No. 186

January 1997

Discussion Papers

Statistics Norway

Statistisk sentralbyrå

Statistics Norway
Research Department

Discussion Papers

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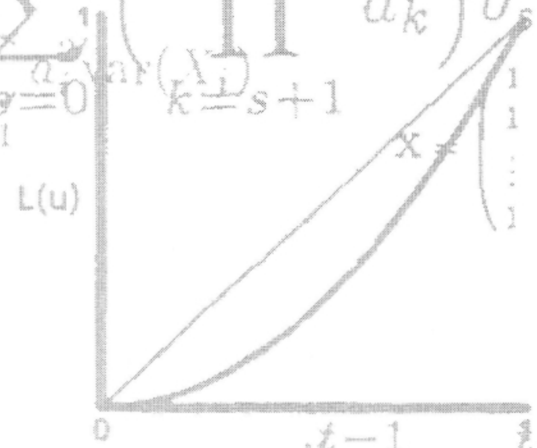

Discussion Papers No. 186 • Statistics Norway, January 1997

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Soil Depletion Choices under Production and Price Uncertainty

Abstract:
This paper studies soil depletion incentives in a dynamic economic model under two different sources of revenue uncertainty (production- and output price risk). The focus is on the long-term effects of risk averse preferences. The land manager is assumed to posses three classes of instruments to control natural topsoil fertility over time. Each instrument is also assumed to have implications for expected short-run production. The analysis shows that the forces at play are different across the three agricultural activities considered and varies for the two sources of risk analysed. In order to predict how risk aversion may influence soil conservation incentives detailed information is needed about input use and cultivation practices and the farmers' perception of their risk implications. If higher output is associated with higher levels of soil degradation, risk averse preferences will strengthen the incentives for soil conservation under output price uncertainty, and the same outcome is likely under production uncertainty. If higher levels of outputs is associated with lower levels of soil degradation, risk averse preferences will induce a farmer to conserve less soil under output price uncertainty, while the likely outcome of production uncertainty is the opposite

Keywords: Risk aversion, farm behaviour, resource management.

JEL classification: Q12, Q20, D81.

Acknowledgement: I am indebted to K.A. Brekke, K. Moene and D. Lund for valuable comments.

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1. Introduction

An important feature of agricultural resource decisions are their risky nature. Uncertainty is considered more of a problem for agricultural production than for most other production activities due to climatic variability and natural hazards such as the occurrence of pest and plagues. In addition a land manager is exposed to uncertainties through market fluctuations and policy interventions which also have consequences for prices, revenues and land values. Climatic uncertainty is especially pervasive and serious for tropical farmers due to extreme rainfall variability, because other changes in climate tend to be severe in their impact on crop yields, and due to the lack of well developed markets.

In this paper we will study the implications of risk aversion for the resource management of land, with particular reference to Third World smallholders and the problem of stochastic production revenues. The presence of uncertainty raises interesting questions about the behaviour of farmers. Knowledge about risk-induced behaviour should act as benchmark for judging whether or not there is a rationale for policy intervention, and is important for the design of policies intended to provide insurance markets, credit markets, and when implementing soil conservation programs. However, the attractiveness of various policies can not only be judged in relation to risk behaviour only, it is also important to address other factors which can cause the private paths of soil degradation processes to deviate from the socially optimal one. Examples on such factors mentioned in the literature are short planning horizons, discounting and off-farm externalities (see Griffin and Stoll, 1984; Rausser, 1983; and Griffin and Bromley,1982). Furthermore, the implications from revenue uncertainty and risk preferences on resource management are not only relevant for agriculture in less developed regions but also for capital-intensive agriculture in industrialised regions since crop insurance in general is not widespread in agriculture.

In the ongoing debate on risk and resource management incentives, in particular on developing countries, risk, uncertainty, and poverty are often mentioned as factors behind resource degradation. However, the causal links and the underlying assumptions are seldom explicitly stated. One of several possible interpretations is that risk averse attitudes contribute to a poorly management of natural resources. The aim of this paper is to explore how risk behaviour can affect the incentives for soil conservation in agricultural production systems. Arguments have long persisted that farmers are of necessity risk averse and empirical studies by Friend and Blume (1975), Hansen and Singleton (1983) and Wolf and Pohlman (1983) have supported this hypothesis. Strong evidence is also found against the hypothesis of risk neutrality for poor farmers in less developed regions (see Lipton, 1968; Schluter and Mount, 1976; Dillon and Scandizzo, 1978; and Binswanger and Sillers, 1983). While risk aversion seems to describe well risk preferences for farmers, another issue is the structure of such preferences. Here, the evidence is not conclusive. In studies by Hamal and Anderson (1982) and Schluter and Mount (1976) the risk premium was found to increase in poverty (income). A study by Binswanger and Sillers (1983) concluded that farmers are risk averse, but found that risk aversion stayed constant as income increases.

There are studies in the soil conservation literature which discuss and/or present evidence on the importance of risk and risk preferences. Kramer, McSweeney and Stavros (1983) apply a Mean-Variance approach to analyse how risk affect farm level soil conservation decisions when uncertainty both in revenues and input costs are considered. However, as pointed out by Shortle and Stefanou (1986) they fail consider the dynamic nature of soil conservation issues. Shortle and Stefanou suggest a dynamic extension of the model to be applied in numerical examinations of interlinkages between risk, risk aversion and conservation incentives. Other studies in the literature apply Stochastic Dominance analysis, a useful method for evaluating risky choices, on related issues. Klemme (1985) applies this approach on experimental plot yield data from Indiana to compare different tillage systems and found that risk averse farmers are more likely to adopt conservation tillage systems than risk neutral farmers. The same method is applied by Williams (1985) for the Central Great plains presenting related findings. A different approach is applied in Ervin and Ervin (1982) who present results from a multiple regression analysis. They found that soil conservation practices decrease as the level of risk aversion rises. An opposing result is presented in Reinhardt (1987) in a study of Colombian farmers. Here, farmers were categorised as risk averse which made them reluctant to adopt new cultivation practices that ignored soil conservation. McSweeney and Kramer (1986) find that an unintended side effect of governmental support programs have been that farmers have brought additional, often fragile lands, into production. A conclusion which seems to support the notion of risk aversion acting as an incentive to conserve soil. Anderson and Thampapillai (1995) surveys issues and evidence on soil conservation incentives when discussing the importance of risk and risk aversion and find that the evidence is mixed and that the role of risk should be subjected to further analysis. Williams and Johnson (1985) find that there is a fine line between whether risk aversion is positive or negative factor in adopting conservation measures.

Most of the studies mentioned above do not relate their findings explicitly to results arrived at in the theory. One reason for this may be that theoretical studies on soil conservation incentives and risk in general are lacking. One exception is a paper by Ardila and Innes (1993) where soil depletion choices of a risk averse farmer have been explored in a purely theoretical framework. Their analysis is conducted within two related models, where land degradation is output-induced in that higher outputs is associated with higher levels of soil degradation, and the farmer has DARA (decreasing absolute risk aversion) preferences. First, they present a two-date model with uncertainty both in revenues and in end-of-land price, where the land manager is to decide upon output and consumption. Second, they present a three-date model in which production and consumption choices are made in both periods, but there is uncertainty only in the second period. Here, the attention is primarily focused on the case of revenue risk. Ardila and Innes relate their results to standard results from the literature on production under uncertainty, in which output is risky and where an increased level of risk aversion makes the farmer to produce less. Ardila and Innes identify two conditions for which risk-averse farmers will produce more (exploit the soil more intensively) than risk neutral counterparts; i) when land risk «dominates» production revenue risk (in the two period model) ii) In the three-period model, considering production revenue risk only, risk aversion will under certain conditions induce the farmer to exploit soil more intensively (higher output) on a short-term (in period 2), while the long-term effect of risk aversion still yields less exploitation of the soil (less output). As a consequence, standard conclusions from literature on production under risk are reversed. However, if the two conditions are not fulfilled, a declining wealth (higher risk aversion) will induce the farmer to produce less thus soil conservation incentives are improved.

The purpose of this paper is much the same as in Ardila and Innes (1993). I want to investigate relationships between farming decisions and risk behaviour by using a theoretical model in order to derive qualitative properties of optimal choices of soil conservation in an uncertain environment. The findings can help us to understand the forces at play and provide us with a framework which enables us to predict the outcome of risk behaviour in agricultural production systems. A dynamic model of soil degradation will be applied for this purpose, but the aim of this analysis is more restricted than is the case for study by Ardila and Innes, since only revenue uncertainty is considered. Unlike Ardila and Innes (1993) we pay explicit attention to the random features of weather in a multi-period framework and two different types of revenue uncertainty is considered at a time; output uncertainty and crop price uncertainty. Furthermore, we will relax the assumption of output-induced soil degradation. This is done by focusing on input decisions rather than output itself as the decision variable. Such a disaggregated approach makes it natural also to discuss risk implications of various production factors. I assume that peasants have well-defined property rights on a given area of land with an infinite horizon, facing only one source of uncertainty, revenue risk. I rule out the case where the evolution of the soil stock is stochastic.

