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Consumer Demand with Unobservable Product Attributes

Part II: Inference

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Abstract:
In this paper we discuss statistical inference associated with the theoretical model developed in Part I. Specifically, we demonstrate how the relationship between the distribution of prices and unit values can be exploited to estimate some of the structural parameters. These estimates are essential for constructing price indexes that account for unobservable taste-shifters and quality/location attributes. Finally, the remaining structural parameters can be estimated from data on demand by inserting the price indexes into the corresponding demand system. Finally, we discuss the estimation procedure in the discrete case when consumers choose one unit of a variant at a time.

Keywords: Price indexes, differentiated products, quasi-maximum likelihood, quality and location attributes.

JEL classification: C25, C43, D11

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1. Introduction

In part I of this paper we developed a particular aggregation theory of consumer demand which is intended to account for products that are differentiated with respect to unobservable quality and location attributes. Our motivation is that many commodities are differentiated with respect to quality, and also that prices vary across locations of the stores, possibly due to differences in transportation costs, quality of service, etc. Thus, we may describe a consumer's choice setting as one in which he faces a variety of feasible product variants/locations represented by a distribution of prices and quality attributes. In Part I of this paper, we derived a demand model under particular assumptions about the preferences and the distribution of prices and nonpecuniary attributes. It was demonstrated that under particular assumptions, the demand model can be represented by a standard demand model in which conventional prices are replaced by particular price indexes (virtual prices). The virtual prices are price indexes that account for unobservable quality/location attributes and taste-shifters. They are unobservable random variables and we shall in this paper consider the problem of estimating parameters of the c.d.f. of the virtual prices, as well as the estimation of the corresponding demand system. To this end we shall exploit the particular relationship between the distribution of unit values and prices which was derived in part I.

The organization of the paper is as follows: In Section 2 we discuss how the relationship between the distribution of unit values and the prices can be exploited to estimate some of the parameters of the model. In Section 3 we discuss the implications from specific assumptions about the distribution of prices and in Sections 4 and 5 we consider the estimation of the remaining parameters in the case where the demand model is a (modified) linear expenditure system or a (modified) AIDS system, respectively. These systems are modified in the sense that prices are replaced by virtual prices. In the final section we consider the discrete case in which the consumer only buys one unit at a time.

2. The distribution of prices and unit values

Recall from Part I that the consumer (household) faces a distribution of feasible products characterized by quality/location attributes and price. There are m observable categories (types) of goods indexed by j, j=1,2,...,m. Within each category, z=1,2,..., indexes an infinite set of stores (location of the stores) and variants of products that are offered for sale in the market. The quantity of observable type j and unobservable location and variant z is $Q_j(z)$, and $T_j(z)$ is the unobservable quality attribute associated with good (j,z), $j \leq m$. The variable $P_j(z)$ is the price of variant/location z of type j.

Recall also that $g_j(p)$ is the density of prices within commodity group j and $\hat{g}_j(p)$ is the corresponding density of unit values. Under particular assumptions about preferences and the distribution of feasible quality/location attributes and prices discussed in Part I we have by Corollary 3 in Part I that

$$\hat{g}_j(p) = \frac{p^{-\alpha_j}\lambda_j(p)\, g_j(p)}{\int_{R_+} x^{-\alpha_j}\lambda_j(x)\, g_j(x)\, dx} \tag{2.1}$$

where

$$\lambda_j(p) = E\left(T_j(z)^{\alpha_j} | P_j(z) = p\right) \tag{2.2}$$

and α_j is a positive constant that is associated with the variance of the random taste-shifters in the utility function. The function $\lambda_j(p)$ can be interpreted as an aggregate quality index conditional on the price level, and it represents the "amount" of information about the relationship between $T_j(z)$ and $P_j(z)$, z=1,2,..., that can be identified from observations on prices and unit values. As discussed in part I, $p^{-\alpha_j}\lambda_j(p)$ can be identified apart from a multiplicative constant. Although it is possible to develop a nonparametric estimator for $p^{-\alpha_j}\lambda_j(p)$ we shall in this paper assume that $\lambda_j(p)$ is known apart from a set of parameters.

Recall that by Theorem 1 in Part I the virtual prices, $\{\hat{R}_j\}$, are Weibull distributed with parameter K_j that can, according to Corollary 5, be expressed as

$$K_j = c_j E\left(P_j(z)^{-\alpha_j}\lambda_j\left(P_j(z)\right)\right) = \frac{c_j}{E\left(\hat{P}_j^{\alpha_j}/\lambda_j\left(\hat{P}_j\right)\right)} = \frac{c_j E\left(T_j(z)^{\alpha_j}\right)}{E\left(\hat{P}_j^{\alpha_j}\right)}. \tag{2.3}$$

where $\hat{P}_j$ is the unit value associated with commodity type j. Eq. (2.3) expresses a very important relationship because it tells us how to obtain the distribution function of the virtual prices. Specifically, if α_j and $\lambda_j(\cdot)$ are known we can obtain an estimate of K_j (apart from an arbitrary positive constant c_j which can be determined at the analyst's convenience) from observations on prices, or alternatively, from observations on unit values.

