence of say, changes in the tax system.

To keep g_1 constant in the simulations may appear more controversial. If g_1 is kept unchanged when say, tax rates are changed, this means that the individual sets of feasible market matches are unaffected by the tax rate changes. This assumption thus implies that the total number of jobs increases (decreases) with increasing (decreasing) labor supply. Again, this stresses the fact that this is not an equilibrium model but a labor supply model. This fact should be kept in mind in the interpretation of the simulation results.

One purpose of the simulation experiments is to examine the influence of certain tax reforms on labor supply, income levels and income inequality among households (married couples with or without children). The basic income concepts are gross income (Y) and disposable income (equal to consumption C) defined as;

$$(5.2) \quad Y = w_F h_F + w_M h_M + I_1 + I_2,$$

and

$$(5.3) \quad C = Y - S(w_F h_F, w_M h_M, I_1),$$

where I_1 and I_2 are taxable and non-taxable non-labor family income and S is the tax function.

Income inequality is examined by employing a transfer sensitive inequality measure. This measure of inequality, denoted the A-coefficient, is discussed in Aaberge (1986). The A-coefficient has a similar geometric interpretation as the Gini-coefficient, but gives more weight to transfers that occur in the lower part of the distribution. The maximum attainable value of the A-coefficient is 1, which corresponds to the distribution where one family has all income, while the minimum attainable value is 0, which corresponds to perfect equality. The mathematical definition and some other relevant information are given in Appendix 4. As a supplement to the information from the A-coefficient, corresponding results for the Gini-coefficient are given in Appendix 4.

Simulations of the model can be carried out as follows: First draw whether a match is a market match for one or both adults of the household according to the "probabilities", g_{11}, $g_{10}\kappa_F$, $g_{01}\kappa_M$, $g_{00}\kappa_{FM}$. Second, draw n points,

$$\{H_F(z), H_M(z), W_F(z), W_M(z), \varepsilon(z)\}, \qquad z = 1,2,\ldots,n.$$

Here $\{H_F(z)\}$ and $\{H_M(z)\}$ are drawn from uniform distributions with full- and part-time peaks, $\{W_F(z)\}$ and $\{W_M(z)\}$ are drawn from lognormal distributions according to the wage equations and $\{\varepsilon(z)\}$ are drawn from the extreme value distribution, $\exp(-e^{-\varepsilon})$. Third, find the realized hours and wages $(H_F(\hat{z}), H_M(\hat{z}), W_F(\hat{z}), W_M(\hat{z}))$ by maximizing

$$\Psi(H_F(z), H_M(z), W_F(z), W_M(z)) + \varepsilon(z)$$

with respect to z = 1,2,...,n. Repeat this procedure for every household in the sample. When n is large this procedure yields results that are close to an "exact" simulation of the model.

The simulation procedure we have followed in the present paper is a refinement of the one described above and it is unbiased for finite n and more efficient. This procedure will be described and analyzed elsewhere.

5.1. Proportional taxes on gross earnings and lump-sum taxation

Two major changes of the tax system are considered in this section. Tables 5 and 6 give the results of three different simulations. Simulation 1 is based on the actual 1979 tax rules. In simulation 2 the 1979 rules are replaced by proportional taxes on gross earnings. The proportional tax rate is derived under the constraint that total tax revenue should be as under the actual 1979-system (simulation 1). This tax rate is found to be 21.5 per cent. In simulation 3 the alternative system considered is lump-sum taxes. The lump-sum amounts are obtained from the conditions that each of the households should have utility levels as in simulation 1 ("1979 rules").

We start with commenting on the lump-sum case. Although it is impossible to practice this system, it is of some interest to study the results of lump-sum taxation. By definition all distortive effects of taxation are removed and it should therefore bring forward the labor supply potential in the economy. From Table 5 we observe that the participation rate among females increases from 0.73 under 1979 rules to 0.94 under lump-sum taxes. Annual hours supplied in the total population increase by 106 per cent in the case of females and by 25 per cent in the case of males. If we only consider those who participate, annual hours increase by 41 per cent among females and by 16 per cent among males. Relative to the 1979 rules lump-sum taxation increases the gross incomes by 63 per cent which indicates the potential increase in earnings from this type of tax reforms.