The paper is organised as follows. In the next section a deterministic dynamic model of land degradation is presented together with a justification of the functional relationships assumed. In section 3, the analysis is conducted in order to ascertain whether risk averse farmers faced with production uncertainty have incentives to deplete their resource base. In section 4 the same analysis is conducted but now the focus is on output price uncertainty. Last section summarises the findings.

2. An economic model of soil degradation

In this chapter an economic model of soil degradation is presented. The model is intended to capture some important features associated with agricultural production systems and how they interact with the soil base. A model of this kind must be a simplification of the complex interlinkages between farming choices and soil dynamics. In addition there is tremendous variation across the world as concerning cultivation practices, input use, and the importance of physical factors such as climate, topography, and soils, all factors which makes it difficult to model human-soil interactions in simplistic models. However, most decisions made by a land manager have consequences both for short-term output and the long-term fertility of the soil resource, even though the effects may vary in importance across regions. The model presented below considers the production of a single crop (or a given crop mix) and consists of three functions; a crop production function, an input costs function, and a function describing how soil evolves over time. In the following each of the functions and their properties will be presented. In addition the model will be compared with other economic models on soil management together with a more detailed justification of the relationships assumed.

There are three variable inputs or types of agricultural activities in the model. Each input (or activity) is classified according to its' effect both on short-term output and the future fertility of soil. Z(t) is denoted productive inputs or cultivation intensity, which when applied in larger quantities are assumed to increase output and degrade the soil. W(t) is denoted win-win inputs, which increases output and saves the soil when applied in larger quantities. C(t) is denoted soil conservation measures or conservation intensity and is assumed to decrease output and save soil. In addition soil fertility, S(t), is a capital stock and can be said to represent the stock of top soil fertility (soil base). The agricultural production function is a function of soil fertility and the three variable inputs, $\psi(S_t,Z_t,C_t,W_t)$ and assumed to be strictly concave in S, Z, C, and W. The input cost function, $h(Z_t,C_t,W_t)$, is assumed to be increasing in each of the three variable inputs. Soil is considered a renewable resource and it's evolution depends on all three variable inputs. The soil dynamics equation is represented by the following equation;

$$(1) \quad \dot{S} = M - n(Z_t, C_t, W_t)$$

where M is a constant representing the natural rate of soil fertility regeneration, and n(Z,C,W) the fertility loss function[1]. Even though the effects of soil regeneration processes like soil formation and nutrient recycling may be minor in any one year they become important over time. All functions are assumed continuos and twice differentiable.

The technological properties assumed so far can be summarised as follows;

$$(2) \quad \begin{pmatrix} \Psi_S > 0, \Psi_{SS} < 0, \Psi_Z > 0, \Psi_{ZZ} < 0, \ \Psi_C < 0, \Psi_{CC} < 0, \ \Psi_W > 0, \Psi_{WW} < 0; \\ h_Z > 0, h_C > 0, h_W > 0, n_Z > 0, n_C < 0, n_W < 0. \end{pmatrix}$$

Like other economic models on soil degradation and/or soil erosion appearing in the literature this model lets soil fertility be represented by one stock variable only, intended to represent various characteristics of the soil like organic matter, mineral content, soil depth and water holding capacity. Furthermore, to limit the number of control variables, cultivation practices and/or inputs with similar features are collapsed into one decision variable, and can be said to represent a vector or an index. Apart from win-win inputs, the structure of this model share similarities with other deterministic dynamic models on land degradation and soil erosion. In a study by LaFrance (1992), the land manager has two means at his disposal to influence the soil base; an index of crop increasing/land degrading inputs and an index of soil conserving/crop reducing inputs. Barbier (1990) applies a soil erosion model first presented by McConnel (1983) with a cultivation input package and a conservation input package. Barrett (1996) also present a model on the optimal control of soil erosion, in which there is; a trade-off variable between immediate output and soil depth (cultivation intensity), a variable representing soil conservation measures, but also an index of non-soil inputs which are assumed to increase output but have no effect on future soil depth. Another feature which distinguishes the model of LaFrance (1992) from the ones of Barbier (1990) and Barrett (1996) is that LaFrance assumes conservation inputs to reduce immediate output.

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[1] There is considerable uncertainty associated with rates of soil regeneration (Johnson et al., 1987). Here we choose to follow Barrett (1996), Ardila and Innes (1993) and McConnel (1983) by assuming an autonomus growth in the soil stock.

The empirical justification of the functional relationships differs to some extent across the models. This matters in particular for the decision variable representing a trade-off between immediate output and future soil productivity (Z). In the literature there are several studies which refer to output-induced soil degradation processes. Inputs that increase crop production can contribute to land degradation and many cultivation practices tend to degrade soil over time (see e.g. Burch, Graetz and Nobel, 1987; Lutz, Pagiola and Reicher, 1994). LaFrance (1992) argues in favour of fertiliser, irrigation and ploughing to increase output and degrade land when employed in larger quantities (see LaFrance, 1992 and the numerous references therein). Ardila and Innes (1993) apply similar arguments when justifying that higher production is associated with more degradation, and also mentions fallowing as an example of output-induced soil-depletion. Barbier (1990) refers to productive inputs (and labour), crop varieties, and cropping patterns and techniques as examples, while Barrett (1996) direct attention to cultivation practices such as strip-cropping and the extent and nature of crop rotation.

The trade-off between output and land degradation also describes important sides of agricultural production in farming systems with a low intensity of external inputs. Examples here are seedbed preparation and tillage practices, weeding, timing decisions and cropland expansion. A farmer may increase output by conducting more intensive tillage practices (deep ploughing) instead of no or minimum tillage practices, by smoothening the seedbeds with secondary tillage, by using plough and animal traction instead of tillage by hand and planting in small pockets. All decisions which may increase short-term output but also cause more erosion thus imposing future productivity losses[2]. Lal (1987) finds erosion losses from ploughed croplands to be 5 to 400 times higher than for no-tilled and Lal (1986) reports that erosion is more severe on ploughed and harrowed land than on reduced- and no tilled. Furthermore, Lutz, Pagiola and Reiche (1994) find that repeated tillage can weaken the soil structure. Timing decisions as delayed planting and waiting to perform tillage operations will shorten the growing season, but protect soils if implemented in periods where the most erosive storms are concentrated (Barber, 1984; Wiscmeier and Smith, 1978). Finally, a farmer can put more land into cultivation by cultivating marginal lands and clearing hillsides (McConnel, 1983).

A farmer can also maintain natural soil fertility or arrest the rate at which land are degrading, by investing in structural soil conservation measures such as terraces, ditches, windbreaks, bundles, hedge rows and stonewalls. All such measures are in this model represented by the intensity of conservation, C, where a higher value means that more resources are devoted to this activity. We follow LaFrance (1992) by assuming that a higher C reduces current production, since structures themselves take up productive land. Lal (1982) and Lutz, Pagiola and Reiche (1994) find that structures reduce the effective area by 10-15 %.

The third group of activities considered in this analysis is inputs and/or farming practices which when implemented increase both immediate output and dampen land degradation processes. Such win-win strategies (W), or overlapping technologies in the terminology of Reardon and Vosti (1992), seem to

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[2] Several studies find that there is a yield penalty associated with minimum tillage methods (see e.g. Walker and Young, 1986).

be ignored in many economic studies on land degradation and soil conservation. Recently, however, we have witnessed an increasing awareness of such technologies in the economic literature. Several studies on Africa stress the importance of external inputs in both arresting soil erosion and increasing short run output (see Brekke et al., 1996; Aune et al., 1994; and Aune and Lal, 1995, Alfsen et al., 1995). The application of fertilisers, irrigation and agrochemicals will provide land with a denser vegetation and improve the root structure of plants, both factors which provide a better protection of soils, thus reducing both water run-off and erosion (Roose, 1977). In addition higher yields also imply additional crop residues which supplies the soil with nitrogen if left on land in-between harvesting and sowing. Logan and Lal (1990), in a study on the Dominican Republic, found that erosion and runoff were lower on fertilised plots compared to unfertilised ones. Other examples could be crop residue management including mulching, liming, and cropping combined with legumes. Reardon and Vosti (1992) mention tied ridges as yet another important example.

The relevance of the assumption of the cost function being monotone in each of the three activities is obvious with respect to marketed inputs, but also seem to capture essential features with other farming decisions as well. Soil conservation structures are costly to construct both in terms of input and labour use (see e.g. Lal, 1987; and Lutz, Pagiola and Reiche, 1994). More intensive cultivation will in general imply that additional resources need to be devoted to cultivation activities. Increasing the area of land under cultivation and extending the crop season will bring about the need for additional inputs and more labour effort. Practising more intensive tillage operations and secondary tillage imply additional labour requirements. Reicosky et al. (1977) for example, find that minimum tillage reduces labour requirements. The application of crop residues involves opportunity costs since stalks and straws have an alternative value as fuel and/or fodder.

