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Modelling Consumers' Energy Demand

by

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Abstract

The modelling of consumers demand for energy in a general consumer demand system is discussed. Electricity, fuel-oil, the stock of electricity using durables and housing are assumed to be separable from other consumer commodities. This lower level demand system is modelled using a Gorman Polar form. The linear expenditure system is a nested hypothesis of the more general Gorman Polar form and the two systems are estimated and tested against each other. A dynamic version of the linear expenditure system is also estimated. As expected the results indicate that the Engel elasticities for electricity and fuel-oil differ considerably, and that the stock of electricity using durables contributes to explain the use of electricity and fuel-oil over the period.

1 Introduction

As a part of the research programme "Energy and Society" the Central Bureau of Statistics (CBS) is developing a long term general equilibrium macroeconomic model for analyses of problems concerning energy demand and supply and the environment. Modelling households demand for energy constitutes one of the subprojects in this research programme. The main aim with this paper has been to discuss models which can be implemented in the overall model structure.

In the last 15 years the relative price between electricity and fuel-oil has been fluctuating due to large changes in the fuel- oil price. From 1980 this relative price has been falling and at the same time we have experienced an increase in households use of electricity. The increase in the use of electricity has then been independent of relative energy prices, and we have to search for other variables than relative prices to be able to explain the households' demand for electricity and fuel-oil.

Due to increasing concerns about energy use and environmental problems, energy taxation is an important political issue. To be able to study the effects of taxation on energy commodities it is of great interest to develop models which incorporates the main features of what determines demand for different energy commodities, (B. Bye and Mysen (1991)).

Consumers' demand for energy commodities is best viewed within a household production framework where the underlying demands are for services such as heat, light and refrigeration, etc. To obtain these requires two things: a durable appliance to produce the service and a fuel to power the appliance. Demand for energy is thus a joint demand for an appliance stock and for its rate of use. Changes in energy prices should affect both the rate of use of an appliance and the decision about durable ownership. The decision to invest in new capital equipment should be determined by a trade off between capital and operating costs reflected by expectations about future energy prices. A model which incorporates the investment decision and price expectations in addition to the rate of use of the appliances would be appropriate for modelling households demand for energy.

There have been some studies of the United States energy demand which try to model the joint determination of appliance demand and use with micro data on households, Goett and McFadden (1982) and Dubin and McFadden (1984). The same model framework has been used to study the demand for gas in some European countries, Bartlett et al. (1988). Bartlett and Aaheim (1990) uses the theory to analyse data from the Norwegian Expenditure Survey. Using this kind of model framework arises many problems especially from the large data requirements. It is difficult to find prices on capital equipment. In addition it is also necessary to make assumptions about price expectations. Baker et al (1989) have avoided some of these problems in their study of energy demand by analysing the demand for different fuels conditional on durable ownership using microdata from the British Family Expenditure Survey. The demand for energy commodities is determined in a two-stage process using the framework of the Almost Ideal Demand System, Baker et al (1989) and Deaton and Muellbauer (1980b). Rødseth (1983) incorporates elements from household production theory in his analysis of consumers demand for energy in Norway conditional on durable ownership. The consumers are divided into different groups depending on what kind of heating equipment and type of housing they have chosen, but the choice of durables is not incorporated in the model.

A complete static demand system is the framework for modelling consumer demand for energy in CBS's macroeconomic models MODAG W and MSG-5. MSG-5 is a general equilibrium model mostly used for long term policy analyses. MODAG W is based on a more demand determined theory and mostly used for medium term policy analyses and forecasting. In both models the consumers demand for domestic energy commodities are determined in a two-stage process. At the upper level the total domestic demand for energy is determined in an ordinary Linear Expenditure System (LES), Cappelen and Longva (1987), Magnussen and Skjerpen (1990) and Aasness (1990). Energy is an aggregate of fuel-oil and electricity. By making assumptions about the properties of the utility function with respect to separability and homogeneity, the composition of electricity and fuel-oil is determined independently of prices of other commodities at the lower level. The utility of total domestic energy use is described with a Constant Elasticity of Substitution function (CES), where electricity and fuel-oil are assumed to be substitutes. The relative demand for electricity and fuel-oil is determined by relative prices between electricity and fuel-oil. The elasticities have been estimated using aggregate time series.

There are some disadvantages with using a linear expenditure system with a separable energy aggregate as done in MODAG W and MSG-5. It can be shown that the energy commodities in the energy aggregate have the same expenditure elasticities. This does not seem reasonable with respect to the different "services" they provide the consumer. The expenditure elasticity for fuel-oil for heating purposes is not necessarily the same as the one for electricity used as "input" in other electric appliances. The assumption about separability from the other commodities can also be questioned because of the dependence between the stock of durables and use of energy.

The present paper concentrates on modelling consumers' demand for energy using aggregate time series, but specifies a more general model framework which makes it possible to analyse some of the special elements concerning energy demand. The demand for energy commodities is determined in a two stage process where the demand for a "Housing and Heating" aggregate is determined at the upper level in an ordinary LES and the demand for the different energy commodities is determined at the lower level. The stock of energy using durables is one of the commodities in this aggregate. This stock variable is treated as an ordinary commodity on the line with the other commodities and the investment decision is not explicit modelled. This approximation seems acceptable because the main purpose with this study is to explain long run properties of energy demand. Then it is the long term growth rate in the stock of appliances and the corresponding energy use which is important, not the annual fluctuations in purchase. The model also exhibits the property of different expenditure elasticities for electricity and fuel-oil.

Section 2 presents a model of consumer demand for energy, using a Gorman Polar utility function and section 3 summarizes the results from the estimation of this model, and the static Linear Expenditure System (LES) which is a nested hypothesis of the more general model in section 2. In section 4 a dynamic specification of the LES is presented and tested against the static models. Section 5 shows the simulation properties of the models in sections 3 and 4 and section 6 concludes the analyses.

2 An Econometric Model of Energy Demand

Assume the following utility function

$$U(X_1,.,X_n,X_A) = \prod_{i=1}^{n}(X_i - \gamma_i)^{\beta_i}(X_A - \gamma_A)^{\beta_A} \tag{2.1}$$

where $i = 1,....,n, A$ $\beta_i > 0$ and γ_i is a parameter which if it is positive can be interpreted as the level of minimum necessary consumption. The β_i-parameters have to fulfill the following restriction.

$$\sum_{i=1}^{n}\beta_i + \beta_A = 1$$

The utility function (2.1) is additively separable in all the commodity groups X_i. If the group utility functions are homothetic or take a generalized Gorman Polar form, i.e. are quasi-homothetic, the utility maximizing process can be described in a two-stage process. This implies that the lower level utility X_A which denotes the "Housing and Heating" aggregate, can be maximized by only taking into consideration within group prices and group expenditure, and at the upper level utility can be maximized with group price indices representing the different groups. Such a two-stage process is described in Gorman (1959), Deaton and Muellbauer (1980a, Ch. 5) and Aasness (1992).

