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Theoretical and Econometric Modeling of Disequilibrium *

by

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Abstract

This paper surveys some recent work in the theoretical and econometric disequilibrium literature. The first part of the paper gives an overview of some the main concepts used in the theory of non-Walrasian economics. These concepts give the theoretical background for a discussion of recent developments in the econometric disequilibrium literature. Particular emphasis is given to two recent developments. One is the virtual price approach of Lee (1986) and the other is the stochastic aggregation (smoothing by aggregation) approach used in Lambert (1988).

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1 Introduction

In the following I will look at recent developments in both theoretical and econometric modeling of disequilibrium. Theoretical work on disequilibrium has been done both within macroeconomics and microeconomics. In a macroeconomic analysis Barro and Grossman (1971) explicitly analyzed how the macroeconomy was affected by non-clearing markets by combining the analysis of the firm under disequilibrium in Patinkin (1956) with the analysis of the household under disequilibrium in Clower (1965). They thereby developed what has become the prototype *neo-Keynesian* model, where there are three macro commodities (a consumption good, labor, and money) and three agents (a consumer, a producer, and government). Malinvaud (1977) introduced the now familiar concepts of Keynesian and classical unemployment. These models were attempts at obtaining Keynesian results in models where economic agents are rational maximizing decision-makers.

Parallel to the above macroeconomic developments, the general equilibrium framework associated with Arrow and Debreu was generalized to the case where there is no Walrasian auctioneer to clear the markets. Most of these attempts were based on Hick's (1946) notion of *temporary equilibrium.* In temporary equilibrium market behavior is thought of as a process taking place sequentially in time with economic agents continually learning about their economic environment. See for example the collection of papers in Grandmont (1988). At each date trade takes place, but the agents' plans for the future are not coordinated and there may be quantity rationing. Such a framework allows for price movements between dates (often thought of as a very short time period) and therefore allows the explicit modeling of price processes. Both the Walrasian market-clearing situation, where prices continually clear markets, and the three commodity neo-Keynesian model are special cases of temporary equilibrium models.

The temporary equilibrium literature has so far been mainly concerned with the existence of temporary equilibrium and the role of expectations in equilibrium.

There has been done relatively little work on the dynamic adjustment process of prices and expectations. Even so, the conceptual framework on which temporary equilibrium models are based makes it clear that such processes are very important. To get a satisfactory picture of the workings of an economy, we must understand how agents form expectations about prices and quantities, how they signal their wishes to the market, and how the market responds to these signals by setting prices and assigning quantities to each agent.

Fixed-price models will in the following denote models where prices are assumed fixed without any explicit modeling of price processes. During the last couple of decades a number of different theories such as efficiency wages and menu costs have been advanced trying to explain the rigid prices and wages of fixed-price models, but these theories have had a tendency to be good stories instead of a compelling general theory. Still, it is the fixed-price model which has been investigated most thoroughly and has been used almost exclusively as a basis for econometric models. Some of the most promising work on explaining disequilibrium is the monopoly models of, among others, Benassy (1976, 1987) which we will briefly discuss later in the paper. Present work on dynamics and wage and price formation will hopefully lead to models with an explicit modeling of price dynamics.

The two main equilibrium concepts which have emerged from the literature are Drèze equilibria due to Drèze (1975) and K-equilibria due to Benassy (1975,1982). They are characterized by utilizing different specifications of the agents' *effective demands*. Effective demands are the demands that arise after the agents have taken rationing into account. Drèze (1975) uses *Drèze demands* which are the result of utility maximization subject to the budget constraint and all quantity constraints that exist. Benassy (1975,1982) uses *Clower demands* which are the result of utility maximization subject to the budget constraint and all quantity constraints except the quantity constraint of the good one is calculating the demand for. Both these concepts have their weaknesses. With Drèze demands there is no difference between effective demand and actual trades and therefore the agents' demands do not signal to the markets whether they are rationed or not. Clower demands are the result of a utility maximization which does not take into account all the quantity constraints simultaneously and therefore do not necessarily satisfy the budget constraint.

The first section of this paper consists of a short exposition of the standard two-market neo-Keynesian model based on quantity rationing. This model serves as an introduction to some of the concepts used in disequilibrium models and has inspired much of the research within non-Walrasian economics. It is still the framework within which most multi-market models are set.

After introducing the standard neo-Keynesian model there follows a presentation of some of the concepts used in the temporary equilibrium literature, including a more thorough discussion of the concepts mentioned above. The emphasis is on explaining the different concepts and not on mathematical rigor. Existence results are not covered. I will use the terms *disequilibrium economics* and *non-Walrasian economics* interchangeably as general terms covering the whole field of inquiry, while *temporary equilibrium* will be used to denote non-Walrasian models where the economy is looked upon as a sequence of markets in the sense of Hicks (1946).

The development of theoretical disequilibrium models has led to a large literature on the estimation of such models, starting with the seminal article of Fair and Jaffee (1972), which examined a single market under fixed prices (the market for housing starts). Later on, estimation methods for two-market models, such as the prototype neo-Keynesian model, were developed by among others Ito (1980). Ito's approach is based on Cobb-Douglas utility functions and Clower demands. An extensive survey of this earlier literature on disequilibrium econometric modeling and of the many problems encountered is given by Quandt (1982). In the present paper I will briefly discuss some of the work done on estimating single-market disequilibrium models and Ito's specification of multi-market models. The methods used in estimating such a single-market disequilibrium model provide a good illustration of the latent variable problems involved. Such problems arise because transacted quantities under rationing carry a limited amount of information about the structure of demand and supply in the market.

The last part of the paper discusses two recent developments in the econometric literature of disequilibrium. The first is the *virtual price* approach advocated by Lee (1986). Virtual prices (shadow prices) are often used in dealing with a situation with rationing. They are the prices that would induce an unrationed consumer to purchase the rations exactly (see Deaton and Muellbauer (1980) pp. 109-114). By using virtual prices, Lee (1986) shows that in fixed-price models the assumption that only one side of a market can be rationed at one time (often referred to as the *min* condition) implies that different specifications of effective demand will lead to observationally equivalent expressions. Effective demand is the demand expressed by an agent after taking into account the constraint he faces. Lee also derives a method for estimating multi-market disequilibrium models in the special case of two representative agents. By using the notion of virtual prices, he overcomes earlier difficulties in finding a computationally tractable method for estimating models with more than two markets.

The second is the *smoothing by aggregation* approach first suggested by Muellbauer (1978) and used in among others Lambert (1988). Many other recent papers estimating empirical disequilibrium models use this approach, which takes as a starting point the specification of supply and demand in micro markets. These micro markets are taken to be small efficient markets where it seems reasonable to postulate that only one side of the market is rationed. These papers then assume that supply and demand in these markets can be modeled as consisting of a structural and a stochastic component. Assuming that the stochastic components are distributed in the same manner in all markets, aggregation to the macro level is achieved by integrating over micro markets. At any one time some micro markets will be in excess demand while others will be in excess supply, so that at the macro level both sides of the market may be partially rationed at the same time. In the neo- Keynesian macro model this means that there can be Keynesian unemployment in some sectors while there is classical unemployment in others. The smoothing by aggregation approach has been used in fairly large and complex macroeconomic models such as the model presented by Lambert (1988).

2 The basic Keynesian disequilibrium model

The idea of disequilibrium modeling is in most economists' minds associated with the neo-Keynesian models of Barro and Grossman (1971) and of Malinvaud (1977). These models are based on utility and profit maximization within a general equilibrium framework and can be seen as a justification for and an embellishment on earlier Keynesian models. What separates these models from neoclassical models is that they assume that prices do not necessarily clear the markets. When markets fail to clear, agents will find themselves rationed in the sense that they cannot buy or sell as much as desired at the prevailing non-clearing prices. The different markets are interdependent, and therefore rationing in one market will spill over to other markets.

While this disequilibrium approach at first was concerned with giving a more satisfying microeconomic foundation to Keynesian economics, it was closely connected with work being pursued in mathematical economics on non-Walrasian equilibrium. Even though it is the Keynesian aspects of the theory which have remained uppermost in many economists' minds, I believe the most important part has been the attempt to generalize and make more realistic the general equilibrium framework of Arrow-Debreu. A very important aspect of this work is the realization that such attempts also are attempts at macroeconomic modeling. Drèze argues in his presidential address to the European Economic Association (Drèze (1987)) that:

> *...general equilibrium with rationing covers macroeconomics automatically, sparing us the need to develop two separate fields.*

Drèze sees this approach as a long overdue integration of general equilibrium theory and macroeconomics. The development of this theory has come a long way during the last twenty years, even though there are important areas which are not well enough developed, especially an understanding of price dynamics. Before discussing the concepts used in generalizing the Arrow-Debreu model, we will give a brief description of the neo-Keynesian model as developed by Barro and Grossman (1971) and Malinvaud (1977).

We start by considering the traditional utility maximization problem of a single consumer who buys a good, X, sells his labor, L, and holds money, M. The price of the good i p, the wage is w, and the consumer's initial endowment is R. Money held initially is included in R. The maximization problem can be written:

$$\max_{X,L,M} U(X, L, M)$$

$$\text{s.t.} \quad pX - wL + M = R. \tag{1}$$

As we will see later, a utility function including money can be viewed as an indirect utility function incorporating the intertemporal aspects of the consumer's behavior. Solving this maximization problem we get demand and supply functions f_1, f_2, and f_3 for the good X, labor L, and money M:

$$\begin{aligned} \tilde{X} &= f_1(p, w, R) \\ \tilde{L} &= f_2(p, w, R) \\ \tilde{M} &= f_3(p, w, R) \end{aligned} \tag{2}$$

These functions are the consumer's *notional* demand and supply functions.

We now assume that the consumer is rationed in the labor market, and can only supply $\bar{L}$ of labor. For the constraint to be binding we must have that $\bar{L} < \tilde{L}$. The consumer's maximization problem when the constraint is binding now becomes:

$$\max_{X,M} U(X, M)$$

$$\text{s.t.} \quad pX + M = R + w\bar{L} \tag{3}$$

leading to the following demand functions:

$$\begin{aligned} X^e &= g_1(p, R + w\bar{L}), \\ M^e &= g_2(p, R + w\bar{L}). \end{aligned} \tag{4}$$

The demand function g_1 gives the consumer's effective demand for the good and g_2 gives the effective demand for money. These *effective* demand functions take into account the constraints faced by the consumer. A more thorough discussion of the concepts of notional and effective demand will be given later.

One should note that the notional or unrationed demand and supply functions in equation (2) depend only upon the initial endowment R, and the prices w and p. In contrast the effective or rationed demand functions in equation (4) also depend upon the quantity rationing in the labor market. Quantity constraints therefore add considerable complexity to the interactions between markets.

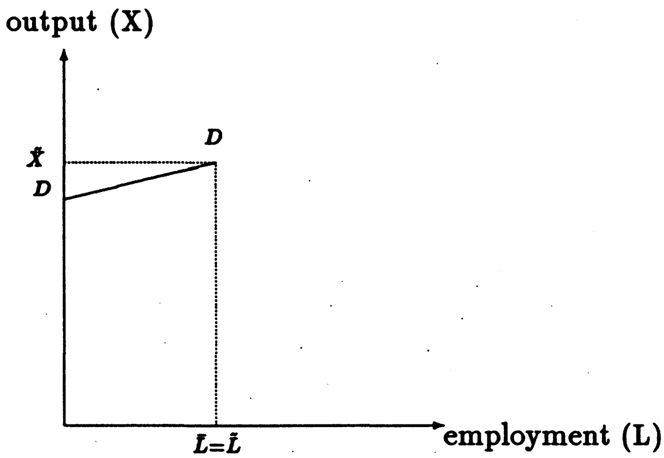

Figure 1. The Consumer's Demand

As long as we are not dealing with inferior goods we have that $\partial X^e/\partial \bar{L} > 0$ when $\bar{L} < \tilde{L}$. In other words, the lower the ration faced by the consumer in the labor market, the less he will wish to buy of good X. In figure 1 this relationship is given by the line DD (for convenience sake we draw the line straight). The line stops at the point where $\bar{L} = \tilde{L}$, because to the right of this point the constraint is not effective. Even when employment is zero, the consumer can buy some output by drawing down his reserves of money. By analogy we assume that the relationship in figure 1 also applies at the macro level with $\bar{L}$ being a constraint on aggregate labor supply. The DD line then denotes how aggregate demand in the economy drops as rationing in the labor market becomes more pervasive.

We now introduce the aggregate production function $X(L)$ for the economy. It is an increasing function of aggregate labor, L, and exhibits decreasing returns to scale. If we add the production function and a line, ww, denoting the exogenously fixed relative wage w/p to figure 1 we get figure 2.

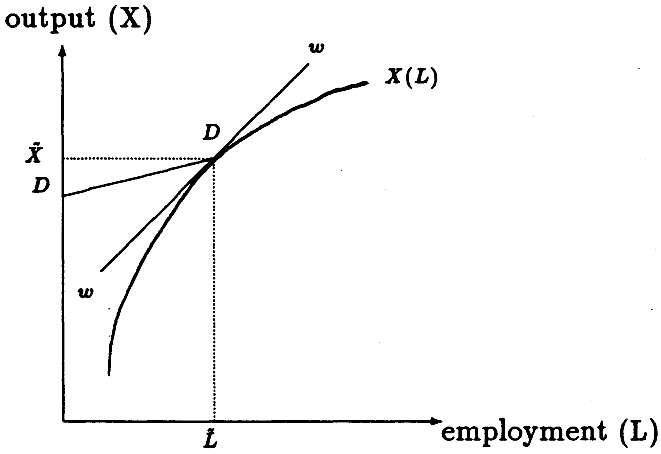

Figure 2. Equilibrium

Figure 2 illustrates a situation with equilibrium in both the labor and the goods market. Output $\tilde{X}$ and employment $\tilde{L}$ are the result of profit maximization, implying that the marginal product is equal to the relative wage. In figure 2 this is the point where the ww line is tangent to the production function $X(L)$. These quantities are also at the point on the DD curve where the consumers' notional demands are satisfied. One should note that a change in wages and prices will change the location of the DD curve.

In neo-classical models the relative wage (the ww line) will always adjust so that we get an equilibrium such as that in figur 2. Neo-Keynesian models assume that this does not necessarily happen. With an exogenously fixed wage rate and price level we can get four different types of fixed price equilibria, which Malinvaud (1977) denoted as Classical Unemployment, Keynesian Unemployment, Repressed Inflation, and Underconsumption.

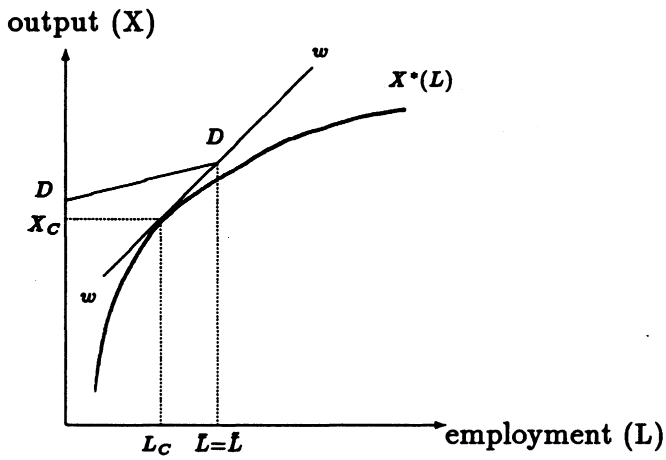

Figure 3. Classical Unemployment

We now assume that wages and prices are such that the markets do not necessarily clear. Consider a production function $X^*(L)$ lying under the earlier function $X(L)$ such as that drawn in figure 3. Now the relative wage given by the line ww no longer clears the market. Profit maximization leads to an output of only X_C and an employment of only L_C, while utility maximization still leads the consumers to wish to buy $\tilde{X}$ and sell $\tilde{L}$. To resolve this inconsistency, we assume that the short side of each market decides the quantity to be sold. This means that the quantity of goods Q_X and of labor Q_L actually transacted are given by:

$$\begin{aligned} Q_X &= \min(X_C, \tilde{X}) \\ Q_L &= \min(L_C, \tilde{L}) \end{aligned} \tag{5}$$

These conditions are often referred to as the *min* condition and imply that all transactions are voluntary. If the firms are willing to trade less than the consumers in both markets then the *min* condition implies that their notional supply X_C and demand L_C are satisfied. The consumers will be rationed in both markets and unemployment will be $\tilde{L} - L_C$. This equilibrium is referred to as the case of *Classical unemployment*. In this situation Keynesian demand management will have no effect on unemployment, because a shift in the DD curve will not induce the firms to produce more. A decrease in unemployment can only come about by lowering the relative wage.