3. Production uncertainty and the incentives for soil conservation

This section investigates how risk behaviour affects soil depletion choices under production uncertainty and tries to identify factors which are of importance for determining such behaviour. This is done by introducing production uncertainty due to climatic variations and/or the occurrence of natural hazards into the model. To investigate the role of risk in agriculture, it is important to apply a specification of the production function with reasonable risk implications. In agriculture, it is not necessarily a unique relationship between the variance of output and the quantity applied of all inputs. Just and Pope (1978) find it likely that additional land use and chemical thinning processes will increase the variance of production attributable to weather, insects and crop diseases, while the effects of pesticides, irrigation, frost protection, and decease-resistant seeds may have opposite effects. Here I will apply a specification suggested by Just and Pope (1978) to take account of such relationships. Let the stochastic production function be as follows;

$$\textbf{(3a)}\quad \Psi(S_t,Z_t,C_t,W_t,\theta_t) = f(S_t,Z_t,W_t,C_t) + g(S_t,Z_t,C_t,W_t)\theta_t$$

It follows from (3a) that the production function consists of two components. First, a deterministic component, f, for which the technology assumptions are similar to the ones assumed for ψ in (2). Second, an additive stochastic component, g, which depends on the same arguments as f, but for which the first derivatives are allowed to be signed differently. θ_t represents the stochastic disturbance and enters the stochastic component mulitplicatively. The first derivative of g with respect to each of the production factors determines what the risk property of each production factor is. In the following we will denote inputs risk-increasing if the first derivative of g with respect to the same activity is positive, risk-neutral if the first derivative is zero, and risk-decreasing if the first derivative is negative. The assumptions made on the overall production function ψ in (2) still remain valid.

The land manager is assumed to be risk averse, and the farmers' Von Neumann-Morgenstern utility function, U, is assumed to be twice differentiable, increasing and concave in current net returns, Π_t, where (the output price is set equal to one);

$$\textbf{(3b)}\ \Pi_t = \Psi(S_t, Z_t, C_t, W_t, \theta_t) - h(Z_t, C_t, W_t)$$

The objective of a rational farmer under production uncertainty is to maximise the discounted expected utility of net returns of crop production where the constraint on this decision problem is (1). The timing and actions over time are as follows. The farmer makes decisions about input use and level of activities before the outcome of the stochastic event is known. As a consequence the farmer does not know the value of θ_t when decisions are made. The sequential nature of the decision process suggests a discrete-time modelling approach. However, I have chosen to solve this problem in continuos-time by using deterministic controls since such an approach is a more powerful device for solving dynamic optimisation problems as compared to discrete-time techniques, especially in a multi-period formulation. This can be done since the outcome of a stochastic event in our problem does not influence actions and since the soil stock at any time can be predicted exactly from current stock level and input use. Furthermore, it is assumed that $\theta_t = dB_t$, where B_t is a Brownian motion, thus θ_t is normally distributed.

The maximisation problem for our problem given an infinite horizon is

$$\textbf{(4)}\quad \begin{aligned} &\underset{Z,C,W}{Max}\, \lambda_0 \int_0^{\infty} E\{U[f(S_t, Z_t, C_t, W_t) + g(S_t, Z_t, C_t, W_t)\theta - h(Z_t, C_t, W_t)]\} e^{-rt}\, dt \\ &s.t.\ \dot{S} = M - n(Z_t, C_t, W_t),\ S(0) = S_0 > 0,\ Z(t) > 0,\ C(t) > 0,\ and\ W(t) > 0. \end{aligned}$$

where E is the expectation operator with respect to θ and r the utility discount factor. Let Q denote the current value Hamiltonian for this optimal control problem (time references are in the following omitted)

$$\textbf{(5)}\quad Q = E\{U[f(S,Z,C,W) + g(S,Z,C,W)\theta - h(Z,C,W)]\} + \lambda[M - n(Z,C,W)]$$

where λ is the current value shadow price for the soil state equation. Since the Hamiltonian is strictly concave in Z, C, and W, respectively, $\lambda_0 = 1$ and assuming interior solutions, the sufficient conditions for an optimal solution are (Seierstad and Sydsæter, 1987);

(6a) $Q_Z = E\{U_\Pi[\Pi][f_Z(S,Z,C,W) + g_Z(S,Z,C,W)\theta - h_Z(Z,C,W)]\} - \lambda n_Z(Z,C,W) = 0$

(6b) $Q_C = E\{U_\Pi[\Pi][f_C(S,Z,C,W) + g_C(S,Z,C,W)\theta - h_C(Z,C,W)]\} - \lambda n_C(Z,C,W) = 0$

(6c) $Q_W = E\{U_\Pi[\Pi][f_W(S,Z,C,W) + g_W(S,Z,C,W)\theta - h_W(Z,C,W)]\} - \lambda n_W(Z,C,W) = 0$

(6d) $Q_S = E\{U_\Pi[\Pi][f_S(S,Z,C,W) + g_S(S,Z,C,W)\theta]\} = r\lambda - \dot{\lambda}$ $\qquad \lim_{t\to\infty} S(t) \geq 0$

(6e) $\dot{S} = M - n(Z,C,W)$ $\qquad \lim_{t\to\infty} e^{-rt}\lambda(t) = 0$

Eqs.(6a-c) say that along the optimal path the expected marginal utility-increase (decrease) associated with higher input use minus the marginal change in input costs generated from the same increase must equal the change in the fertility loss function that goes with the same change in input evaluated by the shadow price of soil (λ). Eq.(6d) determines the adjustment in λ along the optimal path. The following expression for the shadow price of soil in optimum can be derived;

(7) $$\lambda(t) = e^{rt}\mathrm{E}\left\{\int_t^{\infty} U_\Pi(\Pi_\tau)[f_{S(\tau)}(S_\tau, Z_\tau, C_\tau, W_\tau) + g_{s(\tau)}(S_\tau, Z_\tau, C_\tau, W_\tau)\theta]e^{-r\tau}\,d\tau\right\}$$

The shadow price of soil is in optimum equal to the expected utility-decrease caused by a marginal reduction in soil fertility at time t for all future periods. It is further noted that the shadow price of soil not only reflects the expected value of future output losses (gains) due to a reduction (increase) in the stock of soil fertility associated with level changes in each of the three activities, but also incorporates risk preferences of the land manager through the curvature of the utility function in current income. This finding is important and emphasises the importance of a multi-period formulation of optimal soil conservation decisions.

A convenient way to study risk-averse farmers' long term reactions to the presence of output uncertainty is to analyse the model in steady state. Steady state equilibrium is attained when dλ/dt=dS/dt=0. Letting bars denote steady-state equilibrium values, imposing the stationary conditions on (6a-d), and (1), and combining the same equations, the optimality conditions can be presented as follows

(8a) $f_i(\bar{S},\bar{Z},\bar{C},\bar{W}) + \alpha_i\gamma - h_i(\bar{Z},\bar{C},\bar{W}) = \dfrac{n_i(\bar{Z},\bar{C},\bar{W})}{r} f_S(\bar{S},\bar{Z},\bar{C},\bar{W})$ $\qquad i = Z, C, W$

(8b) $M = n(\bar{Z},\bar{C},\bar{W})$

where

(9a) $\gamma \equiv \dfrac{COV\{U_\Pi(\bar{\Pi}),\theta\}}{E\{U_\Pi(\bar{\Pi})\}}$

(9b) *and* $\alpha_i \equiv g_i(\bar{S},\bar{Z},\bar{C},\bar{W}) - \dfrac{1}{r}n_i(\bar{Z},\bar{C},\bar{W})g_s(\bar{S},\bar{Z},\bar{C},\bar{W})$ $\qquad i = Z, C, W$

From (8a-b) and (9a-b) it is observed that the structure of risk preferences in our model is now isolated to the second term on the left hand side of each of the three optimality conditions described in (8a), represented by γ. γ can be said to represent the *security equivalent* for θ and reflects to what degree θ contributes to uncertainty in profits. It is further noticed that there is one more factor appearing in the second term of (8a). In each optimality condition which corresponds to a certain input there is a α_i associated with the same input. Such factors will in the following will be denoted *risk-factors* and can be said to represent an overall risk effect. From (9b) it is seen that the *risk-factor* for a variable input i depends on the *risk properties* of the variable input itself (g_i) and the stock the variable (g_s). In addition the *risk factor* for an input i, depends on the first derivative of the fertility loss function (n_i) and the utility discount rate. The first derivatives of the stochastic component of the production function with respect to a production factor i determines how the variability in output at an instant of time is affected by changes in the same factor at the same instant. This term can be said to represent an immediate-risk effect. The second term present in the *risk*-factor represents a long-term risk effect. Remember that changing the quantity applied of a variable input have implications for the stock of soil fertility being available in the future. The direction of such changes are determined by the first derivative of the soil loss function, while the first derivative of the stochastic component of the production function with respect to soil stock reflects what the implications are for future variability in output for the same change. The utility discount factor appearing in the denominator of the long-term risk effect simply ensures that changes in the risk implications for the rest of the horizon are evaluated at current values. We have shown that the magnitude and sign of the second term of each optimality condition presented in (8a) depends on the structure of risk preferences (*security equivalent*) and possible immediate - and long-term risk effects (*risk factor*).

