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Abstract:
This paper deals with optimal consumption over time. The starting point is a dynamic utility function which is exponential where the exponent is quadratic in the observable consumption outlays. The approach is shown to be a generalization of Hall's formulation of the consumption relation. While Hall's structural form of consumption is independent of the income process, we show that this no longer holds. On the contrary, parameters of the income process are shown to affect the parameters of the consumption process in an essential way. The paper also argues for a stochastic maximum principle. In addition to generating the optimal current decisions, this principle produces simultaneously optimal estimates of the future values of the decision variables. This interplay of optimization and prediction is interesting. The paper terminates with statistical testing procedures which compare the testing of hypotheses deduced by Hall with testing of those derived in the present paper.

Keywords: Behavior under uncertainty, Non-separable dynamic utility, Optimal consumption, A stochastic maximum principle, Statistical testing.

JEL classification: E21

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1 Introduction

The present paper deals with the specification of the dynamic consumption process based upon the optimal behaviour of a forward-looking consumer. In the stochastic case it appears to be almost impossible to give an explicit solution to this problem of any generality. However, by successively extending existing results we hope to be able to gain further insight into the specification of the consumption relation.

In the sections to come we intend to extend the analysis of Hall (1978). This interesting paper by Hall (op.cit.) deals with optimal consumption behaviour of consumers possessing time-additive, separable quadratic utilities. Thus, subject to a linear stochastic budget equation, the Hall consumer maximizes the present discounted value of the future expected utility, conditional on his/her information set at time zero. This is the standard certainty equivalent (CEQ) model, but Hall managed to show that this model had interesting implications for regression analyses of time-series consumption data. Specifically, he showed that of all the information available at time t, only c_t was effective in predicting c_{t+1}. Previous incomes and wealth variables are via the optimization procedure eliminated from the equation predicting c_{t+1}. We shall call this hypothesis the "Euler equation hypothesis". He also showed that if the rate of time preference is equal to the rate of interest, the consumption process becomes a martingale. This hypothesis we shall call the "Martingale hypothesis". Both these hypotheses have inspired numerous empirical studies (see Hall (1989) for a recent review). Although, the empirical evidence is mixed, only a few support Hall's two hypotheses.

Rather than embarking on yet another empirical study, we shall first study the consumer's allocation problem within a more general model, and finally supplement our theoretical results with an empirical study. Our motivation for writing this paper has been the fact that Hall's (op.cit.) (CEQ) model has behavioural implications which are not a priori convincing. For example, the criterion, being the expectation of a sum of discounted one-period quadratic utilities, will exclude prudent consumer behaviour since the third derivative of the aggregate utility with respect to c_t at any time is zero. Prudent behaviour is continuously belived to be an essential feature of any optimal dynamic consumption process (see Leland (1968) or Blanchard and Fischer (1989)). If this is true any specification which ignores this fact should not be successful in empirical research.

The present extension of Hall satisfies the following reasonable requirements. Firstly, Hall's hypotheses appear as special cases in our specification. Secondly, it allows the consumers to have prudent behaviour. Thus, we are able to deal with serious shortcomings of Hall's certainty equivalent model.

The plan of the paper is the following. In the next section we specify and discuss the aggregate utility to be applied, and state the problem to be analysed. Section 3 is devoted to the forward-going optimization approach to be used to solve our consumption problem. Following the terminology of Whittle (1990), we call it a stochastic maximum principle (SMP). This principle is particularly interesting in the present application since it delineates clearly the effect on the optimal solution of prudent behaviour (see remark 3). In section 4 we show the optimality of this approach. The infinite horizion case is clarified in section 5. The implications of our theory for time-series regressions are derived in section 6. There we show that Hall's (op.cit.) specification has very restrictive consequences for the consumption process. Implications which make us doubtful about its empirical relevance. Finally, section 7 is devoted to empirical analysis and testing on our Swiss consumption data.

2 Dynamic consumption without time-additive separable aggregate utility

In a standard dynamic optimization model the criterion to be optimized is typically a sum/integral of some additively separable utility. When the one-period (immediate) utilities are quadratic the analysis becomes particularly tractable. But also other specifications are manageable. Zeldes (1989) supplemented this approach by replacing the one-period quadratic utilities with one-period CRRA (constant relative risk aversion) utilities, and showed that this implied several modifications of the previous certainty equivalent results.

However, there is no strong reason beyond analytical convenience to assume time-additive separability. Recent work on dynamic demand systemes and labour supply suggest that this assumption may not be tenable. In dynamic behaviour under uncertainty an important consequence of this assumption is that it links up the measure of relative risk aversion and the elasticity of intertemporal substitution (see Blanchard and Fischer (1989) p. 40). In a dynamic stochastic analysis intertemporal substitution and risk aversion are distinct aspects of consumers' preference we should like to separate. But truely, dynamic stochastic optimization without additive separability of the criterion function is, generally, difficult to analyse. However, it turns out that this can be done in important special cases. One such case will be studied in the present paper. We shall relax the assumption of additive separability, and use as our criterion the expected value of:

$$U = -\exp\Big\{\theta \sum_{j=0}^{h} \beta^j \frac{(c_j - c_j^*)^2}{2}\Big\}, \qquad 0 \le c_j \le c_j^* \tag{2.1}$$

This criterion forms the basis for our extension of Hall's analysis. In this respect it has properties which we should like such a generalization to have.

Firstly, consider the function:

$$g(c_0, c_1, \ldots, c_n; \theta) = \frac{1 - \exp\{\theta U(c_0, c_1, \ldots, c_n)\}}{\theta}, \qquad \theta > 0 \tag{2.2}$$

We observe that $\lim_{\theta \to 0} g(c_0, c_1, \ldots, c_n; \theta) = -U(c_0, c_1, \ldots, c_n)$. Hence, if we put

$$U(c_0, c_1, \ldots, c_n) = \sum_{j=0}^{h} \beta^j \frac{(c_j - c_j^*)^2}{2},$$

it is obvious that Hall's criterion will emerge as a limit case by letting the parameter θ tend to zero in (2.2). Furthermore, maximizing the expected value of $g(c_0, c_1, \ldots, c_n; \theta)$ is equivalent to maximizing the expectation of $U(\cdot)$ given by (2.1). Therefore, all results attained by Hall (op.cit.) will come out as particular cases in our analysis.

Secondly, the criterion can also be motivated within the general framework developed by Koopmans et.al. (1964) for representing aggregate utility. Under certain conditions they showed that aggregate utility can be expressed by a recurrent relation which they write in the form:

$$U_t(c_t, c_{t+1}, \ldots) = V(u(c_t); U_{t+1}(c_{t+1}, c_{t+2}, \ldots)), \qquad \forall t \tag{2.3}$$

where the aggregator V is increasing in its two arguments u and U_{t+1}. Equation (2.3) says that at any time t the aggregate utility U_t is a function of the immediate utility u_t and the prospective aggregate utility U_{t+1}. Note that if the aggregator V is linear this recursion will imply that the aggregate utility is additively separable. In order to show that (2.1) can be generated by a backward recursion of this form, we initiate the iteration by putting:

$$U_h(c_h) = -\exp\Big\{\theta \frac{(c_h - c_h^*)^2}{2}\Big\} \tag{2.4}$$

$$D_h(U_h) = -(-U_h)^\beta \tag{2.5}$$

Since, $u(c_t) = -\exp\{\theta(c_t - c_t^*)^2/2\}$ we attain:

$$U_{h-1}(c_{h-1}, c_h) = -\Big(-\exp\Big\{\theta \frac{(c_{h-1} - c_{h-1}^*)^2}{2}\Big\} D_h(U_h)\Big) = V(u_{h-1}, U_h) \tag{2.6}$$

Then, putting $D_{h-1} = -(-U_{h-1})^{\beta}$ and continuing this backward iteration we will finally attain (2.1). We also note that in this case the aggreator $V(\cdot)$ is increasing and linear in the first variable (u) and increasing and concave in the second argument (U) when $0 < \beta < 1$.

Thirdly, under uncertainty the aggregate utility does a double duty. Not only do we wish that it should reflect the consumer's attitude to risk, but also that it should indicate the degree of substitution between consumption in different time periods. A serious objection to an additively separable aggregate utility is that these two attitudes are closely related (see Blanchard and Fischer (1989) ch. 6). The non-separability of aggregate utility cuts this linkage.

Also, in evaluating temporal prospects under uncertainty Kreps and Porteus (1978) have shown a representation theorem (th. 1, p. 192) which is recursive and in spirit similar to those obtained by Koopmans et. al. (op.cit.) for deterministic prospects. Hence, by similar arguments we can embed our aggregate utility into the Kreps-Porteus framework.