When all individual lump-sum taxes are aggregated, we get a total tax revenue of NOK 60 700 which is 59 per cent higher than the tax revenue under the 1979 rules. Thus, the excess burden of taxation, when 1979 rules are compared to lump-sum taxes, is 59 per cent. This burden is rather high and indicates severe losses from collecting taxes through the 1979 tax system. Moreover, Table 6 also demonstrates that lump-sum taxes will reduce income inequality among households. The main explanation is that the gross income inequality is less under lump-sum taxes than under the 1979 rules. Lump-sum taxation reduces the differences in hours supplied, particular among females.

A tax reform of more practical interest is to replace the 1979 rules by proportional taxes on gross earnings (simulation 2 in Tables 5 and 6). From Table 5 we observe that labor supply increases by introducing this reform, but not to the same extent as under lump-sum taxes. Labor supply (total hours of work) is 52 per cent higher for females and 19 per cent higher for males when proportional taxes replace the 1979 rules. Gross household income increases by 40 per cent, or approximately 63 per cent of the increase obtained when the 1979 rules are replaced by lump-sum taxes. Thus, 63 per cent of the potential increase in this part of GNP can be achieved by proportional taxes on wage earnings.

Table 6 suggests that the inequality in the distribution of disposable income is lower under proportional taxes than under the 1979 rules. One reason for this rather striking result is that the introduction of a proportional tax on wage income leads to a considerable reduction in gross income inequality. In order to understand this result it is important to recall that the deduction opportunities under the 1979 tax system have undermined the progressive and redistributive effects of the rather steep tax schedule in Norway. The most common deductions are related to interest payments on loans. The last column of Table 6 gives the ratio between the A-inequalities of the distributions of disposable and gross income and can be interpreted as an aggregate estimate of the degree of progression.

Thus, the conclusion is that if the 1979 rules are replaced by proportional taxes, this reform will stimulate the economy without the costs of increasing income inequality.

Table 5. Participation rates, annual hours of work, gross income, taxes and disposable income (NOK) for couples under three different tax regimes. Means

	Partici-pation rates		Annual hours of work		Gross earnings		Gross income	Taxes	Dis-posable income
	F	M	F	M	F	M	Households		
Simulation 1:									
1979 tax rules	0.73	0.93	1 000	2 038	32 000	88 300	131 000	38 100	92 900
Simulation 2:									
Proportional taxes[1)] earnings	0.83	0.99	1 519	2 428	54 300	119 500	183 500	38 100	145 400
Simulation 3:									
Lump-sum taxes[2)]	0.94	1.00	2 061	2 550	75 200	128 700	213 700	60 700	153 000

1) The proportional tax rate (21.5 per cent) on gross earnings is derived from simulation on the model under the restriction of a constant tax revenue equal to the revenue under the 1979 rules.