The assumption of risk aversion, $U_{\pi\pi}<0$, implies that $Cov[U_\pi,\theta]<0$[3] and consequently $\gamma < 0$. Hence, the optimality conditions becomes different from the same conditions in the situation in which attitudes to risk are absent, $U_{\pi\pi}=0 \Rightarrow \gamma=0$. A first and rather trivial conclusion is that climatic uncertainty does influence a risk averse farmer's incentives for soil conservation, thus causing fertility losses to deviate from the risk neutral path. If the social value attached to the soil stock is different from that of the farmer, the private optimal path is different from the social optimal one. This will be the case if governments are risk neutral which implies that there is a rational for policy intervention in the resource management of cultivated land. This conclusion, however, does not always remain true. Remember that all effects which are generated from the modelling of output uncertainty in this model occur in one term only (see 8a). If each of the three *risk-factors* equals zero then the structure of risk preferences does not influence optimal behaviour. This will matter if all production factors in our model are defined as risk-neutral, $g_i=0$ for $i = S,Z,C,W$ (e.g. additive risk). Given these assumptions the optimal path for risk averse farmer will coincide with the path of a risk neutral farmer, and our problem can be said to degenerate to a deterministic one. A rational farmer will now at the margin balance changes in immediate output (revenue) arising from additional input use with the changes in variable input costs and the shadow value of soil that goes with the same change. If however, at least one of the production factors is risk increasing or -decreasing, risk preferences will play a role for

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[3] There exists probability distributions for which this result not necessarily is valid (see Lund, 1993).

optimal behaviour. How, risk aversion affects the optimal soil conservation incentives still remains a question and such issues will be pursued below.

A general problem when analysing the role of risk preferences in dynamic stochastic models is that changing the structure of risk preferences have consequences for the utility function in the sense that the intertemporal elasticity of substitution changes. However, the shape of the utility function does not affect the steady in this model. To see this, disregard for a moment uncertainty in the model, then by combining (6a-e), the optimality conditions in steady state can be described as follows;

$$\textbf{(10a)} \qquad U_{\Pi}(\overline{\Pi})[f_i + g_i - h_i] = \frac{n_i}{r} U_{\Pi}(\overline{\Pi})[f_S + g_S] \qquad i = Z, C, W.$$

From (10a) it is noticed that the curvature of the utility of net income has no impact on endogenous variables.

As mentioned above, the implications of an uncertain environment are in our model isolated to one term in (8a), represented by the product of the *security equivalent* and the *risk-factor* for each variable input. Furthermore, I have argued for that the presence of risk averting attitudes compared to risk neutral ones can be interpreted as a shift in γ. In order to derive what the effects on soil conservation incentives are from risk averse preferences one approach could be to undertake comparative statics in steady state with respect to γ. However, a higher value γ is not only reflecting changes in the structure of risk preferences but also measures the degree of uncertainty. To see this we can apply a theorem by Rubinstein (1976). Since Π is normally distributed γ can be written as follows

$$\textbf{(10b)} \qquad \gamma \equiv R(\overline{\Pi})COV(\Pi,\theta) \;\; = R(\overline{\Pi})g(\overline{S},\overline{Z},\overline{C},\overline{W}) \qquad where \;\; R(\overline{\Pi}) = \frac{E\{U_{\Pi\Pi}(\overline{\Pi})\}}{E\{U_{\Pi}(\overline{\Pi})\}}$$

From (10b) we notice that γ is the product of R and a covariance. R can be said to measure the degree of risk aversion and Varian (1992) denotes R - global risk aversion. A higher R means that an individual becomes more risk averse for all levels (global) of net returns, while the covariance term measures the degree of uncertainty. Conducting a shift in R instead of γ would be a better approach to analyse the role of risk preferences in this model. Here, however, I will choose a slightly different approach which simplifies the analysis in several respects. Since Π is normally distributed choices among net returns can be reduced to a comparison on their means and variances and expected utility (mean-variance utility function). Let expected utility be as follows

$$\textbf{(11)} \;\; E\{U(\Pi)\} = E(\Pi) - \beta VAR(\Pi)$$

From (11) it follows that expected utility can be expressed as a linear function in the mean and the variance of net returns. β has a clear interpretation as the degree of risk aversion. Now, replace the utility function in problem (4) with the one in (11). Above, Var(Π) was set equal to g(S,Z,C,W.)^2, now we assume that var(Π) is g(S,Z,C,W). As will be seen from below, steady state for this problem coincides with the one in (8) if $\beta=-\gamma$. The Hamiltonian for this problem, still denoted H, is

$$\textbf{(12a)}\qquad H = f(S,Z,C,W) - h(Z,C,W) - \beta g(S,Z,C,W) + \lambda[M - n(Z,C,W)]$$

The assumptions made for problem (4) still remain valid, thus the optimality conditions for this problem are;

$$\textbf{(12b)}\qquad H_i = f_i(S,Z,C,W) - h_i(Z,C,W) - \beta g_i(S,Z,C,W) - \lambda n_i(Z,C,W) = 0 \quad i = Z,C,W.$$

$$\textbf{(12c)}\qquad H_S = f_S(S,Z,C,W) - \beta g_S(S,Z,C,W) = r\lambda - \dot{\lambda}$$

$$\textbf{(1)}\quad \dot{S} = M - n(Z,C,W)$$

An investigation of the effect of a higher degree of risk aversion on soil conservation incentives can now be undertaken by conducting changes in β. Imposing the stationary conditions on the optimality conditions in (12bc) and (1) and differentiating the system w.r.t S, Z, C, W and β, yields after some tedious algebra, the following expression for the impact on steady state soil fertility (all arguments are evaluated in steady state);

$$\begin{aligned}
&\frac{d\bar{S}}{d\beta} = \frac{1}{D}\{n_Z[\alpha_Z(R_{CC}R_{WW} - R_{CW}R_{WC}) - R_{ZC}(\alpha_C R_{WW} - \alpha_W R_{CW}) + R_{ZW}(\alpha_W R_{CC} - \alpha_C R_{WC})] \\
\textbf{(13)}\qquad &\quad + n_W[\alpha_Z(R_{CZ}R_{WZ} - R_{CC}R_{WZ}) - R_{ZZ}(\alpha_C R_{WC} - \alpha_W R_{CC}) + R_{ZC}(\alpha_C R_{WZ} - \alpha_W R_{CZ})]\} \\
&\quad - n_C[\alpha_Z(R_{CZ}R_{WW} - R_{CW}R_{WZ}) - R_{ZZ}(\alpha_C R_{WW} - \alpha_W R_{CW}) + R_{ZW}(\alpha_C R_{WZ} - \alpha_W R_{CZ})]\}
\end{aligned}$$

where α_Z, α_C, and α_W are risk-factors for this problem and defined in (9b) while all R-terms are defined in Appendix 1. D<0 follows from the saddle path condition for this problem (see Appendix 1).

When studying (13) in more detail it follows that the total impact of a higher degree of risk aversion (or risk averse preferences compared to risk neutral ones) on steady state soil fertility can be said to depend on three categories of effects. First, the partial derivatives of the fertility loss function, n_i, for each of the three variable inputs, which are already signed from (1). Second, from the *risk-factor* for each input, α_i, whose signs depend on assumptions made on the risk properties of all production factors. Third, the R-terms which all are functions of second order derivatives of the Hamiltonian function, and depend on cross partial derivatives of the production function, the cost function and the fertility loss function with respect to different pair wise combinations of the production factors (hereafter denoted *indirect effects*). The complexity of the numerator of (13) makes it difficult to comment on the various forces at play. Instead, I choose to focus on partial models rather than the general one, an approach which provides us with a better understanding of the forces at play.

Let X be a single input variable which can represent Z, C, or W, while S is still the stock of soil fertility. The maximisation problem for a mean-variance utility function now becomes

$$(14)\quad \underset{Z,C,W}{Max}\ \lambda_0 \int_0^{\infty} \{F(S_t,X_t) - T(X_t) - \beta G(S_t,X_t)\} e^{-rt}\, dt$$

$$s.t.\ \dot{S} = M - N(X_t),\ S(0) = S_0 > 0,\ X(t) > 0.$$

Which input X is meant to represent depends on the assumptions made on technology. The following three partial models is considered where capital letters now describe functional forms;

	Cultivation model:	$F_X>0$, $N_X>0$. (C and W are ignored)
(15)	Win-win model:	$F_X>0$, $N_X<0$. (Z and C are ignored)
	Conservation model:	$F_X<0$, $N_X<0$. (Z and W are ignored)

The assumptions on variable costs are the same for all partial models ($T_X>0$), while G(S,X) will be discussed below. The Hamiltonian for this problem (still denoted H) is assumed to be strictly concave in S and X. Following the same procedures as above, the following expression for the impact on steady state soil fertility from a higher degree of risk aversion is derived (tilde now denote steady state values)

$$(16)\quad \frac{d\tilde{S}}{d\beta} = \frac{\alpha_X}{D} = \frac{(G_X - \frac{N_X}{r} G_X)}{[H_{XS} - \frac{N_X}{r} H_{SS}]}$$

The saddle path condition for this problem imply that the denominator in (16), D, is positive for $N_X>0$ (Cultivation model) and negative for $N_X<0$ (Win-win model and Conservation model). To be able to sign (16) we need to sign the numerator (α_X where X= Z, C,W) which is the risk-factor for each of the three partial models.