Finally, we have to interpret the parameters β and θ appearing in (2.1). In order to get better insight into β we consider the infinite horizon case. Then the recursion (2.3) reads:

$$U(\tilde{c}_t) = V(u(c_t), U(\tilde{c}_{t+1})) = u(-U)^{\beta} \tag{2.7}$$

where $\tilde{c}_t$ and $\tilde{c}_{t+1}$ denote the infinite vectors $\tilde{c}_t = (c_t, c_{t+1}, \ldots)$, $\tilde{c}_{t+1} = (c_{t+1}, c_{t+2}, \ldots)$. Koopmans et. al. (op.cit. p. 97) have shown that the quantity (in their notation):

$$\left(\frac{\partial V(u, U)}{\partial U}\right)_{u=W^{-1}(U)}$$

can be interpreted as a discount factor. Applying this definition to (2.7) we attain:

$$\beta(U) = \beta = \left(\frac{\partial V(u, U)}{\partial U}\right)_{u=U} \tag{2.8}$$

The fact that this rate is independent of U in our case is perhaps a little surprising, but is very reasonable, indeed. In the deterministic case maximizing (2.1) is equivalent to minimizing the exponent of (2.1), and there β certainly appears as a discount factor. As a matter of fact our result (2.8) shows that Koopmans' definition is very appropriate. The parameter β thus reflects a time perspective in aggregate utility, and is therefore an interesting parameter which should be included in the criterion. This should be noted since criteria similar to (2.1) for some time have been used in the operation research (OR) literature but ignoring the β parameter.

(OR) writers for obvious reasons always interpret θ as a risk parameter (Whittle (1982), (1990)). Although θ reflects attitudes to risk, it is clear from the limiting process above leading to the Hall case that θ also mirrors an effect caused by the non-separability of the aggregate utility. In the stochastic case we shall see that a value of θ different from zero (actually we assume $\theta > 0$) has important implications.

Hall (op.cit.) considered the certainty equivalent (CEQ) consumer. Therefore, the present approach, beeing an extension of Hall, will be labeled "The generalized certainty equivalent consumer" (GCEQ).

By the (GCEQ) consumer we mean the specification:

$$\max_{c_0, c_1, \ldots, c_n} \mathrm{E}\Big(-\exp\Big\{\theta \sum_{\tau=0}^{h} \beta^{\tau} \frac{(c_\tau - c_\tau^*)^2}{2}\Big\}\Big|\Omega_0\Big) \tag{2.9}$$

subject to the budget equation:

$$w_\tau = a w_{\tau-1} + y_\tau - c_\tau \tag{2.10}$$

$$y_\tau = z_\tau + x_\tau \tag{2.11}$$

$$x_\tau = \rho x_{\tau-1} + \varepsilon_\tau, \qquad \varepsilon_\tau \sim \mathrm{N}(0, \sigma^2) \tag{2.12}$$

with the initial value w_{-1} given and terminal condition:

$$w_h = 0 \tag{2.13}$$

We use the following definitions:

$a = (1 + r)$

r – the real rate of interest, supposed constant.

β – a constant discount factor.

θ – a risk/non-separability parameter.

z_τ – a known deterministic function of time.

x_τ – an AR(1) process.

c_t – consumption outlays in period t.

$c_\tau^{(t)}$ – the estimate of c_τ at current time t, $\tau > t$.

h – the consumer's horizon.

$w_\tau^{(t)}$ – the estimate of w_τ formed in period t.

w_t – observable wealth at time t.

x_t – stochastic labour incomes in period t.

$x_\tau^{(t)}$ – the estimate of x_τ, formed in period t.

y_τ – observable incomes in period τ.

Ω_t – the "large" information set containing everything known at time t.

REMARK 1. For the sake of clarity we note. The non-separability of the criterion in (2.9) has consequences for the stochastic, but not for the corresponding deterministic case. In this respect our criterion parallells previous parametrizations of non-separability criteria by Epstein (1988), Farmer (1990), Weil (1990), etc. However, this relaxation of the standard time-additive, separability assumption on aggregate utility is sufficient to separate the risk aversion and intertemporal substitution.

REMARK 2. The aggregate utility (2.1) has the unattractive property that the marginal aggregate utility with respect to any c_t is finite at zero consumption ($c_t = 0$), which means that part of the optimal consumption sequence could be negative. Problem

3 The Stochastic Maximum Principle (SMP) Applied to the (GCEQ) Consumer Problem

In the present section we solve the optimization problem specified by (2.9)–(2.13) by a maximum principle set forth by Whittle ((1982), (1990)). Compared to the usual dynamic programming principle (DDP) which is based on backward induction, the (SMP) goes forward. Therefore, (SMP) simultaneously produce current decisions but also estimates of the future optimal decisions as well as estimates of the future values of the endogenous and exogenous variables not observed at time t. Hence, in economics/econometrics this principle is particularly noteworthy since it generates information economists would like to see, but which tends to be suppressed in the (DP) approach. In addition, (SMP) can be simple to apply to models in which (DP) are quite prohibitive. In fact, that was the case in the present application.

However, to be convinced that the (SMP) provides us with the optimal decisions we shall show that they coincide with the corresponding (DP) decisions.

Before we start the following comments may be helpful. At any operating time t, s will denote the number of periods to go, $s = h - t$. It is intuitive, since (SMP) is a forward going approach, to take account of the constraints on the paths given by the state equations by introducing Lagrange multipliers. We also note that since the random incomes $\{x_\tau\}$ is a Markov process, at any time t the joint density of $\{x(t+1), x(t+2), \ldots, x(h)|x(t)\}$ can be written:

$$f(x(t+1), x(t+2), \dots, x(h)|x(t)) = \prod_{j=1}^{h-t} f(x(t+j)|x(t+j-1)) \tag{3.1}$$

Then we state:

PROPOSITION 3.1. *Let us consider the consumer model given by eqs. (2.9)–(2.13). Then, at time t the current consumption strategy $c_t = c_t^{(t)}$ and the estimates $c_\tau^{(t)}$ of the future optimal consumption outlays derived by the (SMP) are given by:*

$$c_t = c_t^* + \frac{(\beta a)^s \left\{a^{s+1} w_{h-(s+1)} + M(s) x_t + \sum_{\tau=0}^{s} a^\tau (z_{h-\tau} - c_{h-\tau}^*)\right\}}{D_s - \beta\gamma G_s} \tag{3.2}$$

$$c_\tau^{(t)} = c_\tau^* + \frac{(\beta a)^{h-\tau} \left\{a^{s+1} w_{h-(s+1)} + M(s) x_t + \sum_{\tau=0}^{s} a^\tau (z_{h-\tau} - c_{h-\tau}^*)\right\}}{D_s - \beta\gamma G_s} \tag{3.3}$$

where:

$$D_s := \sum_{\tau=0}^{s} (\beta a^2)^\tau = \frac{1 - (\beta a^2)^{s+1}}{1 - \beta a^2} \tag{3.4}$$

$$G_s := H_1(s) - H_2(s) \tag{3.5}$$

$$H_1(s) := \frac{1 - (\beta a^2)^s}{(1 - \beta a \rho)(1 - \beta a^2)(1 - a^{-1}\rho)} - \frac{(\beta a^2)^s (a^{-1}\rho)(1 - (a^{-1}\rho)^s)}{(1 - \beta a \rho)(1 - a^{-1}\rho)^2} \tag{3.6}$$

$$H_2(s) := \frac{(a^{-1}\rho)(1 - (\beta a \rho)^s)}{(1 - a^{-1}\rho)(1 - \beta\rho^2)(1 - \beta a \rho)} - \frac{(\beta a \rho)^s (a^{-1}\rho)^2 (1 - (a^{-1}\rho)^s)}{(1 - \beta\rho^2)(1 - a^{-1}\rho)^2} \tag{3.7}$$

and the sequence $\{M(j)\}$, $j = 1, 2, \dots$ is given by the recursion

$$M(j) = a^j + M(j-1)\rho \tag{3.8}$$

with initial value $M(0) = 1$.