Consider now the problem of estimating the parameters of $\lambda_j(p)p^{-\alpha_j}$ when λ_j is assumed to have a particular parametric functional form. Assume that data on unit values, $\hat{P}_{ij}$, i=1,2,...,N, is available for a sample of N consumers. In addition, assume that observations on prices are available. Then we can estimate the parameters of $\lambda_j(p)p^{-\alpha_j}$ without imposing functional forms on $\hat{g}_j(\cdot)$ and $g_j(\cdot)$. According to the interpretation in part I the unit values are assumed to be obtained from single purchases — or from means of purchases made in a short period of time. Let $\varphi_j(p) \equiv p^{-\alpha_j}\lambda_j(p)$, and recall that we assume that $\varphi_j(\cdot)$ is known apart from an unknown parameter vector. We shall now consider a quasi-maximum likelihood procedure. By (2.2) the likelihood of the unit values equals

$$L_j = \sum_{i=1}^{N} \log \hat{g}_j\left(\hat{P}_{ij}\right) = \sum_{i=1}^{N} \left(\log \varphi_j\left(\hat{P}_{ij}\right) + \log g_j\left(\hat{P}_{ij}\right)\right) - N \log\left(\int_{R_+} \varphi_j(x)\, g_j(x)\, dx\right) \tag{2.4}$$

where N is the sample size. If $g_j(\cdot)$ were known one could maximize L_j with respect to the unknown parameters. Unfortunately, $g_j(x)$ is not known. However, if price observations are available we can obtain an estimate of the last term in (2.4). For notational convenience, let $P_{kj} \equiv P_j(z_k)$, k=1,2,..., M, denote price observations of type j. Define a quasi-likelihood function by

$$\tilde{L}_j = \sum_{i=1}^{N} \left(\log \varphi_j\left(\hat{P}_{ij}\right) + \log g_j\left(\hat{P}_{ij}\right)\right) - N \log\left(\frac{1}{M}\sum_{k=1}^{M} \varphi_j\left(P_{kj}\right)\right). \tag{2.5}$$

When M increases $\tilde{L}_j$ will converge towards L_j with probability one due to the strong law of large numbers.

For simplicity we shall next discuss the quasi-maximum likelihood estimator in the special case when Assumption A4 in Part I holds. i.e., when $\lambda_j(\cdot)$ has the structure

$$\lambda_j(p) = \frac{p^{\alpha_j \kappa_j} E\left(T_j(z)^{\alpha_j}\right)}{E\left(P_j(z)^{\alpha_j \kappa_j}\right)} \tag{2.6}$$

where κ_j is a constant. It is clear that more general cases can be treated analogously. In this case $\log\varphi_j(p) = n_j \log p$, where $n_j = \alpha_j\kappa_j - \alpha_j$. Hence (2.5) reduces to

$$\tilde{L}_j = \sum_i \left(n_j \log \hat{P}_{ij} + \log g_j\left(\hat{P}_{ij}\right)\right) - N \log\left(\frac{1}{M}\sum_{k=1}^{M} P_{kj}^{n_j}\right). \tag{2.7}$$

By differentiating (2.7) with respect to n_j we obtain that the quasi-maximum likelihood estimate, $\tilde{n}_j$, is determined by

$$\frac{1}{N}\sum_{i=1} \log \hat{P}_{ij} = \frac{\sum_{k=1}^{M} P_{kj}^{\tilde{n}_j} \log P_{kj}}{\sum_{k=1}^{M} P_{kj}^{\tilde{n}_j}}. \tag{2.8}$$

We can now prove the following result:

Theorem 1

Suppose that A1 to A4 in Part I hold and assume that $\theta = \lim_{N\to\infty}(N/M)$ *exists. Then the quasi-maximum likelihood estimate* $\tilde{n}_j$*, which maximizes (2.7), is determined by*

$$\frac{1}{N}\sum_{i=1}^{N} \log \hat{P}_{ij} = \frac{\sum_{k=1}^{M} P_{kj}^{\tilde{n}_j} \log P_{kj}}{\sum_{k=1}^{M} P_{kj}^{\tilde{n}_j}}, \tag{2.9}$$

and has the property

$$\sqrt{N}\left(\tilde{n}_j - n_j\right) \xrightarrow{w} N\left(0, \sigma_j\right)$$

where

$$\sigma_j^2 = \frac{1}{Var\left(\log \hat{P}_j\right)} + \frac{Var\left(P_j(z)^{n_j} \log P_j(z)\right)\theta}{\left(Var\left(\log \hat{P}_j\right)\right)^2 \left(E\left(P_j(z)^{n_j}\right)\right)^2}. \tag{2.10}$$

A proof of Theorem 1 is given in the appendix.