In the following I will discuss the signs of the *risk-factors* in more detail. As mentioned above their sign will depend on the assumptions made about the risk properties of each production factor in the three partial models. Below we present a table where all possibilities are summarised. Some of our earlier conclusions are easily confirmed in Table 1. First, if all production factors are risk neutral all *risk-factors* become equal to zero. Second, if some production factors are risk-increasing or risk decreasing the *risk factors* are in general different from zero. It is further noticed that there are some sets of assumptions which make it impossible to sign the *risk-factor* for a variable input. For productive inputs (Z), this matter if productive inputs and soil are assumed to have opposite risk properties. For soil conservation measures(C) and win-win inputs (W) this matter if they have similar risk properties. For all other sets of assumptions, the *risk-factors* can be signed.

Table 1. Signing risk factors under different assumptions about the risk properties of production factors

	Productive inputs (Z)			Win-win activities (W) and Soil conservation measures (C) i=W,C		
Soil (S)	Risk increasing	Risk neutral	Risk decreasing	Risk increasing	Risk neutral	Risk decreasing
Risk increasing	$\alpha_z=?$	$\alpha_z<0$	$\alpha_z<0$	$\alpha_i>0$	$\alpha_i>0$	$\alpha_i=?$
Risk neutral	$\alpha_z>0$	$\alpha_z=0$	$\alpha_z<0$	$\alpha_i>0$	$\alpha_i=0$	$\alpha_i<0$
Risk decreasing	$\alpha_z>0$	$\alpha_z>0$	$\alpha_z=?$	$\alpha_i=?$	$\alpha_i<0$	$\alpha_i<0$

Sufficient conditions for the *risk-factor* of Z being negative ($\alpha_X <0$) are that productive inputs are risk-decreasing ($G_X<0$), while the stock of soil fertility is risk-increasing or risk neutral ($G_S\geq 0$). Given that productive inputs are risk-decreasing the immediate-risk effect for productive inputs is negative. Additional productive inputs will cause the stock of soil fertility to be lower in all future periods since productive inputs are defined as land-degrading ($N_X>0$). Given that soil quality is categorised as risk-increasing, $G_S>0$, the long-term risk effect also becomes negative. The two effects pull in the same direction and the *risk-factor* can be signed. If soil quality is considered risk-neutral, $G_S = 0$, the sign of the *risk-factor* will be determined by the immediate-risk effect only and vice versa. The above discussion illustrates that those table elements in Table 1 which can not be signed arises from opposing immediate - and long-term risk effects.

So far it is shown that signs of the *risk-factors* can in principle go either way. As a consequence it is not possible to sign (16) independent of which variable input (partial model) is considered. A natural next step is to apply evidence which can help rule out some of the table-elements in Table 1. To my knowledge there is no systematic research done on the risk properties of various inputs and farming practices or the soil stock itself. However, in the literature there are some statements which can aid us in signing the *risk-factors*. Colacicco, Osborn and Alt (1989) claim that soil erosion (lower soil depth) may increase the variability of production regardless of its effect on average yields. Reinhardt (1987), in a Colombian study, found that farmers resisted modernisation programs which ignored soil conservation, since increased losses due to soil erosion and fertility losses would increase crop risk. Reicovsky et. al (1977) claim that conservation practices to some extent modify future risks of crop failure by conserving moisture. One the basis of the above references it seems that soil depth or soil fertility best can be described as a risk-decreasing production factor in agriculture. A higher stock of soil fertility is often associated with deeper soils, better chemical and structural properties of the soil which again improves the soils' moisture holding capacity and imply higher infiltration rates. All properties which are most likely to reduce the variability in output in seasons with both rainwater deficiency and excessive precipitation. Applying this information about the properties of soil limits the number of possible outcomes in Table 1. The *risk-factor* for a productive input is positive if the input is not risk-decreasing. The *risk-factor* for a conservation input and a win-win input are negative as long as these inputs are not risk-increasing.

The evidence that exists on the risk properties of variable inputs seems to support a classification of productive inputs as being risk-increasing, and win-win inputs and soil conservation measures as being risk-decreasing. Bishop and Allen (1989) find that there are often important secondary benefits associated with structural soil conservation measures, due to improved moisture retention and increased infiltration rates besides the actual reduction in soil and fertility losses experienced over time. The risk properties of productive inputs and win-win inputs are more complex to predict, partly because their classification depends on which degradation processes are considered. If the focus is on soil erosion, many external inputs can be viewed as win-win inputs due to their land cover effects. Just and Pope (1978) classifies both irrigation, pesticides, and herbicides as risk-decreasing inputs, while they refer to cropland expansion (more intensive cultivation), as a risk-increasing strategy. On the basis of the above discussion it seems reasonable to sign the *risk-factor* of productive inputs positively, while win-win and soil conservation inputs have negative *risk-factors*.

By applying the conclusions arrived at with respect to the signs of three *risk-factors,* it follows that (16) becomes positive for all three models. As a consequence the incentives for soil conservation are strengthened in each model for a higher degree of risk aversion. This finding needs to be accompanied with some intuition. Let us first consider productive inputs (cultivation model). Given that productive inputs are risk-increasing it becomes optimal, ceterius paribus, for a risk averse farmer at the margin to apply less of this input due to the increase in output variability that arises from such input use. A risk averse decision maker will give weight to the risk properties of productive inputs and not only consider changes in expected profits as will be the situation for a risk neutral decision maker. In addition risk preferences encompass future events, in the sense that risk implications which evolves over time from changes in productive input use (long-term risk effect) are paid attention to. Remember that less use of degrading productive inputs will increase the stock of soil for all future periods and thus represent not only future expected production gains but also involve risk implications perceived as costly or beneficial depending on how soil stock changes affect future output variability. Since soil is a classified as a risk-decreasing production factor, less use of productive inputs will at the margin reduce output variability in all future crop seasons, events which will considered as beneficial for a risk averse farmer. I have shown that the immediate risk effect and the long-term risk effect pulls in the same direction, as a consequence a risk averse farmer compared to a risk neutral one will devote less resources to land degrading activities, thus the incentives for soil depletion which is spurred from productive inputs have weakened. The same conclusion apply if productive inputs are risk neutral, since in this case the sign of the long-term risk effect is decisive for the signing of the *risk-factor*.

If risk-decreasing conservation measures and win-win technologies are considered (Win-win model and Conservation model), it becomes optimal for a risk averse farmer at the margin to apply more of both inputs as compared to a risk neutral farmer (immediate risk effect). More of both inputs will increase the stock of soil fertility being available for future crop seasons which again have future risk implications. Since soil is risk-decreasing, such a change will be considered as desirable from a risk averse farmers point of view. The immediate- and long-term risk effects pulls in the same direction for these two partial models, thus improving the incentives for devoting resources to soil-conserving activities (soil conservation measures and win-win activities). If win-win inputs and soil conservation measures are considered risk- neutral the total effect is determined by the long-term risk effect only, and the same conclusions matter.

From the above discussion it follows that the implications for variable input use due to risk behaviour in agricultural production systems are asymmetric. Higher risk aversion tends to change input-mix choices. In each of the two partial models this means more use of win-win inputs and soil conservation measures and less resources invested into productive degrading activities. However, the implications for soil conservation incentives arising from such input changes are symmetric. Less use of degrading inputs and more use of conservation measures and win-win inputs will all strengthen the incentives for soil conservation. The conclusions arrived at above depend on quite restrictive assumptions on technology. Considering each variable input at a time (partial models) means that numerous indirect effects present in (13) are ignored. In order to investigate the role of such effects we return to the general model, however a simplifying assumption is introduced. In the following win-win inputs will be ignored (W is kept constant throughout the planning horizon) and only productive inputs and soil conservation measures are considered. As a consequence the model structure of this model coincides with the model suggested by LaFrance (1992). Now (13) becomes as follows

$$\textbf{(17)}\quad \frac{d\overline{S}}{d\beta}=\frac{n_Z\alpha_Z R_{CC}-n_Z\alpha_C R_{ZC}-n_C\alpha_Z R_{CZ}+n_C\alpha_C R_{ZZ}}{D}$$

where D is negative. Since win-win inputs are ignored, we are now left with only four R-terms. From (2) we already know that n_Z is positive while n_C is negative. Furthermore, we have already made appealing assumptions on the signs of the *risk-factors* (α_Z is positive and α_C is negative). As a consequence, the R-terms present in the numerator are the only ones remaining indeterminate.

In the following we will adopt some assumptions on technology which is applied in deterministic studies on the optimal control of soil degradation, in order to see whether they can help in signing the R-terms. The assumptions on technology which will be used are as follows;

I) Additional conservation mitigates the soil degrading effects of more intensive cultivation, $n_{ZC}<0$ (LaFrance, 1992; Barrett, 1996).

II) A higher cultivation intensity (additional use of productive inputs) increases the marginalproductivity of soil, $f_{SZ}>0$ (Barbier, 1990; LaFrance, 1992).

III) Conservation inputs (higher conservation intensity) reduce the marginal productivity of both soiland productive inputs (more intensive cultivation), $f_{ZC}<0$, $f_{SC}<0$ (LaFrance, 1992).