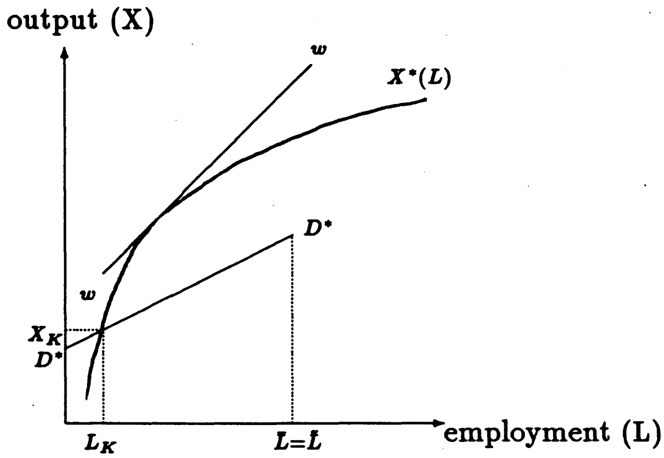

Figure 4. Keynesian Unemployment

We get another type of equilibrium if we assume that the consumers' demand curve lies much lower than the DD line we have looked at until now. In figure 4 we assume that the wage rate and price level induce the demand curve D^*D^*. Knowing that we must lie on the production function and using the *min* condition, we see that the transacted quantities of goods and labor become X_K and L_K. In this situation there is excess supply in both the goods and the labor markets. At the going price and wage the firms wish to supply more goods than X_K. If they try to produce more than this and thereby increase employment, they will be unable to sell all of the extra output because the rise in consumers' demand along the D^*D^* curve is less than the rise in production along the production function $X^*(L)$. The equilibrium given by (X_K, L_K) is referred to as the case of *Keynesian unemployment.* In this situation Keynesian demand management can be effective. By increasing demand (shifting the D^*D^* curve), the government can increase production and reduce unemployment.

Classical and Keynesian unemployment are the most frequently occurring cases in most economies. The two other cases that theoretically can occur are *Repressed* *inflation* and *Underconsumption.* When households are rationed in the goods market and firms are rationed in the labor market we have Repressed inflation. A rise in either the wage rate or the price level (or both) will reduce excess demands. When the wage rate is so low that the firms cannot sell all they wish to produce and at the same time wish to employ more labor than they can get, we have Underconsumption. This last type of equilibrium is considered very rare.

We see that in a neo-Keynesian model employment is not uniquely associated with the real wage. The effective demand for labor can vary even with the real wage fixed. A criticism of the model is that it seems unrealistic to assume that the whole economy will either experience Classical unemployment or Keynesian unemployment. It seems more realistic to assume that at any time some sectors of the economy will be constrained by lack of general demand while others will be constrained by unfavorable prices and wages. Aggregating across these sectors will give a macro picture somewhere between the extremes of Classical and Keynesian unemployment. An empirical approach to this problem is the "smoothing by aggregation approach" of Lambert (1988), which we will discuss later.

The model sketched above is a first tentative step to building macroeconomics on the foundation of microeconomics, but is fairly rudimentary and has a number of shortcomings. A more detailed approach, such as the work done on temporary equilibrium in mathematical economics, is needed to make the model more realistic. It is necessary to develop a better understanding of the role of time, space, and information in the workings of an economy. In the next section we will discuss some concepts which may help to clarify what type of problems we are dealing with.

3 Temporary equilibrium

Walrasian general equilibrium theory is a theory of the equilibrium resource allocation of a market economy where equilibrium is achieved solely through the price mechanism. There is no use of quantity signals and no agent actually sets prices. The determination of prices is left to an implicit Walrasian auctioneer. While this framework is only concerned with allocations at equilibrium prices, it has proved a valuable reference point in economic theory, especially when discussing issues concerning efficiency.

The assumptions behind Walrasian general equilibrium theory are fairly extreme. It is theoretically unsatisfactory that there is no description of price determination, and the theory is unable to explain the observed fact that some adjustments take place, at least for a period of time, by means of quantitative rationing.

Non-Walrasian theory attempts to build a theory of markets where market clearing is not assumed. The main goal of this theory is to generalize and extend the Walrasian framework. It aims to explain how allocations are determined at given prices (fixed-price equilibria) and how these allocations adjust over time. In a situation where rationing may appear, both quantity signals and price signals become important. Most of the research which has been done recently has concentrated on determining allocations at given prices and less on incorporating price dynamics. During the last decade the most used non-Walrasian framework has been the temporary equilibrium approach. An early work on non-clearing markets and price dynamics is Frisch (1949), where he considers how price changes influence quantities when markets are out of equilibrium. The now much used *min* condition (which says that transacted quantity is the minimum of supply and demand) is employed in the analysis.

In the following I will discuss some general concepts from the temporary equilibrium literature before we proceed to discuss the econometric literature. The following discussion is in large part based on the surveys by Grandmont (1982) and

Benassy (1990) and on the monograph by Böhm (1989).

3.1 A basic framework

At the basis of the temporary equilibrium approach is the idea, originating with Hicks (1946), that in modeling an economy one can "treat a process of change as consisting of a series of temporary equilibria". At each point in time agents make forecasts of the economy based on current information and they trade among themselves. One way of viewing the series of markets that arise in this process is as a succession of competitive equilibria. Viewing the process as such a *temporary competitive equilibria* assumes that in each period the agents trades are made compatible by price adjustments. In contrast to the Walrasian situation, the agents plans for the future are not coordinated and may well be incompatible.

Another way of viewing the process is as a succession of markets where prices do not move fast enough to match supply and demand. In such a *temporary equilibrium with quantity rationing* it is important to model how the quantitative constraints encountered by the agents are perceived. In the following we will mainly be concerned with the more general case of temporary equilibrium with quantity rationing, but the general concepts we discuss below, will apply in both cases.

We shall attempt, within the temporary equilibrium framework, to give a general formulation of the decision-making process of a single agent. The focus will be on discussing the concepts involved rather than on mathematical rigor. A more mathematical rigorous discussion is given in Grandmont (1982) and Böhm (1989). Our discussion will assume that the agent's decision-making only takes into account two discrete periods of time, but the analysis is valid for an arbitrary number (finite or infinite) of discrete periods. In the first period, period 1, the agent receives information, often called a *signal*, about the economy and his own situation, which we denote s_1. This information, together with earlier information and his expectations about the future, is what he bases his decision-making on. The information the agent has received in previous periods will not be explicitly mentioned in the following, as it is fixed in the analysis. In Walrasian general equilibrium theory the signals received by the agents are the equilibrium prices, set by the mythical Walrasian auctioneer. The agent will also receive a signal, s_2, in the next period. In the present period he assigns a probability to each possible signal he might receive. This amounts to a *forecast* of his future environment.

Given a signal in period 1, the agent will undertake trading, or more generally *actions*, a_1, in period 1. His actions depend upon and may be constrained by the information he has received. If the agent for example receives information that he is rationed in a market, he will not attempt to trade more than the perceived quantity constraint. The agent also makes *plans*, a_2, for the next period which consists in deciding for each possible signal s_2 what action he will take. These plans will depend on the signals s_1 and s_2, and the action a_1 he chooses in period 1.

What matters to the agent is, of course, the *consequences* of his choices. The consequences will in general depend upon his environment and thereby upon the signals he has received and on the actions he has taken. In general the consequences will not be known with certainty. These factors are taken into account by modelling the consequences in each period as a function of actions, signals, and two stochastic variables ω_1 and ω_2. The stochastic variables are included to take into account that the agent might be uncertain about what the consequences of his actions are. In the following we denote the consequences in period 1 as $\gamma_1(a_1, s_1, \omega_1)$ which depend upon the situation in period 1 (as described by the signal s_1), the action a_1 taken in period 1, and the random term ω_1. We denote the consequences in period 2 as

$$\gamma_2(a_1, a_2, \gamma_1(a_1, s_1, \omega_1), s_2, \omega_2),$$

which depend upon the actions taken in period 1 and period 2, the consequences experienced in period 1, the situation in period 2 and the random term ω_2.

The concepts of signals, actions, plans, and consequences are the basic concepts of temporary equilibrium theory as for example presented in Grandmont (1982). We now assume that the agent has a von Neumann-Morgenstern utility function,

$$u\left(\gamma_1(a_1, s_1, \omega_1),\ \ \gamma_2(a_1, a_2, \gamma_1(a_1, s_1, \omega_1), s_2, \omega_2)\right). \tag{6}$$

Expected utility $U(a_1, a_2 \mid s_1)$ for a given signal s_1 can now be written as

$$\begin{aligned} U(a_1, a_2 \mid s_1) \ =& \\ \iiint & u\left(\gamma_1(a_1, s_1, \omega_1),\ \ \gamma_2(a_1, a_2, \gamma_1(a_1, s_1, \omega_1), s_2, \omega_2)\right) \\ & \cdot \Psi(s_2 \mid s_1) \cdot \Upsilon(\omega_1, \omega_2)\, ds_2\, d\omega_2\, d\omega_1, \end{aligned} \tag{7}$$

where $\Psi(s_2 \mid s_1)$ is the subjective probability density of receiving signal s_2 given that the agent has received signal s_1, and $\Upsilon(\omega_1, \omega_2)$ is the joint probability density of the stochastic variables ω_1 and ω_2.

The agent observes s_1 and then maximizes his expected utility subject to the constraints $a_1 \in B_1(s_1)$ and $a_2 \in B_2(a_1, s_1, s_2)$, where B_1 is a subset of the set of all possible actions in period 1 and B_2 is a subset of the set of all possible actions in period 2. This maximization problem can be written:

$$\begin{aligned} \max_{a_1, a_2} U(a_1, a_2 \mid s_1) \quad \text{s.t} \quad & a_1 \in B_1(s_1), \\ & a_2 \in B_2(a_1, s_1, s_2). \end{aligned} \tag{8}$$

It is important to note that the above maximization problem involves a separate constraint for each period, in contrast to problems which only involve constraints across the whole optimization period. Under certain regularity conditions (see Grandmont (1982)), it is possible to employ a standard dynamic programming formulation. This procedure entails first maximizing the von Neumann-Morgenstern utility function u with respect to a_2 to give us an indirect utility function. We then maximize the expected utility of this indirect utility function with respect to a_1, which is a maximization problem involving only contemporary variables. As we shall see later this procedure also allows leads to the introduction of money into the utility function.

In deriving the indirect utility function we ask, for a given combination of signals s_1, actions a_1, and the stochastic variable ω_1, what plan would be chosen if the agent knew for sure the signal s_2 and the ω_2 which will be realized in the next period. The maximum utility acquired in this situation is denoted as:

$$u^*(a_1, \gamma_1(a_1, s_1, \omega_1), s_1, s_2, \omega_2) \tag{9}$$

being the solution to the problem

$$\begin{aligned} &\max_{a_2} u\left(\gamma_1(a_1, s_1, \omega_1),\ \ \gamma_2(a_1, a_2, \gamma_1(a_1, s_1, \omega_1), s_2, \omega_2)\right) \\ &\text{s.t.}\ \ a_2 \in B_2(a_1, s_1, s_2). \end{aligned} \tag{10}$$

The indirect utility function $u^*(a_1, \gamma_1(a_1, s_1, \omega_1), s_1, s_2, \omega_2)$ says how maximum utility varies with the signal received and the action taken in the first period, the signal received in the second period, and the ω's. The expected indirect utility $V(a_1 \mid s_1)$ is then derived by integrating out the second period signal and the random terms:

$$\begin{aligned} V(a_1 \mid s_1) &= \\ &\iiint u^*(\gamma_1(a_1, s_1, \omega_1), a_1, s_1, s_2, \omega_2) \cdot \Psi(s_2 \mid s_1) \cdot \Upsilon(\omega_2, \omega_1)\, ds_2\, d\omega_1\, d\omega_2. \end{aligned} \tag{11}$$

We have thereby reduced the problem of maximizing expected utility over two periods in equation (8) to maximizing an expected utility function involving only the contemporary variables a_1, and s_1:

$$\max_{a_1} V(a_1 \mid s_1) \quad \text{s.t.} \quad a_1 \in B_1(s_1). \tag{12}$$

This technique of reducing a complex problem involving several periods to a single-period decision problem has interesting economic implications. It allows the introduction of financial assets such as money into the utility function even though they may have no intrinsic value. Current signals also enter the utility function because they are instrumental in forming the agent's expectations of the future. These aspects will be discussed more closely in the following.

3.2 A monetary economy

In a monetary economy money has three roles, as a unit of account, as a store of value, and as a medium of exchange. The role of a medium of exchange is especially important when considering economies in disequilibrium. In disequilibrium transaction and coordination costs become important and can to some extent be most efficiently met by the use of a costless recording device such as money.

One of the characteristics of the Walrasian framework is that it cannot account for fiat money. The general equilibrium framework including time-dated and event-contingent commodities developed by Arrow and Debreu assumes that all contingent futures markets exist. At a single date all transactions take place, both of current goods and contracts for future delivery of goods. There is no need for trading at later dates, because all desirable trades have already been arranged. The agents in this model are only constrained by their life-time budgets. In this setting money has no function because all intertemporal allocations are performed through the futures markets.

In the temporary equilibria approach there is a need for a commodity which can transfer purchasing power from one period to another. Futures markets are generally inactive and markets reopen over time. Agents face a sequence of budget constraints that must be fulfilled at each trading date. Money, in such a situation, is an efficient means of transferring purchasing power over time (by saving or borrowing). While other durable goods could perform such a task, they would generally incur real resource costs not associated with money.

The sequential nature of trades associated with temporary equilibria makes greater demands on information than the Arrow-Debreu model. In a spatial economy where exchange can take place at a number of locations the need for information and coordination is even greater. In a barter economy where m goods are exchanged, there would be $m(m-1)/2$ markets, one for each pair of goods. Without a medium of exchange it is very difficult to coordinate trades while ensuring that all agents stay within their budget constraints. Introducing money reduces the number of markets to m (there is no market for money) where each good is exchanged against money. Money is also an efficient device to record one's trading history, ensuring that one keeps within the budget constraint. Ostroy and Starr (1990) surveys recent developments in incorporating the transactions role of money into a general equilibrium framework. Grandmont (1983) gives an excellent discussion of monetary theory using the temporary equilibrium approach (without quantity rationing).

We assume that there are m goods in the economy. At the beginning of a period an agent holds a quantity of money $\bar{M}$ and a vector of non-monetary goods $\bar{X}$ where all components are positive or zero. The agent trades during the period at the price vector p. His net purchase of goods is denoted by the vector X. The agent's final holdings of a vector of goods, Z, and of money, M, will then be:

$$\begin{aligned} Z &= \bar{X} + X, \\ M &= \bar{M} - pX \end{aligned} \tag{13}$$

where the last equation is the budget constraint.

