No. 149 | December 1995 | Discussion Papers

Statistics Norway
Research Department

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Discussion Papers

Statistics Norway
Statistisk sentralbyrå

Discussion Papers No. 149 • Statistics Norway, December 1995

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Distribution of Preferences and Measurement Errors in a Disaggregated Expenditure System [+]

Abstract: A complete system of consumer expenditure functions with 28 commodity groups is modelled and estimated by means of Norwegian household panel data. Measurement errors are carefully modelled. Total consumption expenditure is modelled as a latent variable, purchase expenditures on different goods and two income measures are considered as indicators of this basic variable. The distribution of individual differences in preferences, represented by individual, time invariant latent variables in the expenditure functions, is structured by means of a two level utility tree which permits a parsimonious parameterization. The usual assumption of no measurement error in total expenditure is clearly rejected. The standard assumption in factor analysis of uncorrelated measurement errors is also clearly rejected. In particular, we find positive correlation between measurement errors (purchase residuals) of food groups which may be explained by rational shopping behavior of the households. The purchase residuals for automobiles show negative serial correlation and positive correlation with the volatile components of latent total expenditure, which is reasonable for such a durable good. The first and second order moments of the observed variables, which are the input in the analysis, consist of 2015 elements which are modelled by means of 213 structural parameters in our reference model. The maximum likelihood estimates of the latter have, with only a few exceptions, the expected sign and a reasonable size.

Keywords: Consumer demand, Engel functions, panel data, preference distributions, latent total expenditure, measurement errors, household expenditure surveys.

JEL classification: C4, C5, D1, D3.

Acknowledgement: An earlier version of this paper was presented at the Econometric Society World Congress 1995, Tokyo, and we thank participants for helpful comments. Financial support has been provided by the Norwegian Research Council (Project no. 2141802 and 108527/510). Tone Veiby has performed professional word processing.

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1. Introduction

Systems of expenditure functions for consumption commodities, including systems of Engel functions, have been analyzed in a substantial number of scientific papers over the years. The interest often focuses on Engel elasticities and parameters representing the effect on consumption of demographic and socioeconomic characteristics. The vast majority of existing empirical analyses of systems of Engel functions utilizes cross section data from a sample of households with an income variable considered as observed without error. Often no distinction is made between income and total consumption expenditure. However, following the classical articles of Summers (1959) and Liviatan (1961) [see also Friedman (1957) and Cramer (1966)], the problem of measurement error in total expenditure and in income has been recognized as important in analyzing data from household budget surveys. An adequate modelling of measurement errors in total consumption expenditure seems to be important not only in order to avoid large biases in estimated Engel functions but also to assess the variability of preferences and the 'true' total consumption expenditure in the population from which the sample is drawn.

A main focus of the present paper is on the modelling of *measurement errors* in consumption in making inference on a complete system of Engel functions, with a fairly *disaggregated commodity classification*, from household budget data. The perspective is, in several respects, wider than in the mainstream literature in this field. First, *panel data* with two observations from each respondent are used. It is well known that panel data in general offer far richer opportunities for analyzing individual effects and for controlling for individual 'nuisance' variables than conventional data types [cf Mundlak (1978), Hausman and Taylor (1981), and Griliches and Hausman (1986)]. Second, in order to allow for imperfect measurement of income and consumption, they are considered as latent variables. Third, the *distribution of latent total consumption expenditure* across households, and its evolution over time, is identified and estimated jointly with the expenditure system. Fourth, *individual differences in preferences*, represented by individual, time invariant latent variables, are allowed for. A primary purpose of the investigation is to quantify the distribution of these differences.

The paper represents an extension of previous research by Biørn and Jansen (1982), Aasness (1990, Essay 5), and Aasness, Biørn and Skjerpen (1993a,b). In the first, using panel data, individual differences in consumption are analyzed by means of a complete demand system (including prices) with an error components specification of the disturbance vector, although with errors of measurement in income and consumption disregarded. The second uses cross section data, thus neglecting the panel aspect, but focuses on errors in variables and identifies and estimates a distribution of latent total consumption expenditure across households simultaneously with a system of Engel functions. The third work partly integrates the two approaches, and extends them by, inter alia, incorporating information on observed incomes from tax records, using, however, an aggregated commodity classification, with only 5 groups exhausting total consumption. A primary purpose of the present paper is to extend certain parts of the 5 group analysis of Aasness, Biørn and Skjerpen (1993a,b) to include a considerably more detailed, and for several practical purposes more interesting, commodity classification, including 28 groups which exhaust total consumption. This paper is, to the authors' knowledge, the first work attempting to combine an errors in variables approach and panel data modelling with such a disaggregate classification of consumption.

In order to keep the model transparent and tractable, we have made several simplifying assumptions. In particular, we have assumed linear Engel curves. This assumption is very convenient in our setting with latent variables, inter alia because we can apply the computer program LISREL 7, which turned out to be very efficient for our large scale latent variable model. However, it may be argued that linearity of all Engel curves is not realistic. There exists a large literature with empirical evidence suggesting nonlinear systems of Engel curves, see e.g. Working (1943), Aasness and Rødseth (1983), and Lewbel (1991). But these studies disregard the errors-in-variables problem, and linear Engel curves may well turn out to be a more appropriate assumption in a setting with latent variables. Furthermore, if the true Engel curves are nonlinear, our latent variable approach may estimate consistently least square approximations to these true nonlinear functions, and these linear approximations are well defined and interesting for some purposes, cf Aasness (1990, pp. 221-222). Be this as it may, we regard it as a challenge for future research to extend our analysis to systems of nonlinear Engel curves with latent total expenditure. Blundell et al (1993) use an instrumental variable approach, but this does not provide a satisfactory solution to the problem. Hsiao (1989, 1992) points out that it is far from trivial to combine errors-in-variables and non-linear functions. Hausman et al (1995) have given an interesting contribution to the estimation of polynomial Engel curves in an errors-in-variables context. However, they apply a single equation approach, while we use a model with a system of 60 equations, each explaining one observable, and with an elaborated modelling of distributions of preference variables and measurement errors. Thus, the above mentioned papers do not give a solution to the problem of modelling nonlinear Engel curves within our rather complex setting, although they may give some suggestions.

The rest of the paper is organized as follows. In section 2, we present the basic notation, the general model framework, and specific features under consideration. Next, in section 3, the data and the inference procedure, implemented by means of the computer program LISREL 7, are briefly discussed. Specification of hypotheses and models for our empirical study is presented in section 4. The empirical results are reported in section 5. Section 6 concludes and surveys the main empirical findings. An appendix shows some implications of a model with a two-level utility tree on the distribution of preference variables, which we have exploited in the empirical modelling.

2. Model framework and basic notation

Let consumption be divided into I commodities and assume that a panel of H households is observed over T years. We specify a system of I linear *Engel functions*,

(1) $$\eta_t = a_{At} + b\xi_t + Cz + \mu, \quad t = 1,\dots,T,$$

where η_t is a $I \times 1$ vector of expenditures, at constant prices, in year t, ξ_t is total expenditure, z is a time invariant $M \times 1$ vector of demographic variables, μ is a time invariant $I \times 1$ vector representing individual preferences attached to the I commodities (and other random effects reflecting unobserved time invariant household characteristics), and a_{At}, b, and C are matrices of coefficients of dimension $I \times 1$, $I \times 1$, and $I \times M$, respectively. The vectors η_t and μ and the scalar ξ_t are latent, the vector z is observable. Realizations of (η_t,ξ_t,z,μ) for different households are assumed to be independent and, for simplicity, the household subscript is suppressed. Finally, b and C have the same values for all households and years, while the year subscripts on a_{At} indicates that shifts in the expenditure functions over time are allowed for. Since, by definition,

(2) $$\iota_I'\eta_t = \xi_t, \quad t = 1,\ldots,T,$$

ι_I denoting the $I \times 1$ vector of ones, the coefficient matrices will be subject to the adding-up restrictions $\iota_I'a_{At}=0$, $\iota_I'b=1$, $\iota_I'C=0_{1M}$, 0_{1M} being the $1 \times M$ zero vector, while the preference variables must satisfy

(3) $$\iota_I'\mu = 0.$$

The $I \times 1$ vector of observed expenditures in year t is

(4) $$y_t = \eta_t + a_{Bt} + v_t, \quad t = 1,\ldots,T,$$

where a_{Bt} and v_t are $I \times 1$ vectors representing measurement errors, v_t being a household specific random measurement error component with expectation equal to zero for all households, while a_{Bt} is a non-stochastic «systematic measurement error» with the same value for all households. In other words, a_{Bt} is the time varying expectation of the total measurement error $a_{Bt} + v_t$. (Note that $a_{Bt} + v_t$ may also be interpreted as including a vector of disturbances in the Engel functions (1), which cannot be empirically distinguished from the measurement errors.) In household budget surveys, the observed expenditures (y_t) are typically represented by purchase costs during a relatively short period, while true expenditures (η_t) can be defined precisely with reference to a specific theory of consumer behavior. For a non-durable good, true expenditure could be the value of the consumption flow during the year, $a_{Bt} + v_t$ representing stock changes during the registration period. In case of a durable good, true expenditure could be the service value of its stock during the period, the difference between the purchase value and the service value being a component of the measurement error. For durables, the systematic measurement error will typically be positive in boom periods and negative in recessions.

Equations (1)-(4) imply that the observed $I \times 1$ vector of expenditures satisfies

(5) $$y_t = a_t + b\xi_t + Cz + \mu + v_t, \quad t = 1,\ldots,T,$$

where $a_t = a_{At} + a_{Bt}$, while the observed total expenditure is

(6) $$x_t = \iota_I'y_t = \xi_t + m_t + \nu_t, \quad t = 1,\ldots,T.$$

Here

(7) $$m_t = \iota_I'a_{Bt}, \quad \nu_t = \iota_I'v_t, \quad t = 1,\ldots,T,$$

are the aggregate systematic and random (household specific) measurement errors, respectively. Note that the parameters a_{At}, a_{Bt}, and m_t will not be identifiable without further restrictions. In section 4 and 5, we will present examples of such restrictions and interpret and test them in the concrete setting given by our data.

Formally, (5) says that y_t contains I *indicators* of the latent total expenditure ξ_t. We also assume that K additional indicators exist, represented by the observed $K \times 1$ vector w_t, and formalize the relationship as

$$(8) \qquad w_t = d_t + e\xi_t + Fz + \lambda + \varepsilon_t, \qquad t = 1,...,T,$$

where d_t, e, and F are coefficient matrices of dimension $K\times 1$, $K\times 1$, and $K\times M$, respectively, λ is a latent time invariant $K\times 1$ vector associated with the indicators, and ε_t is a $K\times 1$ vector of error terms. The individual effects λ play formally the same role as μ in (5), but λ, like d_t, e, and F, are unrestricted. Otherwise, (5) and (8) are similar, so that the vector (y'_t,w'_t) may be interpreted as containing I + K indicators of ξ_t.

In the present study, w_t will be specified as including K different measures of household income in year t defined for tax purposes. The interpretation of (8) is not obvious - several interpretations are possible, see Aasness, Biørn, and Skjerpen (1993a, p.1398). It may be considered as a simple representation of the reduced or semireduced form of a (possibly complex) structural model of the income and wealth distribution mechanism, the statutory tax system, and the spending, saving, and tax paying activity of the individual household. In the following, (8) will be referred to as '*income functions*', and λ and ε_t, like μ and v_t, will, for brevity, be denoted as a '*preference vector*' and a '*measurement error vector*', respectively.

When the number of commodities, I, is large, as it is in the present study, the covariance structure of the preference vector μ and the measurement error vectors v_t may easily become overparametrized if their covariance matrices are not restricted in some way. However, assuming full diagonality of these matrices, i.e. no correlation between the preference variables of different commodities and no correlation between their measurement errors, would seem far too restrictive. On the one hand, apart from the fact that (3) implies singularity of the covariance matrix of μ, the elements of this vector may be correlated via the preference structure underlying the system of Engel functions (1). For instance, the preferences for meat may be related to the preferences for vegetables, the preferences for public transport may be related to the preferences for private transport, etc. On the other hand, the purchase and shopping activity of the household may imply positive, or negative, correlation between the measurement errors of different commodities. For instance, customers pay a limited number of visits to their usual shop or shopping centre during the short period in which they are observed, owing to the fixed costs etc. involved. For several commodities, they make purchases for several days, some of which are, strictly speaking, not consumption, but stock increases, which in our context become parts of the measurement errors. This suggests a positive correlation between the measurement errors of goods purchased in the same shop, or even in the same shopping trip. For durable goods, the measurement errors in v_t may, as noted above, represent the difference between the quantity purchased and the service flow 'produced' by the stock of the good. Since a household is very unlikely to make a positive investment in the stock of such a good, say an automobile, in two successive years (assuming that the registration period is one year for durables), this may lead us to expect a negative correlation between the corresponding elements of v_t and v_{t-1} for this kind of goods.

We have tried to take the above considerations into account in modelling the covariance structure of μ and $v_1,...,v_T$ as described below.

We assume that the preferences of a typical household can be represented by a Stone-Geary utility function in two levels. The commodities which are related, either via the preferences or via the measurement errors (since they are purchased more or less simultaneously), are assumed to belong to one aggregate group. The overall utility function is specified as a Stone-Geary function in the utility levels of the aggregate groups. The utility function of each aggregate group is, in turn, specified as a Stone-Geary function in the quantities consumed of the commodities which belong to the group. This parametrization implies that the marginal utilities of all the commodities which belong to the same aggregate group depend on the quantities of all the commodities in the group, while the 'within-group' marginal utilities do not depend on the quantities consumed of any commodity outside the group.

Let G be the number of groups and I_g the number of commodities in group g, $g = 1,\dots,G$, $\sum I_g = I$. In appendix A, it is shown that the preference vector μ can be written as

(9) $$\mu = (I_I - b\iota_I')\alpha,$$

where α is a (stochastic) $I \times 1$ vector and I_I is the $I \times I$ identity matrix. Since $\iota_I'b = 1$, this ensures that (3) is satisfied automatically regardless of which assumptions are made about the distribution of α. Let α_g and b_g be the $I_g \times 1$ subvectors of α and b, respectively, which belong to group g, i.e.,

$$\alpha' = (\alpha_1',\dots,\alpha_G'), \quad b' = (b_1',\dots,b_G').$$

We decompose α_g as (cf eq. (A.22) in appendix A)

(10) $$\alpha_g = \underline{\alpha}_g + b_g\overline{\alpha}_g, \quad g = 1,\dots,G,$$

where $\underline{\alpha}_g$ is a $I_g \times 1$ vector of commodity specific preference components and $\overline{\alpha}_g$ (scalar) is a preference component specific to group g. (We use 'underscore' and 'overscore' to symbolize disaggregate commodities and aggregate groups, respectively.) We assume that

(11) $\underline{\alpha}_1,\dots,\underline{\alpha}_G,\overline{\alpha}_1,\dots,\overline{\alpha}_G$ are uncorrelated,

with zero expectations and

(12) $$E(\underline{\alpha}_g\underline{\alpha}_g') = \Sigma^g_{\underline{\alpha\alpha}}, \quad g=1,\dots,G,$$

(13) $$E(\overline{\alpha}_g^2) = \sigma^g_{\overline{\alpha\alpha}}, \quad g=1,\dots,G,$$

which imply zero correlation between 'necessity consumption' of commodities belonging to different groups, while within group correlation is allowed for. From (10)-(13) it follows that

(14) $$\Sigma^g_{\alpha\alpha} = E(\alpha_g\alpha_g') = \Sigma^g_{\underline{\alpha\alpha}} + b_g b_g'\sigma^g_{\overline{\alpha\alpha}}, \quad g = 1,\dots,G,$$

and that the $I \times I$ covariance matrix of α has the block diagonal form

(15) $$\Sigma_{\alpha\alpha} = E(\alpha\alpha') = \mathrm{diag}(\Sigma^1_{\alpha\alpha},\dots,\Sigma^G_{\alpha\alpha}).$$

Defining the block diagonal matrices

$$\Sigma_{\underline{\alpha\alpha}} = \operatorname{diag}(\Sigma^1_{\underline{\alpha\alpha}},\dots,\Sigma^G_{\underline{\alpha\alpha}}),$$

$$\Sigma_{\overline{\alpha\alpha}} = \operatorname{diag}(\sigma^1_{\overline{\alpha\alpha}},\dots,\sigma^G_{\overline{\alpha\alpha}}),$$

$$B = \operatorname{diag}(b_1,\dots,b_G),$$

of dimension $I \times I$, $G \times G$, and $I \times G$, respectively, we can rewrite (15) as

$$(16) \qquad \Sigma_{\alpha\alpha} = \Sigma_{\underline{\alpha\alpha}} + B\Sigma_{\overline{\alpha\alpha}}B'.$$

From (9) and (16) it follows that

$$(17) \qquad \Sigma_{\mu\mu} = E(\mu\mu') = (I_I - b\iota_I')\Sigma_{\alpha\alpha}(I_I - \iota_I b') = (I_I - b\iota_I')(\Sigma_{\underline{\alpha\alpha}} + B\Sigma_{\overline{\alpha\alpha}}B')(I_I - \iota_I b').$$

In the particular case where $\Sigma_{\underline{\alpha\alpha}}$ is diagonal and $\Sigma_{\overline{\alpha\alpha}} = 0$, the I elements of α are uncorrelated. Note, however, that the preference vector μ will always have a non-diagonal covariance matrix, since its elements will always be related via the household's budget, cf (3) and (9). By imposing suitable restrictions on $\Sigma_{\underline{\alpha\alpha}}$ and $\Sigma_{\overline{\alpha\alpha}}$, we can represent the covariance structure of the preference vectors α and μ in a far more parsimonious way than by letting $\Sigma_{\alpha\alpha}$ be a full unrestricted matrix or a block diagonal matrix with unrestricted blocks. This will be elaborated in more detail in section 4.