The estimation procedure above requires that micro-data on both prices and unit values are available. We shall also consider the case where only observations on unit values are available and, in addition, an estimate of $EP_j(z)$ is available. Under Assumption A4 in Part I it follows from (2.1) and (2.4) that

$$EP_j(z) = \frac{E\left(\hat{P}_j^{\,1-n_j}\right)}{E\left(\hat{P}_j^{\,-n_j}\right)}. \tag{2.11}$$

From (2.11) we can easily form a sample analogue estimator.

<u>Theorem 2</u>

Assume that A1 to A4 in Part I hold and let $\bar{n}_j$ be an estimator defined by

$$\frac{1}{M}\sum_{k=1}^{M} P_{kj} = \frac{\sum_{i=1}^{N} \hat{P}_{ij}^{\,1-\bar{n}_j}}{\sum_{i=1}^{N} \hat{P}_{ij}^{\,-\bar{n}_j}}. \tag{2.12}$$

Then, provided $\theta = \lim_{N\to\infty}\left(\frac{N}{M}\right)$ *exists,*

$$\sqrt{N}\left(\bar{n}_j - n_j\right) \xrightarrow{w} N\left(0, \psi_j\right)$$

where

$$\psi_j^2 = \frac{\theta \, Var P_j(z) + Var\left(\hat{P}_j^{\,1-n_j}\right)\left(E\hat{P}_j^{\,-n_j}\right)^{-2}}{\left(\dfrac{E\left(\hat{P}_j^{\,1-n_j} \log \hat{P}_j\right)}{E\left(\hat{P}_j^{\,-n_j}\right)} - \dfrac{E\left(\hat{P}_j(z)^{1-n_j}\right)E\left(\hat{P}_j^{\,-n_j} \log \hat{P}_j\right)}{\left(E\left(\hat{P}_j^{\,-n_j}\right)\right)^2}\right)^2}. \tag{2.13}$$

The proof of Theorem 2 is similar to the proof of Theorem 1 and it is therefore omitted.

3. Particular distributional assumptions

In this section we shall demonstrate that in some special cases the functional forms and the estimation procedure simplify drastically.

Consider now the implication from Assumption A4 in Part I when prices are lognormally distributed. Under the assumption of lognormality it follows from (2.1) and Assumption A4 that $\hat{P}_j$ also is lognormally distributed, and

$$E \log \hat{P}_j = E \log P_j(z) + n_j \operatorname{Var} \log P_j(z) \tag{3.1}$$

and

$$\operatorname{Var} \log \hat{P}_j = \operatorname{Var} \log P_j(z). \tag{3.2}$$

Moreover, we have that

$$\log E P_j(z) = E \log P_j(z) + \frac{1}{2} \operatorname{Var} \log P_j(z) \tag{3.3}$$

and

$$\log E \hat{P}_j = E \log \hat{P}_j + \frac{1}{2} \operatorname{Var} \log P_j(z). \tag{3.4}$$

From (3.1), (3.2), (3.3) and (3.4) we get

$$\frac{\log E\hat{P}_j - \log EP_j(z)}{\operatorname{Var}\log \hat{P}_j} = n_j \equiv \alpha_j \kappa_j - \alpha_j. \tag{3.5}$$

Eq. (3.5) demonstrates that under Assumption A4 and the assumption of lognormally distributed prices it is possible to obtain estimates for $n_j \equiv \alpha_j\kappa_j\text{-}\alpha_j$ from estimates of $E\hat{P}_j$ and $EP_j(z)$. From Corollary 7 in Part I and the lognormality assumption it follows readily that

$$K_j = c_j E\left(T_j(z)^{\alpha_j}\right)\exp\left(-\alpha_j E\log P_j(z) - \left(\alpha_j n_j + \frac{1}{2}\alpha_j^2\right)\operatorname{Var}\log P_j(z)\right). \tag{3.6}$$

Due to (3.1) and (3.2) we can also express (3.6) as

$$K_j = c_j E\left(T_j(z)^{\alpha_j}\right)\exp\left(-\alpha_j E\log \hat{P}_j - \frac{1}{2}\alpha_j^2 \operatorname{Var}\log \hat{P}_j\right). \tag{3.7}$$

Thus by Corollary 1 in Part I, (3.7) implies a convenient expression for the mean virtual price;

$$E\hat{R}_j = c_j^{-1/\alpha_j}\Gamma\left(1+\frac{1}{\alpha_j}\right)\left(\left(E\left(T_j(z)^{\alpha_j}\right)\right)^{-1/\alpha_j}\exp\left(E\log \hat{P}_j + \frac{1}{2}\alpha_j \operatorname{Var}\log \hat{P}_j\right)\right). \tag{3.8}$$