We now set the agents decision-making problem into the framework developed earlier. All subscripts in the following refer to the two periods we have been discussing. The actions taken by an agent are in this case the net demands he makes. We distinguish between net demands, which are signals to the market reflecting the agent's wishes, and the actual net transactions that take place. These net transactions, Q_1 and Q_2, are defined as the difference between the purchases and sales of each agent. The net transactions will be negative if the agent is a net seller of a good. The only consequences which we assume are important to the agent is his holdings of goods in the two periods. These holdings follow directly from the net transactions which take place. This means that we in this special case can set the consequence functions equal to the holdings of goods:

$$\gamma_1(X_1, s_1, \omega_1) = \bar{X}_1 + Q_1, \tag{14}$$
$$\gamma_2(X_1, X_2, \gamma_1(X_1, s_1, \omega_1), s_2, \omega_2) = \bar{X}_2 + Q_2,$$

when the only actions the agent undertakes are his net demands X_1 and X_2.

We also need to specify more closely the signals the agent receives. In doing this we concentrate exclusively on market signals. We view a market as a mechanism for transforming the different agents' intentions (which may be in conflict with each other) into feasible outcomes. A market will in general consist of two elements, The first consists of the agents signaling their desires to the market and the second is some form of market-making process which translates these signals into transactions between the agents. Such a market-making process can take many forms, and it is possible that a number of agents may be rationed.

We assume that there are n agents in the economy. The vector of actions, a_i, which the agent undertakes in period 1 includes signals to the market (for example bids). The signals received, s_1 and s_2, include information about the situation in the market. If the market consists of a Walrasian auctioneer, then this information will simply be the market-clearing prices. The market signals s are generally functions of the actions of all agents:

$$s_{1i} = F_{1i}(a_{11}, a_{12}, \ldots, a_{1n}), \tag{15}$$
$$s_{2i} = F_{2i}(a_{21}, a_{22}, \ldots, a_{2n})$$

where the first subscript denotes period and the second indexes agents. The actions undertaken by the agent in the market reflect his wishes, while the signal he receives is information about the state of the market. He may for example undertake the action of registering himself at the unemployment office, while at the same time observing the unemployment rate. The decision to register will in general depend upon the unemployment rate he observes. The actions taken in the market can be many, but to be consistent with our earlier assumptions, we assume that the only conditions which influence F are the agents net demands $s_{1i} = F_{1i}(X_{11}, X_{12}, \ldots, X_{1n})$, and $s_{2i} = F_{2i}(X_{11}, X_{12}, \ldots, X_{1n})$. The signal generated by F from the agents' net demands can be prices, rations, etc.

We now introduce the concept of a *rationing mechanism*, ν, as in Böhm (1989). Such a concept was originally developed by Benassy (1975) and expresses the net transactions Q_{1i} and Q_{2i} as functions of the agent's actions and the signals he receives. The net transactions of all the agents must be consistent with each other. The rationing mechanism ν is a way of specifying the consequence functions γ we discussed earlier. In our case there is a very simple relationship between the rationing mechanism in period 1 for a single agent i and the consequence function γ_{1i}:

$$\begin{aligned} \nu_{1i}(X_{1i}, F_{1i}(X_{11}, \ldots, X_{1n}), \omega_1) &= \bar{X}_{11} - \gamma_{1i}(X_{1i}, F_{1i}(X_{11}, \ldots, X_{1n}), \omega_1) \\ &= Q_{1i} \qquad (16) \end{aligned}$$

where ν is such that the transactions in period 1, $Q_{11}, \ldots, Q_{1n}$, are consistent across agents:

$$\sum_{i=1}^{n} \nu_{1i}(X_{1i}, F_{1i}(X_{11}, \ldots, X_{1n}), \omega_1) = 0 \qquad \text{for all possible } X_{11}, \ldots, X_{1n}. \quad (17)$$

The rationing mechanism in period 2 can be written in the same manner as the rationing mechanism for period 1. A rationing mechanism of the above type is called *stochastic* because it includes the random variable ω_1. It takes into account that the agents may not know for certain what the consequences (which rations they receive) of their actions will be. By excluding the random variable ω_1 we get a *deterministic* rationing mechanism, where there is no uncertainty regarding the rations:

$$\sum_{i=1}^{n} \nu_{1i}(X_{1i}, F_{1i}(X_{11}, \ldots, X_{1n})) = 0 \qquad \text{for all possible } X_{11}, \ldots, X_{1n}. \quad (18)$$

Examples of rationing mechanisms are uniform rationing where all agents face the same rations and proportional rationing where the rations are proportional to the expressed demands and supplies. The last type of rationing will be vulnerable to strategic behavior on the part of the agents. They may signal greater demands than they really wish, so as to secure larger rations. In general the rationing mechanisms may lead to complicated interdependencies between the different markets. Rationing mechanisms describe how the market making mechanism affects each individual and therefore are influenced by searching, matching, and information gathering in the markets.

In the preceding we have narrowed an agent i's actions down to only including his net demands, X_{1i} and X_{2i}, and assumed that the only consequences he cares about are the net transactions Q_{1i} and Q_{2i} (more precisely his net holding of goods). In addition to this, we have assumed that the agent's net transactions in period 1 are determined by a rationing function ν_{1i}. We now go back to considering a single agent and resume letting the subscripts only denote time period. We now use the dynamic programming technique discussed earlier. We assume that we have the following von Neumann-Morgenstern utility function,

$$u\left(\bar{X}_{11} - \gamma_{1i}(X_{1i}, F_{1i}),\ \ \gamma_2(X_1, X_2, \bar{X}_{11} - \gamma_{1i}(X_{1i}, F_{1i}), s_2, \omega_2)\right). \tag{19}$$

Expected utility $U(X_1, X_2 \mid s_1)$ for a given signal s_1 can now be written as

$$\begin{aligned} U(X_1, X_2 \mid s_1) \; &= \\ \iiint & u\left(\bar{X}_{11} - \gamma_{1i}(X_{1i}, F_{1i}),\ \ \gamma_2(X_1, X_2, \bar{X}_{11} - \gamma_{1i}(X_{1i}, F_{1i}), s_2, \omega_2)\right) \\ & \cdot \Psi(s_2 \mid s_1) \cdot \Upsilon(\omega_1, \omega_2)\, ds_2\, d\omega_2\, d\omega_1, \end{aligned} \tag{20}$$

where $\Psi(s_2 \mid s_1)$ and $\Upsilon(\omega_1, \omega_2)$ are defined as in equation (7). The standard maximization problem is:

$$\max_{X_1, X_2} U(X_1, X_2 \mid s_1)$$

$$\text{s.t. } p_1 X_1 + M_1 \leq \bar{M}, \tag{21}$$

$$p_2(s_2) \cdot X_2 \leq M_1.$$

The agent faces separate budget constraints each period, and the signal received in period 2 implies a price $p(s_2)$. In the above formulation money is a means of transferring purchasing power from period 1 to period 2.

As before the indirect utility function is the maximum value function we get from maximizing u with respect to X_2:

$$\max_{X_2} u\left(\bar{X}_{11} - \gamma_{1i}(X_{1i}, F_{1i}),\ \ \gamma_2(X_1, X_2, \bar{X}_{11} - \gamma_{1i}(X_{1i}, F_{1i}), s_2, \omega_2)\right)$$

$$\text{s.t. } p(s_2) \cdot X_2 \ \leq\ M_1. \tag{22}$$

The budget constraint for period 2 says that the agent's net purchases in this period may not exceed the amount of money he holds at the beginning of the period. The resulting indirect uttility function can be written:

$$u^*(\nu_1(X_1, s_1, \omega_1), M_1, \bar{X}, s_1, s_2, \omega_2). \tag{23}$$

The indirect utility function u^* contains two arguments which the agent has control over in period 1, X_1 and M_1. The optimal level of M_1 will reflect the agent's intertemporal evaluation of consumption today and consumption tomorrow. The indirect utility function u^* thereby incorporates intertemporal considerations even though it only depends on contemporary variables, the second period signal s_2, and the second period random term ω_2. The expected indirect utility $V(X_1, M_1 \mid s_1)$ is now derived by integrating out s_2 and ω_2:

$$
\begin{aligned}
V(X_1, M_1 \mid s_1) &= \\
&\iiint u^*(\nu_1(X_1, s_1, \omega_1), M_1, \bar{X}, s_1, s_2, \omega_2) \qquad (24)\\
&\quad \cdot \Psi(s_2 \mid s_1) \cdot \Upsilon(\omega_2, \omega_1)\, ds_2\, d\omega_1\, d\omega_2. \qquad (25)
\end{aligned}
$$

The problem of maximizing expected utility over two periods in equation (21) is now reduced to maximizing an expected utility function involving only the contemporary variables X_1, M_1, and s_1:

$$
\max_{X_1, M_1} V(X_1, M_1 \mid s_1) \quad \text{s.t.} \quad p_1 X_1 + M_1 \leq \bar{M} \qquad (26)
$$

where:

$$
s_1 = F_1(X_{11}, X_{12}, \ldots, X_{1n}),
$$

and where we assume that the feasible net demands are such that with probability 1 the agent cannot go bankrupt. This last assumption is important when proving the existence of non-Walrasian equilibria (which will not be covered in the present article), because it ensures that no discontinuities can occur. A simple proof of a proposition similar to our assertion that problem (21) and problem (26) lead to the same result is given in Grandmont (1983).

The above gives us a justification for only considering one period in an intertemporal model. Expectations of future prices and quantities enter the indirect utility function v through the current signal s_1 and it's influence on the agent's forecast of the future. Money also enters the indirect utility function even though money has no intrinsic value. It's importance is as a means of transferring purchasing power over time. Feenstra (1986) discusses in more detail the functional equivalence between using real balances as an argument of the utility function and entering money into the budget constraint. He looks at a broad class of utility functions and a broad class of transactions cost models.

Solving the above maximization problem with either the stochastic or deterministic rationing functions gives us what are often referred to as the agents *effective* *net demands.* We denote the vector of such demands from agent i as X_i^e. It follows from (26) that these effective net demands will be functions of the signal received in period 1 by agent i,

$$X_i^e = G(s_i). \tag{27}$$

The effective demands can be viewed as the demands the agent signals to the market after taking into account the information he has about rationing through the signaling function F and how the rationing affects him through the rationing mechanism ν. Assuming that all agents derive effective demands of the above type we can write the effective demands as:

$$X_i^e = G(F(X_1^e, \ldots, X_n^e)), \tag{28}$$

where agent i's effective demands are written as a function of the effective demands of all the agents in the economy. The rationing mechanism can be written as:

$$\nu_i = \nu_i(X_i^e, F_i(X_1^e, \ldots, X_n^e), \omega). \tag{29}$$

Solving the utility maximization problem in equation (26) without taking the rationing constraints into account gives us the agent's *notional net demands*, which we as before denote $\tilde{X}$. If in fact the agent is not rationed, the effective and notional demands will coincide, otherwise they will in general differ.

We have now reduced an intertemporal model to a single period, where money serves as a means of transferring purchasing power from one period to another. In this transformed model there is an unspecified market mechanism which is such that each agent sends a bid (their effective demands) to the market and as a consequence can undertake transactions given by a rationing mechanism. It is important to remember that price and quantity expectations are incorporated into the indirect utility function.

3.3 Walrasian equilibrium

We will now characterize a Walrasian equilibrium within the framework of the monetary economy discussed above. A Walrasian equilibrium requires all markets to clear within one period. There is no rationing so that effective and notional demand are equal and there is no rationing mechanism. In the following we let the first subscript denote period and the second index the n agents in the economy. The utility maximization problem for agent i can in this case be written:

$$\begin{aligned} &\max_{X_i} v_i(\bar{X}_{1i} + X_{1i}, X_{2i}) \\ &\text{s.t. } p_1 X_{1i} + p_2 X_{2i} = \bar{M}_i, \end{aligned} \tag{30}$$

which yields the notional net demand vectors $\tilde{X}_{1i}(p_1, p_2, \bar{M})$ and $\tilde{X}_{2i}(p_1, p_2, \bar{M})$. A Walrasian equilibrium is given by the price vectors p_1^* and p_2^* which ensure that all markets clear, i.e.

$$\sum_{i=1}^{n} \tilde{X}_{1i}(p_1^*) = 0 \tag{31}$$

and

$$\sum_{i=1}^{n} \tilde{X}_{2i}(p_2^*) = 0.$$

The only signal that the agents need, for the economy to achieve equilibrium, is the price vectors p_1^* and p_2^*. Since a Walrasian equilibrium assumes that all trades take instantaneously place at one period of time, money does not have the role we gave it in our earlier intertemporal discussions. The main difference between the above problem and the intertemporal problem we looked at in the preceding section is that here there is only one intertemporal budget constraint while in the preceding section there was a separate budget constraint for each period. This underlines the well-known fact that in a Walrasian equilibrium the optimal holdings of fiat money will be zero. The vector of agent i's transacted quantities, Q_i, will in a Walrasian equilibrium be equal to both notional demand $\tilde{X}_i(p^*)$ and effective demand X^e.

3.4 Voluntary trading and no short side rationing

Now we go back to the framework of temporary equilibrium. In keeping with our earlier argument for reducing a multiperiod maximization problem to a single period problem we deal in the following with only one period, keeping in mind that each period is only a link in a long chain of markets. Even though we restrict ourselves to looking at a single period, it must be stressed that the following discussion is embedded in an intertemporal structure which is absent from the Walrasian equilibrium setting. When the economy is in disequilibrium the realized price vector, p, will be different from the Walrasian price vector p^*. In this situation the agents observe more than just the price vector. They perceive that they cannot realize all their notional demands and supplies and take this into account. In the terminology we introduced earlier the signal function F gives them information about the rationing and induces them to change their actions. The effective net demands X^e are expressions of their wishes to the market after taking rationing into account (and other relevant signals). The transacted quantities Q need be equal neither to the notional nor the effective demands and supplies.

The actual quantities transacted in disequilibrium will be the result of an exchange process that transforms the inconsistent demands and supplies to a result where actual demand equals actual supply. For this to be possible the process may lead to rationing of some agents in some markets. This is done through the rationing mechanisms defined earlier.

We now impose two conditions on the rationing mechanism in the same manner as Böhm (1989). A more detailed discussion of these conditions can be found in Benassy (1982). Böhm also imposes a number of regularity conditions which are not discussed in the following. The first condition says that all trades are voluntary, meaning that no agents are forced to trade more than they are willing to. The rationing mechanism was defined in equation (16) as being a mechanism $nu_{ij}(X_i, s_i, \omega_i)$ which transformed all the agents' demand vectors $X_1, \cdot, X_n$ into a transaction q_{ij} of good j by agent i which is consistent with all other transactions in the economy. The notation here differs somewhat from equation (16) with i indexing the n agents, j indexing the m different goods. The vector X_i includes all the individual goods $x_{i1} \cdots x_{im}$ demanded by agent i while the transacted quantity q_{ij} is an element of the vector of agent i's transactions Q_i. Under the assumption that the transacted quantity q_{ij} has the same sign as the agent's net demand x_{ij}, then the condition of voluntary trading implies that:

$$q_{ij} = \nu_{ij}(X_i, s_i, \omega_i) \begin{cases} \leq x_{ij} & \text{if } x_{ij} \geq 0 \\ > x_{ij} & \text{if } x_{ij} < 0. \end{cases} \tag{32}$$

This says that agent i can not be forced to buy or sell more than he wishes to. This condition can be used to give a more formal representation of how much the agent can influence the rations he faces. We let $\overline{q}_{ij} = \overline{\nu}_{ij}(X_i, s_i, \omega_i)$ be the maximum possible quantity agent i can buy of good j and let $\underline{q}_{ij} = \underline{\nu}_{ij}(X_i, s_i, \omega_i)$ be the maximum possible quantity he can sell of the good. These upper and lower bounds indicate how much each agent can influence his maximum transactions through his net demands X_i. The condition of voluntary trading implies that the rationing function ν can be written:

$$\nu_{ij}(X_i, s_i, \omega) = \min \left\{ \max \left\{ x_{ij}, \underline{\nu}_{ij} \right\}, \overline{\nu}_{ij} \right\}. \tag{33}$$

This says that if the agent's net demands x_{ij} lies within the bounds $\underline{\nu}_{ij}$ and $\overline{\nu}_{ij}$, then the rationing function allows him to realize this demand. If, on the other hand, his net demands lie outside the bounds then his realized transactions will be equal to the bound which is closest to x_{ij}.