PROOF. Considering the exponential criterion (2.9) at time t, and taking account of the future constraints on the budget equation (2.10) by Lagrangian multipliers, we have to calculate the expectation of an exponential with exponent given by θL_t where:

$$L_t = \sum_{\tau=t}^{h} \beta^{\tau-t} \frac{(c_\tau - c_\tau^*)^2}{2} - \sum_{\tau=t}^{h} \beta^{\tau-t} \lambda_\tau (w_\tau - a w_{\tau-1} - y_\tau + c_\tau) \tag{3.9}$$

Since $x(t)$ is known at time t and the conditional distribution of $x(t+j)$ given $x(t+j-1)$ is normal $\mathrm{N}(\rho x(t+j-1), \sigma^2)$, it follows from (3.1) that we have to intergrate over the infinite domain an exponential with exponent given by θQ_t where:

$$Q_t = \sum_{\tau=t}^{h} \beta^{\tau-t} \frac{(c_\tau - c_\tau^*)^2}{2} - \sum_{\tau=t}^{h} \beta^{\tau-t} \lambda_\tau (w_\tau - a w_{\tau-1} - z_\tau - x_\tau + c_\tau) - \sum_{\tau=t+1}^{h} \beta^{\tau-(t+1)} \frac{(x_\tau - \rho x_{\tau-1})^2}{2\theta\sigma^2} \tag{3.10}$$

Since the exponent Q_t is a negative definite quadratic form in the integrator variables $\{x_{t+1}, x_{t+2}, \ldots, x_h\}$, this integrad can be calculated by maximizing Q_t (3.10) wrt. integrator variables. The exponential with the resultant exponent will, except for an irrelevant constant, give a correct evaluation of the integral (Whittle (1990), lemma 6.11). Hence, we shall extremize the Lagrangian form (3.10) wrt. to $\{c_t, c_{t+1}, \ldots, c_h\}$, $\{w_t, w_{t+1}, \ldots, w_h\}$, $\{x_{t+1}, x_{t+2}, \ldots, x_h\}$ and finally $\{\lambda_t, \lambda_{t+1}, \ldots, \lambda_h\}$. That is, we extremize wrt. all decisions not made and all variables (endogenous and exogenous) variables not observed at time t.

Thus we attain:

$$(c_\tau^{(t)} - c_\tau^*) - \lambda_\tau^{(t)} = 0, \qquad \tau = t, t+1, \ldots, h \tag{3.11}$$

$$\lambda_\tau^{(t)} - \beta a \lambda_{\tau+1}^{(t)} = 0, \qquad \tau = t, t+1, \ldots, h \tag{3.12}$$

$$w_\tau^{(t)} - a w_{\tau-1}^{(t)} - z_\tau - x_\tau^{(t)} + c_\tau^{(t)} = 0, \qquad \tau = t, t+1, \ldots, h \tag{3.13}$$

$$\beta\gamma\lambda_\tau^{(t)} - (x_\tau^{(t)} - \rho x_{\tau-1}^{(t)}) + \beta\rho(x_{\tau+1}^{(t)} - \rho x_\tau^{(t)}) = 0, \qquad \tau = t+1, \ldots, h-1 \tag{3.14}$$

$$\gamma := \theta\sigma^2 \tag{3.15}$$

The terminal conditions are:

$$\beta\gamma\lambda_h^{(t)} - (x_h^{(t)} - \rho x_{h-1}^{(t)}) = 0 \tag{3.16}$$

$$w_h^{(t)} = 0 \tag{3.17}$$

In solving these equations, eq. (3.12) implies, immediately:

$$\lambda_\tau^{(t)} = K(\beta a)^{-\tau}, \qquad \tau = t, t+1, \ldots, h \tag{3.18}$$

where K is some constant.

Solving the predictor eq. (3.14) for the future random incomes, the two constants appearing in the general solution are determined by the initial condition $x_t^{(t)} = x_t$ and the determinal condition (3.16). Thus we attain:

$$x_\tau^{(t)} = \rho^{\tau-t} x_t + \frac{\beta\gamma K}{1 - a^{-1}\rho} \left((\beta a)^{-\tau} \frac{1 - (\beta a \rho)^{\tau-t}}{1 - \beta a \rho} - \frac{1 - (\beta\rho^2)^{\tau-t}}{1 - \beta\rho^2} (a^{-1}\rho)^{h+1} (\beta\rho)^{-\tau} \right) \tag{3.19}$$

Combining eqs. (3.13) and the terminal condition (3.17) we attain ($s = h - t$):

$$a^{s+1} w_{h-(s+1)} + \sum_{\tau=t}^{h} a^{h-\tau}(z_\tau + x_\tau^{(t)} - c_\tau^{(t)}) = 0 \tag{3.20}$$

Combining (3.20) with eq. (3.19) (to eliminate $x_\tau^{(t)}$), eq. (3.11) (to eliminate $c_\tau^{(t)}$), and finally (3.18) (to eliminate $\lambda_\tau^{(t)}$), we, eventually, determine K by:

$$K = \frac{(\beta a)^h \left(a^{s+1} w_{h-(s+1)} + M(s)x_t + \sum_{\tau=t}^{h} a^{h-\tau}(z_\tau - c_\tau^*)\right)}{D_s - \beta\gamma G_s} \tag{3.21}$$

These calculations show that the quantities D_s and G_s are given by eqs. (3.4)–(3.7). Similarly we deduce that the sequence $M(\tau)$ obeys the recursion (3.8). Finally, combining eqs. (3.21), (3.18) and (3.11) we attain (3.2) and (3.3). ■

REMARK 3. Now we should note an interesting implication of the non-separability of the aggregate utility ($\theta > 0$). The estimates at time t, $x_\tau^{(t)}$, of the future incomes x_τ have been risk adjusted. We observe from (3.19) that:

$$x_\tau^{(t)} = \mathrm{E}\{x_\tau | \Omega_t\} + \text{the risk-adjusted term} \tag{3.22}$$

Hence, these predictions deviate from the conditional expectations $\mathrm{E}\{x_\tau|\Omega_t\}$. In the (CEQ) case, (the Hall case), $\theta = 0$ implying $\gamma = 0$ (see 3.15), then we have the standard situation $x_\tau^{(t)} = \mathrm{E}\{x_\tau|\Omega_t\}$.

The following results, distributed over a couple of lemmas for convenience, will be helpful in the sequel.

LEMMA 3.1. *Let us consider the recursion (3.8), i.e. the difference equation $M(j) = a^j + M(j-1)\rho$ with the initial condition $M(0) = 1$. The solution of this equation is given by:*

$$M(j) = \frac{a^j - (a^{-1}\rho)\rho^j}{1 - (a^{-1}\rho)} \tag{3.23}$$

PROOF. Apply the standard machinery. ■

LEMMA 3.2. *The quadratic form Q_t (3.10) has at the optimum the evaluation:*

$$\hat{Q}_t = \frac{K^2(\beta a)^{-2h}\beta^s}{2}\left(\sum_{j=0}^{s}(\beta a^2)^j - \gamma\beta\frac{\sum_{j=0}^{s-1}\beta^j(a^j(1-(a^{-1}\rho)^{j+1}))^2}{(1-(a^{-1}\rho))^2}\right) \tag{3.24}$$

or

$$\hat{Q}_t = \frac{(D_s - \beta\gamma B_{s-1})\beta^s}{2(D_s - \beta\gamma G_s)^2}\left(a^{s+1} w_{h-(s+1)} + M(s)x_t + \sum_{\tau=t}^{h} a^{h-j}(z_\tau - c_\tau^*)\right)^2 \tag{3.25}$$

PROOF. The expressions (3.24) and (3.25) follow by direct substitution. By (3.11) we have $(c_\tau^{(t)} - c_\tau^*)^2 = (\lambda_\tau^{(t)})^2$. The Lagrangian terms of (3.10) vanish because of (3.13). Finally, using (3.19) we attain:

$$(x_\tau^{(t)} - x_{\tau-1}^{(t)})^2 = \left(\frac{K\beta\gamma(\beta a)^{-\tau}(1-(a^{-1}\rho)^{h-1+\tau})}{1-a^{-1}\rho}\right)^2, \qquad \tau = t+1, t+2, \dots, h \tag{3.26}$$

which is used in evaluating the last sum of (3.10). By using (3.21) to substitute for K^2 in (3.24) and using the solution (3.23) for $M(j)$, we recognise the term $(D_s - \beta\gamma B_{s-1})$ (see definitions (3.4) and (4.3)). Then (3.25) follows from (3.24). ■

4 On the Optimality of the present (SMP)

The above application of the (SMP) involves two technical details which are not quite intuitive. Firstly, (3.9) and (3.10) show that the constraints given by the budget equations appear in the quadratic form Q_t, i.e. in the exponent of the exponential. Secondly, (3.10) indicates that our specification assumes a discounting of the future errors ε_τ, $\tau > t$.

Hence, it is not obvious that the (SMP) at any time t, will provide the optimal consumption strategy we seek. However, it is, and to show this fact we shall demonstrate that the (SMP) and the (DDP) generate identical strategies. Since fulfilment of the optimal equation of dynamic programming (DP) is necessary and suficient for optimality in this case, we can then conclude that the (SMP) solution (3.2) is optimal.