Due to (3.4), (3.8) can also be expressed as

$$E\hat{R}_j = c_j^{-1/\alpha_j}\Gamma\left(1+\frac{1}{\alpha_j}\right)\left(E\left(T_j(z)^{\alpha_j}\right)\right)^{-1/\alpha_j} E\left(\hat{P}_j\right)\exp\left(\frac{1}{2}(\alpha_j - 1)\operatorname{Var}\log \hat{P}_j\right). \tag{3.9}$$

For the purpose of policy experiments it is of interest to express $E\hat{R}_j$ in terms of the price distribution. From (3.6), (3.3) and Corollary 1 we get

$$\begin{aligned} E\hat{R}_j &= c_j^{-1/\alpha_j}\Gamma\left(1+\frac{1}{\alpha_j}\right)\left(E\left(T_j(z)^{\alpha_j}\right)\right)^{-1/\alpha_j}\exp\left(E\log P_j(z) + \alpha_j\left(\kappa_j - \frac{1}{2}\right)\operatorname{Var}\log P_j(z)\right) \\ &= c_j^{-1/\alpha_j}\Gamma\left(1+\frac{1}{\alpha_j}\right)\left(E\left(T_j(z)^{\alpha_j}\right)\right)^{-1/\alpha_j} E\left(P_j(z)\right)\exp\left(\left(\alpha_j\kappa_j - \frac{\alpha_j}{2} - \frac{1}{2}\right)\operatorname{Var}\log P_j(z)\right). \end{aligned} \tag{3.10}$$

As discussed in Part I, $E\hat{R}_j$ can be interpreted as a <u>price index</u> of commodities of type j. We realize, by (3.10) that the price index will in general depend both on the mean and the variance in the price distribution. Suppose now that $E\left(T_j(z)^{\alpha_j}\right)$ varies slowly over time. Then we may choose c_j such that

$$c_j E\left(T_j(z)^{\alpha_j}\right) = \Gamma\left(1+\frac{1}{\alpha_j}\right)^{\alpha_j}.$$

Then, from (3.10) we realize that if $E\hat{R}_j$ is replaced by $EP_j(z)$ in a demand system the associate price coefficient will have the right sign but be biased, provided $\mathrm{Varlog}P_j(z)$ varies little over time. The bias depends on the factor

$$\exp\left(\left(\alpha_j\kappa_j - \frac{\alpha_j}{2} - \frac{1}{2}\right)\mathrm{Var}\log P_j(z)\right).$$

Thus, if $2\alpha_j\kappa_j>\alpha_j+1$, the associated price coefficient will be overestimated, while it will be underestimated if $2\alpha_j\kappa_j<\alpha_j+1$. From (3.9) we note that if $E\hat{R}_j$ is replaced by $E\hat{P}_j$ the corresponding bias will depend on the sign of α_j-1.

Consider next the case where prices are Gamma distributed, apart from a scale parameter. That is,

$$g_j(p) = \frac{s_j^{v_j} p^{v_j-1} e^{-s_j p}}{\Gamma(v_j)} \tag{3.11}$$

where $v_j>0$ and $s_j>0$ are unknown parameters (cf. Johnson and Kotz, 1972). Under Assumption A4 and (3.11) we immediately get from (2.1) that $\hat{g}_j(p)$ also is a scaled Gamma density, i.e.,

$$\hat{g}_j(p) = \frac{(p s_j)^{v_j+n_j-1} s_j e^{-s_j p}}{\Gamma(v_j+n_j)}. \tag{3.12}$$

Furthermore, (3.12) yields that

$$EP_j(z) = \frac{\nu_j}{s_j}, \quad VarP_j(z) = \frac{\nu_j}{s_j^2} \tag{3.13}$$

and

$$E\hat{P}_j = \frac{\nu_j + n_j}{s_j}, \quad Var\hat{P}_j = \frac{\nu_j + n_j}{s_j^2}. \tag{3.14}$$

From (3.13) and (3.14) we obtain

$$\frac{E\hat{P}_j\left(E\hat{P}_j - EP_j(z)\right)}{Var\hat{P}_j} = n_j. \tag{3.15}$$

In this case Corollary 5 in Part I yields

$$K_j = c_j E\left(T_j(z)^{\alpha_j}\right)\frac{\Gamma\left(\nu_j + n_j\right)s_j^{\alpha_j}}{\Gamma\left(\nu_j + n_j + \alpha_j\right)}. \tag{3.16}$$

Consider finally the special case with α_j=1 and $c_j E\left(T_j(z)^{\alpha_j}\right) = 1$. In this case Corollary 1 in Part I yields that

$$E\hat{R}_j = \frac{\Gamma\left(\nu_j + n_j + 1\right)}{\Gamma\left(\nu_j + n_j\right)s_j} = \frac{\nu_j + \kappa_j - 1}{s_j} = E\hat{P}_j$$

which demonstrates that the mean unit prices are correct instruments for the virtual prices in a demand system. Note that by (3.13) we can express $E\hat{R}_j$ in this case as

$$E\hat{R}_j = \frac{\nu_j}{s_j} + \frac{\kappa_j - 1}{s_j} = EP_j(z) + \left(\kappa_j - 1\right)\frac{VarP_j(z)}{EP_j(z)}. \tag{3.17}$$