A rationing function is called *non-manipulable* if an agent can't influence his maximum and minimum bounds. This is the case if $(\underline{\nu}_{ij}, \overline{\nu}_{ij})$ are independent of the agent's actions:

$$\begin{aligned} \underline{\nu}_{ij}(X_i, s_i, \omega) &= \underline{\nu}_i^*(s_i, \omega), \\ \overline{\nu}_{ij}(X_i, s_i, \omega) &= \overline{\nu}_i^*(s_i, \omega). \end{aligned} \tag{34}$$

If the agent's expressed net trades influence the maximum limits, then the rationing function is called *manipulable.* If the rationing function is manipulable, then rationed agents may engage in strategic actions to secure themselves as great a quantity as possible. This can lead to an infinite series of overbidding. An example of a manipulable rationing scheme is proportional rationing where the rations are proportional to the agents' announced net demands.

A problem with non-manipulable rationing functions is that they may lead to there being no disequilibrium information in the market. When the agent knows that his expected transactions must lie within his upper and lower bounds he may have no incentive to signal a net demand outside those bounds. Manipulable rationing mechanisms must in most cases be stochastic for there to exist an equilibrium.

The second condition usually imposed on the rationing mechanism says that there shall not be excess demand and excess supply for a good at the same time. This means that there is a certain efficiency in the market and implies that if

$$\left(\sum_i^n x_{ij}\right) \cdot x_{ij} \leq 0$$

then we must have that

$$\mathrm{E}\, q_{ij} = \mathrm{E}\, \nu_{ij}(X_1, s_1, \omega_1) = x_{ij}.$$

The above expression says that if the sign of an agent's demand is different from the sign of aggregate excess demand (implying that the agent is on the short side of the market), then his expected transaction is equal to his net demand and he is not rationed.

3.5 Fixed-price equilibria

We now return to the concept of effective net demands. An agent's effective net demands X^e are the demands the agent signals to the market after taking into account how rationing affects him. If the agent believes he can manipulate the upper and lower transaction bounds he faces, he may even announce effective net demands which are much greater than what he actually wishes to transact. In any case the effective net demands will in general be different from the notional (Walrasian) net demands.

As is apparent from our earlier discussion, the effective demands of each agent will depend on the signal they receive from the market about the actions of all other agents. An equilibrium in the economy requires that the signals in the economy are consistent with the effective net demands expressed by the agents.

A fairly general equilibrium concept is that proposed by Gale (1979), which defines equilibrium as a combination of market demands X_i and signal functions F_i that satisfy the following two conditions:

$$\left.\begin{aligned} X_i &= X_i^e(s_i) \\ s_i &= F_i(X_1, X_2, \ldots, X_n) \end{aligned}\right\} \quad \text{for all } i \tag{35}$$

which admits the use of stochastic rationing mechanisms. In the first condition, the agent takes the signal s as given. He does not believe his actions influence the signal he receives. This may be the case even if his rationing mechanism is manipulable. The first condition seems reasonable as long as the there are large number of agents in the economy. Gale (1979) uses the above equilibrium concept to show the existence of quantity constrained stochastic rationing equilibria.

In the literature the two most heavily used equilibrium concepts are Drèze equilibria due to Drèze (1975) and K-equilibria due to Benassy (1975,1982). Both these concepts assume non-stochastic and non-manipulable rationing mechanisms. They are special cases of the equilibrium concept due to Gale, and are characterized by utilizing different specifications of effective demand. Svensson (1980) compares these effective demand concepts with the more general effective demands one gets with stochastic rationing mechanisms.

We now assume that the signal received by agent i consists of a price vector p and the quantity constraints he faces in all markets. These quantity constraints consist of a list of upper bounds $\overline{s}_{ij}$ and of lower bounds $\underline{s}_{ij}$ for all goods j. These bounds will respectively be functions $\overline{F}_{ij}$ and $\underline{F}_{ij}$ of the actions of all the agents in the economy:

$$\begin{aligned} \overline{s}_{ij} &= \overline{F}_{ij}(X_1^e, \ldots, X_n^e), \\ \underline{s}_{ij} &= \underline{F}_{ij}(X_1^e, \ldots, X_n^e), \end{aligned} \tag{36}$$

where we assume that the only actions undertaken by agent i is the expression of effective demands X_i^e. From equation (33) we know that this implies that the rationing function can be written as:

$$\nu_{ij}(X_i^e, s_i, \omega) = \min\left\{\max\left\{x_{ij}^e, \underline{s}_{ij}\right\}, \overline{s}_{ij}\right\}. \tag{37}$$

The assumption that the actions in the above equations are confined to effective demands (optimization taking all constraints into account) leads to the possibility that these effective demands will be multivalued. It is easily seen that if a $x_{ij}^e \geq \overline{s}_{ij}$ is optimal, then all other effective demands larger than the upper bound will also be optimal. To get around this problem Benassy (1975,1982) used *Clower demands* as effective demands. Clower demand (Clower (1965)) for a good is the result of utility maximization subject to a budget constraint and all quantity constraints except the quantity constraint which applies to the good in question. The utility maximization problem for agent i can in this case for each good x_{ij} be written as:

$$\max_{x_j} V_i(\bar{X}_i + X_i, M_i),$$

$$\text{s.t. } pX_i + M_i = \bar{M}_i, \qquad (38)$$

$$\underline{s}_{ik} \le x_{ik} \le \overline{s}_{ik}, \qquad k \neq j$$

which for good j yields the Clower net demand $x^c_{ij}(p, \underline{s}_i, \overline{s}_i)$. Repeating this maximization for all m goods yields the vector of Clower demands $X^c_j(p, \underline{s}_i, \overline{s}_i)$. This vector of effective demands has the property that whenever a constraint is binding on a market, the corresponding demand is different from the constraint and thereby from the resulting transaction. The Clower demands therefore signal to the market that the agent is constrained. The Clower demands have the weakness that the procedure of maximizing market by market is arbitrary, and that the resulting transactions may violate the budget constraint. Assuming strict quasi-concavity of the utility function however, is sufficient to assure that the budget constraint is satisfied.

Benassy (1975) defines a fixed-price equilibrium using the concept of Clower demands. This equilibrium, called a *K-equilibrium* is a set of demands, X, transactions, Q, and perceived constraints $\overline{s}_i$ and $\underline{s}_i$ such that:

$$\begin{aligned}
&\text{(a)} \quad X_i = X^c_i(p, \underline{s}_i, \overline{s}_i), \qquad i = 1, \ldots, n, \quad j = 1, \ldots, m,\\
&\text{(b)} \quad q_{ij} = \min\left\{\max\left\{x_{ij}, \underline{s}_{ij}\right\}, \overline{s}_{ij}\right\}, \qquad i = 1, \ldots, n, \quad j = 1, \ldots, m,\\
&\text{(c)} \quad \overline{s}_{ij} = \overline{F}_{ij}(X_1, \ldots, X_n), \qquad i = 1, \ldots, n, \quad j = 1, \ldots, m,\\
&\qquad\;\; \underline{s}_{ij} = \underline{F}_{ij}(X_1, \ldots, X_n), \qquad i = 1, \ldots, n, \quad j = 1, \ldots, m,
\end{aligned}$$

where q_{ij} is an element of the transaction vector Q_i. Since condition (b) defines a rationing mechanism, it follows that in equilibrium we will have that $\sum_i q_{ij} = 0$ will be the case for all markets. It should be noted that it is possible to define a K-equilibrium without imposing the condition of no short side trading we discussed earlier. This opens for the possibility of inefficient rationing schemes.

Drèze (1975) contains a definition of equilibrium which is different from the one proposed by Benassy. It utilizes an effective demand concept, Drèze demands, which is theoretically more satisfying than the Clower demands. Drèze demand (Drèze (1975)) for a good is the result of maximizing a utility function subject to a budget constraint and all quantity constraints that exist. The utility maximization problem for agent i can in this case be written:

$$\begin{aligned} \max_{X_i, M_i} \quad & u_i(\bar{X}_i + X_i, M_i) \\ \text{s.t.} \quad & pX_i + M_i = \bar{M}_i, \\ & \underline{s}_{ik} \leq x_{ik} \leq \bar{s}_{ik}, \qquad k = 1, \ldots, m \end{aligned} \tag{39}$$

which yields the vector of Drèze net demands $X_i^d(p, \underline{s}_i, \bar{s}_i)$. A *Drèze equilibrium* is defined as the set of transactions Q and perceived constraints $\bar{s}_i$ and $\underline{s}_i$ that satisfy:

$$\begin{aligned} &\text{(a)} \quad Q_i = X_i^d(p, \underline{s}_i, \bar{s}_i), \qquad i = 1, \ldots, n, \\ &\text{(b)} \quad \sum_i^n Q_i = 0, \\ &\text{(c)} \quad Q_{ij} = \bar{s}_{ij} \text{ for some } i \text{ implies } Q_{kj} > \underline{s}_{kj} \text{ for all } k \neq j, \\ &\qquad\;\; Q_{ij} = \underline{s}_{ij} \text{ for some } i \text{ implies } Q_{kj} < \bar{s}_{kj} \text{ for all } k \neq j. \end{aligned}$$

Condition (a) says that each agent's transactions are equal to his Drèze demands based on individual utility maximization. Condition (b) says that the transactions should balance on each market, and condition (c) says that only one side of the market can be rationed at a time. This is in contrast to the K-equilibrium which in general allows inefficient rationing. The greatest problem with the Drèze equilibrium is that it imposes that each agent's transaction is equal to his demand. The agent has no possibility to signal to the market that he is rationed. The Drèze demands are thereby inadequate as measures of disequilibrium.

Condition (c) of the K-equilibrium specified how the quantity constraints are distributed among agents, while there is no such specification in the Drèze equilibria.

One must therefore make additional assumptions about the distribution of quantity constraints, such as uniform or proportional rationing. If one assumes that all rationing schemes are efficient (no short side rationing) then the two definitions of effective demand yield similar sets of equilibrium allocations for a given price system.

As we have seen, the two equilibrium concepts of Benassy and Drèze each have their weaknesses. The K-equilibrium is based on effective demands which are arbitrary while the Drèze equilibrium does not differentiate between demands and transactions. The most widely used concept in theoretical work has been Drèze demands and Drèze equilibrium because they are explicitly based on utility maximization. Drèze demands have been little used in econometric work because they imply that there can be no discrepancy between actual and effective demands. Most econometric specifications of multi-market disequilibrium models have employed Clower demands. Lee (1986) shows that in fixed-price models the assumption that only one side of a market can be rationed at one time implies that different specifications of effective demand will lead to observationally equivalent expressions. This will be discussed in more detail later.

The definition of equilibrium in a stochastic framework proposed by Gale (1979), which is given in equation (35) above, encompasses the K-equilibrium and the Drèze equilibrium. Weinrich (1984) argues that the concept of stochastic rationing is unavoidable for any satisfactory definition of effective demand and that these rationing schemes ought to be manipulable.

In our discussion of non-Walrasian equilibrium, we have been preoccupied with discussing the most relevant concepts, while overlooking such issues as the existence and uniqueness of the different equilibria. These issues are outside the scope of this paper. It suffices to mention that Gale (1979) shows the existence of quantity-constrained stochastic rationing equilibria in a general framework, while Benassy (1975) and Drèze (1975) show the existence of their respective equilibrium concepts. Weinrich (1984) shows the existence of equilibria with stochastic manipulable rationing schemes.

3.6 Wage and price stickiness

In the above discussion we have looked at the economy as a series of sequential temporary equilibria. Prices and wages are not necessarily constant over time in such models, but are assumed not to instantaneously clear markets. Rationing can therefore occur because the economy is stuck in an equilibrium where prices and wages do not clear the markets or because prices and wages adjust slowly and therefore take time to clear markets. If prices and wages are sluggish, but converge towards an equilibrium, this equilibrium may be either Walrasian or non-Walrasian. In any case there will be a period where the economy is in disequilibrium. If the economy is constantly experiencing unanticipated shocks it seems natural that this will be the case most of the time. Even small unanticipated shocks can bring about shifts in expectations, which may lead to substantial changes in the wage-price vector required to achieve Walrasian equilibrium. This is illustrated in Neary and Stiglitz (1983) using a two-period temporary equilibrium model. Their model implies that even if many markets in an economy clear instantaneously, quantity rationing in other markets can have serious consequences for the economy as a whole. They conclude that:

> *... with limited flexibility of some prices, increasing the flexibility of other prices may reduce rather than increase the ability of the system to return to Walrasian equilibrium.*

Neary and Stiglitz also show that introducing rational expectations (perfect foresight of future constraints) into their model increases the probability of disequilibrium and enhances the effectiveness of anticipated government policy.

Disequilibrium models are of course more relevant, the more prevalent rigidities are in the economy. In the literature there are many theories which try to explain why wages and prices are sticky for at least a short period of time. Implicit contract theory assumes that a lack of insurance markets leads firms to implicitly promise workers a wage that is higher than their marginal product in bad times if they will accept a wage that is lower than their marginal product in good times. This type of implicit contract leads to wages staying fairly constant over the business cycle. Union bargaining over explicit wage contracts can also explain fixed wages over a period of time.

Menu cost is an explanation for sticky prices which focuses on the cost firms incur when changing prices. Firms may also be reluctant to change prices because of signaling effects. A producer of a luxury good will often keep a high price even when demand falls, because a high price is a signal that the good is of high quality.

Efficiency wage models assume that there are some attributes of labor that the firms are unable to observe, such as human capital, moral, or work effort. In order to get a higher quality labor the firms choose to pay high wages. In itself, efficiency wage models do not explain wage rigidity, but provide a reason for markets not clearing. One can also speculate that when wages are not directly connected to excess labor supply, as in the efficiency wage model, they may stay constant longer than they otherwise would. Another theory with similar consequences is insider-outsider models (see for example Lindbeck and Snower (1989)) which assume that workers who are already employed have some monopoly power which they use to ensure themselves higher wages (or other benefits) than they otherwise would get.

Most of the theories mentioned above are concerned with the labor market and are therefore too partial to be thought of as providing a more general price theory which can be used in general models of the economy. At present a promising way of incorporating price setting in such general models is by assuming monopolistic competition.

Monopolistic price setting was incorporated for the first time into a general equilibrium model by Negishi (1961), and one of the first papers to incorporate monopolistic price setting in a disequilibrium framework was Benassy (1976). Both these papers use the concept of *perceived demand curves.* The perceived demand curve gives the maximum quantity that the monopolist thinks he can sell as a function of price, given his market observations. Benassy assumes that quantities adjust infinitely faster than prices, allowing a separation of price and quantity decisions. The price setting process is described in his paper as follows:

> *Assume that all firms have fixed their prices; quantity movements occur, with eventual multiplier effects, then quantities stabilize at what we will call a K-equilibrium, and transactions can actually take place. Firms then observe some price and quantity variables, re-estimate their perceived demand curves, change their prices, and so on .... Monopolistic equilibrium is attained when no monopolist wants to change his price on the basis of what he observes.*

Benassy proves the existence of equilibrium in this model.