Then we are ready to attack the specification (2.9)–(2.13) by (DP) arguments.

PROPOSITION 4.1. *Let us consider the consumer model given by eqs. (2.9)–(2.13). Then the optimal consumption strategies at time $t = h-1$ and generally at time $t = h-s$ are given by:*

$$c_{h-1} = \frac{(1-\beta\gamma)c_{h-1}^* + \beta a(a^2 w_{h-2} + (a+\rho)x_{h-1} + az_{h-1} + z_h - c_h^*)}{1-\beta a^2 - \beta\gamma} \tag{4.1}$$

$$c_t = c_t^* + \frac{(\beta a)^s(a^{s+1}w_{h-(s+1)} + M(s)x_t + \sum_{j=0}^h a^j(z_{h-j} - c_{h-j}^*))}{D_s - \beta\gamma B_{s-1}} \tag{4.2}$$

where in (4.2) D_s is the partial sum defined by (3.4) and B_{s-1} is defined by:

$$B_{s-1} := \sum_{\tau=0}^{s-1} \beta^\tau M^2(\tau) \tag{4.3}$$

$M(\tau)$ is the sequence given by (3.8).

PROOF. At the horizon point h all uncertainty is resolved and the final decision c_h is determined by the terminal condition $w_h = 0$. Then we proceed by the familiar backward induction pattern of (DP). Although this procedure is tedious in the present application, it is well known, and therefore omitted. ■

In order to show that the (SMP) strategy (3.2) is optimal, we have to demonstrate that (3.2) is equal to (4.2) for any time t. By comparing the two equations we observe that we have to show that the partial sum G_s defined by (3.5) is equal to the partial sum B_{s-1} defined by (4.3) for an arbitrary value of s. For clarity we state this as lemma (4.1).

LEMMA 4.1. *The partial sum* $G_s = H_1(s) - H_2(s)$ *defined by (3.5)-(3.7) is identical to* B_{s-1}*, defined by (4.3), for any value of* s.

PROOF. A direct approach is possible, but the following inductive argument is simpler. An easy calculation shows that $G_1 = B_0$. Suppose that $G_{s-1} = B_{s-2}$ then $G_s = B_{s-1}$ if and only if the partial sums G_{s-1} and B_{s-2} are added identical increments ΔG_s and ΔB_{s-1} by proceeding from $s-1$ to s. From (3.6) we deduce that H_1 is to be added $(\beta a^2)^{s-1}(1-(a^{-1}\rho)^s)/(1-(a^{-1}\rho))^2$ by this transition. Similarly, we find from (3.7) that H_2 gets the increment $(\beta a\rho)^{s-1}(a^{-1}\rho)(1-(a^{-1}\rho)^s)/(1-a^{-1}\rho)^2$.

Hence, we attain from (3.5):

$$\begin{aligned}\Delta G_s = \Delta H_1 - \Delta H_2 &= \frac{\beta^{s-1}(a^{2(s-1)} - a^{s-2}\rho^s)(1-(a^{-1}\rho)^s)}{(1-a^{-1}\rho)^2} \\ &= \frac{\beta^{s-1}(a^{s-1} - (a^{-1}\rho)\rho^{s-1})^2}{(1-a^{-1}\rho)^2}\end{aligned} \tag{4.4}$$

From (4.3) we observe that:

$$\Delta B_{s-1} = \beta^{s-1}M^2(s-1) = \frac{\beta^{s-1}(a^{s-1} - (a^{-1}\rho)\rho^{s-1})^2}{(1-a^{-1}\rho)^2} \tag{4.5}$$

where the last equality follows from (3.23). Eqs. (4.4) and (4.5) shows that $\Delta G_s = \Delta B_{s-1}$ and by the induction hypothesis we can, therefore, conclude that $G_s = B_{s-1}$. ■

From the proof above the next proposition follows:

PROPOSITION 4.2. *Let us consider the consumer model given by eqs. (2.9)-(2.13). Then the (SMP) consumption strategy (3.2) is identical to the (DP) strategy (4.2). Since this holds for any operating period* t*, the (SMP) generates the optimal consumption strategy.*

REMARK 4. Now, we could ask: "Although, the (SMP) is shown to determine the current consumption decisions optimally, isn't the (SMP) an awkward detour"? The answer is no! Compared to the (DPP) the (SMP) gives considerably more. It provides estimates of what the optimal decisions will be in the future ($\tau > t$), as well as estimates of the future values of the endogenous and exogenous variables. This is information which is suppressed by the (DPP). Thus, the (SMP) makes explicit the idea of a provisional forward plan. Although we know that the plan will be revised as later observations become available, this conception of a continually revised plan corresponds very well to our intuition and actual economic practice. Also as noted above the (SMP), being, as we have shown, a study of a Lagrangian form, it will often offer tractable solutions in models in which it will be almost impossible to work out the solutions by the (DPP).

5 The Infinite Horizon Case

End-conditions will often blur results we attain in models with finite horizon. However, under appropriate specifications these effects will fade away and eventually vanish if we let the horizon h tend to infinity. Thus, infinite horizon results will often be more transparent and simpler to analyse.

If, at operating time t, we let h or s tend to infinity in (3.2) or (4.2) we attain:

$$c_t = c_t^* + \frac{(1-\beta a^2)(1-a^{-1}\rho)^2}{\beta\gamma - \beta a^2(1-a^{-1}\rho)^2}\Big(aw_{t-1} + \frac{x_t}{1-a^{-1}\rho} + f_t\Big) \tag{5.1}$$

Obviously this is the stationary infinite horizon consumption strategy. However, to be on the safe side we should check that it is just this strategy which satisfies the equilibrium form of the optimality equation of (DP).

We note by lemma 4.1 that $G_s = B_{s-1}$ for all s. Then the quadratic form (3.25) reduces and can be slightly rewritten:

$$\hat{Q}_t(s) = \frac{(\beta a^2)^s}{2(D_s - \beta\gamma B_{s-1})}\Big(aw_{h-(s+1)} + a^{-s}M(s)x_t + a^{-s}\Big(\sum_{\tau=t}^{h} a^{h-\tau}(z_\tau - c_\tau^*)\Big)\Big)^2 \tag{5.2}$$

We shall define:

$$f_t := \lim_{s\to\infty}\sum_{j=0}^{s} a^{-j}(z_{t+j} - c_{t+j}^*) < \infty \tag{5.3}$$

If we let h or s tend to infinity in (5.2) and use definitions (3.4), (4.3) and (3.23) of D_s, B_{s-1} and $M(s)$ we attain:

$$\hat{Q}_t = \lim_{s\to\infty}\hat{Q}_t(s) = \frac{(1-\beta a^2)(1-a^{-1}\rho)^2}{2(\beta\gamma - \beta a^2(1-a^{-1}\rho)^2)}\Big(aw_{t-1} + \frac{x_t}{(1-a^{-1}\rho)} + f_t\Big)^2 \tag{5.4}$$

Hence, if all decisions are taken optimally from time t onwards, then apart from an inessential positive constant, the expected utility at time t is given by:

$$U_t = -\exp(\theta\hat{Q}_t) \tag{5.5}$$

Finally, we have to show:

PROPOSITION 5.1. *The value function* $\{-\exp(\theta\hat{Q}_t)\}$ *where* $\hat{Q}_t$ *is given by (5.4) satisfies the equation of optimality:*

$$\{-\exp(\theta\hat{Q}_t)\} = \max_{c_t} \mathrm{E}\left\{-\exp\left(\theta\left[\frac{(c_t-c_t^*)}{2}+\beta\hat{Q}_{t+1}\right]\right)\right\} \tag{5.6}$$

and the optimal strategy c_t *is identical to (5.1).*

PROOF. Calculating the expectation of (5.6) wrt. the distribution of x_{t+1}, ($x_{t+1} \sim \mathrm{N}(\rho x_t, \sigma^2)$) the righthand side of (5.6) will become an exponential with exponent given by:

$$\theta\left\{\frac{(c_t-c_t^*)^2}{2}+\frac{\beta(1-\beta a^2)}{2(\beta a^2(\beta\gamma-(1-a^{-1}\rho)^2))}\left(\rho x_t+(1-a^{-1}\rho)(aw_t+f_{t+1})\right)^2\right\} \tag{5.7}$$

Minimizing (5.7) wrt. c_t gives after some rearrangement:

$$c_t = c_t^* + \frac{(1-\beta a^2)(1-a^{-1}\rho)^2}{\beta\gamma-\beta a^2(1-a^{-1}\rho)^2}\left\{aw_{t-1}+\frac{x_t}{1-a^{-1}\rho}+f_t\right\} \tag{5.8}$$