IV) The cross partial derivatives of the costs function is zero, $h_{ZC}=0$ (LaFrance, 1992; Barrett, 1996; and Barbier, 1991; - all assume input unit costs).

Below the four R-terms are written out in length

$$\textbf{(18a)}\qquad R_{CC}=H_{CC}-\frac{n_C}{r}(f_{SC}-\beta g_{SC})$$

(18b) $$R_{ZZ} = H_{ZZ} - \frac{n_Z}{r}(f_{SZ} - \beta g_{SZ})$$

(18c) $$R_{CZ} = f_{CZ} - \beta g_{CZ} - \lambda n_{CZ} - \frac{n_C}{r}(f_{SZ} - \beta g_{SZ})$$

(18d) $$R_{ZC} = f_{ZC} - \beta g_{ZC} - \lambda\ n_{ZC} - \frac{n_Z}{r}(f_{SC} - \beta g_{SC})$$

Since the Hamiltonian is strictly concave in Z and C, and by using assumption II and III, it follows that both (18a) and (18b) are negative. Eq. (17) can now be signed if (18c) and (18d) are negative. However, it follows from assumption I-III, that R_{ZC} and R_{CZ} can not be signed due to opposing cross partial derivatives effects of the production function and the fertility loss function.

We have shown that some R-terms can not be signed without making non-appealing assumptions on technology. However, signing all R-terms is not a sufficient condition for reaching a determinant conclusion. We may still be left with opposing effects, and if win-win activities are included the picture becomes even more complicated since additional *indirect effects* are introduced. This feature emphasises the inherently difficulty in arriving at determinant conclusions in economic models on soil conservation focusing on input-mix choices, which seems to distinguish them from other dynamic models on natural resources. First, soil degradation models often contain at least two instruments (variable inputs) which opposes each other with respect to their effect on the stock variable. Second, the soil stock itself is an argument in the criteria function. Both features introduces indirect effects, being one important reason for the inability to sign comparative statics effects. How important the indirect effects are remains a question to be examined by empirical investigation.

Now I will reintroduce win-win inputs into the model, but assume additive separability in the production function, the fertility loss function and the cost function. This assumption implies that indirect effects are ignored (or considered unimportant), and that the focus is on direct effects only. As a consequence the numerator of (13) becomes as follows;

(19) $$\frac{d\bar{S}}{d\beta} = \frac{n_C \alpha_C H_{ZZ} H_{WW} + n_W \alpha_W H_{ZZ} H_{CC} + n_Z \alpha_Z H_{CC} H_{WW}}{D}$$

(where D is positive). From our earlier assumptions it can easily be confirmed that (19) is positive. We are left with three positive direct effects on steady state soil fertility, each for one of the variable inputs in the model. The conclusions arrived at in the partial approach above are confirmed in (19)

In some works which model soil-human interactions the causal links are less complex than is the case for this model. Here, simplifications are undertaken in order to focus on particular features which are considered as important and/or the analyses are adopted to particular regions and/or farming systems. Furthermore, the focus can be on specific land degradation processes which are considered as the limiting factor for particular soils. One observed approach is to relate crop production to the degree of soil degradation experienced. The model suggested by Ardila and Innes (1993) is of this type, in that soil degradation is output-induced. Similar features are found in models on fallowing systems (see e.g. Larson and Bromley, 1990). Other models in the literature, however, portray soil conservation as being output-induced (see Aune et al., 1994, Alfsen et al., 1995, Aune and Lal, 1995, and Brekke et al., 1996). In these models, both erosion processes and nutrient recycling processes are considered, while the role of soil conservation measures are not considered. If we focus on how erosion processes are related to output level in these models, a higher output level implies lower levels of soil losses.

Important features of the two very different modelling approaches appear however as special cases of our model. The structure of the model of Ardila and Innes can be represented by ignoring conservation measures and win-win inputs and focus on productive inputs only (Cultivation model). By applying these assumptions into our framework and let q denote the unit cost of a productive input, profits at date t can be described as follows:

$$(20)\ \Pi_t = f(S_t, Z_t, \overline{C}_t, \overline{W}_t, \theta_t) - qZ_t = F(S_t, Z_t, \theta_t) - qZ_t$$

Let the farmer minimise costs in period t for a given soil stock and a given crop production, $[Y_t{=}F(S_t,Z_t,\theta_t)]$, under the assumption of θ_t no longer being a stochastic parameter but a constant in the crop production function (deterministic cost minimisation approach). Solving this problem gives the following cost function: $C(Y_t, S_t q, \theta)$. Using the envelope theorem, it can be shown that the cost function is increasing in crop production (Y) while decreasing in soil quality (S). The profit function in (20) can now be expressed as follows;

$$(21)\ \Pi_t = Y_t - C(Y_t, S_t; q, \theta)$$

The stochastic production profits of Ardila and Innes (1993) is as follows;

$$(22)\ \Pi_t = Q(Y_t, \varepsilon) - C(Y_t, S_t)$$

Ardila and Innes denote Y the ex-ante production target which yields the ex-post production of Q(Y,ε), where ε is a random variable representing revenue uncertainty, and where Y and S is a positive and a negative argument in the cost function, respectively. We see by comparing (21) and (22) that the structure of the model of Ardila and Innes is similar to the Cultivation model.

If we ignore soil conservation measures and productive inputs in our general model (keeping them constant for the rest of the horizon) and only direct attention to the role of win-win inputs, we arrive at model in which there is a monotone relationship between output and soil erosion (Win-win model). Additional use of a win-win input increases crop production but on the same time reduces soil fertility losses. How soil conservation incentives are affected by risk behaviour in output-induced degradation models and in output-induced conservation model will depend on what the risk properties of crop production (inputs) are. In the way the models are portrayed here, it is likely that conservation incentives under production uncertainty are strengthened in both model types. For the output-induced soil-conservation model, a higher degree of risk aversion induces a farmer to produce more outputs if output is considered to be risk-decreasing or risk-neutral. The presence of risk averse preferences in an output-induced soil degradation model induces a farmer to produce less if output is risk-increasing or risk-neutral.

Ardila and Innes found, considering revenue uncertainty only, that long-term soil conservation incentives are improved if farmers are risk averse. However, comparing the results arrived at in Ardila and Innes under pure revenue uncertainty with our model under production uncertainty - considering productive inputs only - is not straightforward. One important reason for this is seen from comparing (21) and (22). Ardila and Innes model the presence of revenue uncertainty as not having any effect on optimal cost-minimisation behaviour, a feature which distinguishes their analysis from this one. Ardila and Innes stress the fact that they focus uncertainties which are associated with the more distant future. In this paper on the other hand the focus is on near-term uncertainties. There is one assumption, however, which seems to be important for some of the conclusions Ardila and Innes arrive at. They assume that the ex-post revenue function has the following property: $Q_{Y\varepsilon} \geq 0$. If a similar assumption was applied in our analysis, given the specification of the stochastic crop production function which is present in (3a), then risk-increasing properties would be imposed. Another conclusion is that if Ardila and Innes paid attention to input-mix choices instead of output itself, under medium and long-term revenue uncertainty, their conclusions would be less decisive. The possibility of controlling and arresting degradation processes by implementing structural conservation measures like the planting of trees as windshields, constructing terraces, building waterways and drainage systems, and devoting more resources to win-win activities, imply that there are additional forces at play in connection with risk attitudes, compared to those arrived at in output-induced degradation models. In this perspective their results do not appear robust.

Our model of land degradation intends to describe farming systems of relatively complexity, with an array of inputs to choose between all having various effects on future soil quality. However, many farming systems, especially in less developed regions of the world, are more simple in their structure. External inputs are not always available or they are perceived as expensive from the farmers' point of view. The crucial input besides land itself is labour effort. Under such circumstances it can be relevant to describe a farming unit as a system of output-induced soil degradation. One example is various fallow-cultivation systems. In such traditional institutions, soil fertility is maintained by returning cropland to fallowland for a shorter or longer period of time. More intensive cultivation means more labour effort and less land under fallow. Less land under permanent vegetation speeds up soil erosion processes. A trade-off between crop production and the future fertility of soil (Cultivation model), seems to capture the essential features by which such indigenous farming systems can be characterised. If putting more land under cultivation is perceived as an immediate risk increasing strategy, while soil fertility is perceived as a risk decreasing production factor, risk averse preferences induce farmers in fallow-cultivation systems to keep more land under fallow relatively to croplands.

4. Output price uncertainty and soil conservation incentives

In this section I will analyse the role of output price uncertainty in connection with soil conservation incentives, in order to investigate if risk behaviour arising from crop price uncertainty differ from risk behaviour under production uncertainty. The important implication for our model from modelling uncertainty in output prices is that the random disturbance (θ) now will enter the production function in a multiplicative manner as opposed to risk in Section 3.