Benassy (1987) (also described in Benassy (1990)) uses the Nash bargaining mechanism to develop another type of disequilibrium model with monopolistic competition. His model assumes that there are three types of goods: money, different types of labor and consumption goods. The economy consists of a number of consumers and firms. Each consumer sells labor to all the firms and each firm sells goods to all the consumers. Each seller sells only one good and is thereby a monopolist in the market for this good. Benassy assumes that each price maker knows how the economy functions and can compute the constraints which arise in all circumstances in the economy. Each firm sets the price of its product and each consumer sets her wage, taking all other prices and wages as given. The demand curves which arise from this behavior are called *objective demand curves* by Benassy.

Equilibrium in the economy is given as a Nash equilibrium in prices and wages, conditional on these objective demand curves. The concept of objective demand curves implies that all agents take into account how the constraints they face vary with the prices they set. The fact that the agents take their constraints into account ensures the consistency of all transactions.

By combining the concept of objective demand curves and the Nash bargaining mechanism, Benassy has been able to develop a disequilibrium model where prices and wages are endogenously set. The model gives as a result that the economy is inefficient, meaning it is possible to increase production and employment with all agents being better off. Such inefficiencies are reminiscent of Keynesian models, but the model also implies that monetary policy is neutral as in Walrasian models.

Hahn (1978) examines more closely the *conjectures* an agent makes in an economy which, in contrast to Negishi and Benassy, is not intrinsically monopolistic. The economies he studies admit the Walrasian case as a possible equilibrium, but can also include cases where the economy is stuck in equilibria with quantity rationing. Hahn assumes that the conjectures made by agents are such that those who are not quantity constrained take prices as given while agents who are constrained are aware that they must change prices to ease the constraints. These conjectures can be of many types and are thought of as being exogenous to the model. An agent only takes his own constraints into account and is not allowed to consider changing prices when he observes that other agents are constrained. A *conjectural equilibrium* of an economy is a situation where agents accept current prices as optimal. Hahn shows that there can be many such conjectural equilibria including a unique Walrasian equilibrium. In the non- Walrasian conjectural equilibria there is quantity rationing but the agents do not believe that changing prices will make them better off.

As the above discussion shows, disequilibrium models are more than models that simply assume that prices are fixed. It is clear that the formation of expectations and price dynamics are essential ingredients in a theory of non-clearing markets. Still it must be admitted that much further research is needed in trying to model such price dynamics and non-Walrasian equilibria. The lack of a clear theory of price determination over time has lead the econometric literature to largely concentrate on fixed-price models without any explicit modeling of price processes (sometimes an ad hoc price adjustment equation is used). In the next sections we will take a closer look at some the recent work in disequilibrium econometrics.

4 Econometric Disequilibrium Models

In the preceding we discussed the theoretical modeling of disequilibrium within the framework of temporary equilibrium. We now look at some recent developments in econometric disequilibrium modeling. Our main interest is in estimating multi-market models, since this reflects the general equilibrium framework we discussed in the preceding section. Before discussing econometric multi-market models, we briefly discuss the estimation of a single isolated market in disequilibrium. This gives a simple illustration of how the latent structure associated with disequilibrium can be modeled econometrically. Examples of single market econometric models are the seminal work of Fair and Jaffee (1972), which studies the housing market, and that of Rosen and Quandt (1978), which looks at the labor market. We consider a model where the price p_t is considered exogenous and where sample separation is unknown in the sense that we do not have any apriori information about whether it is the demand or supply side which is constrained.

After looking at the estimation of a single market in disequilibrium we discuss the problems involved when estimating multi-market disequilibrium models. The most important problems are related to the specification of spillovers and to computational problems which arise when there are more than two or three markets. The following discussion on how to specify spillovers will focus mainly on Ito's (1980) use of Clower demands. This discussion serves as an introduction to the discussion of the virtual price approach of Lee (1986) and to the smoothing by aggregation approach used by among others Lambert (1988).

4.1 A single market disequilibrium model

A very simple model which illustrates estimation of single market disequilibrium models is the following (which is a very similar specification to one used by Fair and Jaffee (1972)):

$$D_t = \alpha_1 p_t + \beta_1 z_{1t} + u_{1t}, \tag{40}$$

$$S_t = \alpha_2 p_t + \beta_2 z_{2t} + u_{2t}, \quad (41)$$

$$Q_t = \min(D_t, S_t), \quad (42)$$

where z_{1t} and z_{2t} are observed exogenous variables (they can be considered vectors), and u_{1t} and u_{2t} are independently distributed random variables with

$$u_{it} \sim \mathrm{N}(0, \sigma_i^2), \qquad i = 1,2.$$

The variable p_t is the exogenous price of the commodity, D_t is the unobserved demand for the good, S_t is the unobserved supply of the good, and Q_t is the observed realized traded quantity at time t.

The model is fairly ad hoc and is not directly based on utility or profit maximization. Equation (42) is the *min* condition we have discussed earlier, implying that all trades are voluntary. Rosen and Quandt (1978) question the appropriateness of this specification when both sides of the market possess some monopoly power. They speculate that under such circumstances the realized outcome Q_t may lie somewhere between D_t and S_t. One way of generalizing the *min* condition is to add an error term to equation (42). Such a formulation leads to a more general stochastic specification but does not represent a better modelling of the economic processes. There have also been suggestions, originating with Muellbauer (1978) and used in Lambert (1988), that the realized outcome is an aggregation of corresponding quantities from numerous unobservable local micro markets. We discuss this later in greater detail.

The model is often extended by specifying a price adjustment equation, usually a variant of partial adjustment. Such equations are usually not well rooted in choice theoretic considerations but do add structure to the model. As noted earlier we lack a theoretically satisfying theory of price dynamics. Estimation of an ad hoc price adjustment equation (and thereby the speed of adjustment) can be of importance when testing for equilibrium. It makes it possible for a disequilibrium model to encompass an equilibrium model with the equilibrium model being equal to a disequilibrium one with instantaneous price adjustment. That such a specification is possible does of course not mean that it will be a satisfactory way of testing for equilibrium. Without a price adjustment equation the equilibrium model and the disequilibrium model will be two non-nested models. This is discussed in more detail in Quandt (1982). In the following we will only consider the simple model above without a price adjustment equation.

For notational simplicity we let $d_t = \alpha_1 p_t + \beta_1 z_{1t}$ and $s_t = \alpha_2 p_t + \beta_2 z_{2t}$ and have that $\mathrm{E}\, D_t = d_t$ and that $\mathrm{E}\, S_t = s_t$. From our assumption that the random variables are independently normally distributed we easily see that

$$D_t - S_t \;=\; d_t - s_t + u_{1t} - u_{2t}$$

is normally distributed $\mathrm{N}(d_t - s_t, \sigma_1^2 + \sigma_2^2)$ and that the probability of supply exceeding demand is

$$\begin{aligned} \mathrm{P}(S_t > D_t) \;&=\; \Phi\left(\frac{-(d_t - s_t)}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right) \\ &=\; 1 - \Phi\left(\frac{(d_t - s_t)}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right) \;\equiv\; 1 - \Phi_t, \end{aligned} \tag{43}$$

where Φ is the cumulative distribution function of the standard normal distribution. The probability of demand exceeding supply will accordingly be

$$\mathrm{P}(S_t < D_t) \;=\; \Phi_t. \tag{44}$$

In the following ϕ will denote the density function of the standard normal distribution and as above we define

$$\phi_t \;\equiv\; \phi\left(\frac{(d_t - s_t)}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right).$$

The *min* condition of equation (42) implies that when $S_t > D_t$ then $Q_t = D_t$. Using this we get that the expected quantity transacted when the suppliers are off their supply curves while demand is satisfied is given by

$$\mathrm{E}\,(Q_t \mid S_t > D_t) \;=\; \mathrm{E}\,(D_t \mid S_t > D_t)$$

$$
\begin{aligned}
&= d_t + \mathrm{E}\left(u_{1t} \mid s_t + u_{2t} > d_t + u_{1t}\right) \\
&= d_t - \frac{\sigma_1^2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \cdot \frac{\phi_t}{1 - \Phi_t}.
\end{aligned} \tag{45}
$$

In the same manner we get that the expected quantity transacted in the opposite situation is given by

$$
\begin{aligned}
\mathrm{E}\left(Q_t \mid D_t > S_t\right) &= \mathrm{E}\left(S_t \mid D_t > S_t\right) \\
&= d_t + \mathrm{E}\left(u_{2t} \mid d_t + u_{1t} > s_t + u_{2t}\right) \\
&= s_t - \frac{\sigma_2^2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \cdot \frac{\phi_t}{\Phi_t}.
\end{aligned} \tag{46}
$$

We can now find an expression for the expected observed quantity in the case where we do not observe which side of the market is rationed. By using the above equations (45) and (46) we get that

$$
\begin{aligned}
\mathrm{E}(Q_t) &= \mathrm{E}(D_t \mid S_t > D_t) \cdot \Pr(S_t > D_t) + \mathrm{E}(S_t \mid D_t > S_t) \cdot \Pr(D_t > S_t) \\
&= d_t\,(1 - \Phi_t) + s_t\,\Phi_t - \sqrt{\sigma_1^2 + \sigma_2^2}\,\phi_t
\end{aligned} \tag{47}
$$

which implies that the expected observed quantity transacted will be less than a weighted average of expected demand and expected supply. This is illustrated in figure 5.

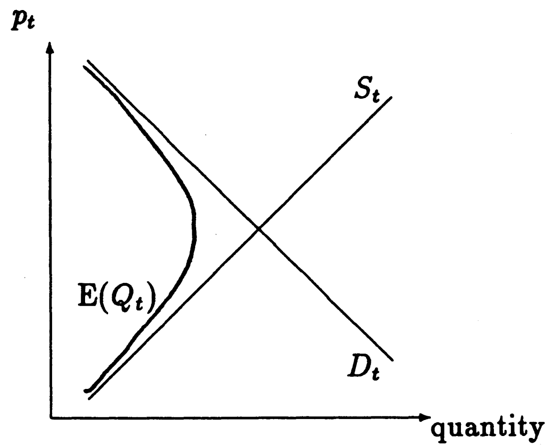

Figure 5. Expected quantity transacted

4.2 Maximum likelihood estimation

We now consider maximum likelihood estimation of the model given in equations (40) to (42). This model implies that there are two possible regimes: One where demand is satisfied and supply is rationed and another where demand is rationed and supply is satisfied (the possibility of both demand and supply simultaneously being satisfied being trivially included in one of the two regimes.) Denote by $g(D_t, S_t)$ the simultaneous density function for D_t and S_t conditional on p_t, z_{1t}, and z_{2t}. We assume, as before, that the residuals u_{1t} and u_{2t} are independently and normally distributed. This makes it possible to write the simultaneous density function as

$$g(D_t, S_t) = \frac{1}{\sigma_1}\phi\left(\frac{D_t - d_t}{\sigma_1}\right) \cdot \frac{1}{\sigma_2}\phi\left(\frac{S_t - s_t}{\sigma_2}\right) \tag{48}$$

where the first part of the right hand side is the density function for D_t and the second part is the density function for S_t. The density function for Q_t conditional on p_t, z_{1t}, and z_{2t} becomes:

$$\begin{aligned} h(Q_t) &= \int_{Q_t}^{\infty} g(Q_t, y)\, dy + \int_{Q_t}^{\infty} g(x, Q_t)\, dx \\ &= \frac{1}{\sigma_1}\phi\left(\frac{Q_t - d_t}{\sigma_1}\right)\left[1 - \Phi\left(\frac{Q_t - s_t}{\sigma_2}\right)\right] \\ &\quad + \frac{1}{\sigma_2}\phi\left(\frac{Q_t - s_t}{\sigma_2}\right)\left[1 - \Phi\left(\frac{Q_t - d_t}{\sigma_1}\right)\right]. \end{aligned} \tag{49}$$

Maximum likelihood estimation entails maximizing the likelihood function $\mathcal{L}$:

$$\mathcal{L} = \prod_{t=1}^{T} h(Q_t)$$

with respect to α_1, α_2, β_1, β_2, σ_1, and σ_2. This type of model can be considered a switching regression model with endogenous switching.

As noted in Quandt (1982,1988) maximum likelihood estimation of models where sample separation is unknown often leads to the likelihood function being unbounded in parameter space, which can result in the computation procedure breaking down. The likelihood will be unbounded from above if for example the likelihood goes toward infinity when one of the variances goes toward zero. Quandt (1988) contains an example of a disequilibrium model where this is the case. This unboundedness is a consequence of the latent structure of our model. Often unboundedness happens at the boundaries of the parameter space (for example where one of the variances are zero). One way to deal with such unboundedness is to employ a constrained maximum likelihood where the constraints are such that the unbounded regions of the parameter space are avoided.

4.3 Multi-market disequilibrium models

We shall now discuss how increasing the number of markets within the above framework greatly increases the complexity of the model and can lead to computational problems. As we have discussed before, when we have more than one market there will be spillovers between the markets. Rationing in one market leads to changes in demand in other markets. An example is the case where consumers are rationed in the labor market and therefore reduce their demand in the goods market. There have traditionally been, as we have mentioned earlier, two ways of implementing such spillovers in econometric disequilibrium models, Drèze demands and Clower demands. Most econometric specifications of multi-market disequilibrium models have employed Clower demands.

A much used specification of Clower demands is the one introduced by Ito (1980), where the effective demand for a commodity is calculated by maximizing a Cobb-Douglas utility function subject to both the budget constraint and quantity constraints on the other markets, ignoring any quantity constraint in the market under consideration. The firms' behavior is taken to be analogous to the consumers'.