The group utility function for the Housing and Heating aggregate is assumed to be of Gorman Polar form. This utility function is quasi-homothetic and hence there are no apriori restrictions on the expenditure elasticities corresponding to the commodities in the aggregate. The "Housing and Heating" aggregate consists of electricity X_E, fuel-oil X_F, housing X_H and the stock of energy using durables X_D. The last variable is incorporated because the demand for electricity is closely related to the stock of energy using durables. If there is a stable linear relationship between the stock of durables and annual purchase, purchase can be used as an approximation to the stock variable. However, purchase fluctuates much more than the stock of such durables. I have therefore chosen to use a stock variable[1]. Earlier analyses of Norwegian data (B.Bye, (1989)) have shown that use of wood fuel is a significant explanatory variable in determining the demand for electricity and fuel-oil, but lack of prices on wood fuel makes it impossible to test demand equations for wood fuel empirically.

The Gorman Polar utility function can be represented by the following cost or expenditure function, Deaton and Muellbauer (1980a, Ch. 5).

$$C(X_A, P_A) = a(P_A) + X_A b(P_A) \tag{2.2}$$

The demand functions can be derived from the cost-function (2.2), and by specifying the functions $a(P_A)$ and $b(P_A)$ we have the exact estimable demand functions. $a(P_A)$ can be interpreted as the cost of subsistence, i.e. the cost of obtaining a minimum consumption level, and hence $b(P_A)$ will have the interpretation as the cost of bliss. This implies that it is only the consumption level in excess of the minimum subsistence level which depends on the income level. P_A is a vector of the commodity prices in the aggregate. Differentiating (2.2) with respect to the price of commodity j gives the compensated (Hicksian) demand functions, Deaton and Muellbauer (1980a, Ch. 2).

[1]This is defined as the stock of all durables except cars, i.e. commodity 40 in the MSG-model. Further comments are given in Appendix C.

$$\frac{\partial C(X_A, P_A)}{\partial P_j} = a_j(P_A) + X_A b_j(P_A) = X_j \tag{2.3}$$

$j = E, F, H, D$. $a_j(P_A)$ and $b_j(P_A)$ denote partial derivatives of the $a(P_A)$ and $b(P_A)$ functions. Let total group expenditure be denoted by Y_A. We then have

$$Y_A = C(X_A, P_A)$$

and from (2.2) follows

$$X_A = \frac{Y_A - a(P_A)}{b(P_A)} \tag{2.4}$$

(2.4) inserted in (2.3) gives the ordinary (Marshallian) demand functions, Deaton and Muellbauer (1980a, Ch. 2).

$$X_j = a_j(P_A) + b_j(P_A)\frac{Y_A - a(P_A)}{b(P_A)} \tag{2.5}$$

$$j = E, F, H, D$$

Multiplying (2.5) by P_j gives total expenditure on commodity X_j.

$$Y_j = P_j a_j(P_A) + P_j b_j(P_A)\frac{Y_A - a(P_A)}{b(P_A)} \tag{2.6}$$

$$j = E, F, H, D$$

If $a_j(P_A) = \alpha_j$ and $\frac{P_j b_j(P_A)}{b(P_A)} = \rho_j$ and α_j and ρ_j are both constant parameters, then equation (2.6) will give the Linear Expenditure System (LES), Deaton and Muellerbauer (1980a, Ch. 3).

The upper level utility function (2.1) is maximized given the following budget constraint

$$\sum_{i=1}^{n} P_i X_i + b(P_A) X_A = Y_C - a(P_A) \tag{2.7}$$

This implies a modified linear expenditure system with the following demand functions for the first n commodity groups at the upper level, Aasness (1992),

$$X_i = \gamma_i + \beta_i \frac{(Y_C - a(P_A) - (\sum_{k=1}^{n} P_k \gamma_k + b(P_A)\gamma_A))}{P_i} \tag{2.8}$$

$$i = 1, ..., n$$

and with the following expenditure function for the commodity group Housing and Heating

$$Y_A = \left(\frac{a(P_A)}{X_a} + b(P_A)\right)\gamma_A + \beta_A \left(Y_C - a(P_A) - \left(\sum_{i=1}^{n} P_i \gamma_i + b(P_A)\gamma_A\right)\right) \tag{2.9}$$

To estimate the demand functions in (2.5), explicit functional forms of the functions $a(P_A)$ and $b(P_A)$ have to be chosen, see Blackorby et al (1978) and Baker et al (1989). The functional forms have to be reasonable approximations to the underlying "true" functions.

The cost of subsistence is likely to be characterized by a flexible functional form which can incorporate substitution effects. A certain level of heating is viewed as necessary for subsistence. Most of the energy substitution is between energy commodities for heating purposes. Hence it is important to have a flexible functional form for the costs of subsistence. The following Generalized Leontief (GL) form for $a(P_A)$ is chosen, Baker et al (1989).

$$a\,(P_A) = \sum_i \sum_j \alpha_{ij} \,(P_i P_j)^{\frac{1}{2}} \tag{2.10}$$

$$i, j = E, F, D, H$$

The following symmetry properties yield for the parameters of the GL-function

$$\alpha_{ij} = \alpha_{ji}$$

For $b(P_A)$ a Stone-Geary model is chosen, Baker et al (1989).

$$\ln b\,(P_A) = \sum_j \rho_j \ln P_j \tag{2.11}$$

An explanation for having other substitution possibilities at the bliss level is that as the income level increases, the propensity to buy and consume services from energy using durables which only use electricity such as driers and fridges, is higher.