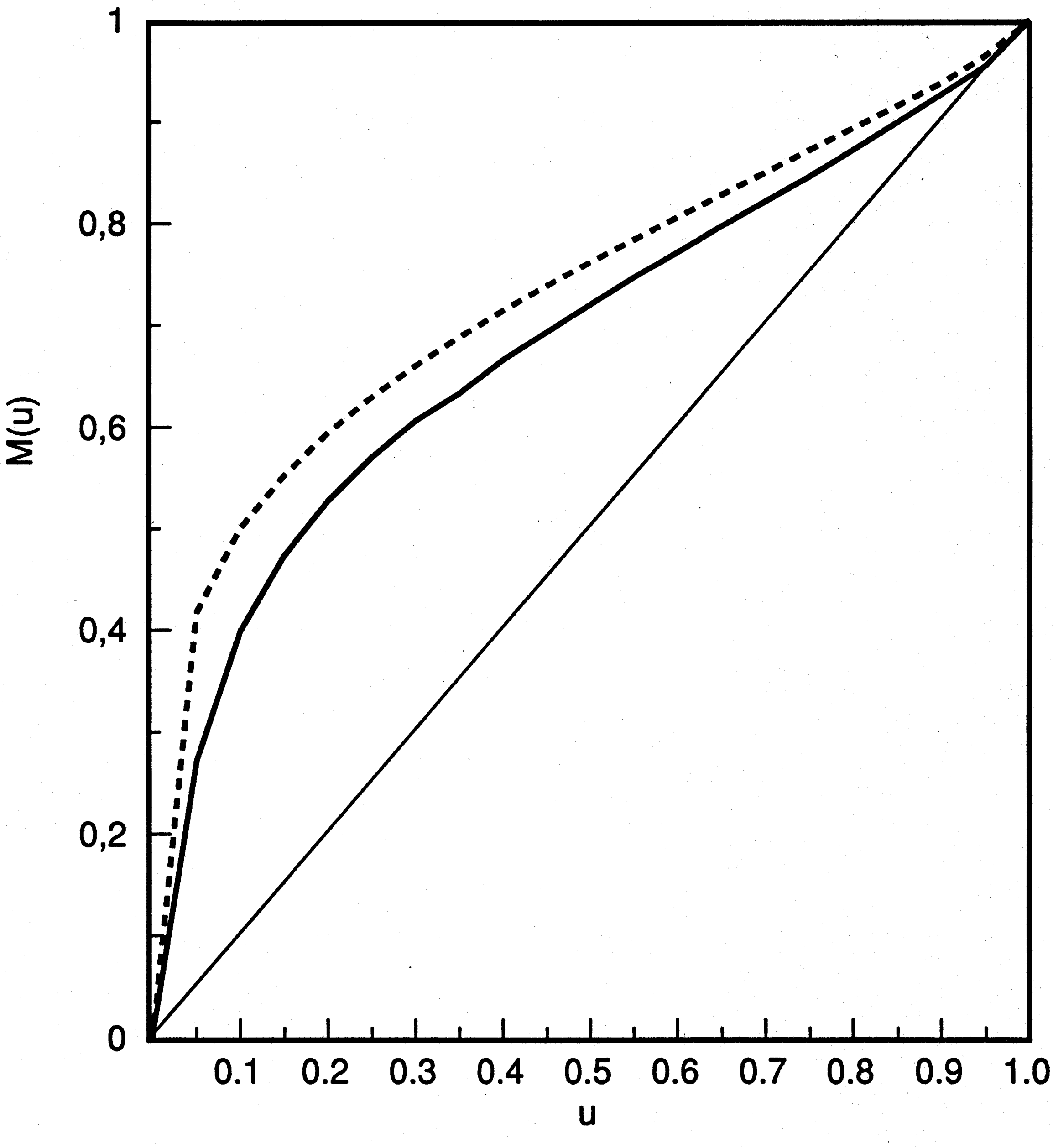
2) Individual lump-sum taxes are derived from simulation on the model given that each household's utility level should be equal to the level under the 1979 rules.

Table 6. A-inequality*) in distributions of annual hours of work, gross, earnings, gross income and disposable income under alternative tax regimes

	Annual hours of work		Gross earnings		Gross income	Dispos-able income	Degree of aggre-gate progres-sion
	F	M	F	M	Households		
Simulation 1:							
1979 rules	.673 (.011)	.318 (.014)	.697 (.010)	.387 (.013)	.313 (.009)	.272 (.009)	.87
Simulation 2:							
Proportional taxes on earnings	.574 (.012)	.169 (.010)	.619 (.010)	.284 (.009)	.263 (.007)	.263 (.007)	1.00
Simulation 3:							
Lump-sum taxes	.438 (.012)	.128 (.004)	.494 (.011)	.247 (.006)	.205 (.006)	.211 (.005)	1.03

*) Standard deviations in parenthesis.

Figure 4. M-curves of the distribution of gross household income under two alternative tax regimes, the 1979 system and proportional taxes.



——— 1979-rules

- - - - - Proportional taxes on wage earning with tax revenue as under the 1979 system

Figure 4 shows how proportional taxes lead to a reduction in gross income inequality among households. The curve displayed in figure 4 is denoted the M-curve. It is a transformation of the income distribution onto [0,1] which is one to one except for scale transformations. The formal definition is given in Appendix 3. A point on this curve, (u,M(u)), gives an estimate of the expected income in the poorest u-fraction of the population relative to the expected income in the total population. The higher up in the diagram the M(u)-curve is located, the more equal is the income distribution. The area between the horizontal line 1 and the M-curve is equal to the A-coefficient reported above.

5.2. Excess burden

In the discussion so far we have neglected the fact that the costs of increased efforts is a reduction in leisure. We therefore now turn to a money measure of the changes in utility and to an estimate of the cost of taxation based on this measure.

This section reports the simulation results of the excess burden of taxation when the 1979 rules are compared to a system of proportional taxes on gross earnings. Excess burden is examined by employing the ratio between the mean level of equivalent variations and the initial mean tax revenue as a summary measure of the cost of taxation.