Ito's specification of spill-over effects can be illustrated in the canonical two-market model. We assume that each consumer has a Cobb-Douglas utility function which depends upon the amount of the consumption good, X, the amount of leisure, $\bar{L} - L$, and the real balance of money, M/p, at the end of a period:

$$U = X^{\alpha}(\bar{L} - L)^{\beta}(M/p) \tag{50}$$

where p is the price of the consumption good. The consumers budget constraint is:

$$R + wL = pX + M \tag{51}$$

where R is endowed income (including initial money balances) at the beginning of a period. Maximizing the utility function (50) with respect to X, M, and L subject to the budget constraint leads to the notional demand and supply functions for the consumer:

$$\tilde{D}_X = \frac{\alpha}{p \cdot (\alpha + \beta + 1)} \left[R + w\bar{L}\right], \tag{52}$$

$$\tilde{S}_L = \bar{L} - \frac{\beta}{w \cdot (\alpha + \beta + 1)} \left[R + w\bar{L}\right], \tag{53}$$

and

$$\tilde{D}_M = \frac{1}{(\alpha + \beta + 1)} \left[R + w\bar{L}\right]. \tag{54}$$

The Cobb-Douglas utility function implies constant budget shares. It is useful to note that combining equations (52) and (53) leads to the following expression for R:

$$R = \frac{p}{\alpha}(\alpha + 1)\tilde{D}_X - w\tilde{S}_L. \tag{55}$$

Effective demand and supply functions of the Clower type are, as mentioned before, obtained by maximizing the utility function subject to both the budget constraint and the quantity constraints on all other markets except the one under consideration. If the consumer is constrained on the labor market we have that the transacted quantity of labor, Q_L, is less than the notional supply, $\tilde{S}_L$. In our case this is the only constraint we need consider when deriving the effective demand for the good X, since we only have two markets. Effective demand of the Clower type for goods, taking into account the spillovers from the labor market, is derived from the optimization problem:

$$\begin{aligned} &\max_{X,M} \; U \\ &\text{s.t.} \quad R + wL = pX + M, \\ &\qquad\quad L = Q_L, \end{aligned} \tag{56}$$

which leads to the effective demand function for good X:

$$\begin{aligned} D_X &= \frac{\alpha}{p \cdot (\alpha + 1)} [R + wQ_L] \\ &= \tilde{D}_X - \frac{\alpha}{(\alpha + 1)} \frac{w}{p} \left[\tilde{S}_L - Q_L\right] \end{aligned} \tag{57}$$

where we have substituted for R from equation (55). We see that the effective demand for goods decreases as the degree of rationing in the labor market increases. We find a similar expression for the effective supply of labor by doing the optimization using the constraint in the goods market ($X = Q_X$):

$$S_L = \tilde{S}_L + \frac{\beta}{(\beta+1)}\frac{p}{w}\left[Q_X - \tilde{D}_X\right]. \tag{58}$$

In the following D and S will denote effective demand and supply while $\tilde{D}$ and $\tilde{S}$ denote notional demand and supply. We let λ_i denote the combinations of coefficients we find in the last two equations:

$$\lambda_1 = \frac{\alpha}{(\alpha+1)}\frac{w}{p}, \qquad \lambda_2 = \frac{\beta}{(\beta+1}\frac{p}{w}.$$

We now assume that the firm maximizes profits with a Cobb-Douglas production function, leading to similar supply and demand expressions as for the consumer. The market we are looking at has only one representative consumer and one representative firm. Taking the consumer's and the firm's supply and demand functions together leads to the following specifications of effective demand in the canonical two-market model:

$$\begin{aligned} D_L &= \tilde{D}_L + \lambda_1(Q_X - \tilde{S}_X), \\ S_L &= \tilde{S}_L + \lambda_2(Q_X - \tilde{D}_X), \\ D_X &= \tilde{D}_X + \lambda_3(Q_L - \tilde{S}_L), \\ S_X &= \tilde{S}_X + \lambda_4(Q_L - \tilde{D}_L), \end{aligned} \tag{59}$$

where the spillovers are proportional to the difference between the actual quantities transacted and the notional demands and supplies. The *min* conditions for the two markets are functions of the effective demands

$$Q_L = \min(D_L, S_L) \tag{60}$$

and

$$Q_X = \min(D_X, S_X). \tag{61}$$

The λ_i's are functions of relative prices and will not be constant unless the demand and supply variables are specified as nominal variables. It is also apparent that the effective demands and supplies need not satisfy the budget constraint.

In concluding this section we will briefly discuss the computational problems involved in increasing the number of markets to more than three or four. As we have just discussed, multi- market disequilibrium models requires one to make assumptions about how the spillovers between markets are. We will in the following assume that an appropriate specification has been made, without going into specific detail. In the two market model above there are four mutually exclusive and exhaustive regimes:

$$\begin{array}{ll} \text{regime 1:} & D_X \geq S_X,\ D_L \geq S_L, \\ \text{regime 2:} & D_X \geq S_X,\ D_L < S_L, \\ \text{regime 3:} & D_X < S_X,\ D_L \geq S_L, \\ \text{regime 4:} & D_X < S_X,\ D_L < S_L. \end{array}$$

For each regime it is possible to derive the relevant part of the probability density function for the observed quantities Q_{Xt} and Q_{Lt} in much the same manner as the single market models (with the added complexity the addition of spillovers entails). The density $g_i(D_{Xt}, S_{Xt}, D_{Lt}, S_{Lt})$ is the conditional density of regime i multiplied by the probability of being in regime i. The density function of the observed variables then becomes:

$h(Q_{Xt}, Q_{Lt}) =$

$$\int_{Q_{Xt}}^{\infty}\int_{Q_{Lt}}^{\infty} g_1(Q_{Xt}, x, Q_{Lt}, y)\, dx\, dy \;+\; \int_{Q_{Xt}}^{\infty}\int_{Q_{Lt}}^{\infty} g_2(Q_{Xt}, x, y, Q_{Lt})\, dx\, dy$$
$$+\int_{Q_{Xt}}^{\infty}\int_{Q_{Lt}}^{\infty} g_3(x, Q_{Xt}, Q_{Lt}, y)\, dx\, dy \;+\; \int_{Q_{Xt}}^{\infty}\int_{Q_{Lt}}^{\infty} g_4(x, Q_{Xt}, y, Q_{Lt})\, dx\, dy. \tag{62}$$

The computation of these integrals is more complicated than in the case of only one market because we cannot directly factor the g_i's as we did in the case of a single market. In the single market model we assumed that each side of the market was independent of the other, but because of spillovers this will not be the case in a multi-market disequilibrium model. In multimarket models the g_i's must include specifications of the interaction between markets, and these spillovers are the main cause of the additional complexity of multimarket models. Computationally it is not easy to do a numerical computation of multivariate normal integrals. Using the specification suggested by Ito (1980) in a two-market model of the type we have sketched above, Goldfeld and Quandt (1979) find that having to compute 2-dimensional rather than 1-dimensional normal integrals increases computing time by a factor of roughly 5 to 1. It is commonly assumed that it is not practically feasible to do computations on anything higher than 3- dimensional normal integrals.

Econometric disequilibrium models are in my view attempts at empirically specifying temporary equilibrium models. Since the temporary equilibrium models encompass in general a large number of markets, the difficulty of extending econometric models beyond two markets is disconcerting. One of the main challenges should therefore be to develop models where there are more than two markets and/or to develop satisfying aggregation methods in such models. In the next two sections we will look at two ways of tackling these issues, the virtual price approach of Lee (1986) and the smoothing by aggregation approach of Lambert (1988).

5 Lee's virtual price approach

Lee (1986) proposes an econometric approach based on virtual prices which makes it unnecessary to choose between different specifications of effective demand. These virtual prices are derived from explicit utility and profit maximization. Under certain stochastic specifications, Lee's virtual price approach leads to a likelihood function which does not require the computation of multiple integrals, even when considering many markets.

The Ito specification discussed earlier (equations (59) to (61)) is based on linear spillover effects, which can be derived from (and only from) Cobb- Douglas utility and production functions, and on *min* conditions in the resulting effective demands and supplies. Lee's approach uses the definition of Drèze equilibrium and derives spillover terms which in a fixed-price equilibrium are observationally equivalent to the Clower demand specifications of Ito (1980) and of Gourieroux et. al. (1980b). The different specifications differ in their modeling of excess demand and excess supply, but their reduced forms are equivalent ways to relate the basic structures to the observations of traded quantities. This is not very surprising when we consider that the Drèze equilibrium is a special case of Benassy's K-equilibrium (which is based on Clower demands). Gales (1979) more general equilibrium concept which opens for stochastic rationing and manipulable rationing schemes has to my knowledge not been used in any econometric models. If a model includes price adjustment relationships which depend on the level of excess demand or excess supply, then Lee's results do not hold, and the relationship between the observed variables will differ according to which spillover specification is chosen.

The Drèze equilibrium we defined earlier and which is used by Lee (1986) can be written as a set of transactions where:

1. transactions balance on each market
2. transactions are utility- and profit-maximizing, taking all constraints into account

3. suppliers and demanders can not be simultaneously rationed in any market

Lee characterizes these requirements using the concept of virtual prices (see Deaton and Muellbauer (1980) pp. 109-114). The consumer's maximization problem in the case of two markets is:

$$\begin{aligned} &\max_{X,L,M} \quad U(X,L,M) \\ &\text{s.t.} \qquad R + wL \leq pX + M, \\ &\qquad\qquad L \leq Q_L, \\ &\qquad\qquad X \leq Q_X, \end{aligned} \tag{63}$$

where an interior solution can be characterized by the Kuhn-Tucker conditions:

$$\frac{\partial U}{\partial L} + \lambda_1 w - \kappa_L = 0, \qquad \kappa_L \geq 0, \tag{64}$$

$$\frac{\partial U}{\partial X} - \lambda_1 p - \kappa_X = 0, \qquad \kappa_X \geq 0, \tag{65}$$

$$\frac{\partial U}{\partial M} - \lambda_1 = 0, \tag{66}$$

and

$$pX - wL + M - R = 0, \tag{67}$$

where λ_1, and the κ's are Lagrange multipliers. Define the virtual prices as

$$\xi_L = -\frac{\partial U(X,L,M)/\partial L}{\partial U(X,L,M)/\partial M} \tag{68}$$

and

$$\xi_X = \frac{\partial U(X,L,M)/\partial X}{\partial U(X,L,M)/\partial M},$$

which are the prices which support the quantities Q_L and Q_X as an unconstrained (notional) utility-maximization solution.

We see that in general (with a quasi-concave utility function) we will have that the Kuhn-Tucker conditions (64) to (66) imply that

$$\xi_L = -\frac{\lambda_1 w + \kappa_L}{\lambda_1} = w - \frac{\kappa_L}{\lambda_1} \leq w \tag{69}$$

and

$$\xi_X = \frac{\lambda_1 p + \kappa_X}{\lambda_1} = p + \frac{\kappa_X}{\lambda_1} \geq p.$$

The producers maximization problem is

$$\begin{aligned} \max_{X,L,M} \quad & \pi = pX - wL - M \\ \text{s.t.} \quad & F(X, L, M) \leq 0 \\ & L \leq Q_L, \\ & X \leq Q_X, \end{aligned} \tag{70}$$

where π is profit and the production function is $F(X, L, M)$. The transacted quantities Q_L and Q_x will be the interior solution to (70) if the following Kuhn-Tucker conditions are satisfied:

$$\lambda_2 \frac{\partial F}{\partial L} - w - \mu_L = 0, \qquad \mu_L \geq 0, \tag{71}$$

$$\lambda_2 \frac{\partial F}{\partial X} + p - \mu_X = 0, \qquad \mu_X \geq 0, \tag{72}$$

$$\lambda_2 \frac{\partial F}{\partial M} - 1 = 0, \tag{73}$$

and

$$F(Q_M, Q_L, M) = 0 \tag{74}$$

where λ_2, and the μ's are Lagrange multipliers. The virtual prices are defined as:

$$\eta_L = \frac{\partial F(X,L,M)/\partial L}{\partial F(X,L,M)/\partial M} \tag{75}$$

and

$$\eta_X = -\frac{\partial F(X,L,M)/\partial X}{\partial F(X,L,M)/\partial M}$$

which are the prices which support the quantities Q_L and Q_X as an unconstrained (notional) profit-maximization solution.

We see that in general (with a quasi-concave production function) we will have that the Kuhn-Tucker conditions (71) to (73) imply that:

$$\eta_L = w + \mu_L \geq w \tag{76}$$

and

$$\eta_X = p - \mu_X \leq p.$$

From the definition of a fixed-price equilibrium above, we know that the producer and the consumer can not be rationed at the same time. This means either κ_L or μ_L must be equal to 0, and that either κ_X or μ_X must be equal to 0. This taken together with equations (69) and (76) means that the following conditions must be satisfied in respectively the labor and the goods market:

$$\xi_L < w,\ \eta_L = w \quad \text{or} \quad \xi_L = w,\ \eta_L \geq w$$

and

$$\xi_X > p,\ \eta_X = p \quad \text{or} \quad \xi_X = p,\ \eta_X \leq p.$$

This enables us to specify the four possible regimes of a two market model in terms of the virtual and market prices:

1. $\left.\begin{array}{l}\xi_L < w,\ \eta_L = w \\ \xi_X > p,\ \eta_X = p\end{array}\right\}$ The consumer is rationed in both markets

2. $\left.\begin{array}{l}\xi_L < w,\ \eta_L = w \\ \xi_X = p,\ \eta_X < p\end{array}\right\}$ The consumer is rationed in the labor market and the producer is rationed in the goods market

3. $\left.\begin{array}{l}\xi_L = w,\ \eta_L > w \\ \xi_X > p,\ \eta_X = p\end{array}\right\}$ The consumer is rationed in the goods market and the producer is rationed in the labor market

4. $\left.\begin{array}{l}\xi_L = w,\ \eta_L > w \\ \xi_X = p,\ \eta_X < p\end{array}\right\}$ The producer is rationed in both markets

There are some in-between cases such as the Walrasian equilibrium but under conventional specifications of the unobservables they occur with probability zero and can therefore be neglected.

The relationships between the virtual prices and the market prices give a complete description of the fixed-price equilibrium as described by Drèze (1975). Lee shows that this fixed-price description is equivalent to the fixed-price specification inherent in both the Ito (1980) and the Gourieroux, Laffont, and Monfort (1980b) spillover specifications, even though the excess demand and supply functions will be different.

Lee's proof of the equivalence of his virtual price approach and the spillover specifications used by Ito (1980) and Gourieroux, Laffont, and Monfort (1980b) are based on four theorems relating the Clower effective demands to the virtual prices. The two theorems for the consumer imply that if and only if the virtual wage is equal to the market wage, will Ito's (and the Clower) specification of effective supply of labor, S_L, be equal to the transacted quantity of labor, Q_L and if and only if the virtual wage (the marginal utility of an extra unit of leisure) is less than the market wage will Ito's effective supply be greater than the transacted quantity of labor. These two theorems can be written:

$$\xi_L = w \quad \Leftrightarrow \quad S_L = Q_L \tag{77}$$

and

$$\xi_L < w \quad \Leftrightarrow \quad S_L > Q_L.$$

The theorems imply similar results for the effective demand for goods,

$$\xi_X = p \quad \Leftrightarrow \quad D_X = Q_X$$

and

$$\xi_X < p \quad \Leftrightarrow \quad D_X > Q_X. \tag{78}$$

Lee's other two theorems imply similar relationships between virtual prices and the Clower effective demands for the producer, namely

$$\begin{aligned} \eta_L = w \quad &\Leftrightarrow \quad D_L = Q_L, \\ \eta_L < w \quad &\Leftrightarrow \quad D_L > Q_L, \\ \eta_X = p \quad &\Leftrightarrow \quad S_X = Q_X, \end{aligned} \tag{79}$$

and

$$\eta_X < p \quad \Leftrightarrow \quad S_X > Q_X,$$

where D_L is Ito's (and the Clower) specification of effective demand. Combining (77), (78), and (79) gives us that:

$$\begin{aligned} Q_L &= \min(D_L, S_L), \\ Q_X &= \min(D_X, S_X). \end{aligned} \tag{80}$$

which proves that the Drèze (1975) definition of a fixed-price equilibrium (characterized by the virtual prices) is equivalent to the observed quantity of goods transacted being the minimum of the Ito (1980) demand and supply. We thereby see that the *min* condition when applied to effective demands of the Clover type is a consequence of the definition of a fixed-price equilibrium. What the econometrician observes and uses in his analysis is therefore independent of the behavioral justification behind the spillover effects. In the end they end up being equivalent ways of characterizing the observed variables Q_L and Q_X as a fixed-price equilibrium. The above results are fairly general and can be extended to many markets.

The virtual price approach of Lee (1986) also makes it tractable to estimate a multi-market model since the likelihood function does not include multiple integrals. In discussing this approach we will use as an example a two-market disequilibrium model of the canonical type where the utility and production functions are Cobb-Douglas:

$$U = X^{\alpha}(\bar{L} - L)^{\beta}(M/p) \tag{81}$$

and

$$F(X, L, M) = AL^{\delta}M - X^{\gamma},$$

and where the parameterization of the production function has been chosen so that the econometric model we discuss later will be as simple as possible. Utility and profit maximization lead to the following virtual prices:

$$\begin{aligned} \xi_L &= -\frac{\partial U/\partial L}{\partial U/\partial M} = \frac{\beta M^c}{(\bar{L}-Q_L)}, \\ \xi_X &= \frac{\partial U/\partial X}{\partial U/\partial M} = \frac{\alpha M^c}{Q_X}, \\ \eta_L &= \frac{\partial F/\partial L}{\partial F/\partial M} = \frac{\delta M^p}{Q_L}, \end{aligned} \tag{82}$$

and

$$\eta_X = -\frac{\partial F/\partial X}{\partial F/\partial M} = \frac{\gamma M^p}{Q_X},$$

where M^c and M^p are the cash balances of the consumer and the producer respectively. These virtual prices are the prices which support the quantities Q_L and Q_X as unconstrained (notional) profit-maximization solutions.