Inserting (2.10) and (2.11) in (2.5) gives

$$X_j = \frac{1}{2}\sum_i \alpha_{ij}\left(\frac{P_i}{P_j}\right)^{\frac{1}{2}} + \frac{\rho_j}{P_j}\left(Y_A - \sum_i \sum_j \alpha_{ij}\left(P_i P_j\right)^{\frac{1}{2}}\right) \tag{2.12}$$

We then have the following expression for the demand equation

$$X_j = \frac{1}{2}\sum_i \alpha_{ij}\left(\frac{P_i}{P_j}\right)^{\frac{1}{2}} - \rho_j \sum_i \sum_j \alpha_{ij}\left(\frac{P_i}{P_j}\right)^{\frac{1}{2}} + \rho_j\left(\frac{Y_A}{P_j}\right) \tag{2.13}$$

Multiplying (2.13) by the price P_j gives the expenditure equation

$$Y_j = \frac{1}{2}\sum_i \alpha_{ij}\left(P_i P_j\right)^{\frac{1}{2}} - \rho_j \sum_i \sum_j \alpha_{ij}\left(P_i P_j\right)^{\frac{1}{2}} + \rho_j Y_A \tag{2.14}$$

To avoid problems with multicollinearity in the estimation, we devide (2.14) by total group expenditure Y_A to derive expenditure shares.

$$s_j = \frac{1}{2}\sum_i \alpha_{ij}\left(\tilde{P}_i \tilde{P}_j\right)^{\frac{1}{2}} - \rho_j \sum_i \sum_j \alpha_{ij}\left(\tilde{P}_i \tilde{P}_j\right)^{\frac{1}{2}} + \rho_j \tag{2.15}$$

$$\tilde{P}_i = \frac{P_i}{Y_A} \quad \text{and} \quad s_j = \frac{P_j X_j}{Y_A}$$

Rearranging (2.15) gives the following equation

$$s_j = \frac{1}{2}\sum_i \alpha_{ij}\left(1-\rho_j\right)\left(\tilde{P}_i \tilde{P}_j\right)^{\frac{1}{2}} - \rho_j \sum_i \sum_{k \neq j} \alpha_{ik}\left(\tilde{P}_i \tilde{P}_k\right)^{\frac{1}{2}} + \rho_j \tag{2.16}$$

The adding up condition implies that the following condition must be satisfied, Deaton and Muellbauer (1980a, Ch. 2).

$$\sum_j \rho_j = 1$$

The stochastic specification and estimation of model (2.16) are described in section 3. By imposing the restriction that $\alpha_{ij} = 0$ for $i \neq j$, the model (2.16) reduces to a share specification of the LES. This specification is also estimated in section 3, and the LES is tested against the more general model (2.16).

3 Estimation

Assume the following stochastic specification of the share equation (2.16)

$$s_{jt} = \frac{1}{2}\sum_i \alpha_{ij}(1-\rho_j)\left(\tilde{P}_i\tilde{P}_j\right)^{\frac{1}{2}} - \rho_j \sum_i \sum_{k\neq j} \alpha_{ik}\left(\tilde{P}_i\tilde{P}_k\right)^{\frac{1}{2}} + \rho_j + u_{jt} \quad (3.1)$$

$$i,j = E,F,H,D$$

where $t = 1963, ..., 1988$[2] u_t is a stochastic error term. s_{jt} is expenditure share of commodity j in year t and P_{jt} is the corresponding price. Y_{At} is total expenditure on the Housing and Heating aggregate, year t.

The parameters in the model (3.1) can be estimated by either quantity demanded X_{jt}, expenditure $Y_{jt} = X_{jt}P_{jt}$ or expenditure shares $s_{jt} = Y_{jt}/Y_{At}$ as the endogenous variables. The stochastic error term u_{jt} in equation (3.1) is assumed to be normally distributed and to satisfy the following properties

$$E(u_{jt}) = 0$$

$$E(u_{jt}u_{it}) = \sigma_{jit}$$

$$E(u_{js}u_{it}) = 0$$

$$s \neq t$$

$$i = 1, ..., n; j = 1, ..., n; t = 1, ..., T; s = 1, ..., T.$$

The expectation is zero and there is no systematic variation in the residuals over time, but it is not necessary to assume constant variances independent of time (Biørn, (1987)). The covariance matrix is also symmetric.

The stochastic error terms in the two other specifications are

$$u^y_{jt} = u_{jt}Y_{At}$$

when expenditure is the endogenous variable, and

[2]The data are described in Appendix C.

$$u^x_{jt} = u^y_{jt}/P_{jt}$$

when demand is the endogenous variable. The corresponding variances are

$$var(u^y_{jt}) = \sigma^2_{jj} Y^2_{At}$$

and

$$var(u^x_{jt}) = \sigma^2_{jj} \frac{Y^2_{At}}{P^2_{jt}}$$

and the stochastic error terms u^y_{jt} and u^x_{jt} are then heteroscedastic. Estimation results give no sign of heteroscedasticity in either of the model specifications, so it does not seem to be important which of the specifications is chosen in respect to this. In the following estimations expenditure share s_j is chosen as the dependent variable, i.e. model (3.1) is estimated.

The conditions from consumer theory impose cross equations restrictions on the parameters in the model. The adding up condition $\sum_j \rho_j = 1$ implies that

$$\sum_j \sum_i \alpha_{ij} (\tilde{P}_j \tilde{P}_i)^{\frac{1}{2}} = 0 \tag{3.2}$$

and

$$\sum_j u_{jt} = 0 \tag{3.3}$$

Multiplying (3.3) with u_{it} and taking the expectation gives the following expression (Biørn (1987)),

$$\sum_j cov(u_{it}, u_{jt}) = 0 \tag{3.4}$$

A necessary condition for adding up is a singular covariance matrix. This problem can be solved by omitting one equation in the estimation, Pollak and Wales (1969). The parameters in the omitted equation can all be derived from the parameters in the other equations, see Appendix A. The system of demand equations are estimated by the Full Information Maximum Likelihood Method (FIML) in TROLL, given the parameter restrictions.

All the explanatory variables in the model (3.1) can be wieved as exogenous variable. Y_{At} is given by

$$Y_{At} = \sum_{i=E,F,H,D} P_{it}X_{it}$$

The linear expenditure system is a nested under-hypothesis of the more general model (3.4). By assuming $\alpha_{ij} = 0$ for $i \neq j$, the function $a(P_A)$ reduces to

$$a(P_A) = \sum_j \alpha_j P_j$$

and $\alpha_j = \alpha_{jj}$. Implementing these restrictions on the general model (3.1) gives a share equation corresponding to the LES.

$$s_{jt} = \alpha_j \tilde{P}_j - \rho_j \sum_k \alpha_k \tilde{P}_k + \rho_j + u_t \tag{3.5}$$
$$j = E, F, H, D$$

The LES is a nested hypothesis of the more general model (3.1). Model (3.5) is then the nil hypothesis H_i and model (3.1) the alternative hypothesis H_j. Models (3.1) and (3.5) can be tested against each other by using the likelihood ratio test.

Let Ω_j denote the estimated value of the likelihood function. Under the assumption that H_i is true we have (Maddala (1983), T. Bye and Frenger (1990))

$$L = -2\ln(\frac{\Omega_{(H_i)}}{\Omega_{(H_j)}}) = T(FCN(\omega_i) - FCN(\omega_j)) \sim \chi^2(l_j - l_i) \tag{3.6}$$

where $l_j - l_i$ represents the number of restrictions which the nil hypothesis H_i imposes on the alternative hypothesis H_j and T is the number of observations. FCN_j is a scaled inverse normalized version of the likelihood function, T. Bye and Frenger (1990). H_i is rejected if the value of the L-statistic exceeds the corresponding fractile in the χ^2-distribution.