To see this I write out the revenue function under output price uncertainty, using our earlier specification of the production function presented in (3). The revenue function Ω may be written as follows;

$$(23)\quad \Omega = \theta[f(S,Z,C,W) + g(S,Z,C,W)] \; = \; \theta\Psi(S,Z,C,W)$$

It follows that the marginal change in the variance of revenues is given by

$$(24)\quad \begin{aligned} \frac{dVar\{\Omega\}}{di} &= 2[f(S,Z,C,W) + g(S,Z,C,W)][f_i(S,Z,C,W) + g_i(S,Z,C,W)]Var\{\theta\} \\ &= 2\,\Psi(S,Z,C,W)\,\Psi_i(S;Z,C,W)\,Var\{\theta\} \qquad where \;\; i = S,Z,C,W \end{aligned}$$

From (24) we can see that a mutiplicative stochastic specification of the overall revenue function imposes an additional constraint. The marginal risk effect from each production factor follows from the assumptions made on the overall production function, Ψ. These properties have implications for the conclusions arrived when output price uncertainty is considered, since risk effects now are determined solely by the relationships of inputs with expected revenues. All production factors which enters the overall production function, ψ, in a positive way become risk-increasing, while those who decrease output when applied in larger quantities become risk-decreasing. For our model such a specification implies that the variance of revenues will increase with stock of soil fertility, productive inputs, and win-win activities, while it will decrease with structural soil conservation measures; $g_S > 0$, $g_Z > 0$, $g_W > 0$ and $g_C < 0$. If we relate these technological properties to Table 1, we find that the *risk-factor* for win-win activities can be signed. Both the immediate and the long-term risk effect pulls in the same direction when win-win activities are considered. However, for structural conservation measures and productive activities this will not be case, here the immediate-risk effects and the long-term risk effects oppose each other, and neither α_Z or α_C can be signed.

Below we will analyse the role of risk aversion in connection with output price uncertainty in partial models. As before I will apply a Mean-Variance utility function, and I normalise expected output price to one and define δ as the variance of the crop price. As a consequence VAR(Π) becomes equal to $(F(S,X))^2\delta$. Let H denote the Hamiltonian for this problem

$$(25)\quad H = F(S_t, X_t) - T(X_t) - \beta[F(S_t, X_t)]^2\,\delta$$

H is assumed to be strictly concave in X and S and the optimality conditions evaluated in steady state are as follows (tildes denote steady state values);

$$(26)\quad F_X(\tilde{S},\tilde{X}) - 2\beta F(\tilde{S},\tilde{X})F_X(\tilde{S},\tilde{X})\delta - T_X(\tilde{X}) = \frac{N_X(\tilde{X})}{r}(F_S(\tilde{S},\tilde{X}) - 2\beta F(\tilde{S},\tilde{X})F_S(\tilde{S},\tilde{X})\delta$$

$$(27)\quad M - N(\tilde{X}) = 0$$

Following the same procedures as in the above section, the following expression for the impact on steady state soil fertility from a higher degree of risk aversion can be derived (arguments are omitted)

$$(28)\quad \frac{d\tilde{S}}{d\beta} = \frac{2F\delta\alpha_X}{D} = \frac{2F\delta[F_X - \frac{N_X}{r}F_S]}{D}$$

From the saddle path condition for this problem it follows that D<0 if $N_X<0$ (Win-win model and Conservation model) while D>0 if $N_X>0$ (Cultivation model). α_X is the *risk-factor* for the three partial models when output price uncertainty is considered. Our earlier conclusion on the role of win-win inputs is confirmed in (28). For the Win-win model $F_X>0$ and $N_X<0$, as a consequence (28) is negative. A higher degree of risk aversion under output price uncertainty weakens the incentives for soil conservation, since less resources are devoted to production activities in the Win-win model. Less output in this model means less soil conservation. However, the risk-factors for the Cultivation model and the Conservation model can not be determined from (28). By rearranging (26) the following expression is derived

$$(29)\quad (F_X - \frac{N_X}{r}F_S) = \frac{T_X}{1-2\beta F\delta} > 0$$

and since $T_X>0$ and since the shadow price of soil fertility in steady state is positive $[\tilde{\lambda} = \frac{1}{r}F_S(1-2\beta F\delta)]$, the *risk-factor,* α_X, is always positive when output price uncertainty is considered. We are now able to sign (28) also for the Cultivation model and the Conservation model, in spite of the presence of opposing risk effects for these two models. This conclusion follows from the fact that changes in the variance of output which follows from marginal changes in any production factor is proportional to the marginal productivity of the same factor. This is seen by studying a situation in which there are no variable costs associated with changing the intensity of one of the activities, $T_X = 0$. From equation (29) we now notice that risk preferences have no effect on steady state soil fertility in all three partial models. Since risk preferences include both immediate- and long-term effects, the absence of variable input costs means that any risk preferences will outweigh each other at the margin. If variable costs do exist, risk preferences will matter since costs are left unaffected by such preferences thus acting as a wedge between immediate- and future risk preferences.

The implication of all *risk-factors* being positive is that less resources are devoted to input use in all three partial models. Output price uncertainty induces a risk averse farmer to cultivate less intensively, but also to invest fewer resources into soil conservation measures and win-win technologies. The implications for soil conservation of this withdrawal of resources is however asymmetric. Less intensive cultivation reduces the rate at which soils are deteriorating, while less investment in soil conservation and win-win technologies, on the other hand, will speed up land degradation.

The conclusion arrived at have similarities to the ones arrived at in standard theory of production under uncertainty where the commonly used multiplicative stochastic specification is applied (see e.g. Sandmo, 1971). The main conclusion from such analyses is that risk averse preferences induce a decision maker to apply less of risky resources compared to risk neutral preferences - for model specifications with one variable input or where output itself is the decision variable. The modelling of uncertainty in the production process of such models coincide with the situation for the land manager in our problem. The outcome of an uncertain event is only known to the producer after the decisions about input use have been made. Thus, the decision about how much resources to devote to an activity is an irreversible decision. However, in spite of the similarity with respect to the conclusions on input use, the forces at play in our analysis are not identical to those of the analyses mentioned above. The mechanisms which drive the results are different for static models as compared to our dynamic analysis. Remember that in standard theory of production all inputs are positive arguments in the production function, as a consequence all inputs become risk-increasing. In our model, on the other hand, structural soil conservation measures are risk-decreasing due to their land ousting effects. In spite of this property we have shown that a risk averse decision maker will find it optimal to apply less of such inputs (Conservation model). This conclusion may appear surprising, but is brought about by the dynamic nature of the model. If a farmer invests in additional conservation measures to reduce the immediate risk, the future stock of soil fertility will as a consequence be higher. Since soil fertility is a positive argument in the production function, a higher stock of soil fertility will increase long-term risk. For soil conservation measures, the long-term risk effect will dominate the immediate risk effect. Whether or not soil conservation measures are assumed to take up productive land does not change this conclusion. The same opposing effects were identified for productive land degrading inputs (Cultivation model). For this input group however, the withdrawal of resources is due to the immediate-risk effect dominating the long-term effect. For win-win activities both effects pulls in the same direction.

The analysis in section 3 have already made it clear that the final effect on soil conservation incentives from risk aversion can not be signed if the general model presented in (12) was analysed under multiplicative uncertainty. As before a general approach will introduce numerous indirect effects which makes the task of signing impossible. In some sense it will be more difficult to arrive at conclusions under output price uncertainty than production uncertainty, since the effects on soil conservation incentives identified in each partial model oppose each other under output price uncertainty. It remains a question whether a higher degree of risk aversion induces the farmer to exploit the soil more or less.

In standard theory of production under multiplicative uncertainty when more than one variable input is considered, Batra and Ulla (1974) and Hartmann (1975) have shown that the assumption of production complementarity is sufficient to ensure that risk averse firms will utilise smaller quantities of all inputs. A similar assumption is not a sufficient condition for securing less use of all inputs in our model. In a multi-period formulation there is a link between each agricultural input and the soil stock via the presence of the soil loss function, where the stock evolution over time depends on the quantity applied of all other production factors. Furthermore, the stock variable (soil fertility) is a direct input in the agricultural production function. As a consequence, additional assumptions on technology are needed, both on the production function and the fertility loss function for arriving at a similar conclusion.

5. Conclusion

In this paper the implications of risk averse preferences are considered. I analyse farmers which operate in an uncertain environment and study how risk preferences influence optimal soil conservation decisions in the absence of effective insurance markets. In doing so a multi-period model is presented where the focus is on input-mix choices, in the sense that a land manager has three instruments at disposal to determine soil evolution over time. The decision maker can choose among multiple inputs and/or cultivation practices which all lies within his/her technological horizon, all having different impacts on short-term crop production and soil degradation processes. Particular attention is given to the importance of the risk properties associated with each agricultural production factor. The focus is on short-term revenue uncertainty arising from climatic variability and the occurrence of natural hazards.