A natural way of defining the spillover from the labor to the goods market for the consumer is to use the difference between the notional demand and the Drèze demand, D_X^*. As an example we look at the consumer's effective demand when the consumer is rationed in the labor market but not in the goods market. The Drèze demand is the result of maximizing the utility function subject to the budget constraint and all other constraints (in this case the rationing in the labor market). In terms of the virtual prices this implies that $\xi_L < w$ and that $\xi_X = p$. The difference between the notional demand and the Drèze demand will be:

$$\tilde{D}_X - D_X^* = \frac{1}{p}\frac{\alpha}{(\alpha+\beta+1)}(w - \xi_L)(\bar{L} - Q_L). \tag{83}$$

The spillovers depend upon the differences between the virtual prices and the market prices in the labor market.

In the above presentation of Lee's model there are two agents and two markets. Lee shows that it is easy to generalize the model to include many markets, but does not extend it to more than two agents. Lee assumes that the parameters in the utility and production functions are stochastic over time in the following manner:

$$\begin{aligned} \alpha &= a \exp(u_1) & \beta &= b \exp(u_2) \\ \delta &= d \exp(u_3) & \gamma &= c \exp(u_4) \end{aligned} \tag{84}$$

where a, b, c and d are fixed parameters and the u's are independent random variables with zero mean. This implies that over time there will be random fluctuations in the utility and production functions. We now derive the likelihood for regime 4, where the firm is rationed in both markets. In this regime Lee (1986) shows that the equations (82) can be written as (doing logarithmic transformations and substituting from the budget constraint for M_c and from the profit expression for M_p):

$$
\begin{aligned}
\ln p &= \ln a + \ln\left(\frac{R + wQ_L - pQ_X}{Q_X}\right) + u_1, \\
\ln w &= \ln b + \ln\left(\frac{R + wQ_L - pQ_X}{\bar{L} - Q_L}\right) + u_2, \\
\ln \eta_L &= \ln d + \ln\left(\frac{pQ_X - wQ_L - \pi}{Q_L}\right) + u_3,
\end{aligned} \tag{85}
$$

and

$$
\ln \eta_X = \ln c + \ln\left(\frac{pQ_X - wQ_L - \pi}{Q_X}\right) + u_4.
$$

The transacted quantities Q_L and Q_X will be equal to the notional demands of the consumer since the consumer isn't rationed. We transform the probability densities for u_1 and u_2 to the corresponding joint probability density for observing Q_L and Q_X when we are in regime 4 leading to:

$$
\begin{aligned}
h_4(Q_L, Q_X) &= |J(Q_L, Q_X)| \cdot f_1\left(\ln p - \ln\left(\frac{R + wQ_L - pQ_X}{Q_X}\right) - \ln a\right) \\
&\quad \cdot f_2\left(\ln w - \ln\left(\frac{R + wQ_L - pQ_X}{\bar{L} - Q_L}\right) - \ln b\right),
\end{aligned} \tag{86}
$$

where f_1 and f_2 are the density functions of u_1 and u_2 and $J(Q_L, Q_X)$ is the Jacobian of the transformation from (u_1, u_2) to (Q_L, Q_X). Equation (86) gives the probability density for our observations if we are in regime 4. We now multiply this density with the probability that we actually are in regime 4 to obtain the unconditional likelihood L_4 of an observation being in regime 4:

$$
\begin{aligned}
L_4 &= h_4(Q_L, Q_X) \cdot \Pr(\eta_L > w) \cdot \Pr(\eta_X < p) \\
&= h_4(Q_L, Q_X) \cdot \left[1 - G_1\left(\ln w - \ln\left(\frac{pQ_X - wQ_L - \pi}{Q_L}\right) - \ln d\right)\right]
\end{aligned}
$$

$$\cdot G_2\left(\ln p - \ln\left(\frac{pQ_X - wQ_L - \pi}{Q_X}\right) - \ln c\right) \tag{87}$$

We can analyze the other regimes in a similar fashion and thereby get expressions for the contribution to the likelihood function of the four regimes: L_1, L_2, L_3 and L_4. We see that this approach avoids the multiple integrals we discussed at the end of the preceding section (see equation (62)). Adding these contributions together gives us the total likelihood. As noted earlier, Lee's approach can be generalized to many markets as long as there are only two agents in the economy. The producer can produce a large variety of goods which are consumed by the consumer, but we can not directly generalize Lee's approach to many consumers and many producers.

5.1 Coherency

The term coherency in an econometric model means that there is an unique solution to a simultaneous equation model. For example in Lee's model we take a transformation from the density function of the random variables, the u's in our case, to the density function of the observed variables, the Q's. This is possible if there for each vector of u's is a unique vector of Q's. If there is a correspondence between the u's and the Q's so that several values of Q correspond to a given value of u, then it is not possible to infer the density function of the Q's from the u's. The problem of coherency occurs in many types of models including Tobit, Probit, and self-selection models. When prices are endogenous in disequilibrium models the problem of coherency can become even more complex.

For the disequilibrium models of Ito (1980) and Gourieroux, Laffont, and Monfort (1980b) necessary and sufficient conditions for coherency are derived. In their models coherency is equivalent to local dynamic stability of the system.

Lee's model is coherent because the solution Q is unique within each regime. It can be calculated for each regime from the virtual price equations which are equal to the market prices (e.g. the two first equations for regime 4 in equations (85)). We therefore have a unique reduced-form equation system which is such that the probability for all regimes add up to unity. The uniqueness of the fixed price equilibrium guarantees that the likelihood is well defined.

6 Macro models using the smoothing by aggregation approach

The models discussed in the preceding section assumed that only one side of the market can experience rationing at one time. This may be a reasonable assumption if each market is fairly small and homogenous, but seems more farfetched when modeling a economy consisting of only two aggregate macro markets as is the case in neo-Keynesian models such as that specified by Ito (1980). Muellbauer (1978) suggested a "smoothing by aggregation" approach where each aggregate market is seen as a continuum of micro markets. Lambert (1988) shows that simple assumptions enables him to specify a macro market which represents an explicit aggregation of micro markets. This makes it possible for some micro markets to be in excess supply while others are in excess demand. Lambert thereby takes a step away from the conventional discrete switching models towards developing an aggregation theory. Lambert's model is a good vehicle for incorporating relevant data into a econometric disequilibrium model, especially that available through business survey data. Gourieroux (1984) derives similar aggregated relationships as those in Lambert (1988).

Lambert (1988) starts by modelling micro markets. These are markets that are so small that we can assume that only one side of the market is rationed, as is reflected in equation (90). This might typically be the case if all transactions in the market take place in only one specific physical location. Lambert assumes that micro demands and supplies are lognormally distributed as in equations (88) and (89),

$$\ln d_j = \lambda^d + \varepsilon_{1j}, \tag{88}$$

$$\ln s_j = \lambda^s + \varepsilon_{2j}, \tag{89}$$

$$\ln q_j = \min(\ln d_j, \ln s_j), \tag{90}$$

where d_j is the notional micro demand in micro market j, s_j is the notional micro supply, q_j is transacted quantity and the λ's are structural relationships (including for example prices). These micro demands and supplies are assumed by Lambert to be effective demands and supplies as defined earlier. The residuals, ε_{1j} and ε_{2j} are bivariate normally distributed:

$$\begin{pmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \end{pmatrix} \sim \mathrm{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \right]$$

and we denote the bivariate density function for ε_1 and ε_2 as $f(\varepsilon_1, \varepsilon_2)$. A $\rho > 0$ implies the reasonable assumption that a larger than average supply corresponds to a larger than average demand. The stochastic residuals are also assumed to be independent across the j micro markets. Under the above assumptions expected demand and supply in each micro market will be

$$\mathrm{E}(d_j) = \exp\left(\lambda^d + \frac{1}{2}\sigma_1^2\right) \tag{91}$$

and

$$\mathrm{E}(s_j) = \exp\left(\lambda^s + \frac{1}{2}\sigma_2^2\right),$$

which can be interpreted as the average supply and demand across a large number of similar micro markets. Lambert introduces the macro variables D and S which are defined as

$$\begin{aligned} D &= N \cdot d_j = N \cdot \exp(\lambda^d + \varepsilon_1), \\ S &= N \cdot s_j = N \cdot \exp(\lambda^s + \varepsilon_2), \end{aligned} \tag{92}$$

and the aggregate transacted quantity Q defined using the *min* condition:

$$Q = \sum_j \min(d_j, s_j). \tag{93}$$

The joint density function of the two variables D and S, $g(D, S)$, can be derived from the density function for ε_1 and ε_2 as:

$$g(D,S) \;=\; \frac{1}{D \cdot S} f(\ln D - \ln N - \lambda^d, \ln S - \ln N - \lambda^s). \tag{94}$$

Expected effective aggregate demand and supply can be written respectively as

$$\begin{aligned} \mathrm{E}(D) &= N \cdot \mathrm{E}(d_j) \\ &= N \cdot \exp\left(\lambda^d + \frac{1}{2}\sigma_1^2\right) \end{aligned} \tag{95}$$

and

$$\mathrm{E}(S) \;=\; N \cdot \exp\left(\lambda^s + \frac{1}{2}\sigma_2^2\right).$$

when there are a large number of micro markets. Aggregate transactions when the *min* condition prevails on each micro market will be the expected transaction on each micro market times the number of markets:

$$\begin{aligned} \mathrm{E}(Q) &= N \cdot \mathrm{E}\left[\min(d_j, s_j)\right] \\ &= \int_{-\infty}^{\infty}\int_{\lambda^d-\lambda^s+x}^{\infty} N \exp(\lambda^d + x)\, f(x,y)\, dx\, dy \\ &\quad + \int_{-\infty}^{\infty}\int_{\lambda^s-\lambda^d+y}^{\infty} N \exp(\lambda^d + y)\, f(x,y)\, dx\, dy \qquad (96) \\ &= \int_0^{\infty}\int_D^{\infty} D \cdot G(D,S)\, dS\, dD + \int_0^{\infty}\int_S^{\infty} S \cdot G(D,S)\, dD\, dS \qquad (97) \\ &= \mathrm{E}(D \mid S > D) \cdot \Pr(S > D) + \mathrm{E}(S \mid D > S) \cdot \Pr(D > S) \end{aligned}$$

Lambert shows that the CES function:

$$\mathrm{E}(Q) \;\approx\; [\mathrm{E}(D)^{-\rho} + \mathrm{E}(S)^{-\rho}]^{-\frac{1}{\rho}} \;\equiv\; Q^* \tag{98}$$

gives a good approximation to equation (96) for $\rho > 0$. He succeeds thereby in getting a fairly simple expression for aggregate transactions. We see that equation (98) implies, as one would expect from Jensen's inequality, that expected aggregate transactions will be less than either expected aggregate demand or expected aggregate supply,

$$Q^* < \min(\mathrm{E}(D), \mathrm{E}(S)). \tag{99}$$

An alternative way of deriving equation (98) is pointed out in Gourieroux (1984). Instead of assuming the lognormal distribution in equations (88) and (89), Gourieroux assumes that the demands and supplies in micro market j are spesified as follows:

$$\begin{aligned} d_j &= \lambda^d \varepsilon_3, \\ s_j &= \lambda^s \varepsilon_4, \end{aligned} \tag{100}$$

where ε_3 and ε_4 are Weibull distributed stochastic variables reflecting the distribution of demand and supply across the many micro markets. In this case equation (98) will apply exactly.

When considering two aggregate markets, Lambert finds it necessary to assume that there is zero correlation between the micro disturbances of these markets. This is necessary to be able to aggregate each market separately and thereby be able to use equations in the form of (98) for each aggregate market. Lambert also shows that the approximation used to get equation (98) leads to a fairly simple expression for the weighted proportion of micro markets in excess demand,

$$\begin{aligned} \mathrm{P}(D \geq S) &= \frac{1}{E(Q)} \int_0^\infty \int_S^\infty g(D,S)\, dD\, dS \\ &\approx \frac{1}{1 + \left(\frac{\mathrm{E}(D)}{\mathrm{E}(S)}\right)^{-\rho}}. \end{aligned} \tag{101}$$

The parameter ρ can be interpreted as a dispersion parameter. A decrease in ρ moves the curve away from the contract lines $E(D)$ and $E(S)$. Taking the derivative of Q^* with respect to $\mathrm{E}(S)$ gives us:

$$\frac{\partial Q^*}{\partial \mathrm{E}(S)} = \frac{Q^*}{\mathrm{E}(S)} \cdot \frac{1}{1 + \left(\frac{\mathrm{E}(D)}{\mathrm{E}(S)}\right)^{-\rho}}. \tag{102}$$

Rearranging this gives us that the elasticity of aggregate transaction (realized quantity) with respect to aggregate supply is equal to the weighted proportion of micro markets in excess demand,

$$\mathrm{El}_S(Q^*) = \frac{\partial \ln Q^*}{\partial \ln \mathrm{E}(S)} \approx \mathrm{P}(D \geq S), \tag{103}$$

and the elasticity of aggregate transaction with respect to aggregate demand is equal to:

$$\mathrm{El}_D(Q^*) = \frac{\partial \ln Q^*}{\partial \ln \mathrm{E}(D)} \approx 1 - \mathrm{P}(D \geq S). \tag{104}$$

The extent of excess demand ($\mathrm{P}(D \geq S)$) reduces therefore the multiplier effect of an autonomous increase in the demand for goods. Lambert (1988) assumes that the extent of excess demand $\mathrm{P}(D \geq S)$ is observable using business survey data. He goes on to show that the relationship between Q^* and P is given by:

$$\begin{aligned} Q^* &= [\mathrm{E}(D)^{-\rho} + \mathrm{E}(S)^{-\rho}]^{-\frac{1}{\rho}} \\ &= \mathrm{E}(S)\left[1 + \left(\frac{\mathrm{E}(D)}{\mathrm{E}(S)}\right)^{-\rho}\right]^{-\frac{1}{\rho}} \\ &\approx \mathrm{E}(S) \cdot [\mathrm{P}(D \geq S)]^{\frac{1}{\rho}} \end{aligned} \tag{105}$$

Rearranging this expression and proceeding similarly for $\mathrm{E}(D)$ we can express the unobservable quantities $\mathrm{E}(S)$ and $\mathrm{E}(D)$ in terms of the observable quantities Q^* and $\mathrm{P}(D \geq S)$:

$$\begin{aligned} \mathrm{E}(D) &\approx Q^* \cdot [1 - \mathrm{P}(D \geq S)]^{-\frac{1}{\rho}}, &(106)\\ \mathrm{E}(S) &\approx Q^* \cdot \mathrm{P}(D \geq S)^{-\frac{1}{\rho}}. &(107) \end{aligned}$$

These equations can now be used to estimate a neo-Keynesian model where some parts of the economy may experience Keynesian unemployment and other parts may experience Classical unemployment.