The results from the estimation of model (3.1) and model (3.5) are presented in Table 3.1. The adding up restriction $\sum_j \rho_j = 1$ and the symmetry restrictions $\alpha_{ji} = \alpha_{ij}$ for all $i \neq j$, are all imposed on the parameters in the estimation.

Table 3.1: Estimation results[1], models (3.1) and (3.5).

Parameter	(3.1)	(3.5)
α_{EE}	0.644	0.300
	(0.342)	(0.077)
α_{EF}	0.101	
	(0.156)	
α_{EH}	1.931	
	(0.565)	
α_{ED}	-0.745	
	(0.201)	
ρ_E	0.176	0.171
	(0.013)	(0.009)
α_{FF}	0.044	0.066
	(0.174)	(0.015)
α_{FH}	0.395	
	(0.181)	
α_{FD}	-0.080	
	(0.065)	
ρ_F	0.028	0.047
	(0.009)	(0.007)
α_{HH}	3.497	1.102
	(0.477)	(0.101)
α_{HD}	-1.429	
	(0.355)	
ρ_H	0.518	0.487
	(0.012)	(0.005)
α_{DD}	1.676	0.227
	(0.398)	(0.056)
ρ_D^*	0.276	0.293
$R^2 - s_E$	0.824	0.723
$R^2 - s_F$	0.369	0.232
$R^2 - s_H$	0.954	0.955
$RSS - s_E$	0.002	0.004
$RSS - s_F$	0.002	0.003
$RSS - s_H$	0.002	0.002
$DW - s_E$	0.497	0.314
$DW - s_F$	0.184	0.153
$DW - s_H$	0.254	0.280
FCN	-31.082	-30.540

1) The standard deviations are given in the parentheses().

*) Follows from the adding up condition.

R^2 is the multiple correlation coefficient, DW is the Durbin-Watson statistic and RSS is the residual sum of squares. The estimation results for model (3.1) show serious problems of autocorrelation. The equation for fuel-oil has a low R^2 compared to the other equations. Three of the parameters are not significant at 5 percent level.

The results from the estimation of model (3.5) show a fall in the model's explanatory power measured by R^2, but all the parameters are significant. The value of the test-statistic given in equation (3.6) is $L = 14.09$ and the corresponding critical level in the Chi-square distribution is 12.59 at a 5 percent level. According to the Likelihood ratio test the hypothesis (3.5) is rejected. This model has nevertheless significant parameters, but this can be due to both downward biases in the standard deviation from the autocorrelated stochastic error terms and fewer parameters to estimate. Since these estimation results give evidence of serious misspecification of the model with respect to lack of dynamic specification, there is a need for further analyses of the system's dynamic properties. In section 4 a dynamic version of the LES is considered.

The expenditure parameters ρ_j's do not differ much in the two model specifications, except for fuel-oil. On the other hand the α_{jj}'s differ significantly between the two models. They are all positive in the LES model (3.5). The commodities in the aggregate are all substitutes according to Hicks' definition of substitutes as commodities with positive compensated cross price elasticity, Deaton and Muellbauer (1980a, Ch. 2). We had expected to find that electricity and durables were complements, but according to Hicks' definition they are not. As we can see from Table 3.2 the Cournot cross price elasticity between electricity and durables is negative, but quite small in both models and hence they are not close substitutes. This implies that an uncompensated increase in the electricity price gives a reduction in the demand for energy using durables (here the stock of energy using durables). The income effect is stronger than the substitution effect when the cross price elasticities are negative. The cross Cournot elasticities are quite small, except the ones between electricity and housing and housing and durables. This can partly be explained by the higher estimated expenditure parameter for housing and durables. There seems though to be a tendency that the Cournot elasticities are smaller in model (3.5) than in model (3.1). The formulas for the elasticities are given in Appendix B.

All the own-price elasticities are negative. The relative Engel elasticity for fuel-oil is approximately half the size of the corresponding elasticity for electricity in model (3.1), but in model (3.5) the relative Engel elasticity for

Table 3.2: Engel[1] and Cournot elasticities[2]

Elasticity	(3.1)	(3.5)
η_{EE}	-0.479	-0.689
η_{EF}	-0.017	-0.016
η_{EH}	-0.149	-0.281
η_{ED}	-0.182	-0.061
ξ_{EY_A}	1.047	1.021
η_{FF}	-0.355	-0.632
η_{FE}	-0.015	-0.072
η_{FH}	0.203	-0.315
η_{FD}	-0.063	-0.068
ξ_{FY_A}	0.689	1.142
η_{HH}	-0.773	-0.745
η_{HE}	-0.016	-0.055
η_{HF}	-0.010	-0.014
η_{HD}	-0.081	-0.052
ξ_{HY_A}	0.932	0.876
η_{DD}	-1.092	-0.817
η_{DE}	-0.475	-0.078
η_{DF}	-0.093	-0.020
η_{DH}	-1.113	-0.345
ξ_{DY_A}	1.178	1.251

1) Relative Engel elasticities for the aggregate, i.e. they are not multiplied by β_A, see the formula in Appendix B

2) All the above elasticities are calculated at the mean of the variables over the estimation period.

fuel oil is larger than the corresponding one for electricity. This is due to the relatively higher expenditure parameter in model (3.5). The stock of durables has the highest relative Engel elasticity.

4 Dynamic specification

The estimation results in the last section gave evidence of serious misspecification of the models with respect to dynamic properties. The DW-statistics for most of the equations are very low, in the range 0.2 to 0.7 for both models. In this section a simple version of a dynamic LES is presented. The minimum necessary consumption of each commodity is assumed to depend on the actual consumption in the previous year, Pollak (1970). This can be interpreted as a habit formation process.

$$\alpha_{jt} = c_{j1} + c_{j2}X_{j,t-1} \tag{4.1}$$

By substituting for α_j in the static demand equation, we get the following short run demand equation for the dynamic LES.

$$X_{jt} = c_{j1} + c_{j2}X_{jt-1} + \rho_j \frac{Y_{At}}{P_{jt}} - \rho_j \sum_k c_{k1} \frac{P_{kt}}{P_{jt}} - \rho_j \sum_k c_{k2} \frac{P_{kt}}{P_{jt}} X_{kt-1} \tag{4.2}$$

The long run demand equation can be found by assuming the steady state solution $X_{jt-1} = X_{jt} = X_j$. Inserting for X_{jt-1} and X_{jt} in the demand equation (4.2) gives the long run demand equation, Pollak (1970).