Let K denote the level of equivalent variation of a household defined by

$$V(f_1,0) = V(f_0,K)$$

where V(f,K) is given by

$$V(f,K) = \max_z U(K+C(z),H_F(z),H_M(z),z)$$

and C is given by (3.2) and (5.3).

f_0 denotes the 1979 rules and f_1 denotes the above mentioned system of proportional taxes on gross earnings with a tax rate approximately equal to 20 per cent.

Recall that the indirect utility is stochastic and its values can be obtained by inserting the values of hours, wages and the taste-shifter that correspond to the optimal match. Since the indirect utility is random, so is K.

The results from this simulation experiment show that no household is loosing by the introduction of a proportional tax system. The expected level of K relative to initial tax revenue is estimated to be 48.4 per cent. This is (by definition) lower than the excess burden when the 1979 rules are compared to lump-sum taxes. If we add initial taxes and the compensation payment, then this sum amounts to 93 per cent of the lump-sum transfers. This clearly demonstrates the potential economic gain from a tax reform along these lines.

Table 8 displays the results on some key characteristics of the households that are worse and better off, respectively, when we switch to a proportional tax system. We observe that all households are gaining from this reform. The 10 per cent households that are worse and better off gain on average NOK 1 600 and NOK 45 400, respectively. Wage rates and labor supply, participation as well as the annual hours worked, are lower among those who are worse off than among the households that are better off.

Tables 9 and 10 give some characteristics of the households in the 10 per cent lower and upper parts of the distribution of disposable income under the 1979 rules. The first line gives the characteristics under the 1979 rules and the second line gives the characteristics of the very same households under a system of proportional taxes on gross earnings.

Table 7. Equivalent variations*) (K); 1979 rules versus proportional taxes on wage earnings

Mean level of K NOK	Mean level of K relative to mean level of tax revenue, per cent	Inequality	
		A	G
18 400 (400)	48.4	.528 (.010)	.369 (.009)

*) Standard deviations in parenthesis.

Table 8. Characteristics of those 10 per cent of the households who gain most and least, respectively, from switching to proportional taxes mean levels

		Mean level of K NOK	Hourly wage rates NOK		Partici-pation rate		Annuals hours of work		Wage earnings NOK		Gross house-hold income NOK	Taxes NOK	Dispos-able income NOK
			F	M	F	M	F	M	F	M			
1979 tax rules	The 10 per cent who gain most	45400	35.0	53.7	.84	.96	1280	2290	46500	123500	182800	65700	117100
	The 10 per cent who gain least	1600	28.4	35.6	.62	.85	633	1617	18300	57300	84500	17500	6700
Propor-tional taxes	The 10 per cent who gain most	-	36.8	63.4	.95	1.00	2037	2629	78800	167800	259400	54200	205200
	The 10 per cent who gain least	-	30.3	38.1	.68	.95	781	1995	24600	77000	110600	22400	88200

Only 17 per cent of the 10 per cent poorest households with respect to consumption under the 1979 rules still stay in this fraction of the population after the tax-change. The mean level of the equivalent variations in this group is NOK 7 100, while the remaining 83 per cent on average gain 19 700 from the change of the tax system.

Approximately one of four households among the 10 per cent richest still stay in this part of the population after the tax change and their mean equivalent variation is NOK 62 500. The remaining 75 per cent of the richest move to the middle part of the consumption distribution and have mean equivalent variation equal to NOK 39 000.

We notice from Tables 9 and 10 that the 10 per cent poorest households increase their labor supply and hence, their gross earnings, far more that the 10 per cent richest when a proportional tax replaces 1979 rules. This is in accordance with the wage elasticity results reported above. An important reason why is that among the 10 per cent poorest households the initial marginal tax rates exceed the flat rate of 21.5 per cent, but the average tax rates are lower than 21.5 per cent. Thus, in this poorer group of the population both the substitution and the income effect have the same sign and imply a higher labor supply. Among the 10 per cent richest the marginal tax rates are higher than the flat rate of 21.5 per cent, but so are also the average rates. In this group the income effect is negative which contributes to a lower total impact on labor supply from lower marginal taxes than is the case for the 10 per cent poorest households.