When $VarP_j(z)$ is kept constant we get from (3.17) that

$$\frac{\partial E\hat{R}_j}{\partial EP_j(z)} = 1 + \left(1 - \kappa_j\right)\frac{VarP_j(z)}{\left(EP_j(z)\right)^2}. \tag{3.18}$$

Eq. (3.17) and (3.18) demonstrate how the bias depend on κ_j when $EP_j(z)$ is applied as instrument for $E\hat{R}_j$. We realize from (3.18) that $E\hat{R}_j$ and $EP_j(z)$ are always positively related when $\kappa_j<1$ and $VarP_j(z)$ is kept constant. If, however, the variation coefficient, $VarP_j(z)/(EP_j(z))^2$, is large and $\kappa_j>1$ then it may happen that $(EP_j(z))^2 < (\kappa_j-1)VarP_j(z)$ in which case $E\hat{R}_j$ decreases when $EP_j(z)$ increases provided $VarP_j(z)$ does not increase. Consequently, if in this case the mean prices are applied as instruments to estimate a demand system the associate price coefficient will turn out to have the wrong sign, (cf. Traijtenberg, 1989). The reason is that when $\kappa_j>1$, high prices signal high quality and consequently high prices will therefore not discourage consumers, and may even attract some consumers. Through the relation (3.17), the quality effect is controlled for in such a way that the value of the mean virtual price for commodity group j decreases as κ_j increases.

4. Identification and estimation when the demand model is a modified linear expenditure system

In this section we assume that the consumer's preferences are represented by a modified Stone Geary utility function given by

$$U(\mathbf{Q},\mathbf{T}) = \sum_{j=1}^{m} \beta_j \log\left(\sum_z Q_j(z)T_j(z)\xi_j(z) - \gamma_j\right). \tag{4.1}$$

where β_j and γ_j are unknown parameters, $j=1,2,...,m$, and $\sum\beta_j=1$. This utility function satisfies Assumption A1 in Part I. From (2.7) in Part I, and (4.1) it follows that the expenditures are given by

$$\bar{x}_j(\hat{\mathbf{R}},y) = \gamma_j\hat{R}_j + \beta_j\left(y - \sum_{k=1}^{m}\gamma_k\hat{R}_k\right) \tag{4.2}$$

provided we exclude the possibility of corner solutions. To focus on the main story of the paper we shall only consider the case with interior solutions.

Recall that since $\hat{\mathbf{R}} \equiv (\hat{R}_1,\hat{R}_2,...,\hat{R}_m)$, is unobservable and is stochastically independent of the unit values we cannot apply the unit values from individual purchases as instrument to estimate the parameters $\{\gamma_j\}$ in the demand equation (4.2) from a single cross-section. Due to Corollary 1 in Part I we can, however, apply aggregate time series data to estimate the model as we shall now discuss.

Provided we are willing to assume that $E\left(T_j(z)^{\alpha_j}\right)$ is constant over time we may, without loss of generality, choose c_j such that

$$c_j E\left(T_j(z)^{\alpha_j}\right) = \Gamma\left(1 + \frac{1}{\alpha_j}\right)^{\alpha_j}. \tag{4.3}$$

Now from Corollary 1 and Corollary 5 in Part I, (2.3) and (4.3) we get

$$E\hat{R}_j = \Gamma\left(1 + \frac{1}{\alpha_j}\right) K_j^{-1/\alpha_j} = E\left(\hat{P}_j^{\alpha_j}\right)^{1/\alpha_j}. \tag{4.4}$$

Thus, it only remains to estimate α_j to obtain estimates for $E\hat{R}_j$. As we have seen above, $E\hat{R}_j$ has a convenient functional form when prices are lognormally distributed, which facilitates estimation in this case. It is, however, possible to estimate $\{\alpha_j\}$ and $\{\gamma_j\}$ without making assumptions about the c.d.f. of prices as we shall briefly indicate below.

Let $X_{ij\tau}$, $\hat{P}_{ij\tau}$ and $\hat{R}_{ij\tau}$ denote expenditure, unit price and virtual price of type j for consumer i in year τ. These observations may relate to single purchases or be time aggregates for each individual across purchases within a year.