In order to pay regard to the purchase and shopping activity etc. of the household mentioned above, while preserving a parsimonious representation of the covariance structure of the error vector v_t, we have tried to 'structure' its distribution by adopting a decomposition related to that of the preference vector α. Using the same grouping of the I goods as above, we let

$$v_t' = (v_{1t}',\dots,v_{Gt}'), \quad h' = (h_1',\dots,h_G'),$$

where v_{gt} is a $I_g \times 1$ subvector containing the elements of v_t which belong to group g, assuming that each group contains goods having similar 'purchase habits', and h_g is a $I_g \times 1$ vector of constants specific to group g. We decompose v_{gt}, in analogy with α_g in (10), as

$$(18) \qquad v_{gt} = \underline{v}_{gt} + h_g\bar{v}_{gt}, \quad g = 1,\dots,G, \quad t = 1,\dots,T,$$

where $\underline{v}_{gt}$ is a $I_g \times 1$ vector of commodity specific measurement error components and $\bar{v}_{gt}$ (scalar) is an error component specific to group g. We assume that

$$\underline{v}_{1t},\dots,\underline{v}_{Gt}, \quad \bar{v}_{1t},\dots,\bar{v}_{Gt} \text{ are uncorrelated,}$$

with zero expectations and

(19) $$E\left(\underline{v}_{gt}\underline{v}_{gt}'\right) = \Sigma^{g}_{\underline{vv}}, \quad g = 1,\ldots, G, \; t = 1,\ldots, T,$$

(20) $$E\left(\bar{v}^{2}_{gt}\right) = 1, \quad g = 1,\ldots, G, \quad t = 1,\ldots, T,$$

which imply zero correlation between measurement errors of commodities belonging to different groups, while within group correlation is allowed for. The variances of all $\bar{v}_{gt}$ are set to unity, otherwise, with no restrictions imposed on h_g, the elements of the latter could not be identified.

From (18) - (20) it follows that

(21) $$\Sigma^{g}_{vv} = E\left(v_{gt}v_{gt}'\right) = \Sigma^{g}_{\underline{vv}} + h_g h_g', \quad g = 1,\ldots,G, \quad t = 1,\ldots,T,$$

and that the $I \times I$ covariance matrix of v_t has the block diagonal form

(22) $$\Sigma_{vv} = E\left(v_t v_t'\right) = \operatorname{diag}\left(\Sigma^{1}_{vv},\ldots,\Sigma^{G}_{vv}\right), \quad t = 1,\ldots,T.$$

The corresponding 'cross covariance' matrices $E(v_t v_s')$, $s \neq t$, may, for reasons stated above, contain some non-zero elements, but we do not formalize this at this stage. (See sections 4.4 and 4.5.) Defining the block diagonal matrices

$$\begin{aligned}\Sigma_{\underline{vv}} &= \operatorname{diag}\left(\Sigma^{1}_{\underline{vv}},\ldots,\Sigma^{G}_{\underline{vv}}\right),\\ H &= \operatorname{diag}(h_1,\ldots,h_G),\end{aligned}$$

of dimensions $I \times I$ and $I \times G$, respectively, we can rewrite (22) as

(23) $$\Sigma_{vv} = \Sigma_{\underline{vv}} + HH'.$$

By imposing suitable restrictions on $\Sigma_{\underline{vv}}$ and H, we can represent the covariance structure of the measurement error vectors $v_1,\ldots,v_T$ in a far more parsimonious way than by letting Σ_{vv} be a full unrestricted matrix or a block diagonal matrix with unrestricted blocks. This will be elaborated in more detail in section 4.

Let $\xi = (\xi_1 \ldots \xi_T)'$, $v = (v'_1 \ldots v'_T)'$, and $\varepsilon = (\varepsilon'_1 \ldots \varepsilon'_T)'$, which have dimensions $T \times 1$, $TI \times 1$, and $TK \times 1$, respectively. We assume that the two composite vectors of 'structural' variables (g) and measurement errors (m),

$$g = (\xi', z', \mu', \lambda')' \text{ and } m = (v', \varepsilon')',$$

are uncorrelated, but we allow for correlation within the vectors, specifying their covariance matrices, in partitioned form, as

$$\Sigma_{gg} = \begin{bmatrix} \Sigma_{\xi\xi} & \Sigma_{\xi z} & 0 & 0 \\ \Sigma'_{\xi z} & \Sigma_{zz} & 0 & 0 \\ 0 & 0 & \Sigma_{\mu\mu} & 0 \\ 0 & 0 & 0 & \Sigma_{\lambda\lambda} \end{bmatrix}, \quad \Sigma_{mm} = \begin{bmatrix} \tilde{\Sigma}_{vv} & 0 \\ 0 & \tilde{\Sigma}_{\varepsilon\varepsilon} \end{bmatrix}, \qquad \Sigma_{gm} = 0,$$

where $\tilde{\Sigma}_{vv} = I_T \otimes \Sigma_{vv}$ and $\tilde{\Sigma}_{\varepsilon\varepsilon} = I_T \otimes \Sigma_{\varepsilon\varepsilon}$.

A minor departure from these assumptions is made in the case of automobiles, cf table 1, in order to get a proper modelling of the dynamics of purchases for this durable good.

From (17) it follows that $\iota_I'\Sigma_{\mu\mu}=0$, regardless of the choice of $\Sigma_{\underline{\alpha\alpha}}$ and $\Sigma_{\overline{\alpha\alpha}}$. Zero correlation between the preference vectors (μ,λ) and the latent total expenditure and the vector of observed demographic variables (ξ,z) is assumed. Correlation between the preference vectors and latent total expenditure, which may be present, but is disregarded here, is discussed in Aasness, Biørn and Skjerpen (1993a, section 4.5) for a more aggregated commodity classification.

We parametrize the distribution of latent total expenditure by assuming

$$\xi_t = q_{0t} + q_t(\chi + u_t), \qquad t = 1,\ldots,T,$$

where (i) χ is a permanent time invariant component of consumption, $E(\chi) = \Phi_\chi$, $\mathrm{var}(\chi) = \sigma_{\chi\chi}$, (ii) u_t are volatile components representing individual mobility in the distribution, $E(u_t) = 0$, $E(u_t u_s) = \delta_{ts}\sigma_{uu}$ (δ_{ts} being the Kronecker delta), and (iii) q_{0t} and q_t are deterministic trend coefficients (where we, by convention and with no loss of generality, set $q_{01}=0$, $q_1=1$). The properties of this process is discussed in Aasness, Biørn, and Skjerpen (1993a, pp.1399, 1410-1412). In matrix notation it reads

$$\xi = q_0 + Q(\iota_T\chi + u),$$

where $q_0 = (q_{01}\ldots q_{0T})'$, $Q = \mathrm{diag}(q_1\ldots q_T)$, and $u = (u_1\ldots u_T)'$. This implies the following restrictions on Σ_{gg}:

$$\Sigma_{\xi\xi} = Q\iota_T\iota'_T Q'\sigma_{\chi\chi} + Q^2\sigma_{uu}, \quad \Sigma_{\xi z} = Q\iota_T\Sigma_{\chi z}.$$

3. Data and inference procedure

The data set is taken from the Norwegian Surveys of Consumer Expenditures for the years 1975-1977, combined with information on incomes from a 'tax file'. Detailed information is given in Biørn and Jansen (1980), and in Aasness, Biørn and Skjerpen (1993a, section 3 and appendix A). We only report some main points here.

The sample consists of H=408 individual households, each of which is observed in two consecutive years (T=2), one half in the years 1975 and 1976 and the other half in 1976 and 1977. A 28 commodity classification, comprising the whole budget, is used (I=28). It can be directly aggregated to give the G=5 commodity grouping used in Aasness, Biørn and Skjerpen (1993a). The households report with an interval of exactly one year. By constructing annual aggregates, we get two annual reports from the 408 households, which we formally treat as if it were a two period balanced panel, although the two time periods are not identical for all households.

The expenditure data are recorded by a combination of bookkeeping and interviews and are collected evenly throughout the year, 1/26 of the households participating in a particular year are observed between 1st and 14th of January, another 1/26 between 15th and 28th of January, and so on. For commodities with a low purchase frequency, expenses during the last 12 months are registered in a concluding interview at the end of the accounting period. Housing expenses are measured by rent (including maintenance and repairs), whereas other durable goods are represented by the value of last year's purchases. These expenditure values are deflated by price indexes constructed from the basic data used in calculating the official Norwegian Consumer Laspeyres Price Index. All expenditures and incomes are measured in 1 000 Norwegian 1974-kroner.

The other indicators of total expenditure are two income variables (K=2) which are taken from a separate 'tax file' giving summary information from the individual tax returns for all personal tax payers in Norway:

w_1 : Taxable income for the central government tax assessment minus taxes.

w_2 : Income base used for calculating social security premiums and pension rights in the public social security system. It includes wages and net enterpreneurial income, but excludes capital income (positive and negative, e.g. interests received and paid) and pensions.

They are aggregated across all the individual tax payers in the household to get household income. Since the two income variables have several components, e.g. net wage income, in common, we expect that their measurement errors (e) are positively correlated, as are also the individual effects (λ), which we take account of in the specification of $\Sigma_{\varepsilon\varepsilon}$ and $\Sigma_{\lambda\lambda}$.

Two demographic variables (M=2) are used to characterize the household size and composition:

z_1 : The number of children, i.e. persons with age $\leq$ 15 years.

z_2 : The number of adults, i.e. persons with age $\geq$ 16 years.

The inference (estimation and testing) procedure is also essentially the same as used in Aasness, Biørn and Skjerpen (1993a), and we only state its main elements here.

Let $s = (y'_1 ... y'_T\ w'_1 ... w'_T\ z')'$ denote the (TI+TK+M) ´ 1 vector containing all the values of the observed variables. The resulting sample mean vector $\bar{s}$ and covariance matrix S, with dimensions (TI+TK+M) ´ 1 and (TI+TK+M) ´ (TI+TK+M) respectively (i.e. 62 ´ 1 and 62 ´ 62), are the basis for our empirical analysis. The realized values are presented in Tables A1 and A2 in Appendix B. Let $\Phi(\theta)$ and $\Sigma(\theta)$ denote the vector of expectations and the theoretical covariance matrix of the observed variables s as functions of the unknown parameter vector θ of our model. The parameter vector θ may be partioned into three disjoint subvectors such that $\theta=(\theta_1',\theta_2',\theta_{12}')'$. The subvector θ_1 contains the parameters which only enter the expression for the first order moments, i. e. the free parameters in a_{At}, a_{Bt}, d_t, q_0, Φ_χ, and F_z. In the same way, the subvector θ_2 contains the parameters which only enter the expression for the second order moments. These parameters are second order moments in the preference and measurement error distributions together with variance and covariance parameters in the multivariate distribution of ξ_1, ξ_2, z_1, and z_2. The last subvector θ_{12} consists of those parameters which enter both the expressions for the first and second order moments of the observed variables, i.e., Engel and demographic derivatives together with the parameter q_2. The realizations of s for the H households in the data set are assumed to be independent. The estimates of $\theta=(\theta_1',\theta_2',\theta_{12}')'$ are the values that minimize the function

$$(24) \qquad F = F(\theta_1,\theta_2,\theta_{12}) = \ln|\Sigma(\theta_2,\theta_{12})| + tr(S\Sigma(\theta_2,\theta_{12})^{-1}) - \ln|S| - (T(I+K)+M) + [\bar{s} - \Phi(\theta_1,\theta_{12})]'\Sigma(\theta_2,\theta_{12})^{-1}[\bar{s} - \Phi(\theta_1,\theta_{12})].$$

Minimization of F is equivalent to maximization of the likelihood function when assuming that s follows a multivariate normal distribution (cf e.g. Anderson (1958, section 3.2)). When the first order moments are unrestricted, which will be the case if the number of elements in θ_1 equals the number of elements in $\bar{s}$, the last term in (24) will be zero and as a result S will be a sufficient statistic for θ_2 and θ_{12} (provided that these parameter vectors are identified). Then θ_2 and θ_{12} can be estimated first by minimizing the sum of the four first terms of (24). The estimation of θ_1 can be made in a second stage by solving the following set of equations with respect to θ_1 (after having inserted the maximum likelihood estimate of θ_{12}, denoted as $\hat{\theta}_{12}$, obtained from the first stage):

$$(25) \qquad \bar{s} = \Phi(\theta_1, \hat{\theta}_{12}).$$

If, however, the dimension of θ_1 is less than the dimension of $\bar{s}$, so that the last quadratic form in (24) is strictly positive, the above two step procedure cannot be used and maximum likelihood estimation requires simultaneous estimation of all parameters from the first and second order sample moments.

Our model can be formalized as a special case of the LISREL model (cf e.g. Jöreskog (1977)), and the numerical minimization of F have been performed by means of the computer program LISREL 7 (cf Jöreskog and Sörbom (1988)), using the Davidon-Fletcher-Powell method. At the minimum of F, the information matrix is computed and used to estimate asymptotic standard errors and t values. LISREL minimizes the function F without imposing inequality constraints on the admissible values of the parameter vector θ. Thus the LISREL estimate of a parameter interpreted as the variance of a latent variable may well turn out to be negative. At a first glance, this may be regarded as a substantial drawback of this computer program. However, if our model and its interpretation is correct the LISREL estimates should turn out to have the expected sign, apart from the sampling errors. Thus, if for a given model all the estimated variances are positive, and all the estimates of the (sub) covariance matrices are positive semidefinite, we will take this as a confirmation that the model has passed an important test. On the other hand, negative estimates of variances, or negative definite "covariance matrices", indicate either that the model is misspecified or that the sampling errors in its estimates are substantial.

Let F_0 and F_1 be the minimum of F under a specific model (labelled 0) and a more general model (labelled 1), respectively, and let r be the difference between their number of parameters. Minus twice the logarithm of the likelihood ratio is equal to $H(F_0-F_1)$. This statistic is thus, according to standard normal theory, approximately χ^2 distributed with r degrees of freedom under the null hypothesis. The χ^2 values given in Table A7 correspond to HF_0, interpreted as the likelihood ratio test statistic when the alternative hypothesis is a saturated model (giving a perfect fit to the sample covariance matrix and accordingly, F_1=0). The test statistic $H(F_0-F_1)$ for an arbitrary pair of models may be computed by simply taking the difference between the corresponding pair of χ^2 values.

The χ^2 statistic HF_0 can be considered as a measure of the goodness of fit of an arbitrary model 0. As an alternative measure of the goodness of fitnious, with more emphasis on parsimon parameterization, we use the Akaike information criterion, which (when disregarding an arbitrary additive constant) can be written as

$$\text{(26)} \qquad AIC = HF_0 + 2p_0,$$

p_0 denoting the number of parameters in model 0. The lower is the value of AIC, the better is the fit (see Akaike (1987)). Some other godness of fit criteria are also reported, cf table A7, so that the robustness of the results with respect to the choice of fit criterion can be assessed. For a discussion of choice of measures of godness of fit, see e.g. Bozdogan (1987).

If one is unwilling to assume normality of the data vector s, which in the present context - considering in particular the detailed commodity classification and the following large tendency to zero expenditure reporting - is a restrictive assumption, then the estimators derived from minimizing F can be labeled *quasi maximum likelihood estimators*. These estimators will be consistent, but their efficiency and the properties of the test procedures are not so obvious. There exists a large literature on the robustness of these type of estimators and test procedures for departure from normality, see e.g. Jöreskog and Sörbom (1988) for an extensive list of references, leading to quite different results depending on the assumptions and methods used. See Aasness, Biørn, and Skjerpen (1993a, section 3, and appendix A) for a discussion of these issues in the present context.

4. Specification of hypotheses and models

An overview of the specified hypotheses and models in this study is given in table 1. We use a disaggregation of total household consumption into I=28 commodities - listed in table 2. An important part of our model formulation is the specification of the covariance matrices of the preference vector μ, denoted as $\Sigma_{\mu\mu}$, and of the measurement error vector ν, denoted as $\Sigma_{\nu\nu}$. In this section, we comment on the hierarchy of specification of these two matrices which has been under consideration in this study, and present the model we use as our reference model in the sequel, see table 1. In the least restrictive case, $\Sigma_{\mu\mu}$ and $\Sigma_{\nu\nu}$ may be specified as positive semidefinite $I \times I$ matrices, and no covariance restrictions, except that $\Sigma_{\mu\mu}$ should satisfy the adding-up restriction $\iota'\Sigma_{\mu\mu} = 0_I'$, i.e. all its column (or row) sums should be equal to zero. With I=28, this extreme case would give a total number of unknown elements equal to I(I-1)/2=378 in $\Sigma_{\mu\mu}$ and equal to I(I+1)/2=406 in $\Sigma_{\nu\nu}$. This specification, requiring 784 unknown elements in these two matrices only, may be characterized as grossly overparametrized, a property which can, however, be tested by means of our data. We use *U (unrestricted)* as abbreviation for this case in the following. At the other extreme, we might specify $\Sigma_{\nu\nu}$ as diagonal, i.e.with I=28 unknown elements. Similarly, we may define $\Sigma_{\alpha\alpha}$ as a diagonal $I \times I$ matrix, with I=28 unknown elements, and let $\Sigma_{\mu\mu}$ be given by (17). The latter specification pays regard to the adding-up restriction on $\Sigma_{\mu\mu}$ which is an integral part of our model [cf eqs. (3) and (17)]. We use *D (diagonal)* as an abbreviation for this case.