$$X_j = B_j + A_j \frac{Y_{At}}{P_{jt}} - A_j \sum_k \frac{P_{kt}}{P_{jt}} B_k \tag{4.3}$$

$$A_j = \frac{\frac{\rho_j}{1-c_{j2}}}{\sum_j \frac{\rho_j}{1-c_{j2}}}$$

$$B_j = \frac{c_{j1}}{1-c_{j2}}$$

For estimation purposes we consider the expenditure shares as the dependent variables to be able to compare the results with the results in section 3. The stochastic specification of the demand system with expenditure shares as the dependent variables is then

$$s_{jt} = c_{j1}\tilde{P}_{jt} + c_{j2}\tilde{P}_{jt}X_{jt-1} + \rho_j - \rho_j \sum_k c_{k1}\tilde{P}_{kt} - \rho_j \sum_k c_{k2}\tilde{P}_{kt}X_{kt-1} + \varepsilon_{jt} \quad (4.4)$$

The stochastic error term ε_{jt} has the following properties

$$E(\varepsilon_{jt}) = 0$$

$$E(\varepsilon_{jt}\varepsilon_{it}) = \sigma_{\varepsilon jit}$$

$$E(\varepsilon_{js}\varepsilon_{it}) = 0$$

$$s \neq t$$

$$i = 1, ..., n; j = 1, ..., n; t = 1, ..., T; s = 1, ..., T.$$

The expectation is zero and there is no systematic variation in the residuals over time, but it is not necessary to assume constant variances independent of time. The covariance matrix is also symmetric.

The short run demand system (4.4) can be estimated with FIML, and the long run parameters A_j and B_j and the elasticities can be derived by using these short run parameters. The estimation results are presented in Table 4.1.

The static system in (3.5) is a nested hypothesis of the dynamic system in (4.4), and hence this model can be tested by using the likelihood ratio test in (3.6).

Table 4.1: Estimation results model (4.4)[1]

Parameter	(4.4)	$(4.4)^2$	$(4.4)^3$
c_{E1}	0.017	0.017	0.017
	(0.022)	(0.028)	(0.025)
c_{E2}	0.977	0.915	0.927
	(0.362)	(0.050)	(0.046)
ρ_E	0.144	0.174	0.170
	(0.080)	(0.067)	(0.071)
c_{F1}	0.024	0.006	0.007
	(0.018)	(0.014)	(0.014)
c_{F2}	0.564	0.591	0.592
	(0.076)	(0.062)	(0.061)
ρ_F	0.285	0.182	0.195
	(0.063)	(0.029)	(0.033)
c_{H1}	-0.027	0.008	0.000
	(0.011)	(0.014)	(0.012)
c_{H2}	1.031	0.98	0.99
	(0.012)	...	...
ρ_H	0.285	0.367	0.358
	(0.063)	(0.040)	(0.041)
c_{D1}	0.020	0.016	0.016
	(0.005)	(0.009)	(0.008)
c_{D2}	0.978	0.924	0.933
	(0.018)	(0.020)	(0.018)
ρ_D^*	0.260	0.2742	0.275
$R^2 - X_E$	0.928	0.926	0.927
$R^2 - X_F$	0.908	0.902	0.905
$R^2 - X_H$	0.995	0.994	0.994
$RSS - X_E$	0.001	0.001	0.001
$RSS - X_F$	0.000	0.000	0.000
$RSS - X_H$	0.000	0.000	0.000
$DW - X_E$	2.210	1.931	1.987
$DW - X_F$	2.081	1.478	1.579
$DW - X_H$	2.351	2.275	2.302
FCN	-35.449	-35.200	-35.270

1) The standard deviations are given in the brackets ().

*) Follows from the adding up condition.

2) $c_{H2} = 0.98$

3) $c_{H2} = 0.99$

R^2 is the multiple correlation coefficient, DW is the Durbin-Watson statistic and RSS is the residual sum of squares. Three of the short run parameters are not significant. Compared to the static model the model's explanation power is increased significantly. DW-statistics in the range 2-2.3 indicates a better dynamic specification. The likelihood ratio test value L is 122.73 which gives rejection of the static LES model (3.5).

The parameter c_{H2} is significantly larger than 1 and hence the model does not have the property of stationarity, Pollak (1970). Stationarity implies that all the c_{j2}'s have to be less than 1. The stability of the parameters is tested by eliminating observations. When the observations 1964-1985 are included, all the c_{j2} parameters are less than 1. The model is then estimated over the whole sample period when the restrictions $c_{H2} = 0.98$ and $c_{H2} = 0.99$ are imposed respectively. These models are nested hypotheses of model (4.4) and they can then be tested against this model by using the likelihood ratio test. The likelihood ratio is 4.48 when $c_{H2} = 0.99$ and 5.99 when $c_{H2} = 0.98$, so both the hypotheses are rejected compared to the alternative hypothesis model (4.4) (the critical level is 3.84). Tested against the nested hypothesis model (3.5), model (3.5) is rejected. In choosing between the two restricted estimations, the value of the likelihood function and other test statistics must be compared. The model with $c_{H2} = 0.99$ has slightly better values of the likelihood function, R^2 and DW when the stability of the parameters was tested than the model with $c_{H2} = 0.98$. The estimation results for both these specifications are given in Table 4.1.

The long run parameters calculated from the estimated short run parameters for the restricted model (4.4) are given in Table 4.2. The B's can be compared to the α's and the A's to the ρ's in the static model.

Table 4.2: Long run parameters.

Model	B_E	B_F	B_H	B_D	A_E	A_F	A_H	A_D
(4.4)	0.793	0.055	0.866	1.727	0.691	0.069	-1.033	0.114
$(4.4)^2$	0.207	0.016	0.400	0.217	0.084	0.018	0.750	0.147
$(4.4)^3$	0.240	0.018	0.09	0.252	0.054	0.011	0.837	0.096

2) c_{H2}=0.98

3) c_{H2}=0.99

The long run parameters from the restricted estimation differs significantly from the parameters in the static model (3.5) given in Table 3.1. The expenditure parameter for housing A_H is much larger than in the static model, and hence the other expenditure parameters have to be smaller to

Table 4.3: Engel[1] and Cournot elasticities[2]

Elasticity	(4.4)[3]	(4.4)[4]
η_{EE}	-0.762	-0.716
η_{EF}	-0.001	-0.001
η_{EH}	-0.050	-0.007
η_{ED}	-0.028	-0.021
ξ_{EY_A}	0.500	0.325
η_{FF}	-0.908	-0.892
η_{FE}	-0.019	-0.013
η_{FH}	-0.043	-0.006
η_{FD}	-0.025	-0.018
ξ_{FY_A}	0.439	0.270
η_{HH}	-0.955	-0.993
η_{HE}	-0.058	-0.076
η_{HF}	-0.005	-0.006
η_{HD}	-0.077	-0.100
ξ_{HY_A}	1.348	1.505
η_{DD}	-0.790	-0.741
η_{DE}	-0.027	-0.020
η_{DF}	-0.002	-0.001
η_{DH}	-0.062	-0.009
ξ_{DY_A}	0.627	0.410

1) Relative expenditure elasticities for the aggregate, i.e. they are not multiplied by β_A, see the formula in Appendix B.