Furthermore, let $y_{i\tau}$ denote consumer i's expenditure in year τ. The corresponding Engel functions can be written as

$$X_{ij\tau} = a_{ij\tau} + \beta_j y_{i\tau} \tag{4.5}$$

where

$$a_{ij\tau} = \gamma_j \hat{R}_{ij\tau} - \beta_j \sum \gamma_k \hat{R}_{ik\tau}. \tag{4.6}$$

The relation (4.5) can be used to estimate β_j, j=1,2,...,m, in a first step. From (4.2) and (4.4) we get

$$Y_{j\tau} = \frac{\gamma_j}{\beta_j} E\hat{R}_{ij\tau} - \frac{\gamma_1}{\beta_1} E\hat{R}_{i1\tau} = d_j E\left(\hat{P}_{ij\tau}^{\alpha_j}\right)^{1/\alpha_j} - d_1 E\left(\hat{P}_{i1\tau}^{\alpha_1}\right)^{1/\alpha_1} \approx d_j\left(\sum_i \hat{P}_{ij\tau}^{\alpha_j}/N\right)^{1/\alpha_j} - d_1\left(\sum_i \hat{P}_{i1\tau}^{\alpha_1}/N\right)^{1/\alpha_1} \tag{4.7}$$

where

$$Y_{j\tau} = \frac{EX_{ij\tau}}{\beta_j} - \frac{EX_{i1\tau}}{\beta_1} \tag{4.8}$$

and $d_j=\gamma_j/\beta_j$. Now α_j and d_j, $j\geq 1$, can, in principle, be estimated from (4.7) by non-linear regression analysis with $Y_{j\tau}$ as dependent variables.

Let us finally discuss informally the identification issue. Let $\mu_{j\tau}\equiv E\hat{P}_{ij\tau}$, and

$$\rho_{j\tau}(\alpha_j) \equiv \left(E\left(\hat{P}_{ij\tau}^{\alpha_j}\right)\right)^{1/\alpha_j} / \mu_{j\tau} = \left(E\left(\frac{\hat{P}_{ij\tau}^{\alpha_j}}{\mu_{j\tau}}\right)\right).$$

With this notation we can express (4.7) as

$$Y_{j\tau} = d_j \mu_{j\tau} \rho_{j\tau}(\alpha_j) - d_1 \mu_{1\tau} \rho_{1\tau}(\alpha_1). \tag{4.9}$$

For simplicity, suppose that $\rho_{j\tau}(\alpha_j)$ can remain constant when $\mu_{j\tau}$ changes (for fixed α_j). Then

$$\frac{\partial Y_{j\tau}}{\partial \mu_{j\tau}} = d_j \rho_{j\tau}(\alpha_j)$$

and

$$\frac{\partial Y_{j\tau_1}}{\partial \mu_{j\tau_1}} \Big/ \frac{\partial Y_{j\tau_0}}{\partial \mu_{j\tau_0}} = \frac{\rho_{j\tau_1}(\alpha_j)}{\rho_{j\tau_0}(\alpha_j)}. \tag{4.10}$$

for $j>1$. Provided $\rho_{j\tau}(\alpha_j)$ varies sufficiently over time, it may be possible to identify α_j, $j>1$, from (4.10). If α_j, $j>1$, are identified, then evidently the remaining parameters are identified.

5. The case with AIDS demans

Let $w_{ij\tau}$ denote the budget share of type j for individual i in year τ. Now assume that (4.2) is replaced by an AIDS type demand model, cf. Deaton and Muellbauer (1980);

$$w_{ij\tau} = h_j + \sum_k \delta_{jk} \log \hat{R}_{ik\tau} + \beta_j \log(y_{i\tau}/q_{i\tau}) \tag{5.1}$$

and

$$\log q_{i\tau} = h_0 + \sum_k h_k \log \hat{R}_{ik\tau} + \frac{1}{2}\sum_{k=1}^{m}\sum_{j=1}^{m} \delta_{jk} \log \hat{R}_{ij\tau} \log \hat{R}_{ik\tau} \tag{5.2}$$

where $\{\beta_j\}$, $\{h_k\}$ and $\{\delta_{ik}\}$ are unknown parameters which satisfy

$$\sum_j h_j = 1, \quad \delta_{jk} = \delta_{kj},$$

and

$$\sum_k \delta_{jk} = \sum_j \delta_{jk} = \sum_j \beta_j = 0.$$

As in Section 4 we rule out the possibility of corner solutions. By Corollary 1 in Part I, we have

$$Ew_{ij\tau} = h_j - \sum_{k=1}^{m} \delta_{jk}\left(\frac{\gamma}{\alpha_k} + \frac{1}{\alpha_k}\log K_{k\tau}\right) + \beta_j E\log y_{i\tau} - \beta_j E\log q_{i\tau} \tag{5.3}$$

where

$$E\log q_{i\tau} = h_0 - \sum_{k=1}^{m} h_k\left(\frac{\gamma}{\alpha_k} + \frac{1}{\alpha_k}\log K_{k\tau}\right) + \frac{1}{2}\sum_{k=1}^{m}\sum_{j=1}^{m} \delta_{jk}\left(\frac{\gamma}{\alpha_j} + \frac{1}{\alpha_j}\log K_{j\tau}\right)\left(\frac{\gamma}{\alpha_k} + \frac{1}{\alpha_k}\log K_{k\tau}\right) \tag{5.4}$$