In view of our remarks in section 2 about (i) preference relations between commodities belonging to the same aggregate group and (ii) possible (positive or negative) correlation of measurement errors of different commodities, owing to the households' shopping and purchasing behaviour, diagonality of $\Sigma_{\alpha\alpha}$ and $\Sigma_{\nu\nu}$, which requires a total number of unknown elements in these two matrices equal to 56, seems too restrictive. This specification can, however, be tested by means of our data.

From these considerations, a strategy leading to a specification between these two extremes, i.e. between 784 and 56 unknown elements, in $\Sigma_{\alpha\alpha}$ and $\Sigma_{\nu\nu}$, seems promising. One such intermediate case is to aggregate the I=28 commodities into a small number of aggregate groups and assume block diagonality of $\Sigma_{\alpha\alpha}$ and $\Sigma_{\nu\nu}$ corresponding to this grouping, i.e. having nonrestricted correlation within groups, but zero correlation between groups. For this purpose, we have defined G=5 aggregate groups, indexed by roman numbers:

I. Food, beverages and tobacco: commodities 01 -- 10.
II. Clothing and footwear: commodities 11 -- 12.
III. Housing, fuel and furniture: commodities 13 -- 17.
IV. Travel and recreation: commodities 18 -- 24.
V. Other goods and services: commodities 25 -- 28.

This coincides with the grouping used in Aasness, Biørn and Skjerpen (1993a, 1993b). If we impose no further restrictions, this reduces the number of unknown parameters in each of the 28×28 matrices $\Sigma_{\alpha\alpha}$ and $\Sigma_{\nu\nu}$ to 111, which is only a little more than one fourth of the corresponding numbers in the U specification. Of these 111 parameters 55 represent group I, 3 group II, 15 group III, 28 group IV, and

10 group V. We use *B (block diagonal)* as an abbreviation for this case in the following. The restrictions imposed by this specification are easily testable with our data.

Still, in view of the two-level specification of the utility maximization, with Stone-Geary utility functions on both levels, discussed in section 2 and appendix A, it seems to be scope for a further parsimony in the specification of the stochastic structure of the preferences and the measurement errors. This, in particular, seems to be the case for groups I, III, and IV, occupying in specification B as many as 55, 15, and 28 parameters, respectively, for each of the matrices $\Sigma_{\alpha\alpha}$ and Σ_{vv}. This brings us to the fourth and final parametrization of $\Sigma_{\alpha\alpha}$ (and thus of $\Sigma_{\mu\mu}$) and Σ_{vv} that we consider in this study. It is an intermediate case between specifications B and D, denoted by *R (restricted)* in the following. This is also our reference specification (cf below), and hence R may also be an abbreviation for reference.

In specification R, $\Sigma_{\alpha\alpha}$ (and thus $\Sigma_{\mu\mu}$) is described by 35 free parameters, i.e. 76 less than in specification B and 7 more than in specification D, and Σ_{vv} is described by 38 free parameters, i.e. 73 less than in specification B and 10 more than in specification D. This is less than one tenth of the number of free parameters in the unrestricted specification, U, and is also a testable hypothesis with our data. In parametrizing $\Sigma_{\alpha\alpha}$ and Σ_{vv}, we exploit (i) the ideas concerning the utility trees of the households described in appendix A and section 2 [cf (10) and (16)], and (ii) the formally similar representation of the measurement errors assumed to follow from the households' shopping and purchasing behavior [cf (18) and (23)]. This gives us a rich framework for formulating interesting hypotheses. In particular, we may, for some groups, model all within group covariances of preferences through one group specific preference variable (α_g), a hypothesis we shall denote as 'utility branch with one common factor' in the following. Correspondingly, we can model all the within group covariances of measurement errors (purchase residuals) through one group specific factor (v_g), a hypothesis to be denoted as 'simultaneous group purchases with one common factor'. But there is no substantial a priori reason to follow this particular specification for all groups. (For two-good groups this model specification is not even identified as regards the parameters describing the distribution of the measurement errors since the h's also must be identified.) One may well combine such a hypothesis for one group with a full covariance matrix for another group and diagonality for a third group, and the modelling of preference variation and measurement errors can be combined in different ways. Thus there are several possibilities for alternative specifications. We have chosen a strongly parsimonious alternative (much closer to D than to B as measured by the number of parameters), but which we think can capture some basic features of preference variation and purchase behavior of Norwegain households. The main ingredients of specification R for the five commodity groups will be described below. All the parameter estimates for this model specification are given in tables A3-A6 in appendix B.

Group I (10 goods): This group is divided into two subgroups: Ia, consisting of the food commodities 01 -- 08, and Ib, consisting of beverages and tobacco, i.e. commodities 09 and 10. We assume zero correlation both in preferences, α, and in measurement errors (purchase residuals), v, between these two groups. For the preference specification within subgroups Ia and Ib, we adhere to relation (10) based on the utility tree in appendix A, and assume that the commodity specific components $\underline{\alpha}_i$ are uncorrelated, i.e. a 'utility branch with one common factor'. This gives a total of 2 parameters more than in specification D (excluding the b parameters, i.e. the marginal budget shares). [Notice that when a group (subgroup) consists of two commodities only, as, for instance, Ib, the hypothesis of a utility branch with one common factor is equivalent to treating the corresponding block in $\Sigma_{\alpha\alpha}$ as unrestricted, which can be seen from a slight reparametrization of the model. The restrictions are effective only when the number of commodities in the group is 3 or more.] For the measurement error specification, we specify block Ia in Σ_{vv} as similar to the corresponding block in $\Sigma_{\alpha\alpha}$, which requires 8 parameters more than in specification D (including the h parameters), allowing for simultaneous purchase behavior for foods. On the other hand block Ib in Σ_{vv} is diagonal, assuming independent purchase residuals for beverages and tobacco.

Group II (2 goods): The blocks in $\Sigma_{\alpha\alpha}$ and Σ_{vv} are left unrestricted, allowing for correlation in preferences and shopping behavior for clothing and footwear. Since there are only two goods in the group this specification is also consistent with 'utility branch with one common factor' and 'simultaneous group purchases with one common factor'. This increases the number of free parameters in each of the two matrices by 1 as compared with specification D.

Group III (5 goods): Here we also assume a 'utility branch with one common factor', allowing for positive correlation between preferences for Housing, Fuel and power, Furniture, Household equipment, and Miscellaneous houshold goods, while increasing the number of parameters of its block in $\Sigma_{\alpha\alpha}$ by 1 only, as compared with specification D. The corresponding block in Σ_{vv} is specified as diagonal except that commodities 15. Furniture and 16. Household equipment have a non-zero error covariance. Thus we allow for simultaneous purchase behavior of these latter goods, due to e.g. fixing up one room in the house, while these purchase residuals are independent of the purchase residual for say Fuel and power which may be mostly influenced by the temperature in the registration period. This also increases the number of parameters by 1 as compared with specification D.

Group IV (7 goods): This group is, like group I, divided into two subgroups: IVa, consisting of the transportation commodities 18 -- 20, and IVb, consisting of recreation commodities 21 -- 24, and we assume zero correlation both in preferences (α) and in measurement errors (v) between these two groups. Within the transportation group, we expect correlations of preferences, but a 'utility branch with one common factor' seems too restrictive since it cannot simultaneously allow for positive correlations between preferences for stock of motorcars and the running cost of these private vehicles, and negative correlations in the preferences for private versus public transportation. To allow for this, we leave the 3×3 block of IVa in $\Sigma_{\alpha\alpha}$ unrestricted, which increases the number of parameters by 3 as compared with specification D. Since we have not found any particularly good reason for expecting non-zero correlations between the preference variables for recreational goods, we specify the 4×4 block belonging to group IVb to be diagonal. The corresponding blocks in Σ_{vv} are both assumed to be diagonal, since we have not found any convincing a priori arguments for these measurement errors to be correlated.

Group V (4 goods): Its blocks in $\Sigma_{\alpha\alpha}$ and in Σ_{vv} are both specified as diagonal matrices since we have no strong arguments against this most simple hypothesis.

5. Empirical results

5.1. Hierarchy of models, goodness of fit, and model selection

Table 1 gives a classification of the hypotheses and models in our empirical investigation. A model is specified as a combination of hypotheses, one from each of the four dimensions. We focus on the first two dimensions: 1. Covariances of preference variables and 2. Contemporaneous covariances of measurement errors. The two other dimensions: 3. Autocovariances of measurement errors and 4. Demand drift and systematic measurement errors are commented upon in section 5.4. Combining our assumptions (hypotheses) in all possible ways, we obtain $4 \times 5 \times 2 \times 4 = 160$ different models. We have estimated 46 of these models, and some characteristics (number of parameters, degrees of freedom, χ^2, AIC and two related information criteria) of all the estimated models are reported in table A7. In figure 1, we have selected 15 models which we find particularly interesting, and for each of these we present two important pieces of information: the number of parameters (p) and the Akaike Information Criterion (AIC). We see that our restricted model ($P_R M_R A_R D_R$, i.e. with the restricted hypothesis R in all four dimensions) has the best AIC score among all the 15 models in figure 1, and also among all the 46 models in table A7. This result is also quite robust with respect to the choice among the three different information criteria in table A7.

This gives strong support to our choice of restricted model, and we use it as a reference model throughout the text. The reference model has p=213 free parameters, DF=1802 degrees of freedom, and AIC=3163. A *saturated model* would have p= 2015 free parameters [which is the maximal number of first order (62) and second order (1953 = 62*63/2) moments of the 62 observed variables in the data set]. Thus its DF=0 and its AIC=2p=4030. In our reference specification, we thus have (i) only between 7 and 8 free parameters per commodity and (ii) a number of free parameters which is only a little more than one tenth of the corresponding number in a saturated model.

The reference model and the other specifications considered can be tested either against the saturated model, or against the unrestricted (U) model, by standard likelihood ratio tests using the c^2 values in table A7 and standard levels of significance (cf section 3). These test will give clear rejection for most of the models, which is not surprising in view of the large number of degrees of freedom involved. The reference model can be looked upon as the specification among those considered which minimizes the AIC, and hence, loosely speaking, gives a useful compromise between a high goodness of fit and a parsimony in parametrization.

Table 1. Classification of hypotheses and models

A specific model is labeled $P_iM_jA_kD_l$, which means that the model is based on hypothesis P_i w.r.t. the covariances of preference variables (μ, α), hypothesis M_j w.r.t. the contemporaneous covariances of the measurement errors (ν), hypothesis A_k w.r.t autocorrelation of measurement errors, and D_l w.r.t. demand drift and systematic measurement errors. Model $P_RM_RA_RD_R$, i.e. with the "Restricted" version in all four dimensions, is used as a reference model throughout the text and is the only model reported with a full set of parameter estimates. Model P_iM_j is a shorthand notation for $P_iM_jA_RD_R$, i.e. with the restricted (or reference) hypothesis in dimensions A and D is subsumed.

1. Hypotheses w.r.t. covariances of preference variables

Label	Interpretation
P_U	Unrestricted, i.e. $\Sigma_{\mu\mu}$ is free except that $\iota'\Sigma_{\mu\mu} = 0$
P_B	Block diagonal, i.e. $\Sigma_{\alpha\alpha} = \text{diag}(\Sigma^1_{\alpha\alpha},\ldots,\Sigma^G_{\alpha\alpha})$, $\Sigma^g_{\alpha\alpha}$ unrestricted, G=5
P_R	Restricted, i.e. $\Sigma_{\alpha\alpha} = \text{diag}(\Sigma^1_{\alpha\alpha},\ldots,\Sigma^G_{\alpha\alpha})$, $\Sigma^g_{\alpha\alpha}$ restricted (see text)
P_D	Diagonal, i.e. $\Sigma_{\alpha\alpha} = \text{diag}(\sigma^1_{\alpha\alpha},\ldots,\sigma^I_{\alpha\alpha})$, $\sigma^i_{\alpha\alpha}$ unrestricted

2. Hypotheses w.r.t. contemporaneous covariances of the measurement errors

Label	Interpretation
M_U	Unrestricted, i.e. Σ_{vv} free
M_N	No measurement errors in total expenditure, i.e. $\iota'\Sigma_{vv} = 0$, otherwise unrestricted
M_B	Block diagonal, i.e. $\Sigma_{vv} = \text{diag}(\Sigma^1_{vv},\ldots,\Sigma^G_{vv})$, Σ^g_{vv} unrestricted, G=5
M_R	Restricted, i.e. $\Sigma_{vv} = \text{diag}(\Sigma^1_{vv},\ldots,\Sigma^G_{vv})$, Σ^g_{vv} restricted (see text)
M_D	Diagonal, i.e. $\Sigma_{vv} = \text{diag}(\sigma^1_{vv},\ldots,\sigma^I_{vv})$, σ^i_{vv} unrestricted

3. Hypotheses w.r.t. autocovariances of measurement errors

Label	Interpretation
A_R	Restricted autocovariation, i.e. autocovariation of purchase residuals of automobiles ($\text{cov}(v_{18,1},v_{18,2})$ = free) and correlation between the purchase residual for automobiles and the volatile component of latent total expenditure ($\text{cov}(v_{18,1},u_1) = \text{cov}(v_{18,2},u_2)$ = free, $\text{cov}(v_{18,1},u_2)$ = free, $\text{cov}(v_{18,2},\mu_1) = 0$) but no such correlations for other goods.
A_N	No autocovariances (i.e. $\text{cov}(v_{18,1},v_{18,2}) = \text{cov}(v_{18,1},u_1) = \text{cov}(v_{18,2},u_2) = \text{cov}(v_{18,1},u_2) = 0$).

4. Hypotheses w.r.t demand drift and systematic measurement errors

Label	Interpretation
D_U	Unrestricted, i.e. either (a_{A1},a_{A2}) unrestricted or (a_{B1},a_{B2}) unrestricted or both.
D_S	Systematic, i.e. systematic measurement errors, but only for durables in the second period ($a_{B1i} = 0\ \forall i$, $a_{B2i} = 0\ \forall i \neq 15,16,18$). No demand drift ($a_{A1} = a_{A2}$)
D_R	Restricted, i.e. no demand drift ($a_{A1} = a_{A2}$), no systematic measurement errors in the first period ($a_{B1} = 0$), systematic measurement errors for automobiles in the second period only ($a_{B2i} = 0\ \forall i, \forall i \neq 18$)
D_N	No demand drift ($a_{A1} = a_{A2}$) and no systematic measurement errors ($a_{B1} = a_{B2} = 0$)

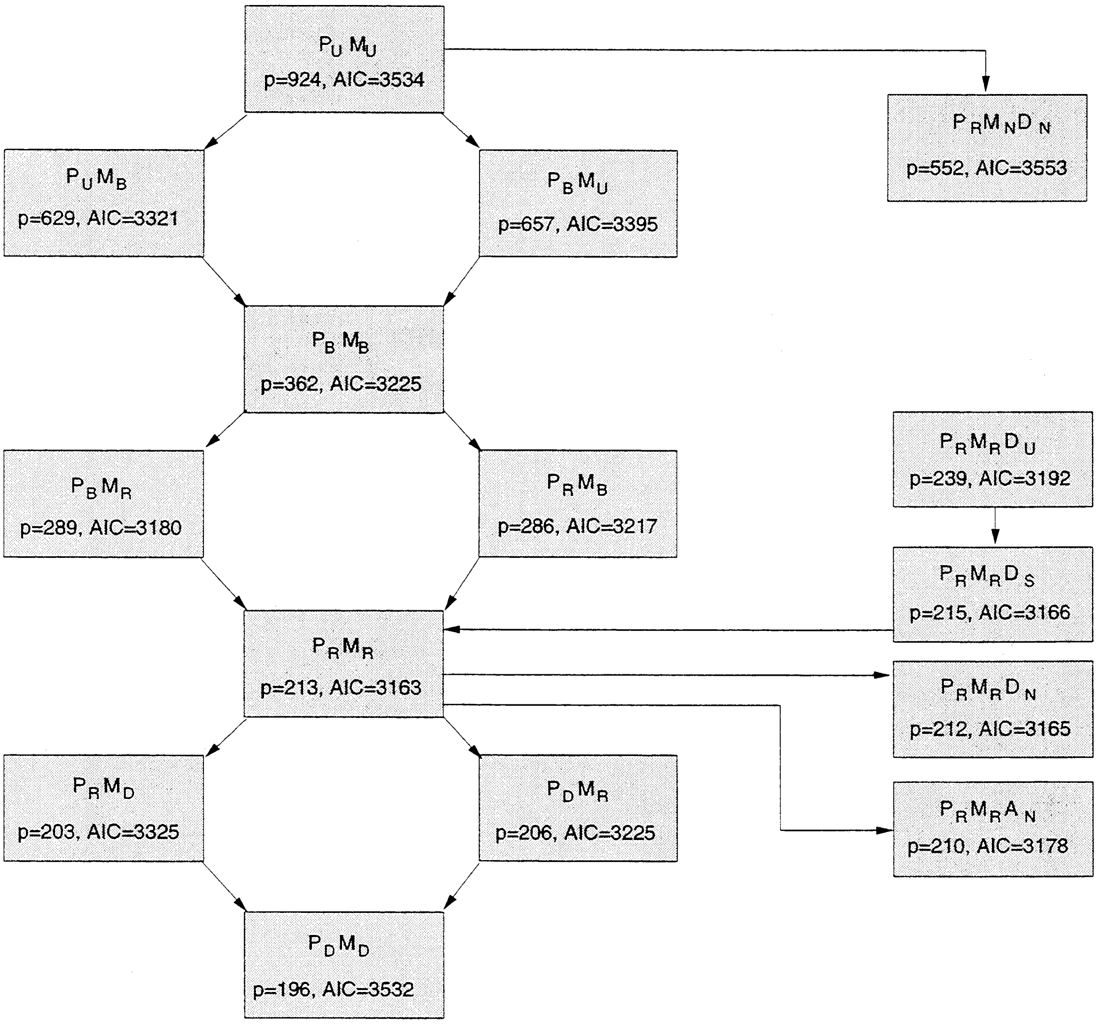

Figure 1. Overview of 15 fitted models with number of parameters (p) and Akaike information criterion (AIC)