Finally, eliminating w_t in (5.7) by using the budget equation, and substituting (5.8) for c_t in the subsequent expression we attain

$$\frac{(1-\beta a^2)(1-a^{-1}\rho)^2}{2(\beta\gamma-\beta a^2(1-a^{-1}\rho)^2)}\left\{aw_{t-1}+z_t-c_t^*+a^{-1}f_{t+1}+\frac{x_t}{1-a^{-1}\rho}\right\}^2 \tag{5.9}$$

From the definition (5.3) of f_t we observe that (5.9) is equal to (5.4). We observe that (5.8) is identical to the limit strategy (5.1). Then we are done. ■

REMARK 5. (Parameter restrictions). In calculating the expectation of the right-hand side of (5.6) wrt. the distribution of x_{t+1}, it is obvious that the coefficient of the quadratic term of the integrator variable (x_{t+1}^2) has to be negative otherwise this expectation will not exist. That is, we must have:

$$\frac{\beta a^2((1-a^{-1}\rho)^2-\beta\gamma)}{2\gamma(\beta\gamma-\beta a^2(1-a^{-1}\rho)^2)} < 0 \tag{5.10}$$

This means that the numerator and the denominator of this fraction must have opposite signs.

If we make the reasonable assumption that there is a positive relationship between consumption c_t and observable wealth w_{t-1}, then we observe from (5.8) and (5.10) that the magnitude of $1-\beta a^2$ will determine which case we have. The denominator of (5.10) will have the same sign as $1-\beta a^2$. In order that (5.8) shall be a sensible consumption function also in the Hall case ($\theta = 0$), (5.8) shows that we must have $\beta a^2 > 1$. We should also note that $\beta\gamma - \beta a^2(1-a^{-1}\rho)^2$ has to be bounded away from zero which put restriction on γ or θ ($\gamma = \theta\sigma^2$).

6 Implications for Time-Series Regressions

The optimal consumption strategies deduced in sections 3 and 5 form the basis for the empirical analysis to come. The strategies (3.2) and (5.1) or (5.8) provide regression equations which can be directly applied to empirical data.

The finite horizon case ($h < \infty$)

Consumers do not live for ever, so it is important to clarify whether finite horizon effects are present in the deduced regression equations.

At operating time t the consumer has $s = h - t$ periods to go. According to (3.2) the optimal strategy is given by

$$c_t = c_t^* + \frac{(\beta a)^s \left(a^{s+1} w_{h-(s+1)} + M(s)x_t + \sum_{\tau=0}^{s} a^\tau (z_{h-\tau} - c_{h-\tau}^*)\right)}{D_s - \beta\gamma G_s} \tag{6.1}$$

Correspondingly, at time $t+1$ we have:

$$c_{t+1} = c_{t+1}^* + \frac{(\beta a)^{s-1}(a^s w_{h-s} + M(s-1)x_{t+1} + \sum_{\tau=0}^{s-1} a^\tau (z_{h-\tau} - c_{h-\tau}^*))}{D_{s-1} - \beta\gamma G_{s-1}} \tag{6.2}$$

Then we use the budget equation $w_t = aw_{t-1} + z_t + x_t - c_t$ and the process equation $x_t = \rho x_{t-1} + \varepsilon_t$ to substitute for w_{h-s} and x_{t+1} in (6.2). Finally, we use the difference equation (3.8), $M(j) = a^j + M(j-1)\rho$, and eventually (6.1) to eliminate the term $a^{s+1}w_{h-(s+1)}$. Thus, we attain:

$$c_{t+1} = c_{t+1}^* + \frac{(\beta a)^{s-1}\left((\beta a)^{-s}(D_{s-1} - \beta\gamma G_s)(c_t - c_t^*) + M(s-1)\varepsilon_{t+1}\right)}{D_{s-1} - \beta\gamma G_{s-1}} \tag{6.3}$$

This is the general finite horizon regression function.

Now, it is instructive to write out the results for the Hall case, i.e. $\theta = 0$ and constant bliss levels $c^* = c_\tau^*, \forall \tau$. Then (6.3) reduces to:

$$c_{t+1} = \frac{(\beta a - 1)c^*}{\beta a} + \frac{1}{\beta a}c_t + \frac{(\beta a)^{s-1}M(s-1)}{D_{s-1}}\varepsilon_{t+1} \tag{6.4}$$

But what happens if the aggregate utility is not additively separable, i.e. if $\theta > 0$. In order to gain further insight in this case we assume constant bliss levels and $\beta a = 1$: Then (6.3) reduces to:

$$c_{t+1} = \pi_0(s-1)c^* + \pi_1(s-1)c_t + \pi_2(s-1)\varepsilon_{t+1} \tag{6.5}$$

where:

$$\pi_0(s-1) = \frac{\gamma\beta^s M^2(s-1)}{D_{s-1} - \beta\gamma G_{s-1}} \tag{6.6}$$

$$\pi_1(s-1) = \frac{D_{s-1} - \beta\gamma G_s}{D_{s-1} - \beta\gamma G_{s-1}} \tag{6.7}$$

$$\pi_2(s-1) = \frac{M(s-1)}{D_{s-1} - \beta\gamma G_{s-1}} \tag{6.8}$$

From (6.6)–(6.8) we attain the limits:

$$\pi_0 = \lim_{s\to\infty} \pi_0(s-1) = \frac{(1-\beta)\beta\gamma}{(1-\beta\rho)^2 - \gamma\beta^2} \tag{6.9}$$

$$\pi_1 = \lim_{s\to\infty} \pi_1(s-1) = \frac{(1-\beta\rho^2) - \beta\gamma}{(1-\beta\rho)^2 - \gamma\beta^2} \tag{6.10}$$

$$\pi_2 = \lim_{s\to\infty} \pi_2(s-1) = \frac{(1-\beta)(1-\beta\rho)}{(1-\beta\rho)^2 - \gamma\beta^2} \tag{6.11}$$

Using the definition of B_s and lemma (4.1) it follows that:

$$G_s = G_{s-1} + \beta^{s-1}M^2(s-1), \qquad \text{for} \quad s = 1, 2, \ldots \tag{6.12}$$

Because of this identity we observe from (6.6) and (6.7) that $0 < \pi_1(s-1) < 1$ and:

$$\pi_0(s-1) + \pi_1(s-1) = 1, \qquad \text{for} \quad s = 1, 2, \ldots \tag{6.13}$$

Evidently we also have (see (6.9) and (6.10)):

$$\pi_0 + \pi_1 = 1 \tag{6.14}$$

The limits (6.9)–(6.11) indicate the magnitudes of $\pi_0(s)$, $\pi_1(s)$ and $\pi_2(s)$ for large values of s. From a statistical point of view we should like to know the rates of convergence of these sequences. If these rates are very fast the variations in $\pi_0(s)$, $\pi_1(s)$ and $\pi_2(s)$ can be ignored, since then hopefully, eq. (6.5) is reasonably well approximated by a standard auto-regression with constant regression coefficients and homoscedastic errors. On the other hand if this convergence is slow we have to be very careful with interpreting and drawing conclusion from empirical applications of this model.

In order to get an idea of these convergence rates several simulations under different specifications were carried out using the equations (6.6)–(6.8) and (6.9)–(6.11). We note the following findings from these experiments.

(i) The convergence rates of $\pi_0(s)$, $\pi_1(s)$ are very sensitive to the magnitude of the θ parameter. For small values of θ the experiments indicated fast convergence of $\pi_0(s)$ and $\pi_1(s)$. However, for larger values of θ as many as $s = 40$ or $s = 50$ periods to go were not enough to bring $\pi_0(s)$ and $\pi_1(s)$ towards their limit values given by (6.9) and (6.10). Hence, in using panel data the assumption of constant regression coefficients might be questionable.

(ii) For a given value of θ the convergence rate of $\pi_2(s)$ appeared to be considerably slower compared to those of $\pi_0(s)$ and $\pi_1(s)$. Although we specified parameter values implying limit values of $\pi_2 > 1$, the sequence $\pi_2(s)$ fluctuated, being less than 1 for spans of periods. Hence, in regression studies studing $\mathrm{E}\{c_t|c_{t-1}\}$ heteroscedasticity is likely to be present. Secondly, the experiments indicated that we should be careful in drawing definite conclusions as to the magnitude of the ratio between the error variances in regression studies treating consumption and income data.

At this point it is instructive to sum up the main results for the two cases; *(i)* $\theta = 0$ and *(ii)* $\theta > 0$.