2) All the above elasticities are calculated at the mean of the variables over the estimation period.

3) $c_{H2} = 0.98$

4) $c_{H2} = 0.99$

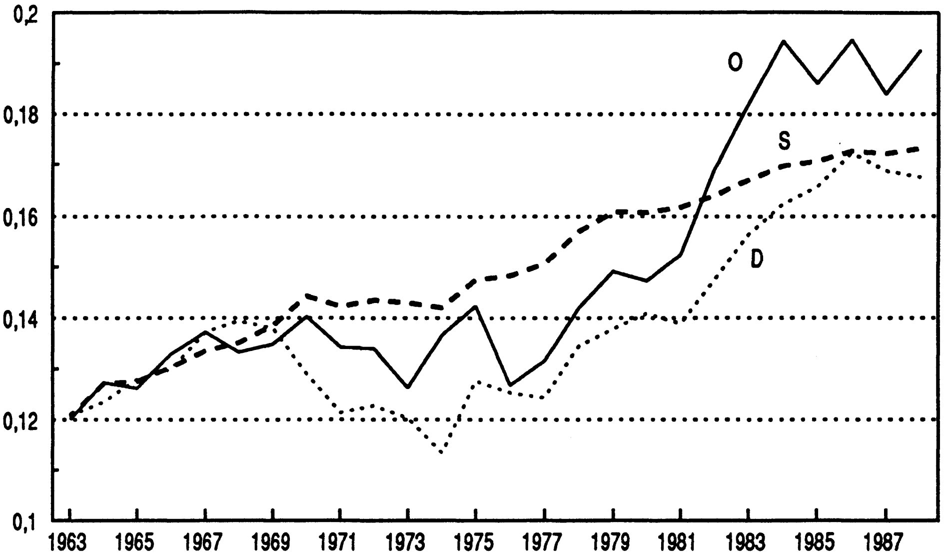
fulfill the adding up condition. The long run Engel- and Cournot elasticities for the restricted estimation calculated from the long run parameters are given in Table 4.3. The formulas are given in Appendix B.

The Engel elasticities for the restricted dynamic model given in Table 4.3 are smaller than the elasticities for model (3.5) in Table 3.2, except for housing. The Engel elasticity for fuel-oil is smaller than the one for electricity, and they are both much smaller than in the static model. In the overall demand system they will be multiplied by the expenditure parameter for the Housing and Heating aggregate times the inverse of the overall expenditure share, see Appendix B, to derive the overall Engel elasticity. The expenditure share for fuel-oil is much smaller than the one for electricity, so the relative size of the Engel elasticities between fuel-oil and electricity will most likely change when the overall elasticities are considered. The results give no common relationship between the cross Cournot elasticities in the dynamic and the static model. The own price elasticities are bigger for all the commodities in the dynamic specification than in the static one except the stock of durables. An explanation for this is that it takes time before the effects of a price change are emptied. The Engel elasticity for durables is much smaller in the dynamic specification, and the opposite is the result for housing. This can be due to the problems with non-stationarity.

5 Simulation Properties

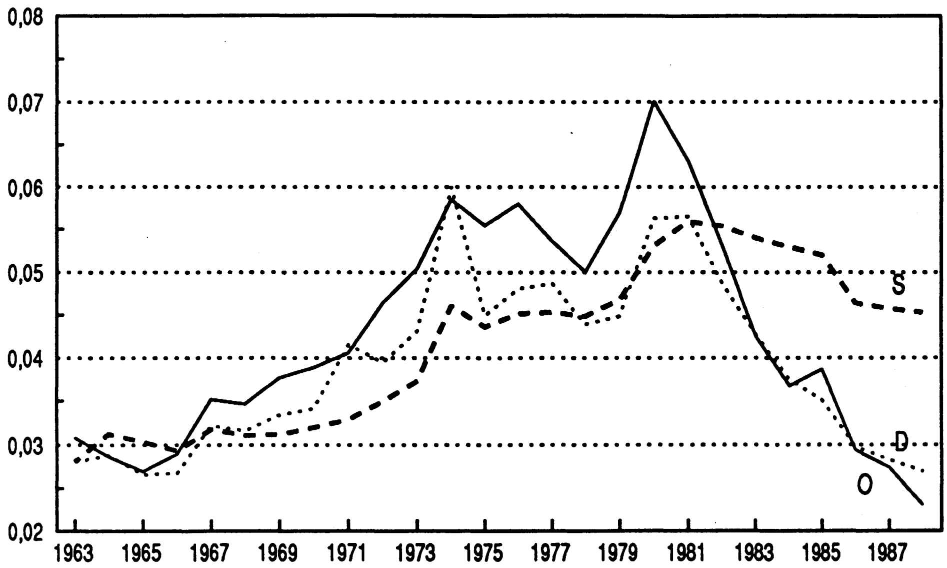
Both the static and dynamic LES models are simulated over the period 1963 to 1988, see Figures 5.1-5.4. In the simulation the stochastic error terms are equal to zero. The difference between the simulated and actual values then reflects the error terms. For all commodities the dynamic model simulates better than the static model, even though the dynamic model has problems with following the relative huge year to year variations especially for fuel-oil but also to a certain extent for electricity. The model has a tendency to underestimate the actual expenditure share. For fuel-oil the expenditure share is considerable underestimated in 1973-74 and 1979-80, both periods with high increases in oil-prices. The expenditure share for electricity is underestimated from the beginning of the seventies. This can be due to the partly unexplained trend towards electricity in this period, especially in the eighties, reflected in the relatively large positive error terms for the period.