Table 9. Characteristics of the 10 per cent poorest (disposable income) households under the 1979 rules

	Participation rate		Annual hours supplied		Earnings		Gross income (1000 NOK)	Taxes	Disposable income
	F	M	F	M	F	M	Households		
Under the 1979 rules	.47	.51	496	892	14 800	28 100	52 500	9 600	42 800
Proportional taxes	.81	.95	1 389	2 187	51 000	98 100	158 700	33 200	125 400

Table 10. Characteristics of the 10 per cent richest households under the 1979 rules

	Participation rate		Annual hours supplied		Earnings		Gross income (1000 NOK)	Taxes	Disposable income
	F	M	F	M	F	M	Households		
Under the 1979 rules	.95	.99	1 957	2 384	75 100	118 600	213 100	72 000	141 000
Proportional taxes	.99	1.00	2 078	2 470	81 600	128 800	229 700	46 400	183 300

6. CONCLUSIONS

In recent years important developments in the estimation of labor supply have taken place. The most well known and widely applied approach is the Hausman type model, Hausman (1980). The contribution made by applying this model was the specification of the budget constraint that allowed for non-convex budget sets. In most countries marginal tax rates are not uniformingly increasing with income which creates a non-convexity in budget sets. The Hausman model is, however, rather restrictive since so far it has proved tractable only for linear and possibly quadratic labor supply curves. Moreover, imperfections in the labor market have been excluded from the analysis.

The labor supply model applied in this paper, described in detail in Dagsvik and Strøm (1990), allows for a detailed specification of complex budget constraints together with rather general specifications of the utility function.

The model allows for a deviation between preferred and offered hours and wages. A deviation of this type occurs if there are market imperfections preventing skills, wages and hours to adjust so that a perfect equilibrium is generated.

The estimated model is applied to simulate the impact of changes in tax rules on labor supply and income distribution. Specifically, the model is applied to simulate the impact of replacing the tax rules as of 1979 by a proportional tax on wage earnings. The simulation results show that a flat tax rate of 21.5 per cent on gross earnings will give the same tax revenue as the existing tax rules. Labor supply is stimulated to a large extent, especially among females. An interesting result is that the labor supply of the poorest individuals is far more responsive than among the richest individuals. The inequality in the distribution of gross income is thus reduced which implies the rather surprising result of almost no impact on the inequality in the distribution of after-tax income from replacing the progressive tax rules as of 1979 by a flat tax rate on gross earnings. It should be noted, however, that liberal deduction rules undermined the otherwise progressive effects of the steep tax schedule as of 1979.

The model is applied to simulate the excess burden of taxation measured as the mean in the distribution of equivalent variation relative to the mean of initial taxes paid. Specifically, it is shown that the excess burden of the 1979 tax rules relative to a system of a proportional tax on wage earnings is as high as 48 per cent. Thus, substantial costs of taxation are indicated and support the view that the gain of tax reforms along the lines analysed here and implemented during the 1980s in some countries could be quite high.

Our analysis also shows that the equivalent variations vary across individuals with the highest amount occuring in the upper part of the income distribution. This result is not inconsistent with the finding that the labor supply is more responsive among the poor than among the rich.

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Appendix 1. Data

The data are obtained from two different data sources with information about couples in Norway that are married (not cohabitating) through 1979. The first source is based on a questionnaire and contains data on hours worked (by intervals), wage rates and socio-demographic variables such as the number and age of children and education level. The other source is based on filled in and approved tax reports and yields detailed information about reported incomes, legal deductions, taxes paid and transfer payments received. The two sets of data are linked on the basis of personal identification numbers. The Central Bureau of Statistics has been responsible for collecting and preparing the data sets. The data based on the tax reports have been used to check the answers on the wage rates and hours worked given in the questionnaire. For around 90 per cent of those working the reported wage rate has been used. Hours worked per year are obtained by dividing the reported labor income per year by the reported wage rate (or the predicted rate in some few cases).

The sample selection rules are as follows. Only couples where the age of the husband is less than 66 years and the age of the wife is between 27 and 66 years are included. Those couples for which one or both spouses have entrepreneurial income that exceeds wage income are excluded. Couples for which the wife or husband have reported hours of work above 3000 hours per year are excluded. When the reported female wage rate below 15 or above 56 NOK it is predicted by a wage equation. The same procedure is followed when the male wage rate is below 24 or above 74 NOK. The resulting sample size is 778. Not working is defined as working less than or equal to 60 hours per year. In table A1 we report sample statistics for some selected variables.