Table 2. Characteristics of the demand model.[a] Standard deviations in parentheses

Commodity group	ω (%)	E	P_1	P_2	Rv_α	RV_v
01) Flour and bread	2.239	0.373	0.472	0.459	0.300	0.468
		(0.055)	(0.057)	(0.089)	(0.024)	(0.014)
02) Meat and eggs	5.862	0.753	0.265	0.106	0.479	0.788
		(0.092)	(0.093)	(0.145)	(0.047)	(0.027)
03) Fish	1.560	0.654	-0.187	0.022	0.742	0.897
		(0.118)	(0.125)	(0.192)	(0.048)	(0.031)
04) Canned meat and fish	0.554	0.671	0.179	0.042	0.526	1.050
		(0.112)	(0.117)	(0.181)	(0.065)	(0.036)
05) Dairy products	2.881	0.188	0.601	0.582	0.289	0.360
		(0.046)	(0.049)	(0.076)	(0.018)	(0.011)
06) Butter and margarine	0.875	0.271	0.457	0.477	0.335	0.747
		(0.075)	(0.080)	(0.123)	(0.046)	(0.024)
07) Potatoes and vegetables	4.237	0.625	0.399	0.129	0.375	0.517
		(0.067)	(0.068)	(0.105)	(0.030)	(0.017)
08) Other foods	3.471	0.527	0.307	0.333	0.312	0.498
		(0.060)	(0.061)	(0.094)	(0.028	0.015)
09) Beverages	2.406	1.733	-0.447	-0.915	0.755	0.976
		(0.144)	(0.139)	(0.218)	(0.059)	(0.035)
10) Tobacco	1.527	0.814	-0.008	-0.071	1.120	0.583
		(0.149)	(0.154)	(0.238)	(0.046)	(0.021)
11) Clothing	8.794	1.147	0.033	0.067	0.442	0.824
		(0.098)	(0.092)	(0.146)	(0.054)	(0.029)
12) Footwear	1.906	1.178	0.243	-0.161	0.122	1.925
		(0.182)	(0.176)	(0.277)	(0.764)	(0.067)
13) Housing	11.325	1.133	-0.055	-0.552	0.580	0.704
		(0.098)	(0.093)	(0.147)	(0.041)	(0.025)
14) Fuel and power	3.435	0.230	0.073	0.147	0.391	0.306
		(0.057)	(0.058)	(0.090)	(0.018)	(0.011)
15) Furniture	5.124	1.365	-0.370	-0.509	0.529	1.466
		(0.152)	(0.146)	(0.229)	(0.115)	(0.051)
16) Household equipment	2.894	1.105	-0.055	-0.177	0.466	1.586
		(0.160)	(0.155)	(0.244)	(0.145)	(0.055)
17) Misc. household goods	2.043	1.013	0.160	-0.738	0.568	1.052
		(0.126)	(0.122)	(0.192)	(0.066)	(0.037)
18) Motorcars, bicycles	7.316	0.740	-0.040	1.066	1.285	2.047
		(0.362)	(0.227)	(0.408)	(0.280)	(0.181)
19) Running cost of vehicles	8.478	1.346	0.135	0.219	0.697	1.066
		(0.133)	(0.123)	(0.195)	(0.065)	(0.038)
20) Public transport	2.223	1.083	-0.602	0.172	0.644	1.408
		(0.166)	(0.158)	(0.249)	(0.096)	(0.050)
21) PTT charges	1.394	0.848	-0.625	-0.271	1.251	2.858
		(0.312)	(0.307)	(0.481)	(0.196)	(0.100)
22) Recreation	6.834	1.344	-0.195	-0.379	0.492	1.250
		(0.134)	(0.128)	(0.201)	(0.096)	(0.044)
23) Public entertainment	3.162	0.763	-0.118	0.512	0.837	1.099
		(0.149)	(0.147)	(0.230)	(0.061)	(0.039)
24) Books and newspapers	1.843	1.016	-0.196	-0.105	0.876	1.027
		(0.151)	(0.149)	(0.233)	(0.058)	(0.036)
25) Medical care	1.407	0.547	0.365	0.290	[b]	2.349
		(0.200)	(0.197)	(0.308)		(0.082)
26) Personal care	1.894	0.976	0.108	0.210	0.449	0.811
		(0.102)	(0.099)	(0.155)	(0.052)	(0.029)
27) Misc. goods and services	1.344	1.712	-0.218	-0.450	0.569	2.035
		(0.210)	(0.205)	(0.321)	(0.201)	(0.072)
28) Restaurants, hotels etc.	2.974	1.904	-0.629	-0.607	0.904	1.121
		(0.166)	(0.160)	(0.252)	(0.067)	(0.040)

[a] ω: Mean budget share, E: Engel elasticity, P_1: Child elasticity, P_2: Adult elasticity, RV_α: Relative variation of preferences and RV_v: Relative variation of measurement errors.

[b] The variance of the preference variable is negative, and hence RV_α cannot be calculated.

5.2 Engel functions

The estimated parameters of the Engel functions are given in appendix B (table A3), while table 2 presents main characteristics in terms of budget shares (ω), Engel elasticities (E), child elasticities (P_1), and adult elasticities (P_2). The child (adult) elasticity is defined as the relative change in household expenditure divided by the relative change in the number of persons in the household, when the number of children (adults) is increased by one. These 'person elasticities' are defined conditionally on the value of (latent) total expenditure, thus their (weighted) sum across all goods is equal to zero owing to the budget constraint. The elasticities are computed at the (global) sample average point (cf table A1). In addition, table 2 includes a measure of the relative variation of preferences (RV_α) for each good, defined as the standard deviation of the preference variable (α) divided by the overall sample mean of the expenditure, and a corresponding measure of the relative variation of the measurement error v (RV_v). Thus these measures are dimensionless numbers similar to coefficients of variation.

Note that all the food groups (01-08) have Engel elasticities that are significantly less than one, and larger than zero, once again confirming Engel's law. The three goods with lowest Engel elastisicity are 05 Dairy products (E=0.19), 14 Fuel and power (E=0.23), and 06 Butter and margarine (E=0.27), while those with the largest Engel elasticity are 28 Restaurants and hotels (E=1.90), 09 Beverages (E=1.73), and 27 Miscellaneous goods and services (E=1.71).

A model assuming a linear homogeneous equivalence scale implies that the person elasticities are positive for necessities (E<1) and negative for luxuries (E>1), see e.g. Bojer (1977,p.183). The empirical results of our less restrictive general model also satisfy this property for most of the goods, in particular for the goods mentioned above, with one exception. This exception is the negative child elasticity for 03 Fish, which may be explained by a tendency of children not to enjoy eating fish as much as adults do.

The goods with the largest estimates of relative preference variation are 18 Motorcars and bicycles ($RV_\alpha = 1.29$), 21 PTT charges ($RV_\alpha = 1.25$), and 10 Tobacco ($RV_\alpha = 1.12$). The goods with the estimated largest relative variation of measurement errors are 21 PTT charges ($RV_v = 2.86$), 25 Medical Care ($RV_v = 2.35$), and 18 Motorcars and bicycles ($RV_v = 2.05$).

5.3 Distributon of preferences

Tables 3 and 4 contain summary characteristics of the distribution of the vectors of preference variables, α and μ, supplementing the relative variation statistics RV_α reported in table 2, column 5. The statistics in table 3 are renormalizations of the elements of the estimated $I \times I$ matrix $\Sigma_{\alpha\alpha}$ while those in table 4 are similar renormalizations of the estimated $I \times I$ matrix $\Sigma_{\mu\mu}$. Recall that the relationship between these two matrices are given by (17). All statistics in these two tables refer to the specification denoted as R (restricted, reference) in section 5.1, all the "basic" free parameter estimates being given in table A5 in appendix B.

Element i on the main diagonal of table 3 (table 4) is the estimated variance of α_i (μ_i) normalized against the squared (global) sample mean of expenditure of commodity i - i.e. it is a sort of squared coefficients of variation of preferences. (The diagonal of table 3 thus contains the square of the entries in the fifth column of table 2.) Below the main diagonal are given the corresponding normalized covariances, i.e. the figure in position (i,j, i > j) is the estimated covariance between α_i and α_j in table 3 and between μ_i and μ_j in table 4, both divided by the product of the expenditures on commodities i and j. Above the main diagonal of table 3 (table 4) are reported the estimated simple coefficients of correlation of α_i and α_j (μ_i and μ_j). Hence, all entries in table 3 and 4 are dimensionless numbers, which is a definite advantage from the point of view of interpretability and stability of the parameters.

It follows from our specification (cf section 4) that the transformed $I \times I$ matrices underlying table 3 are block diagonal matrices, but it is by no means a priori obvious that all covariance elements of each block should have the same sign. However, it follows from our assumptions regarding commodities 01 -- 08, i.e. the food commodities (block Ia), that all the normalized covariances of the preference variables in the α vector of these commodities are positive. This is due, inter alia, to the positivity of all the estimated marginal budget shares, i.e. the corresponding elements of the estimates of the $I \times 1$ vector b [cf (14)]. A positive relationship between the preference variables represented by the μ vector is also indicated for the eight food commodities, cf table 4, although the two $I \times I$ matrices underlying the latter table are not block diagonal matrices (cf (17)) - recall that all columns of $\Sigma_{\mu\mu}$ add to zero. Hence, at least some of its off-diagonal elements must be negative, since its estimated and normalized variances, along its diagonal, are all positive (with one exception, see below).

An interesting result is found for the three transportation commodities, commodities 18 - 20 (block IVa). Here the elements of $\Sigma_{\alpha\alpha}$ are not a priori restricted to have the same sign (its 3×3 submatrix is in fact freely estimated, cf section 5), and our estimates give the quite reasonable result that the preferences for both 18 Motorcars and bicycles and 19 Running costs of vehicles are negatively correlated with the preferences for 20 Public transportation. On the other hand, the preferences for commodities 18 and 19 are positively correlated, as they should probably be. We here obtain the same qualititive result regardless of whether we use the α or the μ vector to represent preference variations.

We find, however, among the 28×28 entries, two anomalous results. First, one preference variable, among the 28, comes out with a negative variance estimate, namely for commodity 25 Medical care. Second, the estimated coefficients of correlation between commodities 11 Clothing and 12 Footwear exceeds one (1.718 for α and 1.187 for μ). Strong positive correlation between the preferences of these two commodities, however, comes as no surprise.

Table 3. Distributional measures of the preference variables α. Relative covariation of preferences in the lower triangle (relative variation along the main diagonal) and correlation coefficients in the upper triangle (excluding the main diagonal)

	α01	α02	α03	α04	α05	α06	α07	α08	α09	α10	α11	α12	α13	α14	α15	α16	α17	α18	α19	α20	α21	α22	α23	α24	α25	α26	α27	α28
α01	0.090	0.244	0.137	0.198	0.101	0.126	0.259	0.263	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α02	0.035	0.229	0.173	0.250	0.128	0.159	0.328	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α03	0.030	0.062	0.551	0.140	0.072	0.089	0.183	0.186	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α04	0.031	0.063	0.055	0.277	0.104	0.129	0.265	0.269	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α05	0.009	0.018	0.015	0.016	0.083	0.066	0.136	0.138	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α06	0.013	0.026	0.022	0.023	0.006	0.112	0.169	0.171	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α07	0.029	0.059	0.051	0.052	0.015	0.021	0.141	0.352	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α08	0.025	0.050	0.043	0.044	0.012	0.018	0.041	0.097	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α09	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.570	0.384	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.325	1.255	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.195	1.718	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.093	0.015	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α13	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.337	0.057	0.248	0.228	0.171	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.013	0.153	0.075	0.069	0.052	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.076	0.015	0.280	0.301	0.226	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.062	0.013	0.074	0.217	0.208	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.056	0.011	0.068	0.055	0.323	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.650	0.359	-0.143	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.321	0.486	-0.211	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	- 0.118	- 0.095	0.414	0.000	0.000	0.000	0.000	*)	0.000	0.000	0.000
α21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.566	0.000	0.000	0.000	*)	0.000	0.000	0.000
α22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.242	0.000	0.000	*)	0.000	0.000	0.000
α23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.701	0.000	*)	0.000	0.000	0.000
α24	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.767	*)	0.000	0.000	0.000
α25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.462	*)	*)	*)
α26	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.202	0.000	0.000
α27	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.324	0.000
α28	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.818

*) The estimated variance of the preference variable is negative, and hence the correlation coefficients can not be calculated.

Table 4. Distributional measures of the preference variables μ. Relative covariation of preferences in the lower triangle (relative variation along the main diagonal) and correlation coefficients in the upper triangle (excluding the main diagonal)

	μ01	μ02	μ03	μ04	μ05	μ06	μ07	μ08	μ09	μ10	μ11	μ12	μ13	μ14	μ15	μ16	μ17	μ18	μ19	μ20	μ21	μ22	μ23	μ24	μ25	μ26	μ27	μ28
μ01	0.089	0.224	0.123	0.193	0.091	0.121	0.246	0.253	0.006	-0.019	-0.012	0.043	-0.073	-0.016	-0.023	-0.012	-0.012	-0.141	-0.092	0.039	-0.011	0.002	-0.019	-0.001	*)	0.023	0.035	0.000
μ02	0.031	0.216	0.148	0.234	0.109	0.147	0.298	0.306	-0.021	-0.033	-0.049	-0.021	-0.123	-0.029	-0.064	-0.046	-0.039	-0.190	-0.148	0.030	-0.023	-0.032	-0.036	-0.017	*)	0.004	0.011	-0.026
μ03	0.027	0.051	0.542	0.129	0.060	0.081	0.164	0.169	-0.011	-0.018	-0.026	-0.009	-0.067	-0.016	-0.034	-0.024	-0.020	-0.104	-0.080	0.017	-0.012	-0.016	-0.019	-0.009	*)	0.003	0.007	-0.014
μ04	0.030	0.057	0.050	0.276	0.095	0.126	0.257	0.264	0.013	-0.017	-0.003	0.063	-0.068	-0.015	-0.015	-0.004	-0.006	-0.142	-0.087	0.045	-0.009	0.011	-0.016	0.002	*)	0.030	0.044	0.007
μ05	0.008	0.015	0.013	0.014	0.083	0.059	0.121	0.124	-0.008	-0.013	-0.019	-0.008	-0.049	-0.012	-0.025	-0.018	-0.015	-0.077	-0.059	0.013	-0.009	-0.012	-0.014	-0.007	*)	0.002	0.005	-0.010
μ06	0.012	0.023	0.020	0.022	0.006	0.112	0.162	0.166	0.006	-0.011	-0.005	0.033	-0.046	-0.010	-0.013	-0.006	-0.006	-0.091	-0.058	0.027	-0.007	0.004	-0.011	0.000	*)	0.017	0.025	0.002
μ07	0.027	0.051	0.045	0.050	0.013	0.020	0.137	0.338	0.001	-0.027	-0.024	0.039	-0.106	-0.024	-0.039	-0.023	-0.022	-0.193	-0.132	0.048	-0.017	-0.006	-0.028	-0.006	*)	0.025	0.039	-0.006
μ08	0.023	0.044	0.038	0.043	0.011	0.017	0.039	0.095	0.009	-0.025	-0.015	0.061	-0.099	-0.022	-0.030	-0.015	-0.015	-0.192	-0.125	0.054	-0.015	0.004	-0.025	-0.001	*)	0.033	0.049	0.001
μ09	0.001	-0.008	-0.006	0.005	-0.002	0.001	0.000	0.002	0.602	0.358	0.028	0.188	-0.090	-0.018	0.009	0.024	0.013	-0.240	-0.123	0.100	-0.007	0.054	-0.016	0.019	*)	0.080	0.113	0.040
μ10	-0.006	-0.017	-0.015	-0.010	-0.004	-0.004	-0.011	-0.009	0.309	1.232	-0.033	-0.035	-0.065	-0.016	-0.040	-0.031	-0.025	-0.089	-0.077	0.007	-0.013	-0.026	-0.020	-0.013	*)	-0.007	-0.007	-0.021
μ11	-0.002	-0.010	-0.008	-0.001	-0.002	-0.001	-0.004	-0.002	0.010	-0.016	0.194	1.187	-0.138	-0.030	-0.029	-0.008	-0.012	-0.289	-0.177	0.093	-0.018	0.023	-0.032	0.005	*)	0.063	0.092	0.015
μ12	0.003	-0.002	-0.001	0.007	0.000	0.002	0.003	0.004	0.029	-0.008	0.104	0.040	-0.193	-0.036	0.081	0.114	0.072	-0.623	-0.280	0.298	-0.003	0.199	-0.022	0.076	*)	0.257	0.358	0.150
μ13	-0.011	-0.030	-0.026	-0.019	-0.007	-0.008	-0.021	-0.016	-0.037	-0.038	-0.032	-0.020	0.275	0.008	0.123	0.125	0.089	-0.271	-0.258	-0.004	-0.050	-0.116	-0.074	-0.055	*)	-0.053	-0.061	-0.093
μ14	-0.002	-0.005	-0.004	-0.003	-0.001	-0.001	-0.003	-0.003	-0.005	-0.007	-0.005	-0.003	0.002	0.151	0.042	0.042	0.030	-0.073	-0.065	0.004	-0.012	-0.024	-0.018	-0.012	*)	-0.008	-0.009	-0.019
μ15	-0.004	-0.015	-0.013	-0.004	-0.004	-0.002	-0.007	-0.005	0.004	-0.023	-0.007	0.008	0.033	0.008	0.266	0.282	0.206	-0.299	-0.199	0.081	-0.024	0.000	-0.041	-0.005	*)	0.046	0.070	-0.002
μ16	-0.002	-0.010	-0.008	-0.001	-0.002	-0.001	-0.004	-0.002	0.009	-0.016	-0.002	0.011	0.031	0.007	0.067	0.215	0.198	-0.265	-0.163	0.083	-0.017	0.019	-0.030	0.004	*)	0.056	0.082	0.012
μ17	-0.002	-0.010	-0.008	-0.002	-0.002	-0.001	-0.005	-0.003	0.005	-0.016	-0.003	0.008	0.026	0.007	0.060	0.052	0.319	-0.201	-0.129	0.059	-0.014	0.008	-0.025	0.000	*)	0.037	0.055	0.004
μ18	-0.051	-0.107	-0.092	-0.090	-0.027	-0.037	-0.086	-0.072	-0.224	-0.119	-0.154	-0.150	-0.171	-0.034	-0.186	-0.148	-0.137	1.453	0.151	-0.296	-0.079	-0.288	-0.109	-0.127	*)	-0.213	-0.276	-0.226
μ19	-0.017	-0.042	-0.036	-0.028	-0.010	-0.012	-0.030	-0.024	-0.059	-0.052	-0.048	-0.034	-0.083	-0.015	-0.063	-0.046	-0.045	0.111	0.376	-0.251	-0.060	-0.156	-0.088	-0.072	*)	-0.083	-0.100	-0.124
μ20	0.008	0.010	0.009	0.016	0.002	0.006	0.012	0.011	0.053	0.005	0.028	0.041	-0.001	0.001	0.028	0.027	0.023	-0.244	-0.105	0.468	0.014	0.112	0.015	0.046	*)	0.114	0.156	0.086
μ21	-0.004	-0.013	-0.011	-0.006	-0.003	-0.003	-0.008	-0.006	-0.006	-0.018	-0.010	-0.001	-0.032	-0.006	-0.016	-0.010	-0.010	-0.119	-0.046	0.012	1.552	-0.011	-0.014	-0.006	*)	0.003	0.007	-0.009
μ22	0.000	-0.007	-0.006	0.003	-0.002	0.001	-0.001	0.001	0.021	-0.015	0.005	0.020	-0.031	-0.005	0.000	0.004	0.002	-0.176	-0.048	0.039	-0.007	0.256	-0.023	0.018	*)	0.087	0.124	0.039
μ23	-0.005	-0.014	-0.012	-0.007	-0.003	-0.003	-0.009	-0.006	-0.010	-0.019	-0.012	-0.004	-0.032	-0.006	-0.017	-0.012	-0.012	-0.109	-0.045	0.008	-0.015	-0.009	0.686	-0.012	*)	-0.002	0.001	-0.019
μ24	0.000	-0.007	-0.006	0.001	-0.002	0.000	-0.002	0.000	0.013	-0.012	0.002	0.013	-0.025	-0.004	-0.002	0.002	0.000	-0.134	-0.039	0.028	-0.006	0.008	-0.008	0.772	*)	0.034	0.049	0.013
μ25	0.005	0.007	0.006	0.010	0.002	0.004	0.008	0.007	0.032	0.005	0.017	0.024	0.002	0.001	0.018	0.016	0.014	-0.062	-0.002	0.030	0.008	0.023	0.006	0.017	-0.445	*)	*)	*)
μ26	0.003	0.001	0.001	0.008	0.000	0.003	0.004	0.005	0.029	-0.004	0.013	0.024	-0.013	-0.002	0.011	0.012	0.010	-0.121	-0.024	0.037	0.002	0.021	-0.001	0.014	0.021	0.225	0.143	0.066
μ27	0.007	0.003	0.003	0.015	0.001	0.005	0.009	0.010	0.056	-0.005	0.026	0.045	-0.020	-0.002	0.023	0.024	0.020	-0.211	-0.039	0.068	0.006	0.040	0.001	0.027	0.039	0.043	0.403	0.094
μ28	0.000	-0.011	-0.009	0.003	-0.003	0.001	-0.002	0.000	0.028	-0.021	0.006	0.027	-0.045	-0.007	-0.001	0.005	0.002	-0.249	-0.070	0.054	-0.010	0.018	-0.014	0.011	0.033	0.029	0.055	0.842