The current study suggests that risk aversion under revenue uncertainty influences the incentives to arrest soil degradation but it remains unclear in what direction. However, the analysis identifies several factors which are crucial when determining optimal risk behaviour. First, it is important to distinguish between two sources of revenue uncertainty - production uncertainty and output price uncertainty. When analysing output price uncertainty, some conclusions became more transparent since the stochastic specification of output price risk imposes restrictions on the risk properties of all production factors. As a consequence, the overall risk effect arising from each variable input can be determined. Second, it is important to assess what kind of inputs which are available to the agricultural production systems considered, and how their use relates to both short-term output and the stock of soil fertility. In particular, whether it is a monotone relationship between the level of output and the degree of soil degradation. Third, knowledge about the risk implications arising from the farming decisions made are important, in particular what farmers perceptions of them are. All the above factors will most likely envisage variations across regions. In order to be able to predict the consequences for soil conservation incentives detailed knowledge about farming systems is needed. However, such information is not necessarily available. Many studies, especially for tropical soils, find that there is an empirical gap as concerning relationships between cultivation practices, input use, and soil degradation processes (see e.g. Aune and Lal, 1994). Due to the site specific character of degradation processes, the huge variation in farming systems across regions, and the possible ambiguity in farmers' perceptions of the risk properties arising from various inputs.

Furthermore, valuable knowledge is obtained by focusing on partial models (or direct effects). Under output price uncertainty, the direct effects from productive inputs on soil conservation incentives are opposing those arising from conserving inputs (win-win inputs and soil conservation measures). Compared to a situation under certainty, risk averse preferences induce a farmer to apply less of all inputs - both degrading - and conserving inputs. Under production uncertainty, by applying reasonable assumptions on the risk property of each production factor, risk aversion induces a farmer to apply less of degrading inputs and more of conserving inputs. As a consequence it seems more likely that soil conservation incentives improve under production uncertainty than is the case for output price uncertainty. If important soil-human interlinkages can be characterised as less complex as those portrayed in the general framework in this paper for example by focusing directly on relationships between crop production and level of soil degradation more decisive conclusions can be derived.

The analysis further stresses the importance of analysing soil conservation incentives in a dynamic setting. If this analysis was undertaken in a static context, all effects which arise from soil base changes would be absent and some conclusions would be reversed. One example is the overall risk effect from soil conservation measures when analysing output price uncertainty. As mentioned earlier, evidence on risk attitudes and soil conservation incentives is not conclusive (see e.g. Williams and Johnson, 1985; Klemme, 1985; Kramer, McSweeney, and Stavros, 1983; Reinhardt, 1987; and Anderson and Thampapillai, 1990, for a discussion of the references). The mixed findings can well be understood within our model. We have identified contradicting direct effects across the three activities when output price uncertainty is considered. In a general setting, the presence of indirect effects makes it difficult to arrive at unique conclusions for both sources of uncertainty. How important such effects are should be a subject of future research.

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Appendix

Necessary and sufficient conditions for a local saddle path equilibrium for problem (12).

One way to derive the necessary and sufficient conditions for a saddle path equilibrium for the dynamic system (12bc) and (1), is to solve the sub system in (12b) for Z, C, and W, simultaneously (Modified Dynamic Hamiltonian System). Hence we arrive at the following equations

$$\begin{aligned} &Z = A(S,\lambda,\beta) \\ \textbf{(A.1)}\quad &C = B(S,\lambda,\beta) \\ &W = E(S,\lambda,\beta) \end{aligned}$$

By inserting the equations in (A.1) into (12c) and (1), respectively, we arrive at a dynamic system for which the saddle path conditions may be derived. We find that the eigen values that correspond to this system (evaluated in equilibrium) are real and opposite, if

$$\begin{aligned} &J = -n_Z[A_S(r - H_{SC}B_\lambda - H_{SW}E_\lambda) + A_\lambda(H_{SS} + H_{SC}B_S + H_{SW}E_S)] \\ \textbf{(A.2)}\qquad &\quad - n_C[B_S(r - H_{SZ}A_\lambda - H_{SW}E_\lambda) + B_\lambda(H_{SS} + H_{SZ}A_S + H_{SW}E_S)] \\ &\quad - n_W[E_S(r - H_{SZ}A_\lambda - H_{SC}B_\lambda) + E_\lambda(H_{SS} + H_{SZ}A_S + H_{SC}B_S)] \qquad < 0 \end{aligned}$$

(see below for a definition of A_i, B_i, and E_i, where $i = S, \lambda$). After extensive manipulation (A.2) may be rewritten as follows

$$\textbf{(A.3)}\quad J \;=\; \frac{r}{F}D < 0$$

where F and D are defined below. If the Hamiltonian is assumed to be strictly concave in (Z, C, W), F becomes negative. For the necessary and sufficient condition for a saddle path equilibrium to be fulfilled D must be positive.

$$A_S = \frac{1}{F}[-H_{SZ}(H_{CC}H_{WW} - H_{CW}H_{WC}) + H_{ZC}(H_{SC}H_{WW} - H_{CW}H_{SW}) - H_{ZW}(H_{SC}H_{WC} - H_{CC}H_{SW})]$$

$$A_\lambda = \frac{1}{F}[n_Z(H_{CC}H_{WW} - H_{CW}H_{WC}) + H_{ZC}(n_W H_{CW} - n_C H_{WW}) - H_{ZW}(n_W H_{CC} - n_C H_{WC})]$$

$$A_\beta = \frac{1}{F}[g_Z(H_{CC}H_{WW} - H_{CW}H_{WC}) + g_C(g_W H_{CW} - g_C H_{WW}) + H_{ZW}(g_C H_{WC} - g_W H_{CC})]$$

$$B_s = \frac{1}{F}[H_{ZZ}(n_C H_{WW} + n_W H_{CW}) + n_Z(H_{CZ}H_{WW} - H_{CW_w}H_{WZ}) + H_{ZW}(n_W H_{CZ} - n_C H_{WZ})]$$

$$B_\lambda = \frac{1}{F}[H_{ZZ}(g_C H_{WW} + g_W H_{CW}) + g_Z(H_{CZ}H_{WW} - H_{CW}H_{WZ}) + H_{ZW}(g_W H_{CZ} - g_C H_{WZ})]$$

$$B_\beta = \frac{1}{F}[H_{ZZ}(g_C H_{WW} + g_W H_{CW}) + g_Z(H_{CZ}H_{WW} - H_{CW}H_{WZ}) + H_{ZW}(g_W H_{CZ} - g_C H_{WZ})]$$

$$E_S = -\frac{1}{F}[H_{ZZ}(H_{SW}H_{CC} + H_{SC}H_{WC}) + H_{ZC}(H_{SZ}H_{WZ} - H_{SW_s}H_{CZ}) + H_{ZS}(H_{CZ}H_{WC} - H_{CC}H_{WZ})]$$

$$E_\lambda = \frac{1}{F}[H_{zZZ}(n_W H_{CC} - n_C H_{WC}) - H_{ZC}(n_W H_{CZ} - n_C H_{WZ}) + n_Z(H_{CZ}H_{WC} - H_{WZ}H_{CC})]$$

$$E_\beta = \frac{1}{F}[H_{ZZ}(g_W H_{CC} - g_C H_{WC}) - H_{ZCV}(g_W H_{CZ} - g_C H_{WZ}) + g_Z(H_{CZ}H_{WC} - H_{WZ}H_{CC})]$$

$$\text{and} \quad F = H_{zz}(H_{cc}H_{ww} - H_{cw}H_{wc}) - H_{zc}(H_{cz}H_{ww} - H_{cw}H_{wz}) + H_{zw}(H_{cz}H_{wc} - H_{cc}H_{wz})$$

The denominator D in (13) written out in length.

$$\begin{aligned} D = n_z\big[&R_{zs}(R_{cc}R_{ww} - R_{cw}R_{wc}) - R_{zc}(R_{cs}R_{ww} - R_{cw}R_{ws}) + R_{zw}(R_{cs}R_{wc} - R_{cc}R_{ws})\big] \\ - n_c\big[&R_{zs}(R_{cz}R_{ww} - R_{cw}R_{wz}) - R_{zz}(R_{cs}R_{ww} - R_{cw}R_{ws}) + R_{zw}(R_{cs}R_{wz} - R_{cz}R_{ws})\big] \\ + n_w\big[&R_{zs}(R_{cz}R_{wz} - R_{cc}R_{wz}) - R_{zz}(R_{cs}R_{wc} - R_{cc}R_{ws}) + R_{zc}(R_{cs}R_{wz} - R_{cz}R_{ws})\big] \end{aligned}$$

where

$$R_{zz} = H_{zz} - \frac{n_z H_{sz}}{r}$$

$$R_{cc} = H_{cc} - \frac{n_c H_{sc}}{r}$$

$$R_{ww} = H_{ww} - \frac{n_w H_{sw}}{r}$$

$$R_{zs} = H_{zs} - \frac{n_z H_{ss}}{r}$$

$$R_{zw} = H_{zw} - \frac{n_z H_{sw}}{r}$$

$$R_{cs} = H_{cs} - \frac{n_c H_{ss}}{r}$$

$$R_{cz} = H_{cz} - \frac{n_c H_{sz}}{r}$$

$$R_{ws} = H_{ws} - \frac{n_w H_{ss}}{r}$$

$$R_{zc} = H_{zc} - \frac{n_z H_{sc}}{r}$$

$$R_{cw} = H_{cw} - \frac{n_c H_{sw}}{r}$$

$$R_{wc} = H_{wc} - \frac{n_w H_{sc}}{r}$$

$$R_{wz} = H_{wz} - \frac{n_w H_{sz}}{r}$$

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ISSN 0803-074X

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