Table A1. Sample Values - Married Couples

	Averages	Standard deviations	Min. values	Max. values
Hours worked per year by wife	919	859	0	3 368
Hours worked per year by husband	2 059	740	0	3 572
Female wage rate, NOK per hour (among those who work)	31.30	6.10	15.50	55.80
Male wage rate, NOK per hour (among those who work)	41.60	9.4	24.00	73.90
Female labor income, NOK per year	30 021	29 914	0	152 497
Male labor income, NOK per year	84 911	35 701	0	185 988
Female pension income, NOK per year	1 247	5 477	0	51 539
Male pension income, NOK per year	2 538	10 410	0	86 988
Other female income, NOK per year	132	1 746	0	34 480
Other male income, NOK per year	802	3 957	0	35 338
Consumption	88 739	26 191	20 554	222 325
Capital income of the household, NOK per year	2 536	7 842	0	162 734
Wife's education in years	10.5	1.7	9.0	17.5
Husband's education in years .	11.4	2.5	9.0	18.0
Age of the wife	43.6	11.3	27	66
Age of the husband	46.1	11.5	25	66
Number of children below 6 ...	0.36	0.66	0	4
Number of children 7-20	1.01	1.55	0	6
Female participation rate, per cent	70.3	45.7	-	-
Male participation rate, per cent	92.8	25.9	-	-

Appendix 2. Norwegian tax rules as of 1979

In a condensed form the tax rules can be described as follows:

Let R_j, Y_j, Q_j denote the net and gross taxable income and deductions for spouse j, j=F,M, respectively. Taxes and levied on net income according to the tax functions $S_1(\cdot)$ when the spouses are jointly taxed, and by $S_2(\cdot)$ when they are taxed separately. Minor parts of the taxes are based on gross income according to the rule denoted by the function $S_G(\cdot)$. Thus, taxes paid by the household, S, is defined as

$$(A.1) \quad S(R_M,R_F,Y_M,Y_F) = \begin{cases} S_1(\sum_j R_j) + \sum_j S_G(Y_j) & \text{when} \quad (R_M,R_F) \in J \\ \sum_j [S_2(R_j) + S_G(Y_j)] & \text{when} \quad (R_M,R_F) \in R_+^2 - J \end{cases}$$

where $R_+^2 = [0,\infty]x[0,\infty]$ and J is defined as the region of R_+^2 for which

$$(A.2) \quad R_J < R_0 \text{ for at least one j,}$$

$$(A.3) \quad R_j = Y_j - Q_j$$

and where R_0 is given by the tax rules.

It is up to the households to decide whether they prefer to be taxed separately or jointly. In 1979 the upper level of R_j that minimized the total taxes paid by the households when they were jointly taxed was NOK 22 000.

Deduction are defined as

$$(A.4) \quad Q_j = \max[Q_{min}, Q_j^*]$$

where Q_{min} is a minimum tax allowance that every taxpayer has the right to deduct. However, expenses such as interest on loans, union fees, travel expenses over and above a given limit are also deductible. Q_j^* denotes the actual deduction legitimately claimed by the taxpayer.

The minimum allowance, Q_{min}, depends on gross income according to rules set out in table A2.

Taxes related to net income follow from the rules reported in table A3.

Taxes on gross income are given by the rule given in table A4.

In addition to the deduction and tax rules outlined so far there are some special transfer payments related to the number of age of children in the household. For children below 17 years of age the parents received (in 1979) NOK 900 per child and NOK 1 200 for children between 17 and 20.

Table A2. Minimum Tax Allowances

Gross income (NOK) Y	Minimum tax allowances (NOK) Q_{min}
0 - 2000	Y
2000 - 9500	.4Y + 1200
9500 - 10000	5000
10000 - 16000	.04Y + 4600
16000 - 17500	.14Y + 3000
17500 - 31000	.10Y + 3700
31000 -	6800

Table A3. Taxation of net income

Separate taxation Intervals for net income (NOK) R_j	Marginal tax rates (per cent) $S_2'(R))$	Joint taxation Intervals for net income (NOK) $R_M + R_F$	Marginal tax rates (per cent) $S_1'(R_M+R_F)$
0 - 7 000	0	0 - 14 000	0
7 000 - 32 000	27.4	14 000 - 48 000	27.4
32 000 - 41 000	33.4	48 000 - 60 000	33.4
41 000 - 58 000	38.4	60 000 - 77 000	38.4
58 000 - 69 000	43.4	77 000 - 88 000	43.4
69 000 - 79 000	49.4	88 000 - 98 000	49.4
79 000 - 89 000	55.4	98 000 - 108 000	55.4
89 000 - 106 000	60.4	108 000 - 125 000	60.4
106 000 - 136 000	65.4	125 000 - 155 000	65.4
136 000 - 186 000	69.4	155 000 - 205 000	69.4
186 000 - 286 000	73.4	205 000 - 305 000	73.4
286 000 -	75.4	305 000 -	75.4