*) The estimated variance of the preference variable is negative, and hence the correlation coefficients can not be calculated.

Table 5. Distributional measures of the measurement errors v. Relative covariation of measurement errors in the lower triangle (relative variation along the main diagonal) and correlation coefficients in the upper triangle (excluding the main diagonal)

	V01	V02	V03	V04	V05	V06	V07	V08	V09	V10	V11	V12	V13	V14	V15	V16	V17	V18	V19	V20	V21	V22	V23	V24	V25	V26	V27	V28
V01	0.219	0.061	0.098	0.109	0.291	0.223	0.156	0.252	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V02	0.023	0.622	0.024	0.027	0.071	0.054	0.038	0.062	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V03	0.041	0.017	0.805	0.042	0.113	0.087	0.061	0.098	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V04	0.053	0.022	0.040	1.103	0.126	0.096	0.067	0.109	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V05	0.049	0.020	0.036	0.047	0.129	0.257	0.180	0.291	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V06	0.078	0.032	0.058	0.075	0.069	0.558	0.138	0.223	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V07	0.038	0.015	0.028	0.036	0.033	0.053	0.267	0.156	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V08	0.059	0.024	0.044	0.057	0.052	0.083	0.040	0.248	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V09	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.953	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.340	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.679	0.206	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.327	3.704	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V13	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.496	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.093	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.150	0.109	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.254	2.516	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.107	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	4.189	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.136	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.983	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.168	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.562	0.000	0.000	0.000	0.000	0.000	0.000
V23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.209	0.000	0.000	0.000	0.000	0.000
V24	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.054	0.000	0.000	0.000	0.000
V25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.516	0.000	0.000	0.000
V26	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.658	0.000	0.000
V27	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	4.140	0.000
V28	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.257

5.4 Distribution of measurement errors

Table 5 contains summary characteristics of the distribution of the vectors of commodity specific measurement errors, v, supplementing the relative variation statistics RV_v reported in table 2, column 6. The statistics in table 5 are renormalizations of the elements of the estimated I × I matrix Σ_{vv}, with different normalizations used below and above the main diagonal. Table 5 is constructed in a similar way as table 3. Note that all statistics in this table, like those in tables 3 and 4, refer to the specification denoted as R (restricted, reference) in section 4. All the basic free parameter estimates characterizing the distribution of measurement errors are reported in table A5 and A6 in appendix B.

Element i on the main diagonal of table 5 is the estimated variance of v_i normalized against the squared (global) sample mean of expenditure on commodity i - i.e. it is a sort of a squared coefficient of variation of measurement errors. (The main diagonal of table 5 thus contains the square of the entries in the sixth column of table 2.) Below the main diagonal are given the corresponding normalized covariances, i.e. the figure in position (i,j, i > j) is the estimated covariance between v_i and v_j, divided by the product of the overall sample mean of expenditures on commodities i and j. Above the main diagonal are reported the estimated simple coefficients of correlation between v_i and v_j (i < j). Hence all entries in table 5, like those in tables 3 and 4, are dimensionless numbers.

According to our R specification of the model, Σ_{vv} is a block diagonal matrix (cf section 4). Observe that all the estimates within each block come out with positive values. This implies that no negative estimates occur in table 5. In particular, we find positive estimated correlation between the measurement errors of all food commodities - with coefficients of correlation varying between 0.04 and 0.25 - which was not imposed as an a priori restriction. This support our purchase behavior argument above for food commodities. The measure of relative variation, reported along the main diagonal of table 5, have quite large values for the three most prominent commodities containing durables, i.e. 15 Furniture, 16 Household equipment, and 18 Motorcars, bicycles (2.150, 2.516, and 4.189, respectively), but we also find large estimates for 12 Footwear (3.704) (which also is often assumed to have some of the characteristics of durables), 21 PTT charges (8.186), 25 Medical care (5.516), and 27 Misc. goods and services (4.140). Medical care also occurred with a negative estimate of the preference variance in tables 3 and 4, which may not be a coincidence.

In several ways, special attention has been devoted to purchase of Motorcars and bicycles (commodity 18), which is the most typical durable good in our data set. (Note that Housing is measured by rent.) An overview of the specific restrictions for Motorcars and bicycles in the R specification are presented in table 1 (part 3 and 4). *First*, we allow for a systematic component of the measurement error (purchase resdiual) in year 2, represented by the parameter $a_{B18,2}$. Macroeconomic statistics indicate that a boom period started in our observation period with a general upswing in the purchase of cars form the first to the last observation. Thus we expect $a_{B18,2}$ to be postive, and this hypothesis is confirmed by the significant estimate presented in table A6. *Second*, for this major durable good, allowance has been made for a non-zero autocovariance ($cov(v_{18,1},v_{18,2})$) between the measurement errors. A priori, this autocovariance is conjectured to be negative since for instance purchase of a car in the first period will most likely not be followed by a another purchase in the subsequent period. Table A6 reveals that a significantly negative autocovariance for the measurement errors is obtained, confirming our conjecture.

Third, we allow for covariation between the measurements error of 18 Motorcars and bicycles and the volatile component of latent total expenditure ($cov(v_{18,t},u_t)$) and $cov(v_{18,2},u_1)$). Our hypothesis here is that a positive surprise in 'income' is associated with an immediate as well as a delayed increase in the purchase of (investment in) Motorcars and bicycles. Again, our hypotheses are supported by significant estimates reported in table A6.

5.5 Distribution of latent total expenditure

Table A6 presents estimates of parameters related to the distribution of latent total expenditure. The ratio between the variance of the permanent and the volatile component of latent total expenditure ($\sigma_{\chi\chi}/\sigma_{uu}$) is almost as high as 25, but the variance of the volatile component is nevertheless significantly positive, indicating some change in the ranking of households according to total expenditure from observation period 1 to period 2. Significantly positive estimates are also obtained for the covariance between the permanent component of latent total expenditure and the number of children and adults respectively ($\sigma_{\chi z1}$ and $\sigma_{\chi z2}$). The time specific intercept term in period 1, q_{01}, has been set to 0 a priori in order to identify the model parameters. For the second period, a significantly negative estimate of the intercept term, q_{02}, is obtained, which means that the coefficient of variation of ξ increases from the first to the second period. The growth factor of latent total expenditure, q_2, significantly exceeds unity and yields, despite the negative shift in the intercept term, a significant growth in expected latent total expenditure ($\Phi_{\xi 2}$ - $\Phi_{\xi 1}$) among Norwegian households in the period 1975-1977. Note, however, that there is a significantly positive discrepancy between the expectation of observed total expenditure and the expectation of latent total expenditure in the second period, corresponding to the estimated systematic component of the measurement error (purchase residual) for 18 Motorcars and bicycles in year 2 ($a_{B18,2}$), cf section 5.4.

6. Conclusions

In this paper, we have analyzed, using a multivarate errors-in-variables approach, a complete system of Engel functions, including household size and composition as covariates, for 28 disaggregate commodities by means of a two wave panel data set for 408 Norwegian households. In each Engel function, a random household specific effect, interpreted as a preference variable is included. The covariance matrix of these preference variables is structured using an approach based on utility trees. Furthermore, each component of the observed consumption expenditure is assumed to contain a random measurement error, mainly interpreted as a purchase residual, i.e. a difference between observed purchase expenditure in the short registration period and the latent consumption defined in the context of a consumer theory with nondurable as well as durable goods. The contemporaneous covariance matrix of these purchase residuals is structured, using a factor analytic approach with an interpretation including joint shopping behavior for groups of goods. Dynamic features of purchases of durables are taken into account. The model also includes, apart from 28 commodity specific consumption expenditure variables, two income variables observed from tax records, used in identifying and estimating the demand system, and analyzing jointly the distribution of (a) latent total consumption expenditure, (b) the latent preference variables, and (c) the measurement errors. Specifying and analyzing the demand system in this way, at such a disaggregate level while using panel data, is, as far as we know, a novel feature of our approach.

The basic data input of our analysis is a 62 dimensional vector of observed variables, giving a number of first and second order sample moments as large as 2015. A data matrix of this size has in fact "worked" in the present context, within the framework of a full information maximum likelihood estimation with errors in variables. Within a class of systematically specified models, a carefully designed parsimonious model with 213 free parameters gave the best fit according to the Akaike Information Criterion (AIC). This is used as a reference model and is completely documented in the paper. Almost all the estimates of the 213 structural parameters have the expected signs and reasonable magnitudes.

The empirical study has thus confirmed our conjecture that this type of latent variable approach is fruitful for econometric analysis of surveys of household expenditures, at a disaggregate level of commodity classification.

Appendix A: Engel functions, preference varation, and two level Stone-Geary utility

The linear expenditure function

Assume that the I commodities in the household's budget are divided into G groups, and let I_g be the number of commodities in group $g \left(g = 1,\ldots,G; \Sigma_g I_g = I\right)$. The utility function U has the following Stone-Geary form in the group specific 'sub-utilities' $U_1,\ldots,U_g$

$$\text{(A.1)} \quad U = \prod_{g=1}^{G} \left(\frac{U_g - \gamma_g}{\beta_g} \right)^{\beta_g}, \quad \beta_g > 0, \quad U_g > \gamma_g, \quad g = 1,\ldots,G,$$

where β_g and γ_g are unknown parameters. Here we can assume, without loss of generality, that

$$\text{(A.2)} \qquad \sum_g \beta_g = 1,$$

since $V = U^{1/\Sigma_g \beta_g}$ is an equally valid (ordinal) representation of the household's preferences as U. The 'sub-utility' functions U_g are also assumed to have the Stone-Geary form

$$\text{(A.3)} \quad U_g = \prod_{i=1}^{I_g} \left(\frac{\eta_{ig} - \gamma_{ig}}{\beta_{ig}} \right)^{\beta_{ig}}, \quad \beta_{ig} > 0, \quad \eta_{ig} > \gamma_{ig}, \quad i = 1,\ldots,I_g, \quad g = 1,\ldots,G,$$

where η_{ig} is the quantity consumed of commodity i in group g, denoted as commodity (i, g) for short, and β_{ig} and γ_{ig} are unknown parameters. We also assume that

$$\text{(A.4)} \quad \sum_i \beta_{ig} = 1, \qquad g = 1,\ldots,G,$$

which imply that all the 'sub-utilities' are homogeneous of degree one in the 'supernumerary consumption' $\eta_{ig} - \gamma_{ig}$ of all commodities. In contrast to (A.2), (A.4) implies restrictions on the demand functions and it will substantially facilitate the model formulation in the sequel. Note also the crucial role played by the non-zero 'minimum consumption' parameters γ_g in the upper utility function (A.1). If all $\gamma_g = 0$, (A.1) and (A.3) would imply an overall utility function having the Stone-Geary form in *all* the I quantities consumed,

$$U = \prod_g \beta_g^{-\beta_g} \left[\prod_i \left(\frac{\eta_{ig} - \gamma_{ig}}{\beta_{ig}} \right)^{\beta_{ig}} \right]^{\beta_g} = \prod_g \prod_i \left(\frac{\eta_{ig} - \gamma_{ig}}{\beta_g \beta_{ig}} \right)^{\beta_{ig}\beta_g}.$$

Note also that (A.1) and (A.3) are equivalent to representing the preferences in the more familiar form

$$U^* = \prod_g \left(U_g^* - \gamma_g^*\right)^{\beta_g}, \quad U_g^* = \prod_i \left(\eta_{ig} - \gamma_{ig}\right)^{\beta_{ig}}.$$

The proof is the following: From the latter equations we have

$$U^* = \prod_g \beta_g{}^{\beta_g} \left(\frac{U_g^* - \gamma_g^*}{\beta_g} \right)^{\beta_g},$$

$$U_g^* = \prod_i \beta_{ig}{}^{\beta_{ig}} \left(\frac{\eta_{ig} - \gamma_{ig}}{\beta_{ig}} \right)^{\beta_{ig}},$$

which when

$$\gamma_g^* = \prod_i \beta_{ig}{}^{\beta_{ig}} \gamma_g,$$

using (A.3), implies

$$U_g^* - \gamma_g^* = \left[\prod_i \beta_{ig}{}^{\beta_{ig}} \right] \left(U_g - \gamma_g\right).$$

Hence, U^* is proportional to U, as defined by (A.1) and (A.3), with a factor of proportionality equal to

$$\prod_g \beta_g{}^{\beta_g} \left[\prod_i \beta_{ig}{}^{\beta_{ig}} \right].$$

When all prices are normalised to one, the budget constraint of the household can be written as

$$\text{(A.5)} \quad \xi_g = \sum_{i=1}^{I_g} \eta_{ig}, \qquad g = 1, \ldots, G,$$

$$\text{(A.6)} \quad \xi = \sum_{g=1}^{G} \xi_g,$$

where ξ is total consumption expenditure and ξ_g is the part of it allocated to group g. A set of necessary conditions for U to be maximised with respect to all η_{it}, subject to (A.5) - (A.6) with ξ given, is that all U_g are maximised with respect to $\eta_{1g}, \ldots, \eta_{I_g g}$, subject to (A.5), with ξ_g given. The solution to this sub-problem, paying regard to (A.4) and assuming an interior solution, is described by the (conditional) within group, linear expenditure functions