and γ=0.5772..., is Euler's constant. Provided we are willing to assume that $E\left(T_j(z)^{\alpha_j}\right)$ is constant over time it is convenient to let

$$c_j E\left(T_{j\tau}(z)^{\alpha_j}\right) = 1 \tag{5.5}$$

which, by (2.3) implies that

$$\log K_{j\tau} = -\log E\left(\hat{P}_{ij\tau}^{\alpha_j}\right). \tag{5.6}$$

Suppose for simplicity that the prices index $q_{i\tau}$ is substituted by a proxy. Then β_j, h_j, α_j and δ_{jk}, $j,k\geq 1$, can be estimated readily from micro-data by means of (5.3) where $\log E\left(\hat{P}_{ij\tau}^{\alpha_j}\right)$ is substituted by

$$\log\left(\sum_i \hat{P}_{ij\tau}^{\alpha_j}/N\right).$$

The identification problem in this case can be treated analogously to the case discussed in Section 4.

When micro-data on each individual purchase are available we may also apply maximum likelihood techniques since by Theorem 1 in Part I, the virtual prices are distributed according to the Weibull law.

6. Discrete choice with observable nonpecuniary attributes

In Section 5 of Part I we discussed the modelling framework for the particular case in which the consumer only buys one unit of a product at a time. We also extended the analysis in this case to accommodate for observable attributes besides price that characterize the product variants. In this section we shall outline how the model in this case can be conveniently estimated by quasi-maximum likelihood techniques. Consistent with Part I, let $X_{j\tau}(z)$ denote a vector of nonpecuniary attributes associated with variant z of type j in period τ. Let $\hat{X}_{ij\tau}$ denote the corresponding attribute vector of the chosen variant of type j by consumer i. In Part I (Theorem 3) we demonstrated that the choice probability of the chosen variant of type j in period τ can, under Assumptions A6 and A7 in Part I, be expressed as

$$H_{j\tau} = \frac{K_{j\tau}}{\sum_{r=1}^{m} K_{r\tau}} \tag{6.1}$$

where

$$1/K_{j\tau} = E\left(\hat{P}_{ij\tau}^{\alpha_j} \exp\left(-\hat{X}_{ij\tau}\theta_j\alpha_j\right)\right). \tag{6.2}$$

Consider next the estimation of α_j and θ_j, j=1,2,...,m. When data in several periods of time are available then it may be possible to estimate all the parameters, α_j, $j\geq 1$, by quasi-maximum likelihood methods. Specifically, if the observations in different periods are independent then the quasi-loglikelihood of the consumers choice equals

$$\tilde{L} = \sum_{\tau} \sum_{j=1}^{m} \tilde{H}_{j\tau} \log\left(\frac{K_{j\tau}^{*}}{\sum_{r=1}^{m} K_{r\tau}^{*}} \right) \tag{6.3}$$

where $\tilde{H}_{j\tau}$ denotes the observed fraction of consumers in period τ that choose product variants of type j and

$$1/K_{j\tau}^{*} = \frac{1}{N} \sum_{i=1}^{N} \hat{P}_{ij\tau}^{\alpha_j} \exp\left(-\hat{X}_{ij\tau}\theta_j \alpha_j\right). \tag{6.4}$$

For notational simplicity, let

$$\hat{X}_{ij\tau}^{*} = \left(\log \hat{P}_{ij\tau}, \hat{X}_{ij\tau}\right)$$

and $\theta_j^{*} = (\alpha_j, \theta_j \alpha_j)$. From (6.3) we obtain the first order condition

$$\frac{\partial \tilde{L}}{\partial \theta_{js}^{*}} = \sum_{\tau} \left(\tilde{H}_{j\tau} - H_{j\tau}^{*}\right) \frac{\partial \log K_{j\tau}^{*}}{\partial \theta_{js}^{*}} = 0 \tag{6.5}$$

where

$$\frac{\partial \log K_{j\tau}^{*}}{\partial \theta_{js}^{*}} = \frac{\sum_{i=1}^{N} \hat{X}_{ij\tau s}^{*} \exp\left(-\hat{X}_{ij\tau}^{*} \theta_j^{*}\right)}{\sum_{i=1}^{N} \exp\left(-\hat{X}_{ij\tau}^{*} \theta_j^{*}\right)} \tag{6.6}$$

and

$$H_{j\tau}^{*} = \frac{K_{j\tau}^{*}}{\sum_{r=1}^{m} K_{r\tau}^{*}}$$

and $X_{ij\tau s}^{*}$ is component s in vector $X_{ij\tau}^{*}$. The elements of the Hessian matrix associated with $\tilde{L}$ are given by