Figure 5.1. Simulated Expenditure Share, Electricity, 1963- 1988



O: Observed S: Static LES D: Dynamic LES, c_{H2} = 0,98

Figure 5.2. Simulated Expenditure Share, Fuel-oil, 1963-1988



O: Observed S: Static LES D: Dynamic LES, c_{H2} = 0,98

Figure 5.3. Simulated Expenditure Share, Housing, 1963-1988

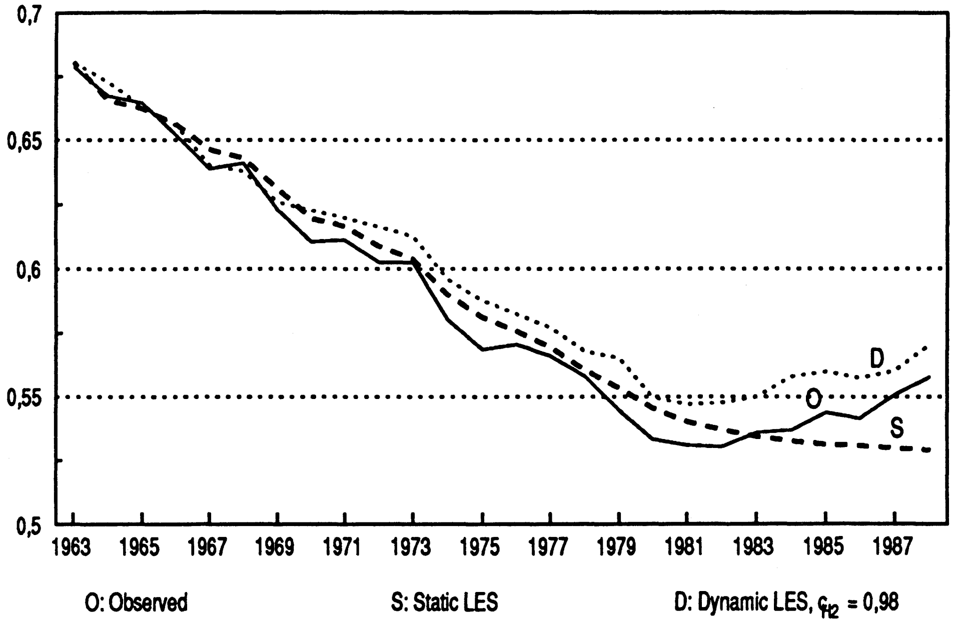
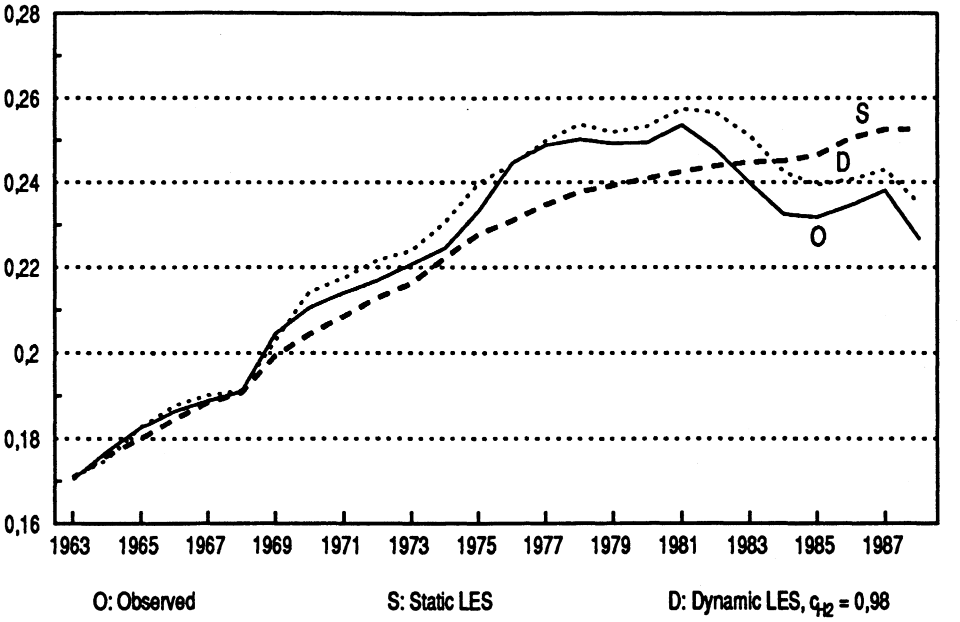


Figure 5.4. Simulated Expenditure Share, Durables, 1963-1988



The dynamic LES model has the property of non-stationarity. From the result in Section 4 it is easy to see that a small change in the crucial parameter value greatly influence the other expenditure parameters and hence all the long run parameters and elasticities. Another aspect is the long run simulation properties of this kind of model. From equation (4.2) it is easy to see that relative large values of the lagged dependent variables other than the one for which we consider the demand, will reduce the level of this dependent variable. If the model is used for long run forecasting the effects of some of the lagged dependent variables can dominate the demand for the other commodities, such that the demand for these commodities will be decreasing during the forecasting period. The opposite will be the situation for the commodities which dominate. Long run simulations with the dynamic model can give this probably unrealistic development.

6 Concluding remarks

The analyses in the previous sections indicates that the stock of energy using durables contributes to explain the use of electricity and fuel-oil over the period. Hence the commodities are not separable. The Engel-elasticities for fuel-oil and electricity also differ significantly. To not take into consideration the differences in Engel-elasticities when building macroeconomic models for analyses of energy-policy topics may result in wrong policy implications in order to reach certain energy policy goals. The static model specifications give evidence of dynamic misspecification. A dynamic LES specification improves the model's simulation properties and explanatory power considerably, but some of the properties with this kind of dynamic model indicates that it is not a proper model to use for forecasting and policy analyses.

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Appendix A

The demand system (3.1) consists of the following share equations (for convinience the stochastic error term is left out)

$$
\begin{aligned}
s_E &= \frac{1}{2}\alpha_{EE}(1-\rho_E)\tilde{P}_E + ((\frac{1}{2}-\frac{3}{2}\rho_E)\alpha_{FE})(\tilde{P}_E\tilde{P}_F)^{\frac{1}{2}} \\
&+ ((\frac{1}{2}-\frac{3}{2}\rho_E)\alpha_{HE})(\tilde{P}_E\tilde{P}_H)^{\frac{1}{2}} + ((\frac{1}{2}-\frac{3}{2}\rho_E)\alpha_{ED})(\tilde{P}_E\tilde{P}_D)^{\frac{1}{2}} \\
&- \rho_E\alpha_{FF}\tilde{P}_F - \rho_E\alpha_{HH}\tilde{P}_H - \rho_E\alpha_{DD}\tilde{P}_D - 2\rho_E\alpha_{HF}(\tilde{P}_H\tilde{P}_F)^{\frac{1}{2}} \\
&- 2\rho_E\alpha_{DF}(\tilde{P}_F\tilde{P}_D)^{\frac{1}{2}} - 2\rho_E\alpha_{HD}(\tilde{P}_D\tilde{P}_H)^{\frac{1}{2}} + \rho_E
\end{aligned} \tag{A.1}
$$