(A.7) $\eta_{ig} - \gamma_{ig} = \beta_{ig}(\xi_g - m_g), \quad i = 1,\ldots,I_g, \quad g = 1,\ldots,G,$

where m_g is 'aggregate minimum consumption' of group g,

(A.8) $m_g = \sum_i \gamma_{ig}, \qquad g = 1,\ldots,G.$

From this it follows that maximal utility of group g is simply equal to the 'supernumerary expenditure' on this group, since (A.3), (A.4), and (A.7) imply

(A.9) $U_g = \xi_g - m_g, \qquad i = 1,\ldots,I_g, g = 1,\ldots,G.$

Substituting (A.9) in (A.1), it follows that the overall utility conditional on group specific utility maximization, for given group expenditures $\xi_1,\ldots,\xi_G$, is equal to

(A.10) $U = \prod_{g=1}^{G} \left(\frac{\xi_g - m_g - \gamma_g}{\beta_g} \right)^{\beta_g}.$

The remarkable property of this 'partially maximised' utility function is that it has exactly the same Stone-Geary form as (A.1) and (A.3), with $m_g + \gamma_g = \Sigma_i \gamma_{ig} + \gamma_g$ now interpreted as the 'minimum consumption of group g'. The upper (overall) utility maximisation of the household is then obtained by maximising (A.10) with respect to $\xi_1,\ldots,\xi_G$ subject to (A.6). The solution to this problem, using (A.4), is, in complete analogy to (A.7), given by the G group specific, linear expenditure functions

(A.11) $\xi_g - m_g - \gamma_g = \beta_g(\xi - m - M), \qquad g = 1,\ldots,G,$

where

(A.12) $m = \sum_g m_g = \sum_g \sum_i \gamma_{ig},$

(A.13) $M = \sum_g \gamma_g.$

Eqs. (A.11) say that the overall 'supernumerary expenditure', defined as $\xi - m - M$, whose components are given by (A.6), (A.12), and (A.13), are allocated on the G groups according to the group specific marginal budget shares β_g. Note also that the overall unconditional maximal utility is equal to the 'supernumerary expenditure', since (A.4), (A.10), and (A.11) imply

(A.14) $U = \xi - m - M = \sum_g \xi_g - \sum_g m_g - \sum_g \gamma_g = \sum_g \sum_i \eta_{ig} - \sum_g \sum_i \gamma_{ig} - \sum_g \gamma_g.$

Using (A.11) to eliminate $\xi_g - m_g$ in (A.7), we find that the commodity specific, linear expenditure functions can be written as

$$\eta_{ig} - \gamma_{ig} = \beta_{ig}\left[\beta_g(\xi - m - M) + \gamma_g\right],$$

or

$$\text{(A.15)} \quad \eta_{ig} - \gamma_{ig} - \beta_{ig}\gamma_g = \beta_{ig}\beta_g(\xi - m - M), \; i = 1,\ldots,I_g, \quad g = 1,\ldots,G.$$

Here, minimum consumption of commodity (i,g) has two additive components, the first, γ_{ig}, representing commodity specific minimum consumption [cf. (A.3)], the second, $\beta_{ig}\gamma_g$, being the share β_{ig} of the group specific minimum consumption [cf. (A.1)]. Likewise, the marginal budget share of commodity (i,g) has two multiplicative components, the first, β_{ig}, being the within group commodity specific marginal budget share, the second, β_g, being the marginal budget share specific to group g.

Demographic specification

In the econometric specification of the model, the minimum consumption parameters γ_g and γ_{ig} are not assumed to be constants, as implied by the above description, but are specified as functions of household characteristics in the following way

$$\text{(A.16)} \quad \gamma_g = \beta_g\left(\bar{a}_g + \bar{c}_g z + \bar{\alpha}_g\right), \qquad g = 1,\ldots,G,$$

$$\text{(A.17)} \quad \gamma_{ig} = \underline{a}_{ig} + \underline{c}_{ig} z + \underline{\alpha}_{ig}, \qquad i = 1,\ldots,I_g; \; g = 1,\ldots,G.$$

Here z is a $M \times 1-$vector of demographic variables, $\beta_g\bar{c}_g$ and $\underline{c}_{ig}$ are $1 \times M-$vectors representing their effect on minimum consumption of group g and of commodity (i,g), respectively, $\beta_g\bar{a}_g$ and $\underline{a}_{ig}$ are corresponding intercept terms, and $\beta_g\bar{\alpha}_g$ and $\underline{\alpha}_{ig}$ are stochastic variables representing (unmeasured) household specific variation in preferences affecting minimum consumption. (We use 'underscore' and 'overscore' to symbolise disaggregate commodities and aggregate groups, respectively).

From (A.16) and (A.17) it follows that the composite minimum consumption parameters in (A.15) can be expressed in terms of the demographic effects and the preference variables as

$$\text{(A.18)} \quad \gamma_{ig} + \beta_{ig}\gamma_g = a^*_{ig} + c^*_{ig} z + \alpha_{ig}, \qquad i = 1,\ldots,I_g; \; g = 1,\ldots,G,$$

and their aggregates as [cf. (A.12) and (A.13)]

$$\text{(A.19)} \quad m + M = \sum_g \sum_i \gamma_{ig} + \sum_g \sum_i \beta_{ig}\gamma_g = \sum_g \sum_i a^*_{ig} + \sum_g \sum_i c^*_{ig} z + \sum_g \sum_i \alpha_{ig},$$

where

$$\text{(A.20)} \quad a^*_{ig} = \underline{a}_{ig} + \beta_{ig}\beta_g\bar{a}_g,$$

$$\text{(A.21)} \quad c^*_{ig} = \underline{c}_{ig} + \beta_{ig}\beta_g\bar{c}_g,$$

$$\text{(A.22)} \quad \alpha_{ig} = \underline{\alpha}_{ig} + \beta_{ig}\beta_g\bar{\alpha}_g, \qquad i = 1,\ldots,I_g; \; g = 1,\ldots,G.$$

Let now b_{ig} be the marginal budget share of commodity (i,g) relative total expenditure, i.e.

(A.23) $b_{ig} = \beta_{ig}\beta_g, \qquad i = 1,\ldots,I_g; \quad g = 1,\ldots,G,$

let a^*, b, α, and η denote the $I \times 1$ vectors of a^*_{ig}, b_{ig}, α_{ig}, and η_{ig} ordered first by group, second by commodity, and let C^* denote the $I \times M$ matrix of c_{ig} similarly ordered. We can then write (A.15) as

(A.24) $\eta = a + b\xi + Cz + \mu,$

where

(A.25) $a = (I - b\iota')a^*, \quad C = (I - b\iota')C^*.$

and

(A.26) $\mu = (I - b\iota')\alpha.$

Eq. (A.24) is identical to (1) in the main text, when the time subscript t is added to η and ξ.

Appendix B. Observed moments and estimated parameters

Table A1. Mean of the observed variables. The y and w variables are measured in 1000 Norwegian 1974 kroner

y1,1	0.9441	y9,1	0.9749	y17,1	0.7891	y25,1	0.5648
y1,2	0.9022	y9,2	1.0093	y17,2	0.8962	y25,2	0.5953
y2,1	2.4093	y10,1	0.6392	y18,1	2.4094	y26,1	0.7505
y2,2	2.4252	y10,2	0.6205	y18,2	3.6250	y26,2	0.8115
y3,1	0.6314	y11,1	3.5680	y19,1	3.3681	y27,1	0.4825
y3,2	0.6552	y11,2	3.6852	y19,2	3.6245	y27,2	0.6260
y4,1	0.2281	y12,1	0.8207	y20,1	0.9605	y28,1	1.1824
y4,2	0.2286	y12,2	0.7512	y20,2	0.8733	y28,2	1.2708
y5,1	1.1714	y13,1	4.4736	y21,1	0.4648	w1,1	38.0961
y5,2	1.2044	y13,2	4.8672	y21,2	0.6846	w1,2	41.9946
y6,1	0.3632	y14,1	1.4001	y22,1	2.6391	w2,1	55.1871
y6,2	0.3587	y14,2	1.4328	y22,2	2.9971	w2,2	58.5761
y7,1	1.7547	y15,1	2.1122	y23,1	1.3293	z1	0.8039
y7,2	1.7400	y15,2	2.1143	y23,2	1.2790	z2	2.2255
y8,1	1.4418	y16,1	1.1186	y24,1	0.7586		
y8,2	1.4214	y16,2	1.2683	y24,2	0.7613		

Table A2. Covariance matrix of the 62 observed variables. The y and w variables are measured in 1000 Norwegian 1974 kroner

	y1,1	y1,2	y2,1	y2,2	y3,1	y3,2	y4,1	y4,2
y1,1	0.427136							
y1,2	0.215846	0.361314						
y2,1	0.447133	0.338968	6.195869					
y2,2	0.330587	0.381913	2.352823	5.927368				
y3,1	0.101367	0.074592	0.484935	0.360822	0.582443			
y3,2	0.079513	0.081344	0.359615	0.311137	0.262870	0.612832		
y4,1	0.052930	0.029663	0.153327	0.106304	0.019004	0.008113	0.064346	
y4,2	0.036874	0.051156	0.166520	0.101980	0.016381	0.038021	0.021663	0.094324
y5,1	0.234593	0.185987	0.502650	0.523929	0.094047	0.070603	0.048213	0.048528
y5,2	0.175702	0.212662	0.364057	0.563827	0.064085	0.098708	0.033098	0.047947
y6,1	0.079471	0.067788	0.135215	0.202977	0.032323	0.028702	0.015086	0.014121
y6,2	0.065777	0.080854	0.172397	0.153049	0.044956	0.051698	0.008166	0.018395
y7,1	0.353961	0.242747	1.227227	1.040389	0.258460	0.138616	0.089245	0.070465
y7,2	0.312377	0.363698	0.870247	0.972351	0.232592	0.196047	0.076412	0.097277
y8,1	0.331429	0.262178	0.913087	0.750300	0.115938	0.054115	0.073660	0.071229
y8,2	0.254850	0.297154	0.682847	0.907944	0.099227	0.146415	0.055003	0.080600
y9,1	0.141597	0.119491	0.634335	0.607859	0.136841	0.170550	0.069735	0.054495
y9,2	0.144206	0.176706	0.760337	0.678932	0.130838	0.266100	0.072587	0.100262
y10,1	0.074411	0.083229	0.363942	0.240242	0.026433	-0.003523	0.040347	0.029372
y10,2	0.075877	0.094914	0.283974	0.274324	0.012733	0.021640	0.038875	0.042145
y11,1	0.608608	0.680887	1.977496	1.756145	0.500960	0.280724	0.069624	0.129389
y11,2	0.720136	0.837372	1.704445	2.405438	0.332580	0.249114	0.177646	0.154668
y12,1	0.198212	0.146368	0.641767	0.247570	0.078971	0.137662	0.045076	0.086069
y12,2	0.150846	0.177070	0.510405	0.747502	0.077012	0.118241	0.058826	0.041654
y13,1	0.367728	0.475860	1.903638	2.137532	0.403810	0.314366	0.078118	0.150814
y13,2	0.636749	0.661392	1.453831	2.287043	0.370971	0.197202	0.146464	0.074140
y14,1	0.089388	0.089491	0.224936	0.335978	0.048241	0.038289	0.017807	0.012949
y14,2	0.056190	0.082186	0.153859	0.324358	0.036292	0.043498	0.013441	0.018260
y15,1	0.260408	0.189862	1.619963	1.676250	0.193207	-0.081491	0.084808	0.062843
y15,2	0.278470	0.288308	1.915437	1.573451	0.098452	0.112788	0.123753	0.165548
y16,1	0.123534	0.121455	0.454470	0.656917	0.152683	-0.035258	0.028269	0.007255
y16,2	0.301779	0.265868	0.895446	1.091201	0.158493	0.063752	0.077476	0.103981
y17,1	0.093732	0.107370	0.407848	0.386707	0.070725	0.067025	0.038174	0.031942
y17,2	0.096436	0.126590	0.359779	0.790188	0.105674	0.094436	0.019434	-0.000075
y18,1	0.310660	0.540626	1.764707	0.514369	0.345963	0.491578	0.120769	0.262373
y18,2	0.686912	0.798792	1.104591	-0.289245	-0.266758	0.048237	0.146226	0.268858
y19,1	0.745686	0.598266	1.255188	1.237085	0.315214	0.034619	0.257504	0.109228
y19,2	0.739685	0.953740	1.540204	2.870788	0.201595	0.254546	0.299850	0.197974
y20,1	0.058885	0.047236	0.614615	0.395892	0.162397	0.044791	0.014374	-0.004469
y20,2	0.175397	0.106720	0.365211	0.527696	0.259743	0.104033	0.058696	0.025904
y21,1	0.019657	-0.055066	0.119160	-0.219854	0.129134	0.034264	0.013684	-0.000886
y21,2	0.035148	0.086750	0.305424	0.403742	0.113313	0.171832	0.061025	0.025854
y22,1	0.401503	0.451754	0.698359	1.109823	0.532398	0.327306	0.086495	0.162542
y22,2	0.488683	0.546365	1.066441	2.177611	0.773086	0.552146	0.111655	0.193024
y23,1	0.199597	0.169699	0.399393	0.524206	0.183446	0.074765	0.051995	0.047499
y23,2	0.296106	0.236096	0.976622	1.055177	0.133732	0.159186	0.057054	0.060495
y24,1	0.075114	0.071795	0.386358	0.127214	0.102322	0.090446	0.016558	0.052296
y24,2	0.123869	0.117612	0.399091	0.435909	0.131327	0.129270	0.035111	0.048885
y25,1	0.134880	0.079410	0.131775	0.122445	0.159581	0.062275	0.032666	0.028516
y25,2	0.082998	0.070279	0.102520	0.228412	0.053773	0.078633	0.018766	-0.010712
y26,1	0.162022	0.137765	0.476244	0.479473	0.060101	-0.001384	0.036935	0.040558
y26,2	0.165238	0.166283	0.306243	0.560734	0.046018	0.038846	0.034788	0.072499
y27,1	0.145432	0.133652	0.190060	0.205906	0.054310	0.068062	0.016744	0.046049
y27,2	0.139223	0.150787	0.354245	0.344939	0.081743	0.123160	0.059581	0.057088
y28,1	0.197866	0.250193	0.531256	0.358649	0.078163	0.150698	0.054675	0.035084
y28,2	0.249810	0.297201	0.597367	0.638176	0.061511	0.137085	0.128271	0.081958
w1,1	3.686286	3.649951	14.747439	11.784499	3.770911	2.580784	0.980665	0.731203
w1,2	4.447128	4.671882	15.633292	14.028107	3.666255	2.610044	0.974740	0.839868
w2,1	8.290483	8.161745	29.708865	22.260621	5.276844	3.741777	2.168640	1.800650
w2,2	9.127864	9.299621	30.644108	24.786522	5.292704	3.649991	2.097247	1.994525
z1	0.324077	0.306573	0.642529	0.853324	0.037979	0.026310	0.062642	0.049138
z2	0.223900	0.205832	0.570418	0.531847	0.110106	0.105979	0.032476	0.052950

Table A2 cont.