Table A4. Taxation of gross income

Intervals for gross income (NOK) Y	Taxes paid (NOK) $S_G(Y)$
0 - 9 000	0
9 000 - 11 500	0.25Y - 2 250
11 599 - 182 400	0.05Y
182 400 -	9 120

Appendix 3. Measurement of inequality

A common approach for measuring inequality in distributions of income is to employ the Gini-coefficient, which satisfies the principles of scale invariance and transfers. The principle of scale invariance states that inequality should remain unaffected if each income is altered by the same proportion and it requires, therefore, the inequality measure to be independent of the scale of measurement. The principle of transfers implies that if a transfer of income takes place from a richer to a poorer person without reversions of the relative positions, the inequality diminishes.

As is wellknown, the Gini-coefficient (G) is related to the Lorenz curve (L) in the followint way.

$$\text{(A.7)} \qquad G = \int_0^1 [1-2L(u)]du.$$

The Gini-coefficient offers a method for ranking distributions and quantifying the differences in inequality between distributions. This strategy, however, suffers from certain inconveniences. Evidently, no single measure can reflect all aspects of inequality of a distribution, only summarize it to a certain extent. Consequently, it is important to have alternatives to the Gini-coefficient. As pointed out by Atkinson (1970), the Gini-coefficient assigns more weight to transfers in the centre of a unimodal distribution than at the tails. As an alternative to the Gini-coefficient, we will employ an inequality measure (the A-coefficient) that assigns more weight to transfers at the lower tail than at the centre and the upper tail. The A-coefficient, see Aaberge (1986), has a similar geometric interpretation and relation to the inequality curve M defined by

$$\text{(A.8)} \qquad M(u) = \frac{E[X|X\leq F^{-1}(u)]}{EX}, \qquad 0\leq u\leq 1,$$

as the Gini-coefficient has to the Lorenz curve. Here X has distribution function F. The A-coefficient is defined by

$$\text{(A.9)} \qquad A = \int_0^1 [1-M(u)]du.$$

If X is an income variable, then M(u) for a fixed u expresses the ratio between the mean income of the poorest 100u per cent of the population and the mean income of the population. As is wellknown, the egalitarian line of the Lorenz curve is the straight line joining the points (0.0) and (1.1). The egalitarian line of the M-curve is the horizontal line joining the points (0.1) and (1.1). Thus, the universe of M-curves is bounded by a unit square, while the universe of Lorenz curves is bounded by a triangle. Therefore visually, there is a sharper distinction between two different M-curves than between the two corresponding Lorenz curves. Note that the M-curve will be equal to the diagonal line (M(u)=u) if and only if the underlying distribution is uniform (0,a) for an arbitrary a. The A-coefficient then takes the value 0.5, while the maximum attainable value is 1 and the minimum attainable value is 0.

Note that M(u) = L(u)/u, which implies

$$\text{(A.10)} \quad A = \int_0^1 [1 - \frac{L(u)}{u}]du.$$

Appendix 4. Estimates of inequality based on the Gini-coefficient

Note that Table G6 below corresponds to Table 6 in section 5.1.

Table G6. A-inequality*) in distributions of annual hours of work, gross, earnings, gross income and disposable income under alternative tax regimes

	Annual hours of work		Gross earnings		Gross income	Disposable income	Degree of aggregate progression
	F	M	F	M	Households		
Simulation 1:							
1979 rules	.494 (.013)	.165 (.009)	.527 (.012)	.231 (.009)	.198 (.006)	.166 (.005)	.84
Simulation 2:							
Proportional taxes on earnings	.386 (.012)	.090 (.004)	.443 (.011)	.182 (.005)	.168 (.004)	.168 (.004)	1.00
Simulation 3:							
Lump-sum taxes	.265 (.009)	.075 (.002)	.325 (.009)	.165 (.004)	.130 (.004)	.136 (.004)	1.05

*) Standard deviations in parenthesis.

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