The Hall case ($\theta = 0$)

(i) The conditional expectation of c_{t+1} given all the information at time t $\{F_t\}$ depends solely on c_t. This is the "Euler equation hypothesis".

(ii) From (6.4) we observe that the parameters of the regression equation $\mathrm{E}\{c_{t+1}|c_t\}$ depend only on the time preference parameter β and the interest rate r $(a = 1 + r)$. When $\beta a = 1$ $\mathrm{E}\{c_{t+1}|c_t\}$ reduces to a martingale. This is Hall's martingale hypothesis on the consumption process.

(iii) There is no finite horizon effects on the regression coefficients of the regression equation $\mathrm{E}\{c_{t+1}|c_t\}$. All finite horizon effects on $\mathrm{E}\{c_{t+1}|c_t\}$ are contained in the random disturbances. A fact which makes this term heteroscedastic.

The generalized Hall case ($\theta > 0$)

*(i)** The "Euler equation hypothesis" is satisfied.

*(ii)** The eqs. (6.6)–(6.7) in the finite horizon case and (6.16) in the infinite horizon case show that the regression coefficients of $\mathrm{E}\{c_{t+1}|c_t\}$ in addition to β and r, also depend on the risk parameter θ and the parameters (ρ, σ^2) of the income process. Thus, a positive risk parameter θ implies a coupling of the consumption and the income process. This we shall call the "Risk sensitivity hypothesis".

*(iii)** The eqs. (6.6)–(6.8) show that the regression coefficients $\pi_0(\cdot)$, $\pi_1(\cdot)$ and the scaling factor $\pi_2(\cdot)$ depend on the number of periods (s) to go.

Thus, the "Euler equation hypothesis" is implied in both cases. Knowing that this hypothesis is a direct consequence of the assumption that the consumers are rational, maximizing agents, the agreement of the two cases on this point is obvious. The income and wealth variables are simply extremized out of the regression model in both models.

However, the two cases differ markedly as regards the other two properties. The implications of Hall's model stated in *(ii)* and *(iii)* are in our opinion counterintuitive and not credible. For example one main finding of Friedman (1957) was that income variability affected the consumers' regression parameters. Friedman (op.cit.) observed that farm families with a large income variability had on the average a lower propensity to consume than non-farm families with less income variability. We observe from *(ii)** that this important property of the conumption process, is compatible with our generalized Hall case ($\theta > 0$).

The implications of the generalized Hall case stated in *(iii)** says that (6.3) is a regression equation with time-varying coefficients and heteroscedastic disturbances. Hence, in contrast to the Hall case, the case ($\theta > 0$) permits consumers to vary the parameters of the regression equation $\mathrm{E}\{c_{t+1}|c_t\}$ over their life-cycle. In particular, it allows for the fact that young and elderly people may have different consumption behaviour. It appears reasonable and intuitive that age should have an effect on consumption behaviour, and the question is often discussed in the life-cycle consumer theory (see Mayer (1972), ch. 5). That our model allows for this effect, emphasises once again that the case $\theta > 0$ is an appropriate extension of Hall (op.cit.).

The empirical application to follow are based upon a number of relatively short time-series, and we are, therefore, unable to control for the aging tendency of the regression coefficients. Our empirical study will be based on the limiting infinite horizon case.

The infinite horizon case

When the horizon h tends to infinity, we deduce from (6.3) that the optimal consumption strategy constitutes a stochastic process given by:

$$c_{t+1} = c^*_{t+1} + \frac{a((1-a^{-1}\rho)^2 - \beta\gamma)}{\beta a^2(1-a^{-1}\rho)^2 - \beta\gamma}(c_t - c^*_t) + \frac{(\beta a^2 - 1)(1-a^{-1}\rho)}{\beta a^2(1-a^{-1}\rho)^2 - \beta\gamma}\varepsilon_{t+1} \tag{6.15}$$

This looks like a simple regression of $(c_{t+1} - c^*_{t+1})$ on $(c_t - c^*_t)$. However, the bliss levels c^*_t are unobservables, a fact which has to be dealt with. In some applications, but certainly not always, it is reasonable to suppose that the bliss levels are constant, i.e. $c^*_t = c^*$, $\forall t$

Then (6.15) becomes:

$$c_{t+1} = \frac{(a(\beta a-1)(1-a^{-1}\rho)^2+\beta\gamma(a-1))}{\beta a^2(1-a^{-1}\rho)^2-\beta\gamma}c^* \\ + \frac{a((1-a^{-1}\rho)^2-\beta\gamma)}{\beta a^2(1-a^{-1}\rho)^2-\beta\gamma}c_t + \frac{(\beta a^2-1)(1-a^{-1}\rho)}{\beta a^2(1-a^{-1}\rho)^2-\beta\gamma}\varepsilon_{t+1} \tag{6.16}$$

which for tidiness can be written:

$$c_{t+1} = \kappa + \alpha_1 c_t + \alpha_2 \varepsilon_{t+1}, \qquad \kappa = \alpha_0 c^* \tag{6.17}$$

The definitions of $\kappa\,(=\alpha_0 c^*)$, α_1 and α_2 are all obvious from (6.16).

We observe that the structural form of (6.16) supplemented by the stochastic income process (2.13) contain 6 structural parameters $\{a, \beta, c^*, \gamma, \rho, \sigma^2\}$. The question is if all these parameters can be determined from the joint distribution of the observable variables. The parameters ρ and σ^2 can be determined from the bivariate distribution of (x_t, x_{t-1}), and the parameter set $\{\pi, \alpha_1, \alpha_2\}$ from the distriution of (c_t, c_{t-1}). In all we are able to identify 5 parameters, which means that not all structural parameters are identifiable. For this to be the case we need an additional independent restriction on the parameters. For instance, the restriction $\beta a = 1$ will do, since we calculate quite easily:

$$c^* = \frac{\kappa}{1-\alpha_1} \tag{6.18}$$

$$\gamma = \frac{(1-\alpha_1)(\alpha_1-\rho)}{(1-\alpha_2)\alpha_2} \tag{6.19}$$

$$\beta = \frac{1-\alpha_2}{\alpha_1-\alpha_2\rho} \tag{6.20}$$

But the assumption $\beta a = 1$ is quite arbitrary and appears to be too restrictive. Hence, we will treat a and β as free parameters. Note that in this case α_1 is not restricted to the interval $(0, 1)$. Although some of the structural parameters are not identifiable, we observe immediately:

$$\alpha_0 + \alpha_1 = 1 \tag{6.21}$$

$$c^* = \frac{\kappa}{1-\alpha_1} \tag{6.22}$$

Finally, on a panel of Swiss family data we shall in the next section test the main hypothesis:

The "Martingale hypothesis":

$$\mathrm{E}\{c_{t+1}|c_t\} = c_t \tag{6.23}$$

The "Euler equation hypothesis":

$$\mathrm{E}\{c_{t+1}|F_t\} = \mathrm{E}\{c_{t+1}|c_t\} = \kappa + \alpha_1 c_t \tag{6.24}$$

The "Euler equation hypothesis":

$$\text{The nullhypothesis} \quad H_0 : \theta = 0 \quad \text{against} \quad H_1 : \theta > 0 \tag{6.25}$$

7 The empirical Investigations

Our statistical tests are based on a sample of 34 Swiss families which over a period of 10 years kept detailed accounts of their consumption outlays, incomes, taxes, etc. (see appendix A). We have made the following considerations. The 34 families are chosen at random from a large number of families of a certain variety. In order to allow for individual family differences, we consider each family's regression line to be a random variable. In order to characterize the whole population of families it is natural to look at the distribution of these lines. The expected values of the stochastic regression coefficients, their variances and covariances are then the parametres of our interest.

The consumption of familiy (i) in period $t+1$ is specified by:

$$c_{it+1} = \kappa_i + \alpha_{1i} c_{it} + \delta_{it+1}, \qquad i = 1, 2, \ldots, 34, \quad t = 1, 2, \ldots, 10 \tag{7.1}$$

We make the following assumptions:

$$\mathrm{E}\{c_{it+1}|c_{it}\} = \kappa_i + \alpha_{1i} c_{it} \tag{7.2}$$

The set of variables $(\kappa_1, \alpha_{11}), (\kappa_2, \alpha_{12}), \ldots$, are independently, identically distributed with means and variances/covariances given by:

$$\mathrm{E}\{\kappa_i\} = \kappa, \qquad \mathrm{E}\{\alpha_{1i}\} = \alpha_1 \tag{7.3}$$

$$\mathrm{var}(\kappa_i) = \lambda_{00}, \qquad \mathrm{var}(\alpha_{1i}) = \lambda_{11}, \qquad \mathrm{cov}(\kappa_i, \alpha_{1i}) = \lambda_{01} \tag{7.4}$$

The random disturbances (δ_{it}) are independently, identically,
normally distributed with means zero and variances ψ. (7.5)

Our estimates of κ and α_1 are calculated as ordinary averages of the least-square estimates $\hat{\kappa}_i$ and $\hat{\alpha}_{1i}$. Similarly we obtained the estimates of the variances λ_{00} and λ_{11}, from which we attained estimates of the standard errors $\mathrm{std}(\hat{\kappa})$ and $\mathrm{std}(\hat{\alpha})$. Standard statistical theory then says that $(\hat{\kappa} - \kappa)/\,\mathrm{std}(\hat{\kappa})$ and $(\hat{\alpha}_1 - \alpha_1)/\,\mathrm{std}(\hat{\alpha}_1)$ are approximately, normally distributed $(0, 1)$.