	y5,1	y5,2	y6,1	y6,2	y7,1	y7,2	y8,1	y8,2
y5,1	0.484955							
y5,2	0.335040	0.528162						
y6,1	0.106057	0.075483	0.104105					
y6,2	0.072855	0.093311	0.035325	0.102894				
y7,1	0.364443	0.317306	0.099052	0.111683	1.809764			
y7,2	0.331181	0.393130	0.092386	0.143412	0.930900	1.736025		
y8,1	0.354440	0.295307	0.110413	0.082914	0.589954	0.571037	0.965470	
y8,2	0.296788	0.359653	0.089301	0.126057	0.406964	0.687808	0.530943	1.076546
y9,1	0.124535	0.083342	0.048788	0.019857	0.320583	0.326421	0.277546	0.206204
y9,2	0.082838	0.103052	0.042193	0.044460	0.508087	0.510123	0.306622	0.329679
y10,1	0.065073	0.044554	0.047463	0.021095	0.066758	0.104331	0.136601	0.136018
y10,2	0.069647	0.053835	0.056163	0.021116	0.046265	0.149589	0.146937	0.184646
y11,1	0.732257	0.679430	0.133420	0.212658	1.172686	1.262116	0.986399	1.175690
y11,2	0.757397	0.807880	0.290056	0.217034	1.275667	1.509832	1.325877	1.476556
y12,1	0.217654	0.205419	0.053381	0.093897	0.368638	0.407375	0.224308	0.337899
y12,2	0.238317	0.227555	0.086333	0.087007	0.188219	0.350962	0.236676	0.275545
y13,1	0.365884	0.493414	0.187954	0.105770	1.086846	1.297630	0.581093	0.512415
y13,2	0.478336	0.435511	0.241425	0.146591	1.341249	1.467293	0.575521	0.461947
y14,1	0.116513	0.128655	0.035929	0.037103	0.244887	0.231712	0.083741	0.146831
y14,2	0.097531	0.085607	0.031577	0.027859	0.180538	0.190753	0.047882	0.107697
y15,1	0.228798	0.226885	0.063631	0.040309	0.642562	0.576306	0.407270	0.430415
y15,2	0.187924	0.312478	0.132675	0.089840	0.763141	0.810256	0.592303	0.851553
y16,1	0.125256	0.058536	0.043718	0.033683	0.271185	0.254564	0.437794	0.298019
y16,2	0.263443	0.193268	0.107034	0.052499	0.627089	0.543818	0.344552	0.378583
y17,1	0.127928	0.108071	0.044101	0.021841	0.326347	0.255686	0.241844	0.193395
y17,2	0.152811	0.175570	0.052882	0.021186	0.357001	0.349445	0.222967	0.202988
y18,1	0.354514	0.484757	-0.047274	-0.029245	1.544741	1.154419	0.256072	0.394121
y18,2	0.815935	0.523891	0.145167	0.045506	0.438943	0.918298	0.606049	1.346081
y19,1	0.765879	0.626910	0.176933	0.075536	1.262196	1.537117	1.256256	1.054028
y19,2	0.776783	0.799447	0.267425	0.254489	1.527395	1.758876	1.132432	1.503112
y20,1	-0.010049	0.085136	0.027632	0.022539	0.149715	0.140380	0.119342	0.062917
y20,2	0.147643	0.174311	0.046835	-0.008325	0.300455	0.229587	0.173785	0.195420
y21,1	-0.049609	-0.035968	-0.021119	-0.028828	-0.119262	-0.086184	0.046299	-0.033715
y21,2	0.046303	0.009390	-0.007712	-0.000989	-0.008311	0.229414	0.044786	-0.040664
y22,1	0.498510	0.518619	0.194265	0.130526	0.492172	1.001870	0.391056	0.868533
y22,2	0.332570	0.283097	0.117525	0.172966	1.278879	1.316864	0.668035	0.844892
y23,1	0.115007	0.168626	0.067537	0.050434	0.359062	0.516310	0.240412	0.307572
y23,2	0.223148	0.258452	0.077690	0.085504	0.420812	0.488760	0.370256	0.483727
y24,1	0.117761	0.080896	0.024541	0.043754	0.160133	0.189680	0.121039	0.150371
y24,2	0.163186	0.144800	0.069045	0.051809	0.212855	0.259567	0.160677	0.201238
y25,1	0.132608	0.098576	0.045481	0.029090	0.259024	0.197265	0.223199	0.086636
y25,2	0.126628	0.073590	0.064476	0.031290	0.083884	0.208498	0.099253	0.035182
y26,1	0.153441	0.123283	0.046656	0.044614	0.295487	0.263973	0.222768	0.253732
y26,2	0.162000	0.162182	0.049600	0.037083	0.258627	0.281841	0.279300	0.265188
y27,1	0.057110	0.080569	0.046400	0.051381	0.251174	0.345190	0.231959	0.213433
y27,2	0.082204	0.127059	0.031841	0.047827	0.271875	0.243825	0.156681	0.289718
y28,1	0.145724	0.153209	0.028506	0.070240	0.376303	0.410499	0.365480	0.267437
y28,2	0.152292	0.158076	0.014421	0.041444	0.723554	0.733123	0.509301	0.418074
w1,1	4.204280	4.207836	1.062758	1.078043	9.670069	8.845030	4.589695	5.398347
w1,2	5.084901	4.906662	1.407518	1.195794	10.492962	11.436743	5.832009	6.469330
w2,1	9.547230	8.562005	2.380721	2.040070	20.454936	17.848346	10.758241	11.703213
w2,2	10.843207	9.902843	2.747409	2.323112	21.031683	20.812516	12.525737	13.375907
z1	0.448809	0.431143	0.116758	0.107830	0.635200	0.593469	0.417774	0.403947
z2	0.258218	0.270234	0.071444	0.081094	0.335901	0.379850	0.303321	0.364366

Table A2 cont.

	y9,1	y9,2	y10,1	y10,2	y11,1	y11,2	y12,1	y12,2
y9,1	1.812462							
y9,2	1.048725	2.227675						
y10,1	0.402626	0.343629	0.689307					
y10,2	0.343783	0.350653	0.552991	0.691208				
y11,1	1.336439	1.373546	0.293627	0.334665	16.084716			
y11,2	1.201851	1.728756	0.530458	0.450259	6.859487	16.067601		
y12,1	0.079275	0.277110	0.053041	0.069869	2.027074	0.870097	2.553365	
y12,2	0.312127	0.373275	0.187347	0.149672	1.592858	2.461253	0.234057	2.533371
y13,1	1.768395	1.919007	0.822483	1.005203	2.498082	3.995185	0.365755	0.815958
y13,2	1.496548	2.092350	0.461524	0.516046	1.765580	4.595734	0.260626	0.859875
y14,1	0.073141	0.105546	0.005101	0.037009	0.509808	0.481135	0.040732	-0.031063
y14,2	0.025744	0.070372	0.028001	0.068108	0.416885	0.260059	0.104693	-0.031191
y15,1	0.755208	0.990030	0.242024	0.252259	1.950618	2.799270	0.252609	0.742307
y15,2	0.607205	0.748948	0.042837	0.098836	1.698142	3.055021	0.285779	1.173184
y16,1	0.248369	0.458641	0.093007	0.054180	0.732659	1.501512	0.197610	0.356419
y16,2	0.548872	0.352137	0.124210	0.058991	1.427828	2.385702	0.028658	0.262306
y17,1	0.181657	0.220968	0.059415	0.040224	0.439596	0.835013	0.082261	0.132580
y17,2	0.243297	0.186730	-0.018306	0.036584	0.788759	1.001329	-0.019740	0.176534
y18,1	0.236730	0.404940	0.119704	0.154422	2.760425	2.316395	0.020046	-0.333032
y18,2	0.139230	0.689378	0.705800	0.506322	2.597394	7.090926	1.302234	0.829487
y19,1	0.444133	0.874229	0.499738	0.508530	4.340117	4.640512	0.689917	1.323512
y19,2	1.126975	1.677848	0.561609	0.715293	3.060600	5.837455	0.630641	1.406015
y20,1	0.340283	0.338778	0.110338	0.167184	0.982390	0.129856	0.049013	0.107437
y20,2	0.197674	0.186574	0.122654	0.152536	0.910046	1.168430	-0.026576	0.297776
y21,1	-0.021626	0.036382	0.117238	0.155086	-0.266515	-0.177794	-0.018857	-0.049729
y21,2	0.277385	0.308627	0.094730	0.119040	0.475883	0.101224	-0.029920	0.314454
y22,1	0.991696	0.990037	0.093846	0.177945	2.996817	3.625141	0.940683	1.757860
y22,2	1.107621	1.785226	-0.029612	0.033376	3.989144	5.126129	0.923942	1.418599
y23,1	0.193025	0.027792	0.212303	0.253489	1.123786	0.458085	0.229465	0.476598
y23,2	0.135059	0.530040	0.208094	0.236842	0.901609	1.309826	0.079682	0.446902
y24,1	0.143957	0.240610	0.076607	0.044409	0.499117	0.537713	0.287179	0.131811
y24,2	0.249703	0.410541	0.164482	0.179564	0.725978	0.829845	0.282345	0.290185
y25,1	-0.034928	0.219427	0.104829	0.043426	0.503914	0.321482	0.168781	0.017228
y25,2	0.144162	0.347858	0.039337	0.005882	0.468712	0.563971	0.024331	0.099671
y26,1	0.270671	0.256374	0.104194	0.081410	1.180518	0.916275	0.210210	0.132396
y26,2	0.283987	0.328194	0.130453	0.121732	1.115104	1.236383	0.196247	0.301877
y27,1	0.240424	0.389938	0.079623	0.070408	1.296183	1.284193	0.262961	0.401716
y27,2	0.212182	0.349199	0.076729	0.093717	0.745267	1.272247	0.300994	0.211781
y28,1	0.576820	0.603313	0.309592	0.379014	1.342466	1.302965	0.237965	0.632334
y28,2	0.637231	1.106493	0.342646	0.464224	1.257032	2.271035	0.216652	0.489025
w1,1	6.516722	7.570098	2.359725	2.301551	28.630329	25.744187	3.918987	6.122210
w1,2	6.785842	8.328030	3.287922	3.648857	32.067728	32.591347	3.590983	8.170610
w2,1	13.598981	15.665262	5.105025	5.225756	54.123168	54.377213	10.308591	12.719236
w2,2	13.164078	15.681642	6.470530	6.866261	59.306892	62.784618	10.460111	15.988889
z1	0.111255	0.150909	0.115328	0.107336	0.976635	1.024995	0.342344	0.264107
z2	0.185081	0.172618	0.106555	0.130044	1.129936	1.131239	0.204552	0.208609

Table A2 cont.

	y13,1	y13,2	y14,1	y14,2	y15,1	y15,2	y16,1	y16,2
y13,1	18.377960							
y13,2	10.970467	25.731114						
y14,1	0.462409	0.609248	0.561221					
y14,2	0.484263	0.496947	0.355142	0.522497				
y15,1	3.677958	2.555219	0.514514	0.547260	12.464296			
y15,2	2.607869	3.330039	0.316922	0.279522	2.724955	12.108100		
y16,1	1.303095	1.136984	0.062969	0.080167	1.659189	0.713237	2.892305	
y16,2	0.963832	0.689831	0.285469	0.261408	1.485110	1.487957	0.695272	5.657513
y17,1	1.030103	0.785381	0.106482	0.125018	0.643226	0.582223	0.379210	0.142305
y17,2	1.243407	0.868575	0.072518	0.061305	0.236826	0.323061	0.156472	0.355231
y18,1	2.346457	0.028898	0.635238	0.529393	0.898633	1.083444	0.159250	0.654645
y18,2	2.479082	2.395723	0.222403	0.104322	5.863044	2.414495	1.217064	0.975675
y19,1	3.796453	5.615823	0.535896	0.353072	2.179635	3.117533	1.551524	1.397527
y19,2	3.214735	5.867746	0.717492	0.577115	2.249649	2.850781	1.493061	1.981300
y20,1	1.287252	1.307587	0.070694	-0.035670	0.235167	0.311829	0.240898	0.120747
y20,2	1.184726	0.875138	0.108540	0.052046	0.586984	0.329629	0.025017	0.283031
y21,1	0.717671	0.688264	0.012337	0.019490	0.174436	0.077318	0.097543	-0.013753
y21,2	0.590204	0.183180	0.046101	0.012679	0.206960	-0.010012	-0.005716	-0.217927
y22,1	2.919130	1.722117	0.250761	0.446815	2.730675	1.198363	0.900066	0.847710
y22,2	4.687050	3.385340	0.231744	0.297263	3.176651	1.885656	1.153861	1.431837
y23,1	0.362323	0.050502	-0.002713	0.114845	0.301574	0.442641	0.145825	0.098973
y23,2	0.708421	0.865132	0.145932	0.109152	-0.128443	0.840438	0.176238	0.031885
y24,1	0.427244	0.550466	0.034071	-0.000290	0.337469	0.125237	0.112418	0.078441
y24,2	0.853486	0.764415	0.075441	0.025926	0.545775	0.284474	0.107481	0.438469
y25,1	0.462977	0.556901	-0.005517	0.020256	0.228135	0.083470	0.228910	0.367177
y25,2	0.775843	1.066878	0.017120	0.000655	0.061462	0.482628	-0.019484	-0.064129
y26,1	0.669397	0.683559	0.079289	0.066557	0.463467	0.473610	0.217408	0.277903
y26,2	0.899365	0.701640	0.103233	0.080433	0.256202	0.638077	0.183816	0.322863
y27,1	0.689010	0.654250	0.058157	0.023033	0.141740	0.377014	0.312419	0.951832
y27,2	0.833525	0.608419	0.079995	0.134958	0.650007	0.808923	0.307119	0.607137
y28,1	2.598127	1.736825	0.127657	0.149552	0.634707	0.742338	0.374912	0.347007
y28,2	2.564543	2.359979	0.272991	0.139255	1.158624	1.187184	0.301534	0.724400
w1,1	31.002630	32.088318	4.046238	3.220699	14.627533	13.419459	6.318074	8.670588
w1,2	30.714094	30.913966	4.776388	4.078605	17.798661	15.385928	6.394412	10.454714
w2,1	70.195567	73.590001	8.244607	6.516796	38.284618	28.454616	15.308700	18.458664
w2,2	69.327956	71.372646	8.656623	7.069177	40.164896	30.725505	14.684609	20.347327
z1	0.760777	1.205321	0.151190	0.113973	0.144571	0.274262	0.164620	0.345005
z2	0.543604	0.733229	0.161687	0.123757	0.413849	0.430484	0.181818	0.372113

Table A2 cont.

	y17,1	y17,2	y18,1	y18,2	y19,1	y19,2	y20,1	y20,2
y17,1	0.865051							
y17,2	0.367976	1.454571						
y18,1	-0.550311	0.185462	42.321240					
y18,2	0.768963	-0.013830	-3.549517	70.497476				
y19,1	0.673258	0.645238	7.909460	5.207505	20.608124			
y19,2	0.638114	1.030357	6.135592	6.658274	10.405217	29.357034		
y20,1	0.006267	0.147462	0.583955	0.466577	0.447015	0.285553	2.347898	
y20,2	0.168145	0.391481	-0.006621	0.363893	0.956465	0.977820	0.600307	2.261795
y21,1	0.106903	0.088494	-0.320137	0.824378	0.503760	-0.072699	0.302099	0.035396
y21,2	-0.006190	-0.042335	0.683535	-0.527489	0.711323	0.568597	0.105872	0.100482
y22,1	0.371131	0.121386	2.766209	6.025767	2.342478	3.401348	0.351722	0.447360
y22,2	0.792467	0.958758	1.278136	4.281799	3.241626	5.111525	0.659953	0.769313
y23,1	-0.089305	0.094599	2.030875	0.283300	1.916684	1.725576	0.160994	0.259270
y23,2	0.159395	0.288974	1.026545	0.778947	0.891752	1.258133	0.244824	0.573857
y24,1	0.057953	0.075814	0.180508	0.370818	0.541892	0.967244	0.168476	0.113241
y24,2	0.126044	0.122431	-0.389258	0.404276	0.504638	1.130257	0.311695	0.345185
y25,1	0.118537	0.084199	-0.172216	0.506836	0.830379	-0.020134	0.230650	0.231495
y25,2	0.088990	0.014838	0.338749	-0.692842	0.362984	0.295944	0.096539	0.020525
y26,1	0.148857	0.176590	0.508778	0.802064	0.740666	0.644328	0.228909	0.184961
y26,2	0.132706	0.188919	0.310640	0.847627	1.160204	0.934825	0.185623	0.245007
y27,1	0.161987	0.115415	0.155463	0.624217	0.868595	1.083557	0.232944	0.128018
y27,2	0.166882	0.106315	0.853925	-0.258747	0.591342	1.322526	0.070131	0.308471
y28,1	0.036594	0.249342	1.944943	1.375464	2.114427	1.948220	0.978443	0.630812
y28,2	0.234949	0.395951	1.616977	1.554452	1.950235	2.639448	0.651421	1.018454
w1,1	3.118751	3.530035	39.384934	33.261552	41.497216	30.824930	10.483736	9.320607
w1,2	3.692583	3.926261	37.403273	48.277211	45.492534	40.375783	11.899276	10.293927
w2,1	8.872891	6.607613	71.482399	64.223625	86.841590	72.276786	15.268546	14.209903
w2,2	8.428382	7.348459	59.695613	83.538204	90.558061	83.995284	17.065770	16.845313
z1	0.248429	0.243189	0.240442	0.710125	1.154147	1.482923	-0.124171	0.002141
z2	0.009855	0.092973	1.552026	1.355742	1.490222	1.352088	0.247418	0.316117

	y21,1	y21,2	y22,1	y22,2	y23,1	y23,2	y24,1	y24,2
y21,1	2.001218							
y21,2	0.568514	4.513210						
y22,1	0.418977	0.203978	17.019875					
y22,2	0.265664	0.103799	4.401599	17.339128				
y23,1	-0.028076	0.586164	0.584499	0.706120	3.485275			
y23,2	0.176551	0.359944	0.455756	0.634920	1.509828	3.708271		
y24,1	0.014315	0.156367	-0.010384	0.446875	0.335907	0.229959	1.165016	
y24,2	0.029727	0.154654	0.443645	0.817023	0.132213	0.381762	0.560366	1.194208
y25,1	0.202059	0.054662	0.243428	0.408966	0.115674	0.196247	-0.027870	0.037229
y25,2	0.054765	0.277609	0.642708	0.132192	-0.034847	0.176379	0.047068	0.233752
y26,1	-0.058248	0.012654	0.447052	0.732792	0.213112	0.190956	0.063144	0.117082
y26,2	-0.010104	0.131641	0.395347	0.841063	0.302820	0.262988	0.030181	0.179268
y27,1	0.002356	-0.032276	1.092529	1.250773	0.262252	0.262281	0.187264	0.265382
y27,2	-0.062922	0.044349	0.600663	1.111477	0.141626	0.147606	0.142658	0.085008
y28,1	0.247618	0.224882	1.284871	1.383912	0.857122	0.642892	0.218008	0.540904
y28,2	0.311985	0.252732	0.051693	1.877583	0.499322	0.674620	0.307359	0.579492
w1,1	0.582622	2.950945	22.680083	24.247857	9.627798	10.253033	3.399450	5.453724
w1,2	0.485522	4.195854	23.951243	28.502319	9.736279	11.102947	4.320699	6.714760
w2,1	-0.954281	4.498567	40.636344	50.339330	18.488781	17.734729	7.094430	10.739445
w2,2	-1.159432	6.228433	41.308530	56.367129	19.475650	19.511045	8.625426	12.183053
z1	-0.196622	0.030881	0.468999	0.595097	0.078154	0.242472	0.078209	0.108924
z2	0.008769	0.136294	0.659226	0.661151	0.424406	0.439685	0.143195	0.199206

	y25,1	y25,2	y26,1	y26,2	y27,1	y27,2	y28,1	y28,2
y25,1	1.390736							
y25,2	-0.096901	2.131410						
y26,1	0.118172	0.042011	0.688810					
y26,2	0.196806	0.120300	0.306136	0.744907				
y27,1	0.201570	-0.016360	0.150642	0.218601	1.371087			
y27,2	-0.035738	0.016922	0.171231	0.245634	0.310263	1.775229		
y28,1	0.252655	0.060884	0.226463	0.375307	0.359730	0.226697	3.521809	
y28,2	0.556137	0.060456	0.420032	0.457922	0.336235	0.345063	2.147500	4.718570
w1,1	5.097891	2.628634	6.075730	6.030144	4.166949	3.459036	14.647653	16.070765
w1,2	5.282209	3.088428	6.128593	6.663512	5.067085	3.452541	15.896009	17.710449
w2,1	7.784461	4.162092	11.859704	12.804665	9.293750	6.687505	29.022080	26.857067
w2,2	8.517956	4.325240	12.098310	14.043105	10.209319	6.748893	32.054987	30.116651
z1	0.096989	0.273937	0.225201	0.213226	0.089884	0.196257	0.128466	0.076684
z2	0.118844	0.146145	0.238436	0.243888	0.161902	0.179226	0.317542	0.426257

	w1,1	w1,2	w2,1	w2,2	z1	z2
w1,1	487.977561					
w1,2	467.259940	573.080053				
w2,1	766.034038	774.003609	1626.371358			
w2,2	752.773693	890.587960	1624.143500	1851.203952		
z1	2.281886	4.351582	9.495276	12.433148	1.579200	
z2	12.660054	13.595467	20.156360	21.558213	0.078527	0.826605