The smoothing by aggregation approach we have discussed above leads to a deterministic solution, since the micro disturbances have been aggregated out. Lambert introduces therefore a further stochastic structure at the macro level (equations (111)) and extends the model to two aggregate markets, one for labor and the other for goods. He uses the Ito (1980) spillover specification and assumes the error terms to be jointly normally distributed. Lambert's full model derived for the Belgian economy is:

$$\begin{aligned} Q_L &= [\mathrm{E}(D_L)^{-\rho} + \mathrm{E}(S_L)^{-\rho}]^{-\frac{1}{\rho}}, \\ Q_X &= [\mathrm{E}(D_X)^{-\rho} + \mathrm{E}(S_X)^{-\rho}]^{-\frac{1}{\rho}}, \end{aligned} \tag{108}$$

$$\begin{aligned} \mathrm{P}(D_L \geq S_L) &= \frac{1}{1 + \left(\frac{\mathrm{E}(D_L)}{\mathrm{E}(S_L)}\right)^{-\rho}}, \\ \mathrm{P}(D_X \geq S_X) &= \frac{1}{1 + \left(\frac{\mathrm{E}(D_X)}{\mathrm{E}(S_X)}\right)^{-\rho}}, \end{aligned} \tag{109}$$

$$\begin{aligned} \ln \mathrm{E}(D_L) &= \ln \tilde{D}_L + \lambda_1(\ln Q_X - \ln \tilde{S}_X), \\ \ln \mathrm{E}(S_L) &= \ln \tilde{S}_L + \lambda_2(\ln Q_X - \ln \tilde{D}_X), \\ \ln \mathrm{E}(D_X) &= \ln \tilde{D}_X + \lambda_3(\ln Q_L - \ln \tilde{S}_L), \\ \ln \mathrm{E}(S_X) &= \ln \tilde{S}_X + \lambda_4(\ln Q_L - \ln \tilde{D}_L), \end{aligned} \tag{110}$$

$$
\begin{aligned}
\ln \tilde{D}_L &= \alpha_1 \ln p_L + \beta_1 \ln z_1 + u_1, \\
\ln \tilde{S}_L &= \alpha_2 \ln p_L + \beta_2 \ln z_2 + u_2, \\
\ln \tilde{D}_X &= \gamma_1 \ln p_X + \delta_1 \ln z_3 + u_3, \\
\ln \tilde{S}_X &= \gamma_2 \ln p_X + \delta_2 \ln z_4 + u_4,
\end{aligned}
\tag{111}
$$

$$
\ln DUC = \zeta_X + \ln\left(\frac{Q_X}{\tilde{S}_X)}\right) + u_5, \tag{112}
$$

and

$$
\ln UNR = \ln\left(\frac{Q_L}{\tilde{S}_L)}\right) \tag{113}
$$

where time subscripts have been dropped. The variables D_i and S_i are the macro variables defined in equations (92) and $\tilde{D}_i$ and $\tilde{S}_i$ are notional demands and supplies. The variable UNR is the unemployment rate, DUC is the degree of capital utilization, and ζ_X is a parameter in the capital utilization equation (112). The error terms, $u_1, \ldots, u_5$, are jointly normally distributed. The vectors z_i consist of observed macroeconomic variables.

Equations (108) and (109) are based on the smoothing by aggregation approach we have discussed above. As mentioned above, equations (108) assume that the micro disturbances for the labor and goods markets are uncorrelated.

Equations (110) are a variant of the Ito (1980) specification of spillovers we looked at in equations (59). Lambert assumes, in contrast to Ito, that the spillovers are log-linear. Lambert's actual model is generalized to also include the Portes (1977) specification of spillovers. His estimates and theoretical considerations lead him to prefer the Ito specification. Lambert assumes that $\lambda_2 = 0$, implying no spillovers from the goods market to labor supply. He argues that the open character of the small Belgian economy and large substitution possibilities among goods leads to households never expecting to be rationed on the goods market seriously or long enough to induce them to alter their labor supply.

The empirical counterpart to the variables $\mathrm{P}(D_L \geq S_L)$ and $\mathrm{P}(D_X \geq S_X)$ are assumed to be observed without error. Including error terms for these variables would make the model much more complicated. They are observed using business survey data. Equations (111) specify the notional variables $\tilde{D}_i$ and $\tilde{S}_i$ as log-linear functions of a set of explanatory variables and an error term. The functional form of equations (111) can be derived by explicit utility and profit maximization, and the total stock of capital is included as one of the explanatory variables in the vectors z_1 and z_4.

Equation (113) gives the definition of the unemployment rate, but is not used by Lambert because he doesn't have data for it. It is included here only for the sake of generality. The degree of capacity utilization, DUC, is theoretically defined in the same manner as the definition of UNR. Lambert's data from business surveys for the Belgian economy do not correspond exactly to this definition, so he has posited a stochastic specification for DUC.

By substituting the notional trade offers $\tilde{D}_L$, $\tilde{S}_L$, $\tilde{D}_X$, and $\tilde{S}_X$ in equations (111) and the unobservable demand and supply variables $E(D_L)$, $E(S_L)$, $E(D_X)$, and $E(S_L)$ in equations (110) into the remaining equations (using equations (106) and (107) to simplify the expressions) and rearranging, Lambert's full model (excluding the UNR relationship) is transformed into five equations which are log-linear in the five observable variables Q_L, Q_X, $P(D_L \geq S_L)$, $P(D_X \geq S_X)$, and DUC and the five error terms. In this manner the smoothing by aggregation approach is incorporated into a full model suitable for estimation. For the exact specification of the model the reader is referred to Lambert (1988).

There have been many empirical studies based on the smoothing by aggregation approach, either based on Lambert's model or on related approaches. Some of the most recent are a study of Belgium by Sneessens and Drèze (1986), of Switzerland by Stalder (1989), of Germany by Franz and König (1990) and a summary of ten country studies by Drèze and Bean (1990).

The main advantage of the smoothing by aggregation approach is that it leads to a model which is fairly simple to estimate and which opens for the possibility of a varying degree of excess demand or excess supply on all aggregate markets. This is an important step away from the previous discrete switching regression type models. The smoothing by aggregation approach incorporates the use of business survey data and leads to a macro model which can be flexibly specified through the choice of variables which are included in the z-vectors

A characteristic of this approach is that it is not explicitly micro based. Building the model on micro markets is only a half- way solution. It would have been much more satisfying if it were possible to do the micro specifications at the level of the individual agent instead of the micro market. It is also theoretically unsatisfying to postulate independence between the micro disturbances in the goods market and the labor market.

7 Some thoughts on future research

The above discussion has focused on the specification of econometric multi-market disequilibrium models and on the theoretical basis for such models. The temporary equilibrium literature provides a very general theoretical platform on which econometric models can be based. In extending the Arrow-Debreu framework to include situations where the economy is outside of equilibrium, this literature highlights how important it is to understand how markets actually work, instead of simply assuming that the outcome will be of a market-clearing nature. In the temporary equilibrium theory observed prices and traded quantities depend on how agents form expectations about prices and quantities, how they signal their wishes to the market, and how the market responds to these signals by setting prices and assigning quantities to each agent. Even though we still lack a thorough understanding of markets, the temporary disequilibrium literature captures the important insight that a market consists of processes taking place over time and involves search, matching and information gathering. The current literature shows that there exists temporary equilibrium under fairly general conditions, but also leaves many questions unanswered. As our earlier discussion of Clower and Drèze demands shows, there are problems connected with specifying satisfactory definitions of effective demand and with specifying the resulting equilibrium conditions. There has also been done little work on the dynamic aspects of modeling a series of markets over time. Understanding such dynamics is especially important when doing empirical work using time series data.

The concept of effective demand is important in theory, but as discussed in section 5, the choice of specification is not as important in econometric work. Lee (1986) shows that different specifications of effective demand lead to the same characterization of the observed quantities in a fixed-price equilibrium. The unobserved excess demands and supplies will be different, but the relationship between the basic structure and the observations of traded quantities will be the same. Lee's paper demonstrates the usefulness of formulating microeconomic disequilibrium models in terms of virtual prices, even when such prices are unobservable. Virtual prices constitute a direct and intuitive way of characterizing different regimes. It would probably be valuable to use this approach also in theoretic work, and doing so might make it easier to incorporate theoretical advances into econometric models.

It is important to note that even though most disequilibrium models in the economic and econometric literature are of the two-market neo-Keynesian fixed-price type discussed in section 2, the neo- Keynesian model is just a special case within the more general framework of temporary equilibrium models. It is possible to increase the number of markets and to consider more general price formation schemes ranging from full rigidity to full flexibility.

The temporary equilibrium approach can be used to derive macroeconomic relationships based on explicit microeconomic modeling while taking into account that there is not necessarily a full utilization of resources at all times. Much of the short term dynamics of an economy can be viewed as the result of economic activities being spread over time and space making coordination and information gathering difficult. Understanding such dynamics requires that they be studied at the micro level and that the computational and informational constraints the agents face be taken into account.

Aggregating from micro to macro is of course far from easy. In addition to the usual problems associated with aggregation, a disequilibrium situation implies that prices and quantities only indirectly reveal the agents preferences. Wages will for example not necessarily be equal to the workers marginal products nor their marginal utility of labor. That such aspects of the agents decision-making are unobservable in disequilibrium makes aggregation more difficult than in an equilibrium situation. If we observe a worker working 40 hours a week, we do not know whether this equals his notional demand, whether he would like to work more at the given wage, or whether he would like to work less. We also have to deal with corner solutions and self-selection, especially in the labor market. A worker's opportunity set will for example usually include a number of types of work which he chooses not to take, and the observed wage distribution will for example not be the same as the underlying wage distribution which confronts the workers. The list of unobservables can easily be extended to include the agent's information sets, their expectations, and to include all manner of unobserved heterogeneity.

The problem of unobserved or latent variables has been at the center of most econometric disequilibrium models. A typical latent variable problem in the econometric disequilibrium literature is the problem of estimating which side of a market is rationed. At the beginning of section 4 we saw how the early literature on estimating single market disequilibrium models dealt with this problem. We also discussed how the problem of estimating multi- market models was dealt with by Ito (1980). Ito's specification of spillovers is based on Clower demands and a Cobb-Douglas utility function. Many two-market models have been estimated using Ito's specification, but expanding the approach to more than two markets has proved difficult due to computational difficulties.

Being able to estimate models with more than two markets would increase the applicability of econometric disequilibrium models. It would, among other things, make it possible to estimate spillover effects between the markets for different types of labor employed in different industries. One might, for example, wish to study how easily unskilled labor can replace different types of skilled labor in different industries. Such a study may indicate how serious it would be if a large part of the labor force stays unskilled while the relative size of different industries changes. Econometric models with more than two markets would also make it possible to study bottlenecks due to shortages of different types of skilled labor or shortages of other inputs.

Lee's use of virtual prices offers a partial solution to the problem of estimating multi-market models. Lee (1986) considers only two agents dealing in many markets and his approach is only applicable under fairly stringent functional and distributional assumptions.

As we saw in section 6, Lambert (1988) uses a stochastic aggregation approach to aggregate across micro markets. Using this method to specify aggregate labor and goods markets, leads to an important generalization of the neo-Keynesian macro model. In the early neo-Keynesian models the whole economy either experienced classical unemployment, Keynesian unemployment or repressed inflation. The "smoothing by aggregation approach" makes it possible for parts of the economy to experience one regime while other parts experience another regime. An economy can thereby experience a combination of for example classical and Keynesian unemployment. Lambert's approach takes as a starting point micro markets instead of starting out with optimizing agents, and does not capture spillover effects between markets.

Instead of aggregating across micro demand and supply functions in the manner of Lambert (1988), it is an open question if is is possible to use the virtual price equations of Lee (1986). On the basis of virtual prices it should, at least in principle, be possible to integrate out unobservable variables to get aggregate relationships. We saw in section 5 that using virtual prices gives a simple interpretation of spillovers as functions of the difference between the virtual prices and the market prices. Stochastic aggregation based on the virtual prices should therefore be able to include spillover effects. To illustrate how such an approach might look, assume that the economy consists of a number of consumers each selling their labor to all firms in the economy, as in the monopolistic competition model of Benassy (1987,1990). The firms also sell goods to the consumers, but for simplicity we only consider the labor market. This implies that each combination of worker and firm is a separate micro market. Benassy goes on to consider the case where each worker has monopoly power in his labor markets and each firm has monopoly power in its product markets. By letting these monopolists set prices he succeeds in getting a disequilibrium model with endogenous prices. We will not discuss further the monopolistic structure of such a model, but will briefly consider how one might aggregate across such micro markets.

As we have seen earlier the virtual price of labor supplied to firm j from person i, ξ_{ij}, and the firm j's virtual price for labor supplied by consumer i, η_{ij} will both be functions of the transacted quantity of labor l_{ij}. These virtual prices are different for each person and firm and are assumed unobservable to the econometrician. Let us assume that utility and profit maximization lead to the following virtual prices for labor:

$$\begin{aligned} \xi_{ij} &= h_1(l_{ij}) + \varepsilon_{1ij} \\ \eta_{ij} &= h_2(l_{ij}) + \varepsilon_{2ij} \end{aligned} \quad (114)$$

where we have assumed that all heterogeneity across individuals and firms is represented by the stochastic variables ε_{1ij} and ε_{2ij}. We make the standard *min* condition assumption that for each micro market the seller and the buyer will not be simultaneously rationed. This means for example that in an interior solution we never can have both $\xi_{ij} < w_{ij}$ and $\eta_{ij} > w_{ij}$ at the same time. Under this condition and assuming an interior solution ($l_{ij} > 0$) there are two mutually exclusive regimes in the micro market for l_{ij}:

regime I	$\xi_{ij} = w_{ij}$	$\eta_{ij} > w_{ij}$
regime II	$\xi_{ij} \leq w_{ij}$	$\eta_{ij} = w_{ij}$

Regime I is the case where producer j is rationed and regime II is the case where consumer i is rationed. Note that the case of no rationing ($\xi_{ij} = \eta_{ij} = w_{ij}$) is included in regime II.

The regimes above will only apply in one micro market as long as $l_{ij} > 0$. Since we specify a supply of labor from each worker to all firms and a demand for each worker's labor from all firms, there will be many corner solutions. Most workers hold only one or two jobs, and will be uninterested in many other jobs. If for example $\xi_{ij} > w_{ij}$ then person i in uninterested in working for firm j at wage w_{ij} even if the firm is interested in hiring. We must therefore also take into account situations where $l_{ij} = 0$. A corner solution, $l_{ij} = 0$, occurs when one of the following are true:

1. $\xi^l_{ij} > w_{ij}$, which implies that person i is uninterested in working in firm j at wage w_{ij}

2. $\eta^l_{ij} < w_{ij}$, which implies that firm j is uninterested in hiring individual i at wage w_{ij}

3. the special case when we have $\eta^l_{ij} = w_{ij}$ or $\xi^l_{ij} = w_{ij}$ at the point $l_{ij} = 0$, which in the first case implies that the firm is exactly indifferent to hiring or not and in the second case implies that the consumer is exactly indifferent to working or not.

The first two cases are corner solutions on one or both sides of the market, while the last is a special case of an interior solution. One should note that they also cover the case where there is a corner solution on one side of the market while the other side is rationed at the point $l_{ij} = 0$ (for example when the consumer is rationed, $\xi^l_{ij} < w_{ij}$, and the firm is uninterested in hiring, $\eta^l_{ij} < w_{ij}$).

From the stochastic specification in equation (114) we can in principle calculate the probability density for l_{ij} and thereby find the aggregate variables $\mathrm{E}\, l_{ij}$ and $\mathrm{P}(l_{ij} > 0)$. We might thereby manage in principle to go from micro relationships based on utility and profit maximization to aggregate macro expressions.

At present, it is unknown whether there exist functional forms and stochastic specifications which make the above approach tractable for empirical analysis. There is also the question of how much of the original micro structure can be identified from the macro variables. Still, the above approach deserves investigation and should at least lead to a better understanding of the problems involved when aggregating in the presence of rationing. One point of interest is that the above approach simultaneously takes into account rationing and corner solutions. Self-selection can also be taken into account by assuming that the wage is stochastically distributed across micro markets.

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