Testing the "Martingale hypothesis"

From (6.23) we observe that this hypothesis is equivalent to the following hypotheses on the population characteristics κ and α_1:

$$H_0 : \text{(a)} \ \ \kappa = 0 \quad \text{and} \quad \text{(b)} \ \ \alpha_1 = 1 \tag{7.6}$$

We reject H_0 if we reject either (a) or (b) of (7.6).

$$\begin{aligned} \hat{\kappa} &= 2048 & \hat{\alpha}_1 &= 0.9112 \\ \text{std}(\hat{\kappa}) &= 655 & \text{std}(\hat{\alpha}_1) &= 0.0482 \end{aligned} \tag{7.7}$$

From what has been said above we conclude that the "Martingale hypothesis" can be rejected straightaway at any sensible significance level.

Testing the "Euler equation hypothesis"

A simple direct test of the hypothesis (6.24) is to run the regression

$$c_{t+1} = \kappa + \alpha_1 c_t + \nu y_t + \delta_{t+1} \tag{7.8}$$

(y_t – current income at time t), and then test the nullhypothesis

$$H_0 : \nu = 0 \quad \text{against} \quad H_1 : \nu \neq 0 \tag{7.9}$$

A version of this procedure has been carried out by many writers. Since we have time-series data for a sample of different families we can apply this test to each family separately. Therefore, our test should be more reliable than the usual ones which are based on aggregate or som kind of pooled data. No doubt, aggregation or pooling of data will usually bring in noise which are difficult to control for in regression studies.

Thus, for each family we regressed c_{t+1} on previous consumption (c_t) and previous incomes (y_t). In all these regressions the (LS) estimates of α_{1i} and ν_i turned out to be very instable, signalling a high degree of collinarity between c_t and y_t. In order to investigate this fact we applied the diagnostic test of collinarity proposed by Belsley et. al. (1980). That is, after having normalized the regressors to unit length giving the matrix P, we calculate the eigenvalues λ_j of the symmetric, non-negative matrix $(P'P)$. Finally, we calculate the proxy $k = \lambda(\max)/\lambda(\min)$, where $\lambda(\max)$ and $\lambda(\min)$ denote the largest and smallest eigenvalues of $(P'P)$. Belsley et. al. (op. cit.) conclude from numerical experiments that values of k in excess of 20 suggest potential problems. On this problem see also Silvey (1969). The following table shows the proxy k values calculated for our sample.

Table 7.1

k-values	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99	100–139	140→
No. of families	6	3	5	7	3	2	2	3	1	2

Hence, this diagnostic test indicates that collinarity between c_t and y_t is a potential problem in all regressions.

It is frequently believed that the life-cycle theories of the consumption function always dissociate current consumption and incomes. This opinion is superficial. Equation (5.8) shows that this is not a necessary implication of the dynamic optimal behaviour. In order to explain the present observed collinarity between c_t and y_t we can argue as follows. Let us suppose that the bliss levels are constant (i.e. $c_t^* = c^* \forall t$). Our theory (5.8) then predicts that c_t is a linear combination of w_{t-1}, x_t and f_t, where $f_t = z_t - c^* + a^{-1}f_{t+1}$ (see (5.3)). It is reasonable to assume that $(1 - a^{-1}\rho)$ is a small number in the interval $(0, 1)$. If the variability of aw_{t-1} and $a^{-1}f_{t+1}$ are moderate which certainly can be the case for many families, the variability of $\{aw_{t+1} + x_t/(1 - a^{-1}\rho) + f_t\}$ is to a large extent accounted for by the variability of x_t and z_t—i.e. by deterministic and stochastic components of observable income (see (2.11)). This means that for many families there can be an approximately linear relation between consumption (c_t) and observable incomes (y_t). For these families collinearity between c_t and y_t will cause a serious problem in the regression (7.8). It is this problem we face in our application of the Swiss data.

As to the test of the hypothesis (7.9) we conclude. Owing to multicollinearity between c_t and $y_t$we are unable to neither reject nor accept the nullhypothesis H_0 of (7.9). The collinearity problem we face can be explained by our theory by using perfectly sensible arguments. Thus there is no reason to reject the "Euler equation hypothesis" on this data-set.

Testing the "Risk sensitivity hypothesis"

As we have noted above, the presence of risk, $\theta > 0$, implies that optimal consumption and observable income are coupled via the regression coefficients. Although, this kind of copuling appears to be very sensible, we should also like to test its strength statistically. Again we shall apply the random coefficient regression model.

Hence, for any familiy (i) we consider the regression model:

$$c_{t+1} = \kappa + \alpha_1 c_t + \delta_{t+1}, \qquad (\kappa = \alpha_0 c^*) \tag{7.10}$$

where the random regression coefficients α_0 and α_1 are given by (see (6.16))

$$\alpha_0 = \frac{(\beta a^2 - a)(1 - a^{-1}\rho)^2 + \beta a(a-1)}{\beta a^2(1 - a^{-1}\rho)^2 - \beta\gamma} \tag{7.11}$$

$$\alpha_1 = \frac{a((1 - a^{-1}\rho)^2 - \beta\gamma)}{\beta a^2(1 - a^{-1}\rho)^2 - \beta\gamma} \tag{7.12}$$

By combining (2.11)–(2.12) we can write the income process $\{y_t\}$:

$$y_{t+1} = (z_{t+1} - \rho z_t) + \rho y_t + \varepsilon_{t+1}, \qquad |\rho| < 1 \tag{7.13}$$

The term $(z_{t+1} - \rho z_t)$ is a deterministic component of observable income. From (7.13) we observe that the variance of $\{y_t\}$ is an increasing function of ρ. That is, the larger ρ the more uncertain (risky) will the incomes $\{y_t\}$ turn out to be.

It is reasonable that uncertainty about the future income process in one way or another should be reflected by the regression coefficients of the consumption process. When $\theta > 0$ we observe from (7.11)–(7.12) that this is, indeed, the case.

But what happens in the Hall caxe ($\theta = 0$)? In this case we observe directly from (7.11) and (7.12) that:

$$\alpha_0 = \frac{\beta a - 1}{\beta a} \tag{7.14}$$

$$\alpha_1 = \frac{1}{\beta a} \tag{7.15}$$

($\gamma = \theta\sigma^2$, so that $\theta = 0$ implies $\gamma = 0$)

In order to test the hypothesis:

$$H_0 : \theta = 0 \tag{7.16}$$

we shall argue as follows. If H_0 is true, neither α_0 nor α_1 depend on ρ, which implies that $\partial\alpha_0/\partial\rho = \partial\alpha_1/\partial\rho = 0$. Therefore, the hypothesis (7.16) in particular implies the hypothesis:

$$H_1 : \omega = \partial\alpha_1/\partial\rho = 0$$

Since the hypothesis H_1 is implied by the hypothesis H_0, we should reject H_0 if we reject H_1. On the other hand if we accept (7.17) then α_1 is independent of ρ, and (7.15) is a possible value of α_1. Hence, if we don't reject H_1 (7.17) we should also abstain from rejecting H_0 (7.16).