Table A3. Engel functions. Marginal budget shares (b), effect of an additional child (c1), effect of an additional adult (c2), and intercept term (a).[a] Standard deviations in parentheses

Commodity group	b (‰)		c1		c2		a_A	
01. Flour and bread	8.36	(1.23)	143.79	(17.48)	139.99	(27.01)	156.07	(55.91)
02. Meat and eggs	44.17	(5.40)	211.19	(74.17)	84.94	(115.47)	262.24	(236.50)
03. Fish	10.20	(1.83)	-39.61	(26.53)	4.74	(40.87)	249.66	(85.01)
04. Canned meat and fish	3.71	(0.62)	13.49	(8.85)	3.18	(13.67)	59.42	(28.33)
05. Dairy products	5.42	(1.32)	235.59	(19.26)	228.36	(29.62)	269.96	(61.74)
06. Butter and margarine	2.38	(0.66)	54.40	(9.54)	56.88	(14.70)	94.01	(30.56)
07. Potatoes and vegetables	26.50	(2.85)	230.16	(38.99)	74.14	(60.76)	319.59	(124.30)
08. Other food	18.31	(2.08)	145.16	(28.66)	157.26	(44.58)	220.25	(91.46)
09. Beverages	41.68	(3.47)	-146.35	(45.45)	-299.59	(71.48)	81.40	(144.41)
10. Tobacco	12.44	(2.28)	-1.68	(31.96)	-14.82	(49.53)	158.38	(102.10)
11. Clothing	100.90	(8.58)	40.04	(110.73)	80.51	(174.83)	-688.40	(350.84)
12. Footwear	22.46	(3.47)	63.06	(45.68)	-41.83	(71.80)	-85.02	(144.78)
13. Housing	128.29	(11.15)	-85.29	(142.89)	-851.19	(225.96)	1415.85	(452.29)
14. Fuel and power	7.91	(1.95)	34.18	(26.95)	68.86	(41.90)	914.04	(85.89)
15. Furniture	69.97	(7.80)	-257.93	(101.56)	-355.08	(160.01)	265.19	(321.72)
16. Household equipment	31.98	(4.63)	-21.82	(61.19)	-69.69	(96.09)	65.63	(194.05)
17. Misc. household goods	20.70	(2.57)	44.61	(34.03)	-205.23	(53.41)	421.64	(108.05)
18. Motorcars, bicycles	54.13	(26.51)	-40.26	(226.26)	1062.07	(406.38)	-2085.16	(698.42)
19. Running cost of vehicles	114.13	(11.25)	155.96	(141.75)	252.51	(225.06)	-1832.64	(447.36)
20. Public transport	24.07	(3.70)	-182.06	(47.74)	52.04	(75.38)	-31.58	(150.92)
21. PTT charges	11.81	(4.35)	-118.64	(58.24)	-51.40	(91.17)	303.98	(184.89)
22. Recreation	91.83	(9.14)	-181.50	(118.64)	-352.43	(187.05)	13.71	(375.81)
23. Public entertainment	24.14	(4.72)	-50.59	(63.34)	220.47	(99.10)	-127.56	(201.25)
24. Books and newspapers	18.72	(2.78)	-49.13	(37.35)	-26.26	(58.42)	96.42	(118.73)
25. Medical care	7.70	(2.82)	69.92	(37.65)	55.62	(58.98)	86.96	(119.48)
26. Personal care	18.48	(1.92)	27.83	(25.40)	54.21	(39.88)	-113.48	(80.65)
27. Misc. goods and services	23.01	(2.83)	-39.97	(37.47)	-82.41	(58.80)	-165.90	(118.97)
28. Restaurants,hotels etc.	56.63	(4.95)	-254.54	(64.96)	-245.84	(102.14)	-324.63	(206.30)
Adding up	1000.03		0.01		0.01		0.03	

[a] The estimated parameters have been multiplied by 1000. Thus c1, c2, and a are measured in kroner and b is measured in per thousand. (Since the input data are measured in 1 000 NOK so are the estimated parameters, but in this table, they are rescaled by 1 000 for convenience.)

Table A4. Income-consumption relations.[a] Standard deviations in parentheses

	Parameters				
Income concept	e	f1	f2	d1	d2
Income measure 1	0.514	-1.385	9.483	-2.427	0.746
	(0.051)	(0.662)	(1.043)	(2.113)	(2.129)
Income measure 2	1.099	-0.062	11.272	-13.760	-11.924
	(0.094)	(1.243)	(1.950)	(3.966)	(3.995)

[a] Confer equation (8). d1 and d2 are intercept terms in period 1 and 2 respectively.

Table A5. Parameters of the distribution of preference variables and measurement errors. Standard deviations in parentheses

Commodities and groups			
Ia Food	**var(α)**	**var(v)**	**h**
01. Flour and bread	0.062	0.139	0.217
	(0.012)	(0.012)	(0.024)
02. Meat and eggs	0.924	3.578	0.233
	(0.250)	(0.254)	(0.109)
03. Fish	0.206	0.321	0.113
	(0.029)	(0.023)	(0.033)
04. Canned meat and fish	0.012	0.055	0.052
	(0.004)	(0.004)	(0.013)
05. Dairy products	0.111	0.121	0.247
	(0.014)	(0.012)	(0.025)
06. Butter and margarine	0.013	0.058	0.120
	(0.004)	(0.005)	(0.015)
07. Potatoes and vegetables	0.280	0.737	0.280
	(0.062)	(0.055)	(0.052)
08. Other foods	0.128	0.380	0.359
	(0.032)	(0.033)	(0.040)
	var($\bar{\alpha}$)		
Group specific preference variable	212.244		
	(57.415)		
Ib Beverages and tobacco	**cov(α)/var(α)**	**var(v)**	
09. Beverages	0.561	0.938	
	(0.088)	(0.067)	
10. Tobacco	0.498	0.135	
	(0.041)	(0.010)	
Beverages vs Tobacco	0.203		
	(0.042)		
II Clothing and footwear	**var (α)/cov(α)**	**var (v)/cov (v)**	
11. Footwear	2.569	8.926	
	(0.631)	(0.633)	
12. Clothing	0.009	2.288	
	(0.115)	(0.160)	
Footwear vs Clothing	0.264	0.933	
	(0.194)	(0.230)	
III Housing,fuel and furniture	**var(α)**	**cov(v)/ var(v)**	
13. Housing	5.970	10.825	
	(1.059)	(0.773)	
14. Fuel and power	0.301	0.187	
	(0.029)	(0.013)	
15. Furniture	0.840	9.604	
	(0.537)	(0.672)	
16. Household equipment	0.224	3.584	
	(0.191)	(0.251)	
17. Misc. household goods	0.194	0.786	
	(0.053)	(0.055)	
Furniture vs Misc. etc.		0.640	
		(0.233)	
	var($\bar{\alpha}$)		
Group specific preference variable	83.693		
	(34.427)		

Table A5 (Continued)

IVa Travel	cov(α)/var(α)	var(v)
18. Motorcars, bicycles	15.025	38.136
	(6.544)	(6.734)
19. Running cost of vehicles	5.945	13.883
	(1.116)	(0.982)
20. Public transport	0.348	1.667
	(0.104)	(0.117)
Motorcars, bicycles vs	3.391	
Running cost of vehicles	(0.944)	
Motorcars, bicycles vs	-0.327	
Public transport	(0.271)	
Running cost of vehicles vs	-0.303	
Public transport	(2.207)	

IVb Recreation	var(α)	var(v)
21. PTT charges	0.517	2.698
	(0.162)	(0.189)
22. Recreation	1.922	12.409
	(0.747)	(0.873)
23. Public entertainment	1.193	2.056
	(0.174)	(0.145)
24. Books and newspapers	0.443	0.609
	(0.058)	(0.043)

V Other goods and services	var(α)	var(v)
25. Medical care	-0.156	1.856
	(0.084)	(0.130)
26. Personal care	0.123	0.401
	(0.028)	(0.028)
27. Misc. goods and services	0.100	1.272
	(0.070)	(0.090)
28. Restaurants, hotels etc.	1.230	1.892
	(1.183)	(0.135)

Income measures

Income concept	cov(λ)/var(λ)	cov(ε)/var(ε)
Income measure 1	192.504	57.728
	(16.377)	(4.354)
Income measure 2	724.866	94.521
	(57.606)	(8.416)
Income measure 1 vs	277.168	54.292
Income measure 2	(27.125)	(8.416)

Table A6. Parameters related to the distribution of latent total expenditure. Standard deviations in parentheses

Parameter	Symbol	Estimate
Variance of the permanent component of latent total expenditure	$\sigma_{\chi\chi}$	364.583 (33.236)
Variance of the volatile component of latent total expenditure	σ_{uu}	14.825 (3.682)
Covariance of latent total expenditure and the number of children	$\sigma_{\chi z1}$	8.705 (1.378)
Covariance of latent total expenditure and the number of adults	$\sigma_{\chi z2}$	9.906 (1.075)
Expected value of the permanent component of latent total expenditure	$\Phi_{\chi} = \Phi_{\xi 1}$	39.964 (1.130)
Expected value of latent total expenditure in the second period	$\Phi_{\xi 1}$	41.377 (1.212)
Growth in expected latent total expenditure from the first to the second period	$\Phi_{\xi 2} - \Phi_{\xi 1}$	1.413 (0.647)
Intercept term for period 2 in the latent total expenditure process	q_{02}	-3.550 (1.350)
Growth factor of latent total expenditure	q_2	1.124 (0.029)
Expected value of the purchase residual for cars in period 2	$a_{B18,2}$	1.139 (0.541)
Covariance of the volatile component of latent total expenditure and the residual for car purchases in the same period	$\mathrm{cov}(v_{18,t},u_t)$	14.490 (3.460)
Covariance of the volatile component of latent total expenditure and the residual for car purchases in the next period	$\mathrm{cov}(v_{18,2},u_1)$	13.144 (4.510)
Autocovariance of purchase residuals for cars	$\mathrm{cov}(v_{18,1},v_{18,2})$	-20.976 (6.408)

Table A7. Overview of fitted models with characteristics[a]

	D_U	D_S	D_R	D_N
P_UM_U	p=950	p=926	p=924	p=923
	DF=1065	DF=1089	DF=1091	DF=1092
	CHI=1660.74	CHI=1684.58	CHI=1685.70	CHI=1693.04
	AIC=3560.74	AIC=3536.58	AIC=3533.70	AIC=3539.04
	CAIC=8321.44	CAIC=8177.01	CAIC=8164.11	CAIC=8164.44
	CAICF=5979.40	CAICF=5897.13	CAICF=5891.12	CAICF=5896.05
P_UM_B	p=655	p=631	p=629	p=628
	DF=1360	DF=1384	DF=1386	DF=1387
	CHI=2041.25	CHI=2062.45	CHI=2063.32	CHI=2068.77
	AIC=3351.25	AIC=3324.45	AIC=3321.32	AIC=3324.77
	CAIC=6633.63	CAIC=6486.56	CAIC=6473.41	CAIC=6471.85
	CAICF=5073.00	CAICF=4988.41	CAICF=4982.20	CAICF=4985.21
P_BM_U	p=683	p=659	p=657	p=656
	DF=1332	DF=1356	DF=1358	DF=1359
	CHI=2056.03	CHI=2080.29	CHI=2080.96	CHI=2087.88
	AIC=3422.03	AIC=3398.29	AIC=3394.96	AIC=3399.88
	CAIC=6844.73	CAIC=6700.72	CAIC=6687.36	CAIC=6687.27
	CAICF=5153.31	CAICF=5071.63	CAICF=5065.18	CAICF=5069.61
P_BM_B	p=388	p=364	P=362	p=361
	DF=1627	DF=1651	DF=1653	DF=1654
	CHI=2478.72	CHI=2500.00	CHI=2500.77	CHI=2505.98
	AIC=3254.72	AIC=3228.00	AIC=3224.77	AIC=3227.98
	CAIC=5199.09	CAIC=5052.10	CAIC=5038.85	CAIC=5037.05
	CAICF=4251.44	CAICF=4166.90	CAICF=4160.59	CAICF=4163.30
P_BM_R	p=315	p=291	p=289	p=288
	DF=1700	DF=1724	DF=1726	DF=1727
	CHI=2578.84	CHI=2600.86	CHI=2601.71	CHI=2606.13
	AIC=3208.84	AIC=3182.86	AIC=3179.71	AIC=3182.13
	CAIC=4787.39	CAIC=4641.14	CAIC=4627.97	CAIC=4625.38
	CAICF=4004.04	CAICF=3920.33	CAICF=3914.11	CAICF=3916.0
P_RM_B	p=312	p=288	p=286	p=285
	DF=1703	DF=1727	DF=1729	DF=1730
	CHI=2623.34	CHI=2644.50	CHI=2645.43	CHI=2650.01
	AIC=3247.34	AIC=3220.50	AIC=3217.43	AIC=3320.01
	CAIC=4810.86	CAIC=4663.75	CAIC=4650.65	CAIC=4648.22
	CAICF=4023.80	CAICF=3939.21	CAICF=3933.08	CAICF=3935.16

Table A7. (Continued)

	D_U	D_S	D_R	D_N
P_RM_R	p=239	p=215	p=213	p=212
	DF=1776	DF=1800	DF=1802	DF=1803
	CHI=2713.71	CHI=2735.83	CHI=2736.74	CHI=2741.16
	AIC=3191.71	AIC=3165.83	AIC=3162.74	AIC=3165.16
	CAIC=4389.40	CAIC=4243.25	CAIC=4230.14	CAIC=4227.55
	CAICF=3759.96	CAICF=3676.39	CAICF=3670.24	CAICF=3672.16
P_RM_D	p=229	p=205	p=203	p=202
	DF=1786	DF=1810	DF=1812	DF=1813
	CHI=2895.69	CHI=2917.68	CHI=2918.52	CHI=2922.91
	AIC=3353.69	AIC=3327.68	AIC=3324.52	AIC=3326.91
	CAIC=4501.27	CAIC=4354.99	CAIC=4341.81	CAIC=4339.19
	CAICF=3895.17	CAICF=3811.57	CAICF=3805.34	CAICF=3807.24
P_DM_R	p=232	p=208	p=206	p=205
	DF=1783	DF=1807	DF=1809	DF=1810
	CHI=2789.66	CHI=2812.02	CHI=2812.92	CHI=2817.48
	AIC=3253.66	AIC=3228.02	AIC=3224.92	AIC=3227.48
	CAIC=4416.27	CAIC=4270.36	CAIC=4257.24	CAIC=4254.79
	CAICF=3820.53	CAICF=3737.27	CAICF=3731.11	CAICF=3733.21
P_DM_D	p=222	p=198	p=196	p=195
	DF=1793	DF=1817	DF=1819	DF=1820
	CHI=3116.64	CHI=3139.45	CHI=3140.31	CHI=3144.94
	AIC=3560.64	AIC=3535.45	AIC=3532.31	AIC=3534.94
	CAIC=4673.14	CAIC=4527.68	CAIC=4514.52	CAIC=4512.14
	CAICF=4097.57	CAICF=4014.78	CAICF=4008.57	CAICF=4010.76
$P_RM_RA_N$	p=236	p=212	p=210	p=209
	DF=1779	DF=1803	DF=1805	DF=1806
	CHI=2734.50	CHI=2756.72	CHI=2757.62	CHI=2762.47
	AIC=3206.50	AIC=3180.72	AIC=3177.62	AIC=3180.47
	CAIC=4389.16	CAIC=4243.11	CAIC=4229.99	CAIC=4227.83
	CAICF=3777.94	CAICF=3694.46	CAICF=3688.30	CAICF=3690.63
P_RM_N	p=579			p=552
	DF=1436			DF=1463
	CHI=2420.29			CHI=2449.22
	AIC=3578.29			AIC=3553.22
	CAIC=6479.81			CAIC=6319.44
	CAICF=5085.44			CAICF=4998.65

[a] The models are generated from combinations of assumptions in the dimensions P, M, A and D; see Table 1 for definitions. For each model are presented the number of estimated parameters (p), the number of degrees of freedom (DF), the chi square statistics (CHI), the Akaike information criterion (AIC), the Consistent Akaike information criterion (CAIC) and the Consistent Akaike information criterion with Fisher information (CAICF); cf. section 3 and Bozdogan (1987) for definitions.

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ISSN 0803-074X

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