$$
\begin{aligned}
s_F &= \frac{1}{2}\alpha_{FF}(1-\rho_F)\tilde{P}_F + ((\frac{1}{2}-\frac{3}{2}\rho_F)\alpha_{EF})(\tilde{P}_E\tilde{P}_F)^{\frac{1}{2}} \\
&+ ((\frac{1}{2}-\frac{3}{2}\rho_F)\alpha_{HF})(\tilde{P}_F\tilde{P}_H)^{\frac{1}{2}} + ((\frac{1}{2}-\frac{3}{2}\rho_F)\alpha_{DF})(\tilde{P}_F\tilde{P}_D)^{\frac{1}{2}} \\
&- \rho_F\alpha_{EE}\tilde{P}_E - \rho_F\alpha_{HH}\tilde{P}_H - \rho_F\alpha_{DD}\tilde{P}_D - 2\rho_F\alpha_{HE}(\tilde{P}_H\tilde{P}_E)^{\frac{1}{2}} \\
&- 2\rho_F\alpha_{DE}(\tilde{P}_E\tilde{P}_D)^{\frac{1}{2}} - 2\rho_F\alpha_{HD}(\tilde{P}_D\tilde{P}_H)^{\frac{1}{2}} + \rho_F
\end{aligned} \tag{A.2}
$$

$$
\begin{aligned}
s_H &= \frac{1}{2}\alpha_{HH}(1-\rho_H)\tilde{P}_H + ((\frac{1}{2}-\frac{3}{2}\rho_H)\alpha_{EH})(\tilde{P}_E\tilde{P}_H)^{\frac{1}{2}} \\
&+ ((\frac{1}{2}-\frac{3}{2}\rho_H)\alpha_{FH})(\tilde{P}_F\tilde{P}_H)^{\frac{1}{2}} + ((\frac{1}{2}-\frac{3}{2}\rho_H)\alpha_{DH})(\tilde{P}_D\tilde{P}_H)^{\frac{1}{2}} \\
&- \rho_H\alpha_{EE}\tilde{P}_E - \rho_H\alpha_{FF}\tilde{P}_F - \rho_H\alpha_{DD}\tilde{P}_D - 2\rho_H\alpha_{EF}(\tilde{P}_E\tilde{P}_F)^{\frac{1}{2}} \\
&- 2\rho_H\alpha_{DE}(\tilde{P}_E\tilde{P}_D)^{\frac{1}{2}} - 2\rho_H\alpha_{DF}(\tilde{P}_F\tilde{P}_D)^{\frac{1}{2}} + \rho_H
\end{aligned} \tag{A.3}
$$

$$
\begin{aligned}
s_D &= \frac{1}{2}\alpha_{DD}(1-\rho_D)\tilde{P}_D + \left(\left(\frac{1}{2}-\frac{3}{2}\rho_D\right)\alpha_{ED}\right)(\tilde{P}_E\tilde{P}_D)^{\frac{1}{2}} \qquad \text{(A.4)}\\
&+ \left(\left(\frac{1}{2}-\frac{3}{2}\rho_D\right)\alpha_{FD}\right)(\tilde{P}_F\tilde{P}_D)^{\frac{1}{2}} + \left(\left(\frac{1}{2}-\frac{3}{2}\rho_D\right)\alpha_{HD}\right)(\tilde{P}_H\tilde{P}_D)^{\frac{1}{2}}\\
&- \rho_D\alpha_{EE}\tilde{P}_E - \rho_D\alpha_{FF}\tilde{P}_F - \rho_D\tilde{P}_H - 2\rho_D\alpha_{HE}(\tilde{P}_H\tilde{P}_E)^{\frac{1}{2}}\\
&- 2\rho_D\alpha_{EF}(\tilde{P}_E\tilde{P}_F)^{\frac{1}{2}} - 2\rho_D\alpha_{FH}(\tilde{P}_F\tilde{P}_H)^{\frac{1}{2}} + \rho_D
\end{aligned}
$$

Equations (A.1) to (A.3) are estimated simultaneously, and the only unknown parameter in equation (A.4) ρ_D, can be derived by using the adding up condition, $\sum_j \rho_j = 1$, which implies that $\rho_D = 1 - \rho_E - \rho_F - \rho_H$.

Appendix B

The Engel elasticity can be derived from (2.13) by inserting for Y_A from equation (2.9)

$$\xi_{jY_C} = \rho_j \beta_A \frac{Y_C}{Y_j} \tag{B.1}$$

The Engel elasticity of total expenditure on Housing and Heating can also be derived from (2.13).

$$\xi_{jY_A} = \rho_j \frac{Y_A}{Y_j} \tag{B.2}$$

We can derive the direct (η_{jj}) and cross price (η_{ji}) demand elasticities (Cournot-elasticities) respectively

$$\eta_{jj} = \left(\frac{1}{4}(1-\rho_j)\frac{1}{s_j}\sum_i \alpha_{ij}\tilde{P}_i\tilde{P}_j \right)^{\frac{1}{2}} - 1 \tag{B.3}$$

$$\eta_{ji} = \frac{1}{2}\left(\frac{1}{2}(1-\rho_j)\alpha_{ij}(\frac{P_i}{P_j})^{\frac{1}{2}} - \rho_j \sum_j \alpha_{ij}(\frac{P_i}{P_j})^{\frac{1}{2}} \right) \frac{1}{X_j} \tag{B.4}$$

For the LES the expenditure elasticities are similar to (B.1) and (B.2), but the Cournot elasticities have the following expressions.

$$\eta_{jj} = (\alpha_{jj}(1-\rho_j))\frac{1}{s_j}\frac{P_j}{Y_A} - 1 \tag{B.5}$$

$$\eta_{ji} = (-\alpha_{ij}\rho_j)\frac{1}{s_j}\frac{P_j}{Y_A} \tag{B.6}$$

Appendix C

The volume and prices of electricity and fuel-oil are taken from The Energy Data Base (Hetland et al, 1990), and the volume and price of housing (commodity 50) and purchase of durables (commodity 40) are taken from the National Account, database AARDAT. The stock of durables is calculated by an ordinary neo-classical investment approach, Biørn and Jensen (1983) and Magnussen (1990). A corresponding user cost of capital (price) is calculated by using a formula given in Biørn and Jensen (1983). The magnitude of the user cost which the consumers actually face, is very uncertain. The assumptions about depreciation rate and interest rate are crucial for the development of this price. The depreciation rate is taken from Magnussen and Skjerpen (1990) and the interest rate chosen is the average nominal interest on bank loans.

The calculated user cost does not differ much from the purchase price for durables which could be another approximation for the user cost. Durables consist of all household durables except cars. It is not unreasonable to use the total stock of these durables as an approximation for the stock of energy using durables because the stock of these different durables has probably developed more or less proportionally. Another aspect is that waterbeds which use electricity for heating the water, are grouped as furniture in the National Account. The number of such beds has increased rapidly over the last ten years. All volumes are measured in mill. kr. fixed 1988-prices and the prices are price-indices equal to 1 in 1988. The sample period is 1963 to 1988.

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