Our test of H_1 is based on a Taylor expansion of α_1 given by (7.12). From (7.11)–(7.12) we observe that $\alpha_0 + \alpha_1 = 1$ and that:

$$\frac{\partial \alpha_0}{\partial \rho} = -\frac{\partial \alpha_1}{\partial \rho} > 0, \quad \text{if} \quad \beta a^2 > 1 \quad \text{and} \quad 0 < a^{-1}\rho < 1 \tag{7.18}$$

$$\frac{\partial \alpha_1}{\partial \rho} = \frac{2\beta\gamma(1-\beta a^2)(1-a^{-1}\rho)}{(\beta a^2(1-a^{-1}\rho)^2 - \beta\gamma)^2} < 0, \quad \text{if} \quad \beta a^2 > 1 \quad \text{and} \quad 0 < a^{-1}\rho < 1 \tag{7.19}$$

Hence, we have to consider the test problem:

$$H_1 : \omega_1 = \frac{\partial \alpha_1}{\partial \rho} = 0 \quad \text{against} \quad H_A : \omega_1 < 0. \tag{7.20}$$

A first-order Taylor expansion of α_1 (7.12) wrt. the variable coefficients around their respective means is given by:

$$\alpha_1 = \omega_0 + \omega_1(\rho - \mathrm{E}(\rho)) + \omega_2(\beta - \mathrm{E}(\beta)) + \omega_3(\theta - \mathrm{E}(\theta)) + v \tag{7.21}$$

where $\omega_1 = \partial\alpha_1/\partial\rho$, $\omega_2 = \partial\alpha_1/\partial\beta$, $\omega_3 = \partial\alpha_1/\partial\theta$ and v denotes random disturbances.

The expansion (7.21) is based on the following reasoning. According to (7.12) α_1 depends on $(\alpha, \beta, \gamma, \rho)$. We suppose that $a = (1 + r)$ is the same for all families. This is resonable since the data refers to the same time period and the families, therefore, will face the same interest rates. A part from σ^2 which is supposed to be constant, the remaining parameters are free to vary over families, i.e. β, θ, ρ may be different for different families. Also, the coefficients β_i and θ_i describe properties of the aggregate utility of family (i), while ρ_i describes the autocorrelation of the income process. Thus, it appears tenable to suppose that the distribution of (β, θ) is independent of the distribution of ρ. This implies that the regression coefficient ω_1 of eq. (7.21) will coincide with the regression coefficient in the simple regression of α_1 on ρ.

In order to test the hypothesis H_1 against H_A (see (7.20)) we shall apply the following simple approach. First we run the 34 regressions as indicated by (7.10) and (7.13). As a proxy-variable for $(z_{t+1} - \rho z_t)$ we use the trend variable (μt). Having attained the estimates $\hat{\alpha}_{1i}$ and $\hat{\rho}_i$ of α_{1i} and ρ, we, then, regress $\hat{\alpha}_1$ on $\hat{\rho}$ giving an estimator of ω_1. Since we have argued that ρ is independent of β and θ, the net and gross effect of ρ on α_1 are equal to ω_1.

This regression gave the following result.

$$\alpha_{1i} = \underset{(0.8527)}{2.09} \; \underset{(0.8551)}{-1.18\hat{\rho}_i} + \hat{v}_i \tag{7.22}$$

In order to decide on the test problem (7.20) we shall use the criteria: (i) The sign of ω_1 predicted by our theory and (ii) the value of the statistic $(\hat{\omega}_1 - \omega_1)/(\mathrm{std}(\hat{\omega}_1)$ under H_1. Under general conditions this statistic is approximately normally distributed.

From (7.22) we observe that $\hat{\omega}_1(= -1.18)$ turns out to have the sign predicted by our theory (see (7.19)). Under H_1 the value of the statistic is $(-1.18/0.8551) = -1.38$ which corresponds to a p-value ≈ 0.08. Hence, according to the calculated statistic we can reject H_1 at a 0.10 level but not at a 0.05 level. Judging these pieces of information together, we are inclined to reject H_1 and accept H_A. According to our arguments above we should in this case, since H_1 is implied by H_0 also reject H_0. Hence, we conclude that the simple Hall model $(\theta = 0)$ is rejected statistically on our data-set.

The implication (7.19) $(\omega_1 < 0)$ may seem counterintuitive at first glance, but further thought confirms it as sensible. As indicated above we can associate the magnitude of ρ with income uncertainty. Then (7.18)–(7.19) simply say that families with volatile incomes will try to smooth out consumption outlays by adjusting the regression coefficients. They will attempt to keep relatively more of the consumption at a fixed, constant level, and reduce that part contributed by the auto-regressive component. Indeed, this effect of income variability was also noted by Friedman (1957) comparing the marginal propensity to consume by farmers and non-farm families. However, in our model this result comes out explicitly as a consequence of the optimal dynamic behaviour of our forward-looking consumer.

Finally, our empirical findings can be summarized as follows:

(i) Hall's simple martingale hypothesis is rejected.

(ii) There is no ground to reject the "Euler equation hypothesis".

(iii) We reject the simple Hall case $(\theta = 0)$ and accept the more general case $(\theta > 0)$. Thus the dynamic behaviour of the families investigated in our empirical study is characterized by risk sensitivity.

8 Conclusion

Beeing a (CEQ) model Hall's model has the pros and cons of this class of models. Generally, (CEQ) specifications make possible simple and explicit solutions, but from an economic point of view they are often too restrictive to capture essential aspects of economic behaviour. The model studied in the present paper is more general in that the aggregate utility explicitly includes a risk parameter θ. As we have shown this parameter constantly modifies Hall's results for the (CEQ) model.

However, inspired by Hall (op.cit.) we have worked hard to derive a fully specified regression model on the basis of an explicitly stated optimization model. At the same time we hope that the forward-going optimization process described in section 3 will have some independent interest. The fact that it produces simultaneously the current optimal strategy and estimates of what the optimal decisions will be in the future, as well as predictions of the future path of the exogenous variables is very appealing.

Appendix A

A major difficulty in testing the life-cycle theories of the consumption function has been the lack of data giving the household's consumption and income over many years. Owing to the kindness of professor Mayer we recived his data giving annual budgets over a five-year period for 124 Swiss households. A sub-sample of 34 families reported their annual budgets for a ten-year period. It is this data-set we have used. The data states the families' annual expenditures for consumption, insurances specified by types, taxes and various public charges, total income and finally a variable (quets) measuring the size of the households. This data is particularly useful because of its quality (Mayer (1972) p. 379), and the length of time each household has reported their outlays and incomes. Compared to data-sets used in similar studies, these data seems to be quite unique. Mayer (op. cit.) devotes a considerable part of his book to the study of this sample. He describes this data in detail (ch. 13 and appendix 5). Since we have followed Mayer as regards definitions of variables, it is sufficient to refer to Mayer's description (op.cit.).

References:

Belsley, D., E. Kuh and R. Welsch (1980): *Regression Diagonstics, Identifying Influential Data and Sources of Collinearity*, Wiley, New York.

Blanchard, O.J. and S. Fischer (1989): *Lectures on Macroeconomics*, The MIT Press, London.

Epstein, L.E. (1988): Risk aversion and asset prices, *Journal of Monetary Economics* 179–192.

Farmer, R. (1990): RINCE-preferences, *Quarterly Journal of Economics* 105, 574–607.

Friedman, M. (1957): *A Theory of the Consumption Function*, Princeton University Press.

Hall, R.E. (1978): Stochastic implications of the life-cycle permanent income hypothesis, *Journal of Political Economy* 86, 971–987.

Hall, R.E. (1989): "Consumption". In J. Barro (ed.): *Modern Business Cycle Theory,* Harvard University Press.

Koopmans, T.C., P.A. Diamond and R.E. Williamson (1964): Stationary utility and time perspective, *Econometrica* 32, 82–100.

Kreps, D.M. and E.L. Poreus (1978): Temporal resolution of uncertainty and dynamic choice theory, *Econometrica* 46, 185–200.

Leland, H.E. (1968): Savings under uncertainty: The precautionary demand for savings, *Quarterly Journal of Economics* 82, 465–473.

Mayer, T. (1972): *Permanent Income, Wealth and Consmption: A Critique of the Permanent Income Theory, the Life-Cycle Hypothesis, and Related Theories*, University of California Press.

Muelbauer, J. (1986): "Habits, rationality and myopia in the life-cycle consumption function", Manuscript, Nuffield Coll., Oxford.

Silvey, S.D. (1969): Multicollinearity and imprecise estimation, *Journal of the Royal Statistical Society, Series B*, 35, 67–75.

Weil, Ph. (1990): Nonexpected utility in macroeconomics, *Quarterly Journal of Economics* 105, 29–42.

Whittle, P. (1982): *Optimization over Time I*, J. Wiley & Sons, Chichester, New York.

Whittle, P. (1990): *Risk-sensitive Optimal Control*, J. Wiley & Sons, Chichester, New York.

Willassen, Y. (1990): "On the specification of the dynamic consumption relation – an application of the stochastic maximum principle", Manuscript, Department of Economics, University of Oslo, Norway.

Zeldes, S.P. (1989). Optimal consumption with stochastic income: Deviation from certainty equivalence, *Quarterly Journal of Economics* 104, 275–298.

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