$$\frac{\partial^2 \tilde{L}}{\partial\theta_{jr}^*\partial\theta_{js}^*} = -\sum_\tau H_{j\tau}^*\left(1-H_{j\tau}^*\right)\frac{\partial \log K_{j\tau}^*}{\partial\theta_{jr}^*}\cdot\frac{\partial \log K_{j\tau}^*}{\partial\theta_{js}^*} + \sum_\tau\left(\tilde{H}_{j\tau}-H_{j\tau}^*\right)\frac{\partial^2 \log K_{j\tau}^*}{\partial\theta_{jr}^*\partial\theta_{js}^*} \tag{6.7a}$$

and

$$\frac{\partial^2 \tilde{L}}{\partial\theta_{jr}^*\partial\theta_{ks}^*} = -\sum_\tau H_{j\tau}^* H_{k\tau}^*\frac{\partial \log K_{j\tau}^*}{\partial\theta_{jr}^*}\cdot\frac{\partial \log K_{k\tau}^*}{\partial\theta_{ks}^*} \tag{6.7b}$$

for $j \neq k$. Since the maximum likelihood estimator is consistent under standard regularity conditions it follows that

$$\operatorname*{plim}_{N\to\infty}\tilde{H}_{j\tau} = \operatorname*{plim}_{N\to\infty}H_{j\tau}^* = H_{j\tau}^0$$

where $H_{j\tau}^0$ is the choice probability evaluated at the true parameter value. But this means that when N is large then the second term in (6.7a) becomes negligible. Consequently, the Hessian matrix defined by (6.7a,b) becomes negative definite and thus the likelihood function is, asymptotically, strictly concave in a neighborhood of the true parameter values. We therefore conclude that the parameters α_j and θ_j are identified for all j.

7. Conclusions

In this paper we have investigated issues related to identification and inference in a particular model for consumer demand which was developed in Part I of this paper.

It is demonstrated that under particular parametric functional form assumptions it is possible to estimate some of the parameters by a semiparametric quasi-maximum likelihood procedure provided microdata on prices and unit values are available. We outlined how the remaining parameters can be estimated from aggregate time series on demand and micro data on unit values.

In the final section we considered an analogous estimation procedure in the context of demand for indivisible products, where possibly nonpecuniary attributes may be observable.

Appendix

Proof of Theorem 1:

Let

$$Y_1 = \frac{1}{N}\sum_{i=1}^{N} \log \hat{P}_{ij} - E\log\hat{P}_j, \tag{A.1}$$

$$Y_2 = \frac{\frac{1}{M}\sum_{k=1}^{M} P_{kj}^{n_j} \log P_{kj}}{\frac{1}{M}\sum_{k=1}^{M} P_{kj}^{n_j}} - E\log\hat{P}_j \tag{A.2}$$

and

$$X = \frac{\sum_{k=1}^{M} P_{kj}^{n_j}\left(\log P_{kj}\right)^2}{\sum_{k=1}^{M} P_{kj}^{n_j}} - \left(\frac{\sum_{k=1}^{M} P_{kj}^{n}\log P_{kj}}{\sum_{k=1}^{M} P_{kj}^{n_j}}\right)^2. \tag{A.3}$$

By first order Taylor expansion we get from (2.8) that

$$\sqrt{N}\left(\tilde{n}_j - n_j\right) = \frac{\sqrt{N}\,Y_1 + \sqrt{N}\,Y_2}{X} + \Delta_{MN}, \tag{A.4}$$

where

$$\operatorname*{plim}_{N\to\infty} \Delta_{MN} = 0.$$

Note that Y_1 and Y_2 are independent, by Theorem 1 in Part I. Moreover,

$$\operatorname{plim}\left(\frac{1}{M}\sum_{k=1}^{M} P_{kj}^{n_j}\right) = EP_j(z)^{n_j} \tag{A.5}$$

$$\operatorname{plim} X = \frac{E\left(P_j(z)^{n_j}\left(\log P_j(z)\right)^2\right)}{EP_j(z)^{n_j}} - \left(\frac{E\left(P_j(z)^{n_j}\log P_j(z)\right)}{EP_j(z)^{n_j}}\right)^2 = \operatorname{Var}\log\hat{P}_j. \tag{A.6}$$

From (A.2) and (A.5) we get that $Y_2\sqrt{N}$ is asymptotically normal with zero mean and variance

$$\operatorname{Var}\left(P_j(z)^{n_j}\log P_j(z)\right)\theta .$$

Also $Y_1\sqrt{N}$ is asymptotically normal with zero mean and variance $\operatorname{Var}(\log\hat{P}_j)$. Hence, $(\tilde{n}_j-n_j)\sqrt{N}$ is asymptotically normal with variance

$$\sigma_j^2 = \frac{\operatorname{Var}\left(\log\hat{P}_j\right)+\operatorname{Var}\left(P_j(z)^{n_j}\log P_j(z)\right)\theta/\left(EP_j(z)^{n_j}\right)^2}{\operatorname{Var}\left(\log\hat{P}_j\right)}.$$

Q.E.D.

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